# Equations for the entire soil-water characteristic curve of a volume change soil

**Hung Q. Pham and Delwyn G. Fredlund**

**Abstract:** Numerous curve-fitting equations have been proposed for soil-water characteristic curves. While these equations have been of considerable value in geotechnical and geoenvironmental engineering, the equations are not able to adequately fit gravimetric soil-water characteristic curve data over the entire range of soil suction for a soil that changes volume when suction is changed. Two new equations for the soil-water characteristic curve are presented in this paper. One equation has curve-fitting parameters that bear a meaningful relationship to conventional physical soil properties (e.g., air-entry value and residual soil suction), but the equation is somewhat complex. The equation is particularly useful for sensitivity type studies when undertaking computer modeling. The other equation is relatively simple to use and is developed as a conventional curve-fitting equation. The two equations are used to best-fit several soil datasets. Both equations perform well and can be used in research and engineering practice to define the gravimetric water content versus soil suction relationship for a soil exhibiting volume change.

*Key words:* soil-water characteristic curve, soil suction, unsaturated soils, air-entry value, residual conditions, degree of saturation.

**Résumé :** De nombreuses équations de lissage de courbes ont été proposées pour les courbes caractéristiques sol-eau. Quoique ces équations aient eu une valeur considérable dans les problèmes géotechniques et géoenvironnementaux, les équations ne peuvent pas lisser adéquatement les données gravimétriques de la courbe caractéristique sol-eau sur la plage entière de succion du sol pour un sol qui change de volume lorsque la succion change. On présente dans cet article deux nouvelles équations pour la courbe caractéristique sol-eau. Une équation a des paramètres de lissage de courbe qui comportent une relation significative aux propriétés physiques conventionnelles du sol (e.g., valeur d'entrée d'air et succion résiduelle du sol), mais l'équation est quelque peu complexe. L'équation est particulièrement utile pour des études de type sensible lorsque traitant de modélisation par ordinateur. L'autre équation est relativement simple à utiliser et est développée comme une équation de lissage de courbe conventionnel. Les deux équations sont utilisées pour mieux lisser plusieurs ensembles de données. Le deux équations offrent de bonnes performances et peuvent être utilisées en recherche et dans la pratique de l'ingénieur pour définir la relation de la teneur en eau gravimétrique en fonction de la succion d'eau pour un sol exhibant un changement de volume.

*Mots-clés :* courbe caractéristique sol-eau, succion du sol, sols non saturés, valeur d'entrée d'air, conditions résiduelles, degré de saturation.

[Traduit par la Rédaction]

## Introduction

The soil-water characteristic curve (SWCC) of a soil plays an important role in defining the hydromechanical behaviour of an unsaturated soil. The SWCC can be used to estimate unsaturated soil property functions such as hydraulic conductivity, water storage, and shear strength functions. A sensitivity study on the use of various SWCC equations in a seepage model has been presented by Thieu et al. (2001). The study showed that it is important to correctly represent the SWCC. Fredlund (2007) stated that the accuracy of the computer simulations is dependent on the accuracy of the SWCC.

Measurement of a SWCC is time consuming but is generally required for the final design of engineering projects involving unsaturated soils (e.g., soil cover design). It is costly to measure the complete SWCC (i.e., water content at many soil suction values) and generally only a few data points are measured along the desorption curve. A curve-fitting equation is then used to provide an estimation of the entire SWCC from the few measured data points.

The amount of water in a soil has been represented using three variables; namely, (*i*) gravimetric water content; (*ii*) volumetric water content; and (*iii*) degree of saturation. The gravimetric SWCC can be most readily measured in a geotechnical laboratory. The degree of saturation and volumetric water content require a measurement of the volume of the soil specimen. Measurement of volume change of an unsaturated soil is quite demanding and requires more time and experimental care (Mbonimpa et al. 2006).

Soil-water characteristic curves also differ depending on the applied total stress path (Fredlund 2002; Fredlund and

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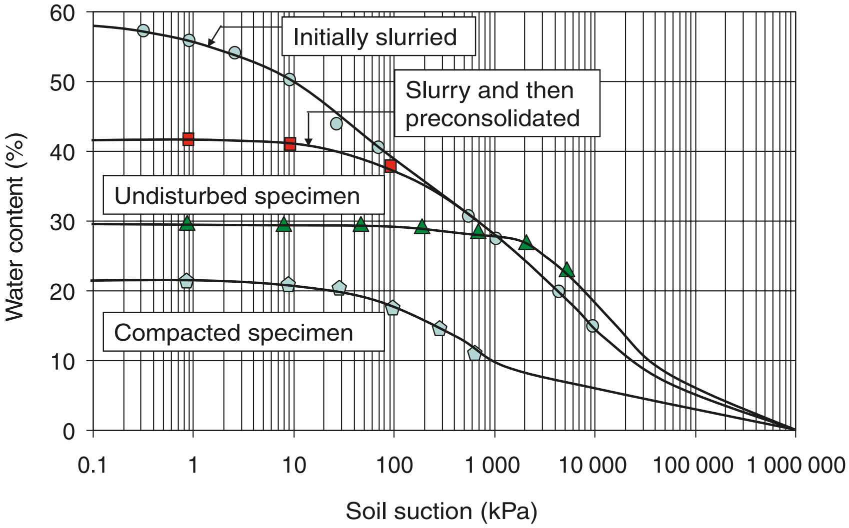
**Fig. 1.** Illustration of the influence of initial state on the soil-water characteristic curve (modified from Fredlund 2002).



Pham 2006). Soil-water characteristic curves measured on soils with different historical stresses are presented in Fig. 1. The gravimetric SWCC of an initially slurried specimen has been selected as the reference state for the development of the new SWCC equations. The initially slurried soil represents the maximum volume change condition and also represents the virgin compression curve up to the air-entry value of the soil. The virgin compression curve has been historically meaningful in geotechnical engineering.

The volume change behaviour has been the focus of several research studies on unsaturated soils. Numerous studies have attempted to model the mechanical and hydraulic behavior of expansive soils (Tuller and Or 2003; Simms and Yanful 2005). Several researchers have attempted to predict the SWCCs of expansive soils (Chertkov 2004; Mbonimpa et al. 2006). However, none of these studies have presented a mathematical equation for the SWCC of an unsaturated soil starting from a slurry condition and continuing to completely dry conditions.

Two new curve-fitting equations for the SWCC are presented in this paper and these equations are referred to as (*i*) the meaningful parameter SWCC equation, and (*ii*) the simplified SWCC equation. The meaningful parameter SWCC equation has curve-fitting parameters that have physical significance. This equation is particularly useful for carrying out sensitivity type studies associated with computer modeling. The simplified SWCC equation is most useful when performing analytical solutions for engineering applications.

## Theory

### Shape of a SWCC

The (gravimetric) SWCC for a soil is typically S-shaped (i.e., for a unimodal curve) (Sillers 1997). When soil suction is less than the entry value of the soil (i.e., all the voids are filled with water), soil suction changes affect the soil in a manner similar to changing the net mean total stress. The first portion of the gravimetric SWCC (i.e., at soil suctions less than the air-entry value of the soil) will have a slope corresponding to the volume change for an isotropically loaded saturated soil. The magnitude of the initial slope depends on the stress history and the compressibility of the soil structure (Fig. 2).

Let us consider the drying or wetting of a soil under zero net mean total stress. When soil suction is less than the air-entry value, the volume change due to soil suction change is equal to the application of an isotropic net mean total stress. The slope of the initial portion of the SWCC at soil suctions less than the air-entry value can be calculated as shown by eq. [1]. The differentiation applies for the virgin compression branch (or the recompression branch) as long as the applied suction is less than the air-entry value.

$$[1] \qquad \frac{\mathrm{d}w}{\mathrm{d}\psi} = \frac{\mathrm{d}(e/G_\mathrm{s})}{\mathrm{d}\psi} = \begin{cases} \dfrac{C_\mathrm{c}}{G_\mathrm{s}} & \text{for } \psi > p_\mathrm{c} \\ \dfrac{C_\mathrm{s}}{G_\mathrm{s}} & \text{for } \psi \leq p_\mathrm{c} \end{cases}$$

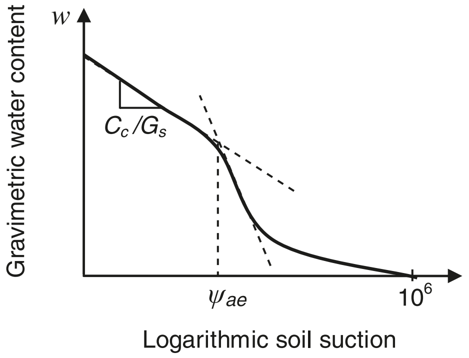
where

$w$ is the gravimetric water content,
$e$ is the void ratio,
$G_\mathrm{s}$ is the specific gravity,
$C_\mathrm{s}$ is the unloading–reloading index,
$C_\mathrm{c}$ is the virgin compression index,
$p_\mathrm{c}$ is the preconsolidation pressure, and
$\psi$ is the soil suction.

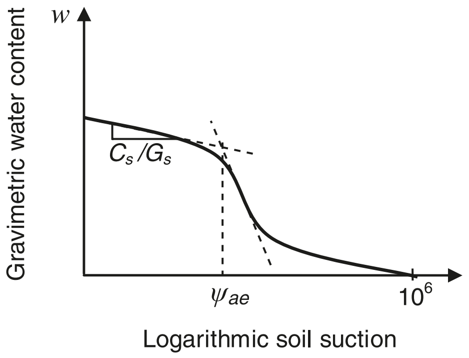
Several theoretical shapes for the (gravimetric) SWCC of a soil are shown in Fig. 2. The same clay soil is subjected to different stress histories, and this gives rise to varying SWCC shapes. It is assumed that the preconsolidation pressure of a slurry soil is approximately 0 kPa (i.e., 0.01 kPa) and the preconsolidation pressure of a highly overconsolidated clay is much higher than the air-entry value of the soil.

There are a number of empirical equations that have been proposed to represent the SWCC (Fredlund 2006). Most equations have been sigmoidal in shape and as such, do not properly represent the SWCC in the region less than the air-entry value (i.e., asymptotic to a horizontal line). These equations are more suitable for best-fitting the degree of saturation and volumetric water content SWCCs. However, SWCC curve-fitting equations are commonly used to fit the gravi-

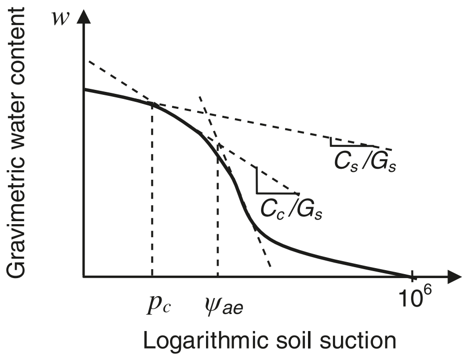
**Fig. 2.** Typical shapes of the (gravimetric) soil-water characteristic curve for a soil subjected to different stress histories.



(a) Slurry soil specimen



(b) Highly consolidated specimen



(c) Normally consolidated specimen

metric SWCCs, particularly in geotechnical engineering. The horizontal line approximation is reasonable for low compressibility soils with a low air-entry value (i.e., sandy soils) but is quite unsatisfactory when fitting SWCCs for clay soils. Therefore, it is prudent to develop new SWCC-fitting equations that are more suitable for soils exhibiting significant volume change at soil suctions less than the air-entry value (Mbonimpa et al. 2006). The following sections present the development of two new SWCC equations that meet the required conditions.

## Meaningful parameter SWCC equation

For engineering applications it is preferable to describe a SWCC in terms of soil parameters that have physical significance (i.e., air-entry value, residual soil suction, residual water content, maximum slope of the SWCC). Such an equation is particularly useful when conducting sensitivity type parametric studies that investigate the influence of various soil properties. Gitirana and Fredlund (2004) proposed an equation for the SWCC with independent physical parameters for each section of the SWCC. The equation was a combination of two rotated and translated hyperboles. The SWCC equation was not intended for high volume change soils (and the gravimetric water content). The development of a SWCC with independent soil properties for high volume change soils is presented in this section.

The basic characteristics of a SWCC are shown in Fig. 3. A SWCC can be represented by three straight lines with slopes of $S_1$, $S_2$, and $S_3$. The water content at a soil suction of $1 \times 10^6$ kPa is assumed to be equal to zero (Fredlund and Xing 1994). Parameters $t_1$ and $t_2$ control the transitions between different portions of the SWCC curve. Three equations corresponding to the three sloping lines on a semilog graph (i.e., $w_3(\psi)$, $w_2(\psi)$, and $w_1(\psi)$) can be written as follows:

$$[2] \qquad w_3(\psi) = S_3 \log\left(\frac{10^6}{\psi}\right)$$

where

$\psi$ is the soil suction, and

$S_3$ is the slope of the SWCC for the portion beyond residual suction.

$$[3] \qquad w_2(\psi) = S_3\log(10^6) + (S_2 - S_3)\log(\psi_r) - S_2\log(\psi)$$

where

$\psi_r$ is the residual soil suction, and

$S_2$ is the slope of the SWCC for the portion between the air-entry value and residual soil suction.

$$[4] \qquad w_1(\psi) = S_3\log(10^6) + (S_1 - S_2)\log(\psi_{ae}) + (S_2 - S_3)\log(\psi_r) - S_1\log(\psi)$$

where

$\psi_{ae}$ is the air-entry value of the soil, and

$S_1$ is the slope of the SWCC for the portion less than the air-entry value.

A mathematical technique is used to connect the above three sloping line equations into a single SWCC equation. To connect two straight lines, a function, *f*, is used that can change its value between 0 and 1. An inflection point at the intersection of the two straight lines is used. This technique has already been implicitly used in several SWCC equations (i.e., Gardner 1958; Feng and Fredlund 1999). A general form for the new SWCC-fitting equation is as follows:

$$[5] \qquad w(\psi) = [w_1(\psi)f_1(\psi, \psi_{ae}) - w_2(\psi)f_2(\psi, \psi_{ae}) - f_3(\psi, \psi_{ae})]f_4(\psi, \psi_r) + w_3(\psi)f_5(\psi, \psi_r) + f_6(\psi, \psi_r)$$

where

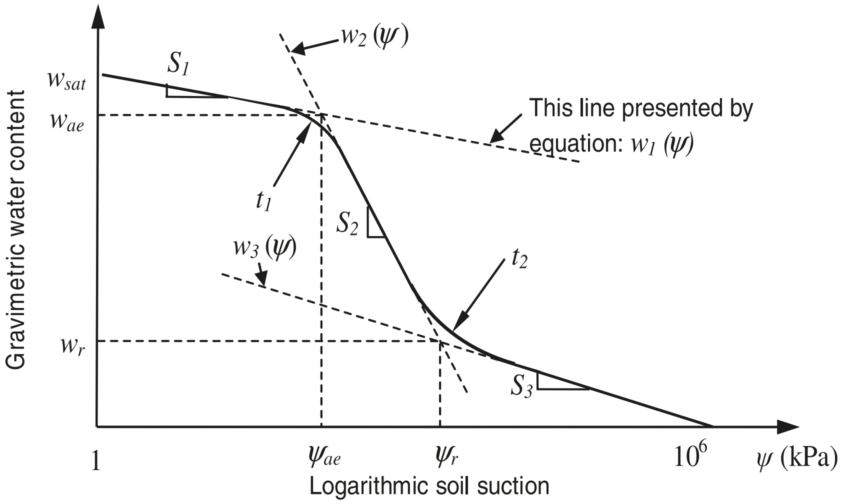
$\psi$ is the soil suction, and functions $f_1$, $f_2$, $f_3$, and $f_4$ need to satisfy the following conditions:

$f_1$ is increased to 1 when $\psi$ is decreased from $\psi_{ae}$ to 0,

$f_1$ is decreased to 0 when $\psi$ is increased from $\psi_{ae}$ to $+\infty$,

$f_2$ is equal to $1 - f_1$,

**Fig. 3.** Typical soil-water characteristic curve for a volume change soil along with an indication of the significance of the variables involved.



$f_4$ is increased to 1 when $\psi$ is decreased from $\psi_r$ to 0,
$f_4$ is decreased to 0 when $\psi$ is increased from $\psi_r$ to $+\infty$,
$f_5$ is equal to $1 - f_4$,
function $f_3$ is added to control the transition at the air entry value, and
function $f_6$ is added to control the transition at residual soil suction.

The following equation can be used to represent functions $f_1$ $f_4$:

$$[6] \qquad f(s, s_1) = \frac{s_1^n}{s^n + s_1^n}$$

where $s$, $s_1$, and $n$ are three variables.

Plots of eq. [6] with $s_1 = 10$ and various $n$ values (i.e., $n$ = 1, 4, and 10) are shown in Fig. 4. A plot of the differentiation of the function, $f(s,s_1)$ with respect to the variable $s$ is presented in Fig. 5 (i.e., with $s_1 = 10$ and $n$ = 1, 4, and 10). Functions $f_1$, $f_2$, $f_4$, and $f_5$ can be described as follows:

$$[7] \qquad f_1(\psi, \psi_{\mathrm{ae}}) = \frac{\psi_{\mathrm{ae}}^{t_1}}{\psi^{t_1} + \psi_{\mathrm{ae}}^{t_1}}$$

$$[8] \qquad f_2(\psi, \psi_{\mathrm{ae}}) = \frac{\psi^{t_1}}{\psi^{t_1} + \psi_{\mathrm{ae}}^{t_1}}$$

$$[9] \qquad f_4(\psi, \psi_{\mathrm{r}}) = \frac{\psi_{\mathrm{r}}^{t_2}}{\psi^{t_2} + \psi_{\mathrm{r}}^{t_2}}$$

$$[10] \qquad f_5(\psi, \psi_{\mathrm{r}}) = \frac{\psi^{t_2}}{\psi^{t_2} + \psi_{\mathrm{r}}^{t_2}}$$

where $t_1$ and $t_2$ are two parameters that are later used for controlling transitions between the different portions of the SWCC.

The function $f_3$ is designated to control the transition at the air-entry value of the SWCC. It is recognized that function $f_3$ can be obtained by differentiating the function $f_1$ on a logarithmic scale.

$$[11] \qquad f_3(s) = \frac{\mathrm{d}[f_1(s)]}{\log(s)} \alpha$$

where $\alpha$ is a scale factor.

Function $f_6$ can be obtained in a manner similar to that of function $f_3$. The following equations are used to present functions $f_3$ and $f_6$:

$$[12] \qquad f_3(\psi, \psi_{\mathrm{ae}}) = \frac{\psi^{t_1}}{\psi^{t_1} + \psi_{\mathrm{ae}}^{t_1}} \frac{\psi_{\mathrm{ae}}^{t_1}}{\psi^{t_1} + \psi_{\mathrm{ae}}^{t_1}} (S_1 - S_2) \frac{\ln(10)}{2t_1}$$

$$[13] \qquad f_6(\psi, \psi_{\mathrm{r}}) = \frac{\psi^{t_2}}{\psi^{t_2} + \psi_{\mathrm{r}}^{t_2}} \frac{\psi_{\mathrm{r}}^{t_2}}{\psi^{t_2} + \psi_{\mathrm{r}}^{t_2}} (S_2 - S_3) \frac{\ln(10)}{2t_2}$$

Substituting eqs. [7]–[10] and [12]–[13] into eq. [5] yields the following SWCC equation:

$$[14] \qquad w(\psi) = \left( \begin{array}{l} A(\psi) \cdot (S_2 - S_1) \left\{ \log\left(\dfrac{\psi}{\psi_{\mathrm{ae}}}\right) - \dfrac{\ln(10)}{2t_1} \left[1 - A(\psi)\right] \right\} \\ +(S_3 - S_2) \left\{ \log\left(\dfrac{\psi}{\psi_{\mathrm{r}}}\right) - \dfrac{\ln(10)}{2t_2} [1 - B(\psi)] \right\} \end{array} \right) B(\psi) + S_3 \log\left(\frac{10^6}{\psi}\right)$$

**Fig. 4.** Plots of eq. [6] used for the representation of functions $f_1$ and $f_4$.

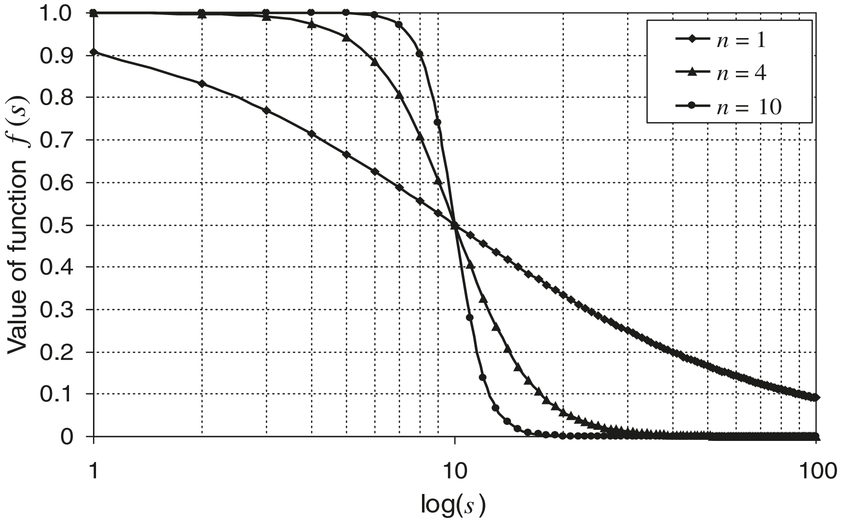
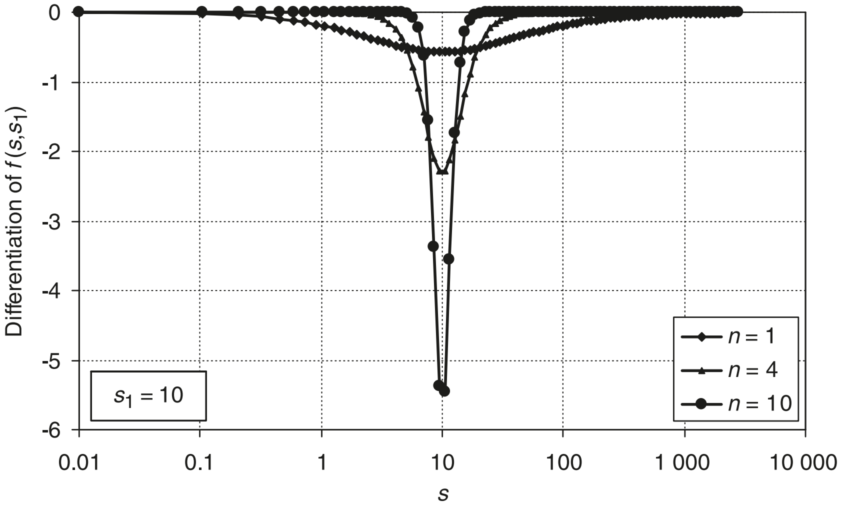


**Fig. 5.** Plots of the differentiation of function $f(s,s_1)$ with $s_1$ = 10 and various $n$ values.



where

$$A(\psi) = \frac{{\psi_{\mathrm{ae}}}^{t_1}}{\psi^{t_1} + {\psi_{\mathrm{ae}}}^{t_1}}$$

$$B(\psi) = \frac{{\psi_{\mathrm{r}}}^{t_2}}{\psi^{t_2} + {\psi_{\mathrm{r}}}^{t_2}}$$

The slope of the curve at and beyond residual soil suction is particularly of interest in geotechnical engineering when considering the prediction of actual evaporation under high soil suction conditions (Wilson et al. 1997). The variable $S_3$ can be replaced by a more meaningful variable $w_{\mathrm{sat}}$ (i.e., gravimetric water content at a soil suction of 1 kPa). The relationship between $S_3$ and $w_{\mathrm{sat}}$ can be written as follows:

$$[15] \qquad S_3 = \frac{w_{\mathrm{sat}} + (S_2 - S_1)\log(\psi_{\mathrm{ae}}) - S_2\log(\psi_{\mathrm{r}})}{\log\left(\frac{10^6}{\psi_{\mathrm{r}}}\right)}$$

where $w_{\mathrm{sat}}$ is the gravimetric water content at a soil suction of 1 kPa.

Equation [14] can now be reduced to the form shown in eq. [16]. Plots of eq. [16] using various soil properties (i.e., curve-fitting parameters) are shown in Fig. 6. The transition parameters used for plotting Fig. 6 are chosen as follows: $t_1$ = 5 and $t_2$ = 5.

$$[16] \qquad \begin{aligned} w(\psi) = {} & w_{\mathrm{sat}}(M_1 + M_3) \\ & + S_1[-\log(\psi_{\mathrm{ae}})(M_1 + M_3) - M_2] \\ & + S_2\left[-M_1\log\left(\frac{10^6}{\psi_{\mathrm{ae}}}\right) - \log\left(\frac{\psi_{\mathrm{r}}}{\psi_{\mathrm{ae}}}\right)M_3 + M_2\right] \end{aligned}$$

where

$w_{\mathrm{sat}}$ is the water content at 1 kPa soil suction,

$\psi_{\mathrm{r}}$ is the residual soil suction,

$\psi_{\mathrm{ae}}$ is the air-entry value,

$S_1$, $S_2$ are slopes of the SWCC of the portion less than the air-entry value and the portion between the air-entry value and the residual soil suction, respectively,

$M_1(\psi)$ is equal to $\dfrac{\left[\log\left(\frac{\psi}{\psi_{\mathrm{r}}}\right) - \frac{\ln(10)}{2t_2}\left(1 - \frac{{\psi_{\mathrm{r}}}^{t_2}}{{\psi_{\mathrm{r}}}^{t_2} + \psi^{t_2}}\right)\right]\frac{{\psi_{\mathrm{r}}}^{t_2}}{{\psi_{\mathrm{r}}}^{t_2} + \psi^{t_2}}}{\log\left(\frac{10^6}{\psi_{\mathrm{r}}}\right)}$,

**Fig. 6.** Sample plots of the meaningful parameter SWCC equation (eq. [16]) using various curve-fitting parameters.

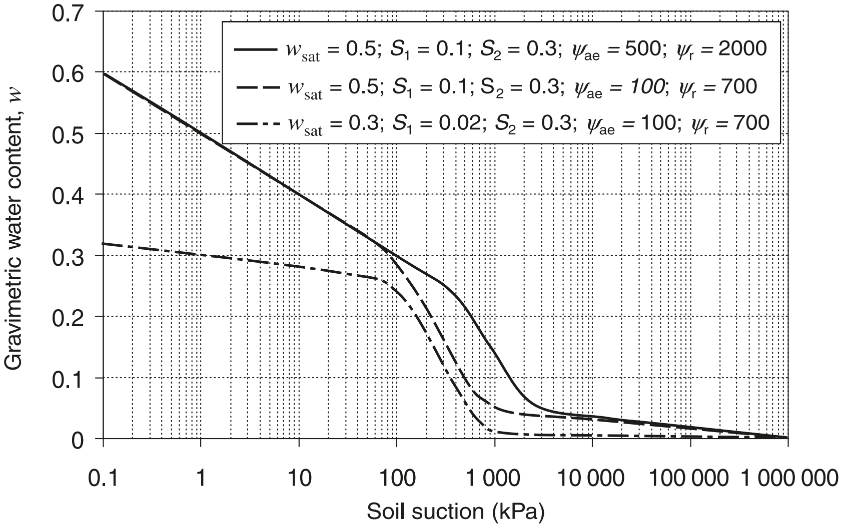
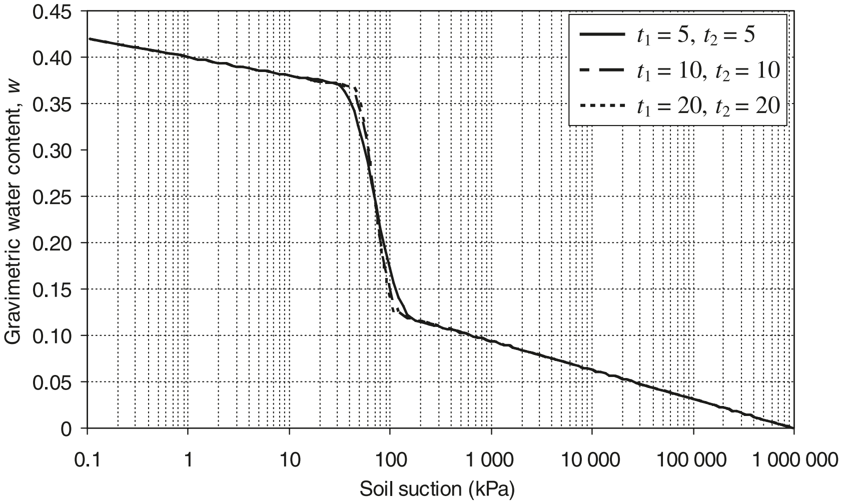


**Fig. 7.** Sample plots of eq. [16] for an artificial silt with $w_{sat} = 0.4$, $S_1 = 0.02$, $S_2 = 0.8$, $\psi_{ae} = 50$ kPa, $\psi_r = 100$ kPa, and various values for the transition parameters.



$M_2(\psi)$ is equal to

$$\left[\log\left(\frac{\psi}{\psi_{ae}}\right) - \frac{\ln(10)}{2t_1}\left(1 - \frac{{\psi_{ae}}^{t_1}}{{\psi_{ae}}^{t_1} + \psi^{t_1}}\right)\right]\frac{{\psi_{ae}}^{t_1}{\psi_r}^{t_2}}{({\psi_{ae}}^{t_1} + \psi^{t_1})({\psi_r}^{t_2} + \psi^{t_2})},$$

$M_3(\psi)$ is equal to $\frac{\log(10^6) - \log(\psi)}{\log(10^6) - \log(\psi_r)}$, and

$t_1$, $t_2$ are curve-fitting parameters that control the transition between portions of the SWCC.

The curve-fitting parameter $t_1$ controls the radius of curvature at the air-entry value, and the curve-fitting parameter $t_2$ controls the radius of curvature at residual suction. These two curve-fitting parameters have little influence on the shape of the remainder of the SWCC (Fig. 7). The transition parameters, $t_1$ and $t_2$, can be chosen as constants prior to the curve-fitting process (e.g., $t_1 = 5$ and $t_2 = 5$). The higher values of the transition parameters $t_1$ and $t_2$ result in sharper transition sections along the SWCC. When best-fitting the degree of saturation (or volumetric water content) SWCC, the curve-fitting parameter $S_1$ can be assumed to be zero. The curve-fitting parameter $w_{sat}$ should be replaced by a constant number (i.e., 100% for the degree of saturation).

## Simplified SWCC equation

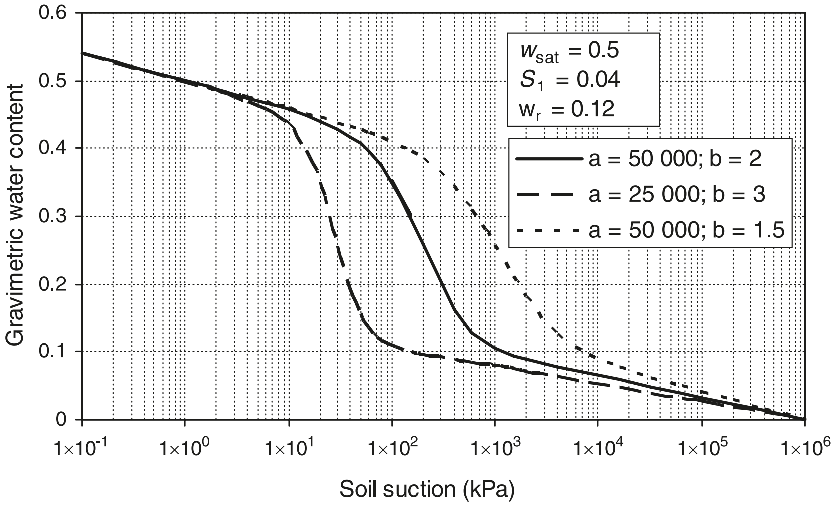
The simplified SWCC equation for high volume change soils also has curve-fitting parameters that are independent of one another, but may not have real physical meaning. A similar mathematical technique to that previously presented is used to build the simplified SWCC equation with independent soil properties. The equation has two parts; namely (*i*) a first part for soil suctions less than residual soil suction (i.e., $\psi_r$), and (*ii*) a second part that makes use of the Fredlund and Xing (1994) correction factor. The proposed equation is as follows:

$$[17] \qquad w(\psi) = \left\{[w_{sat} - S_1\log(\psi) - w_r]\frac{a}{\psi^b + a} + w_r\right\} \times \left[1 - \frac{\ln\left(1 + \frac{\psi}{\psi_r}\right)}{\ln\left(1 + \frac{10^6}{\psi_r}\right)}\right]$$

where

$w_{sat}$ is the gravimetric water content at 1 kPa soil suction,
$S_1$ is the slope of the portion of the curve at low soil suction,

**Fig. 8.** Plots of eq. [17] corresponding to the use of different curve-fitting parameters.



$w_r$ is the residual water content,
$a$, $b$ are curve-fitting parameters, and
$\psi_r$ is the residual soil suction (i.e., can be approximated by $(2.7a)^{1/b}$).

The water content of a soil at a soil suction of 1 kPa will be used as a reference starting point for the SWCC. The slope of the portion of the SWCC at soil suctions less than the air-entry value is also defined at 1 kPa soil suction. Therefore, $w_{sat}$ and $S_1$ can be considered to be known values. It has been found that the actual break point in the SWCC occurs at a soil suction value of approximately $(2.7a)^{1/b}$. The user can choose to use a residual suction, $\psi_r$, equal to $(2.7a)^{1/b}$ or use an independent curve-fitting parameter. If $\psi_r$ is replaced by $(2.7a)^{1/b}$, then the curve has only three additional curve-fitting parameters; namely, $w_r$, $a$, and $b$. Plots of eq. [17] using various soil parameters are shown in Fig. 8.

## Presentation of the equations using measured soil data

The performance of the two proposed equations is illustrated using measured SWCCs for seven soils. A description of the soils is presented followed by the results of the two curve-fitting equations. The *Genfit* function in MathCad 2000 (MathSoft Inc. 1999) was used to best-fit the proposed functions to the measured data. The *Genfit* function is used to find curve-fitting parameters with least-squared errors.

### Soils

Three soils reported in the literature, along with four soils tested in the laboratory at the University of Saskatchewan, Saskatoon, Canada, were analyzed. The soils collected from research literature are: (*i*) Regina clay (Fredlund 1964); (*ii*) Jossigny loam (Fleureau et al. 1995); and (*iii*) kaolinite (Fleureau et al. 2004). The four soils tested in the laboratory program are: (*i*) Beaver Creek sand; (*ii*) processed silt; (*iii*) Saskatchewan silty sand; and (*iv*) Indian Head till (Pham. 2005). The soils used in the testing program were initially slurry soils. The classification properties of the soils tested are presented in Table 1. The grain-size distribution curves for the soils are presented in Fig. 9.

### Presentation of the results using the meaningful SWCC parameter equation

The best-fit results for the seven soils using the meaningful parameter SWCC equation (eq. [16]) are presented in this section. To reduce difficulties associated with the fitting procedure, the curve-fitting parameter $w_{sat}$ was estimated for each soil. A curve-fitting procedure was then used to find the following soil properties: (*i*) the slope of the SWCC at soil suctions less than air-entry value, $S_1$; (*ii*) the slope of the main portion of the SWCC (i.e., in the desaturation zone), $S_2$; (*iii*) the air-entry value of the soil, $\psi_{ae}$; and (*iv*) residual soil suction, $\psi_r$. Best-fitting curves for the SWCC for Regina clay (Fredlund 1964), Jossigny loam (Fleureau et al. 1995), and kaolinite (Fleureau et al. 2004) are shown in Fig. 10. A few data points were added along the desaturation portion for Regina clay to make the dataset more complete. Best-fitting curves for the measured SWCC data for the four soils tested in the laboratory program are shown in Fig. 11. Curve-fitting parameters associated with the seven soils are summarized in Table 2.

### Presentation of the simplified SWCC equation

The best-fit results for the seven soils using the simplified SWCC equation (i.e., eq. [17]) are presented in this section. To simplify the curve-fitting procedure, the slopes of the SWCCs at soil suctions less than the air-entry values for the seven soils were estimated prior to undertaking the fitting procedure. The fitting procedure was then used to find the following curve-fitting parameters: $a$, $b$, $S_1$, $w_{sat}$, and $w_r$. Best-fit SWCC parameters for the three collected soils using the simplified SWCC equation (i.e., eq. [17]) are shown in Fig. 12. Best-fit SWCC parameters for the four soils tested in the laboratory using the simplified SWCC equation (i.e., eq. [17]), are shown in Fig. 13. A summary of the curve-fitting parameters corresponding to the simplified SWCC equation for the seven soils are shown in Table 3.

## Discussions

Both proposed SWCC equations (i.e., eqs. [16] and [17]) make use of the curve-fitting parameters, $w_{sat}$ and $S_1$ corresponding to saturated conditions starting from a suction of

**Table 1.** Soil properties of the four soils tested in the laboratory testing program.

| Soil | Liquid limit, LL (%) | Plastic limit, PL (%) | Specific gravity, $G_s$ |
|---|---|---|---|
| Beaver Creek sand | NA | NA | 2.65 (Bruch 1993) |
| Saskatchewan silty sand | NA | NA | 2.65 |
| Processed silt | 26.8 (Wilson 1990) | 25.4 (Wilson 1990) | 2.70 (Wilson 1990) |
| Indian Head till | 36.1 | 16.4 | 2.73 |

**Note:** NA, not applicable.

**Fig. 9.** Grain-size distributions for the four soils used in the laboratory testing program.

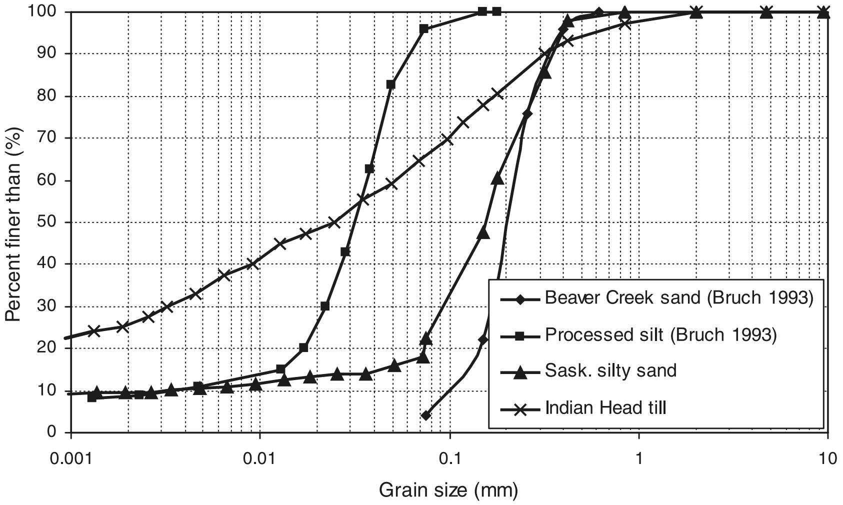
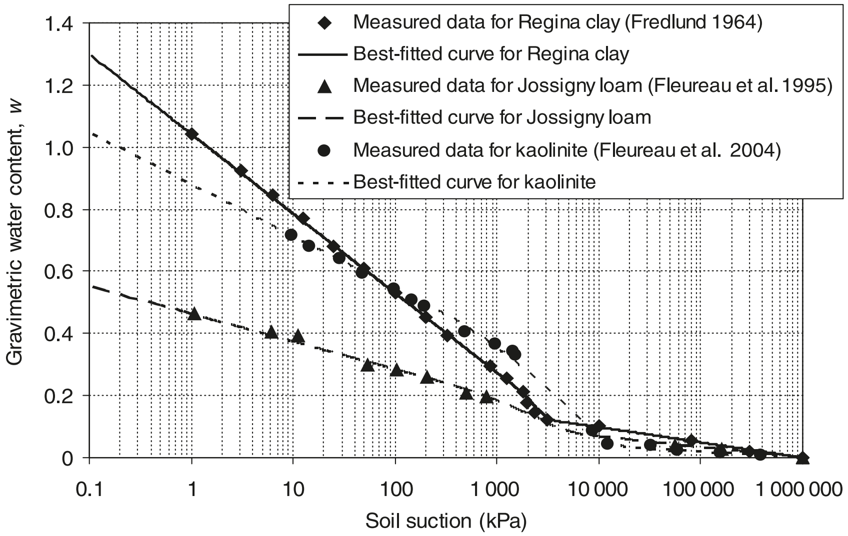


**Fig. 10.** Best-fitting curves using the meaningful parameter SWCC equation (eq. [16]) along with the measured data points for the three soils collected from the research literature.



1 kPa. The variable, $w_{\text{sat}}$ represents the gravimetric water content at a suction of 1 kPa for a soil, provided the soil has an air-entry value greater than 1 kPa. For soils with an air-entry value less than 1 kPa, $w_{\text{sat}}$ represents the value of water content at a suction of 1 kPa on an extension of the line through soil suctions less than the air-entry value on the SWCC. Curve-fitting parameter, $S_1$, represents the slope of the gravimetric SWCC at soil suctions less than the air-entry value. These two soil properties (i.e., curve-fitting parameters) can be estimated or measured using one of several possible testing methods.

Figures 10–13 show that the two proposed SWCC equations fit the measured SWCC data for the seven soils well. The soils have a wide range of grain-size and plasticity properties (i.e., from sand to clay). Both equations make use of the curve-fitting parameters $w_{\text{sat}}$ and $S_1$ (i.e., representing the gravimetric water content at 1 kPa soil suction and slope of the SWCC at soil suctions less than the air-entry value of the soil). It can be seen from Tables 2 and 3 that there is a good agreement between the two equations on the values obtained for the curve-fitting parameters $w_{\text{sat}}$ and $S_1$ for each soil. One exception is the curve-fitting pa-

**Fig. 11.** Best-fitting curves using the proposed meaningful parameter SWCC equation (eq. [16]) along with the measured data points for four soils tested in the laboratory program.

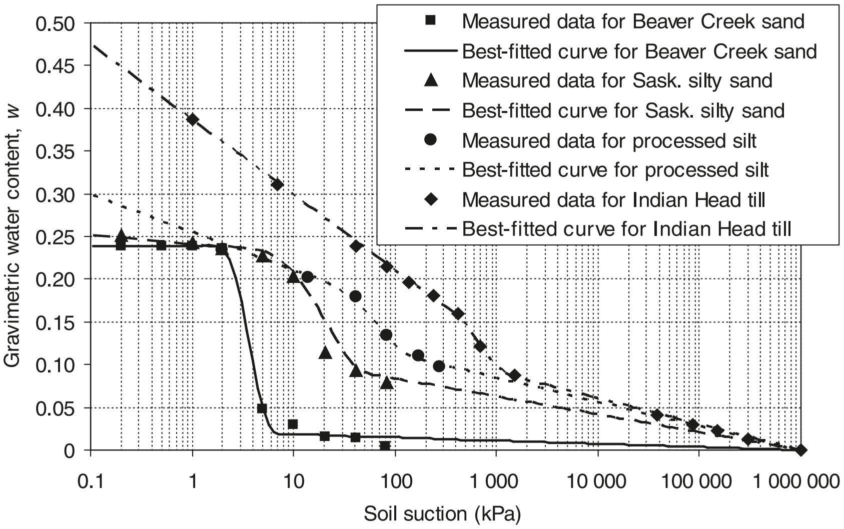


**Table 2.** Curve-fitting parameters for seven soils using the proposed meaningful parameter SWCC equation with independent soil properties (i.e., eq. [16]).

| | | Curve-fitting parameters[a] | | | | |
|---|---|---|---|---|---|---|
| ID | Soil | $w_{sat}$ | $S_1$ | $S_2$ | $\psi_{ae}$ | $\psi_r$ |
| 1 | Regina clay[b] | 1.042 | 0.256 | 0.677 | $2.31\times10^3$ | $2.78\times10^3$ |
| 2 | Jossigny loam | 0.463 | 0.089 | 0.169 | $1.01\times10^3$ | $4.82\times10^3$ |
| 3 | Kaolinite | 0.881 | 0.168 | 0.303 | $8.83\times10^2$ | $1.25\times10^4$ |
| 4 | Beaver Creek sand | 0.239 | 0.003 | 0.652 | $2.46\times10^0$ | $5.33\times10^0$ |
| 5 | Saskatchewan silty sand | 0.241 | 0.010 | 0.200 | $8.01\times10^0$ | $4.02\times10^1$ |
| 6 | Processed silt | 0.255 | 0.044 | 0.140 | $3.37\times10^1$ | $1.18\times10^2$ |
| 7 | Indian Head till[c] | 0.387 | 0.088 | 0.498 | $8.01\times10^2$ | $9.58\times10^2$ |

[a] Parameters $t_1$ and $t_2$ are chosen as constant values prior to the fitting procedure: $t_1 = 4$ and $t_2 = 8$.

[b] Several empirical data points were added along the desaturation part for the Regina clay for ease in curve fitting.

[c] The data points for the soil-water characteristic curve at soil suctions higher than 1500 kPa were measured by Sillers (1997).

rameter, $S_1$, for the sand material where the compression index is extremely small. There are slight differences in the residual soil suctions (i.e., on log scale) using the two curve-fitting equations, but these differences are deemed acceptable.

The meaningful parameter SWCC equation requires seven curve-fitting parameters (i.e., including two parameters for transition points corresponding to different portions of the curve). If the two transition parameters are chosen prior to the curve-fitting exercise, then only five curve-fitting parameters are required. The five curve-fitting parameters are independent and have physical soil property meaning (e.g., water content at 1 kPa soil suction, slope of the curve at soil suctions less than the air-entry value, air-entry value, residual soil suction, maximum slope of the SWCC). An advantage of the meaningful parameter SWCC equation is that the curve-fitting parameters can be estimated directly from several basic soil properties or from soil classification properties. This feature is particularly useful when performing a sensitivity type study. The proposed meaningful parameter SWCC equation may appear to be more complex than previous SWCC equations but the equations have the advantage of more closely representing actual water content versus soil suction conditions for all soil types.

The simplified SWCC equation also requires five curve-fitting parameters. However, only four parameters are truly independent (i.e., the residual soil suction can be approximated using two other curve-fitting parameters: *a* and *b*). Of the four curve-fitting parameters, only two parameters have a physical meaning (i.e., the gravimetric water content at 1 kPa soil suction, and the slope of the SWCC at soil suctions less than the air-entry value). The advantage of this equation is its mathematical simplicity while still retaining an accurate representation of measured SWCC results over the entire soil suction range. The simplified SWCC equation has a defined slope at soil suctions less than the air-entry value and appears to be no more complex than previously proposed equations (i.e., King 1965; van Genuchten 1980; Pereira and Fredlund 1997). The equation is relatively simple to use and can be readily incorporated into numerical models in an analytical form for solving geotechnical engineering problems.

**Fig. 12.** Best-fit SWCCs using the simplified SWCC equation (eq. [17]) along with the measured data points for the three soils collected from the research literature.

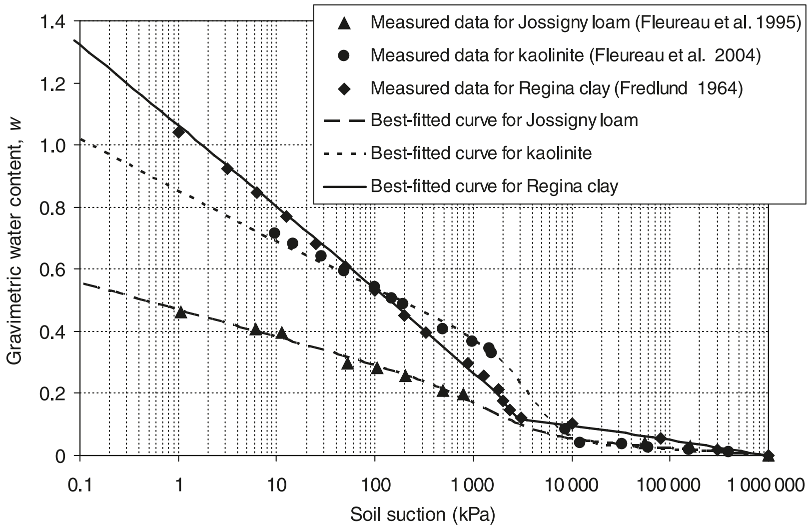
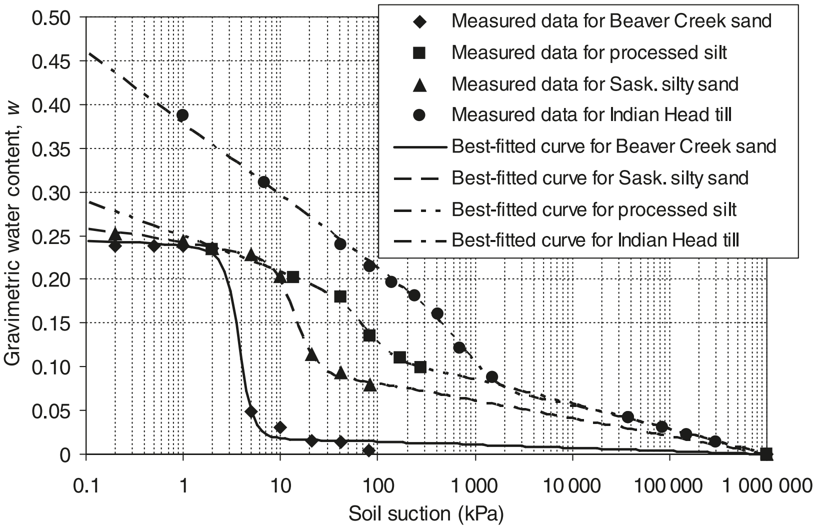


**Fig. 13.** Best-fit SWCCs using the simplified SWCC equation (eq. [17]) along with the measured data points for the four soils tested in the testing program.



## Conclusions

Two new SWCC equations have been proposed for best-fitting the SWCC for a wide range of soils types including high volume change clay. The two equations were developed based on past research that has defined the shape of the SWCC for a volume change soil. Each of the two proposed SWCC equations has its own advantages. Appropriate soil parameters can be determined using experimental data or estimations based on previous experience. The procedure used will depend upon the application at-hand.

The meaningful SWCC parameter equation has five physically meaningful curve-fitting parameters. The curve-fitting parameters for the equation can be obtained from laboratory data or they can be estimated from previous experience based on soil classification properties. This equation is particularly useful for performing sensitivity type studies.

The simplified SWCC equation has the advantage of mathematical simplicity. The equation is of similar complexity to those previously proposed to represent the SWCC. This equation is particularly useful when performing numerical modeling studies.

Both of the proposed SWCC equations define the soil suction versus water content over the entire range of possible soil suctions. The volume change characteristics can be taken into account in both of the proposed equations. Both of the proposed equations adhere to the findings of research on a wide range of soil types. The soil properties for the SWCC equations also have the advantage of being restricted to a particular portion of the suction versus water content relationship. There are now many empirical equations that have been proposed for the SWCC; however, the more recently proposed equations are becoming more closely related to the physical properties of the soil.

**Table 3.** Curve-fitting parameters for seven soils using the simplified SWCC equation (i.e., eq. [17]).

| | | Curve-fitting parameters | | | | | |
|---|---|---|---|---|---|---|---|
| ID | Soil | $w_{\mathrm{sat}}$ | $S_1$ | $a$ | $b$ | $w_{\mathrm{r}}$ | $\psi_{\mathrm{r}}$[a] |
| 1 | Regina clay[b] | 1.057 | 0.261 | $1.07\times10^{33}$ | 9.838 | 0.131 | $2.52\times10^{3}$ |
| 2 | Jossigny loam | 0.467 | 0.086 | $7.13\times10^{4}$ | 1.404 | 0.064 | $5.81\times10^{3}$ |
| 3 | Kaolinite | 0.830 | 0.145 | $1.92\times10^{9}$ | 2.527 | 0.049 | $6.99\times10^{3}$ |
| 4 | Beaver Creek sand | 0.242 | 0.0046 | $3.70\times10^{3}$ | 6.186 | 0.020 | $4.43\times10^{0}$ |
| 5 | Saskatchewan silty sand | 0.244 | 0.015 | $5.30\times10^{4}$ | 4.052 | 0.098 | $1.87\times10^{1}$ |
| 6 | Processed silt | 0.250 | 0.036 | $3.01\times10^{5}$ | 2.824 | 0.114 | $1.24\times10^{2}$ |
| 7 | Indian Head till[c] | 0.382 | 0.084 | $1.47\times10^{8}$ | 2.627 | 0.078 | $1.88\times10^{3}$ |

[a] The residual soil suction is calculated from the curve-fitting parameters *a* and *b*.

[b] Several empirical data points were added along the desaturation part of the Regina clay for ease in curve fitting.

[c] The data points for the soil-water characteristic curve at soil suctions higher than 1500 kPa were measured by Sillers (1997).

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