Unsaturated Soils: Advances in Geo-Engineering – Toll et al. (eds)
© 2008 Taylor & Francis Group, London, ISBN 978-0-415-47692-8

Theoretical modelling of the compaction curve

N. Kurucuk & J. Kodikara
Department of Civil Engineering, Faculty of Engineering, Monash University, Clayton, VIC, Australia

D.G. Fredlund
Golder Associates, Saskatoon, Saskatchewan, Canada

ABSTRACT: Soil compaction is one of the major activities in geotechnical engineering involving earthworks. The compaction curve is used to find the optimum water content that maximizes dry density. Since its introduction by Proctor in 1933, several researchers have provided qualitative explanations for the inverted parabolic shape of the compaction curve. However, fundamental research on the compaction process and the evolution of compaction characteristics are limited, particularly from a quantitative sense. In order to understand the driving mechanisms of soil compaction, this paper investigates the effect of soil suction, stiffness and pore air pressure on the shape of the compaction curve, from an unsaturated soil mechanics standpoint. This paper presents an approach to predict the soil compaction curve during undrained loading. Particular attention is focused on the derivation of the compressibility coefficient due to net stress. Model predictions of the compaction curve are compared with some experimental results from the literature.

1 INTRODUCTION

Soil compaction is widely used in geo-engineering and is important for the construction of roads, dams, landfills, airfields, foundations, hydraulic barriers, and ground improvements. Compaction is applied to the soil, with the purpose of finding optimum water content in order to maximize its dry density, and therefore, to decrease compressibility, increase shearing strength, and in some cases, to reduce permeability. Proper compaction of materials ensures the durability and stability of earthen constructions.

A typical compaction curve presents different densification stages when the soil is compacted with the same apparent energy input but different water contents. The water content at the peak of the curve is called the optimum water content (OWC) and represents the water content at which dry density is at its maximum for a given compaction energy.

Since Proctor's pioneering work in 1933, many researchers have attempted to explain qualitatively the leading mechanisms in the densification stages, mainly on the dry side of optimum water content. The compaction curve was explained in terms of capillarity and lubrication (Proctor, 1933), viscous water (Hogentogler, 1936), pore pressure theory in unsaturated soils (Hilf, 1956), physico-chemical interactions (Lambe, 1960), and concepts of effective stress theory (Olson, 1963). More recently, Barden & Sides (1970) undertook experimental research on the relation between the engineering performance of compacted unsaturated clay and microscopic observations of clay structure. In addition, Lee & Suedkamp (1972) conducted research on the shape of the compaction curve for different soils.

Despite this research work, and the importance and high demand for the compaction process in engineering practice, the compaction of soil is quite complex and not well explained, particularly from a quantitative sense. Theoretical modelling of the soil compaction curve will provide a better understanding of the main parameters that affect the shape of the compaction curve, and understanding the behaviour of compacted materials. Therefore, there is need for research to be undertaken at a fundamental level to understand the compaction characteristics of soil and the inverted parabolic shape of the compaction curve.

This paper presents a theoretical explanation of the compaction curve using unsaturated soil mechanics principles. Particular attention is focused on the prediction of the compressibility coefficient due to net stress. Likely predictions of the model are compared with the experimental results from literature.

2 THEORETICAL BACKGROUND FOR MODELLING

Theoretical concepts utilized for the development of soil compaction curves are presented in this section.

Initially, Hilf's (1948) approach for pore pressure development is presented. This is continued with Fredlund & Morgenstern's (1976) volume change theory for a compacted soil and the derivation of the dry density of soil.

2.1 *Pore pressure development during static compaction*

One of the main simulations for the generation of the compaction curve is that of pore pressure development. Hilf (1948) developed a relationship between pore pressure and applied stress, which is based on one-dimensional K_0 soil compression, Boyle's law, and Henry's law, and is expressed as follows:

$$\Delta u_a = \left[\frac{1}{1 + \frac{(1-S_0+hS_0)n_0}{(u_{a0}+\Delta u_a)m_v}} \right] \Delta \sigma_y \tag{1}$$

where; Δu_a = change in absolute pore air pressure, S_0 = initial degree of saturation, h = coefficient of solubility, n_0 = initial porosity, u_{a0} = initial absolute air pressure, m_v = coefficient of volume change in saturated soil, and $\Delta\sigma_y$ = change in applied vertical stress.

Hilf (1948) developed this equation assuming that air and water phases are undrained, and volume reduction is due to air dissolving in the water and compression of free air. Both liquid and solid parts were considered to be volumetrically incompressible. Hilf also assumed that the change in pore air pressure is equal to the change in pore water pressure, and therefore, matric suction change was insignificant. Experimental results on suction change during compaction can be found in literature (e.g. Li 1995, Montanez 2002). It is shown that matric suction only decreases marginally with a density increase and may be approximated to be constant. Therefore, Hilf's analysis assuming constant suction during compaction appears to be close to the real situation. Further justification for assuming constant matric suction during the compaction test is presented in Kurucuk et al. (2007).

2.2 *Computation of volume change and dry density*

The volume change constitutive relationship as applicable to K_0 loading, which is defined in terms of two independent stress variables as proposed by Fredlund & Morgenstern (1976) for unsaturated soils, is used for the calculation of compaction curves:

$$\varepsilon_v = \frac{\Delta V_v}{V} = m_1^s \Delta \left(\sigma_y - u_a\right) + m_2^s \Delta \left(u_a - u_w\right) \tag{2}$$

where; ε_v = volumetric strain, ΔV_v = overall volume change of soil element, V = initial total volume of soil element, m_1^s = compressibility of soil particles with respect to net stress $(\sigma_y - u_a)$, m_2^s = compressibility of soil particles referenced to matric suction $(u_a - u_w)$, $\Delta(\sigma_y - u_a)$ = change in net stress, and $\Delta(u_a - u_w)$ = change in soil suction.

Since soil particles are incompressible, it is accepted that deformation is primarily due to compression of the pore fluid (i.e., the air and air/water mixture). The independent stress state variable concept is utilized in the derivation; namely, net stress $(\sigma_y - u_a)$ (causes a reduction in volume with compression), and matric suction stress $(u_a - u_w)$ (generally results in volume increase with compression). Once the overall volume change is computed, the corresponding dry density can be easily computed.

3 MODELLING ASSUMPTIONS

Kurucuk et al. (2007) showed that the assumption of constant coefficients of compressibility during compaction does not produce a proper shape of the compaction curve especially on the dry side of the optimum water content. Their analysis showed that it is m_1^s that controls the volume changes during compaction because the associated change in suction may be neglected. The parameter m_1^s was represented as a function of saturation and decreases with decreasing saturation. However, the experimental results presented by Loret et al. (2003) showed that m_1^s decreased with both suction and net stress. Therefore, following the functional form suggested by Sheng et al. (2007), the volumetric strain, ignoring suction change, may be presented as:

$$\varepsilon_v = \frac{dV}{V} = \lambda_{vp} \frac{d\left(\sigma_{net} - u_a\right)}{\left(\sigma_{net} - u_a\right) + s_0} \tag{3}$$

where; ε_v = volumetric strain, $(\sigma_{net} - u_a)$ = mean net stress, u_a = pore air pressure, s_0 = suction, λ_{vp} = slope of the normal compression line (NCL) of the saturated soil, and V = initial total volume of the soil element.

This gives m_1^s as:

$$m_1^s = \frac{\lambda_{vp}}{\left(\sigma_y - u_a\right) + s_0} \tag{4}$$

This assumption will be used and discussed further in the modelling of the compaction curve. It is reasonable to replace m_v in Equation 1 by m_1^s. A numerical example of the variation of m_1^s during compaction process is given in the following section. Equations (1), (3) and (4) were used in incremental forms to compute the incremental and total volume change and the corresponding dry density values during compaction.

4 NUMERICAL EXAMPLES

The performance of the proposed model is demonstrated by comparing the experimental results presented by Montanez (2002) and Kenai et al. (2006). Figures 1 and 2 show the compaction curves for sand-bentonite mixture with bentonite content of 5% and 15% by weight. Montanez's experimental data present values for the Standard Proctor Test (BS, external gross energy input = 637 kJ/m^3 or kPa). In Figures 1 and 2, two model predictions are also shown. The curves shown by dashed lines represent the static compaction curve predicted by the model for undrained (air/water) loading up to external quasi-static pressure, σ_y, of 637 kPa. The curves shown by solid lines are for equal energy input, calculated by integrating the applied stress σ_y with respect to volumetric strain. The actual energy input into the soil was computed on the basis of the values applicable at the optimum water content, which were found to be 16 kJ/m^3 and 18 kJ/m^3 respectively.

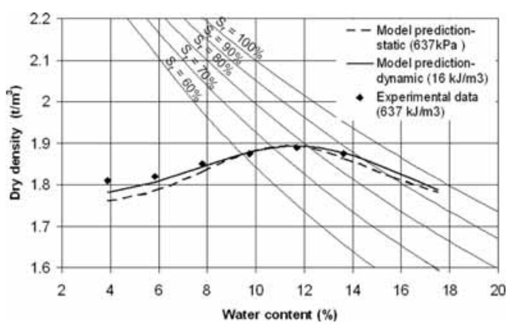

Figure 1. Comparison of predicted and experimental compaction curves for well graded sand with 5% bentonite (after Montanez, 2002).

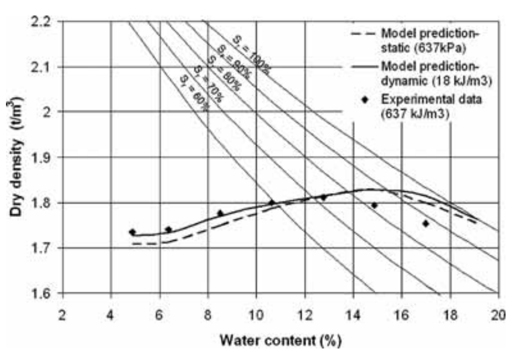

Figure 2. Comparison of predicted and experimental compaction curves for well graded sand with 15% bentonite (after Montanez, 2002).

Figure 3 shows an example of compaction curve for clay sandy soil (liquid limit = 39%, plasticity index = 15%) adopted from Kenai et al. (2006). Experimental results shown in figure are for static (σ_y = 2100 kPa) and dynamic (external gross energy input 3000 kJ/m^3) compaction tests. Both predicted compaction curves are produced from quasi-static compaction up to external pressures, σ_y, of 2100 kPa and 4000 kPa respectively.

Model parameters used for prediction of the above compaction curves are shown in Table 1, 2 and 3. Initial pore air pressure (u_{a0}) is taken to be equal to atmospheric pressure (101.3 kPa). For a certain soil, a lower initial porosity was assumed and the computations were performed for a range of moisture contents which also define the values of initial degree of saturation (S_0). The water solubility value is adopted from Fredlund & Rahardjo (1993). The values of λ_{vp} (slope of the NCL) are selected to best fit the experimental results and compared with the measured values from literature. These values are found to be generally in the range of experimentally measured values. Table 2 shows the initial equilibrium suctions measured for compacted specimens at different moisture contents given by Montanez (2002). They are presented as constant suction contours which are

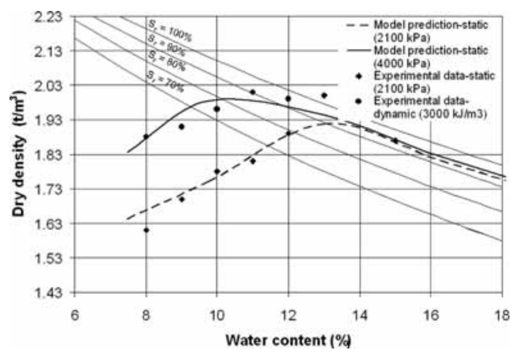

Figure 3. Comparison of predicted and experimental compaction curves for clay sandy soil (after Kenai et al., 2006).

Table 1. Parameter values for the proposed model.

	Well graded sand with 5% bentonite	Well graded sand with 15% bentonite
Parameter	Value	Value
h*	0.02	0.02
λ_{vp}	0.045	0.13
n_0	34 %	36 %
G_s	2.656	2.660

* Water solubility

Table 2. Initial matric suction (s_0) values.

Well graded sand with 5% bentonite content		Well graded sand with 15% bentonite content	
w (%)	s_0 (kPa)	w (%)	s_0 (kPa)
3.9	4630	4.9	17800
5.8	1130	6.4	11500
7.8	350	8.5	2550
9.7	130	10.6	1260
11.7	54	12.7	850
13.6	32	14.9	530
15.6	26	17.0	290
17.5	22	19.1	270

Table 3. Parameter values for the proposed model.

	Clay sandy soil
Parameter	Value
h*	0.02
λ_{vp}	1.3
n_0	46 %
G_s	2.66

generally perpendicular to the water content axis giving approximately constant suction values for a given compaction water content. This ignores the curving of these contours close to saturation towards the left eventually becoming almost parallel to the full saturation line.

Figures 1 and 2 show comparisons between experimental and predicted values of compaction curves for sand-bentonite mixtures. The experiments were performed under dynamic conditions (Proctor compaction), whereas model prediction assumed static undrained conditions for both air and water. Despite these differences, it is clear that reasonable predictions of the shape of the compaction curve can be obtained with the proposed approach.

Figure 3 shows comparison between experimental and predicted values of the compaction curve for sandy clay soil. For this example, experiments were performed under both dynamic and static conditions. It should be noted that in this example, experimental results did not include the initial suction values. Therefore, initial suction values are assumed to be same as well graded sand with 15% bentonite (Table 2).

Differences in the predicted and experimental behaviour can be traced to a number of sources. One possibility is the drainage of air, particularly on the dry side of the optimum, which can lead to higher dry densities. This analysis, however, shows that the

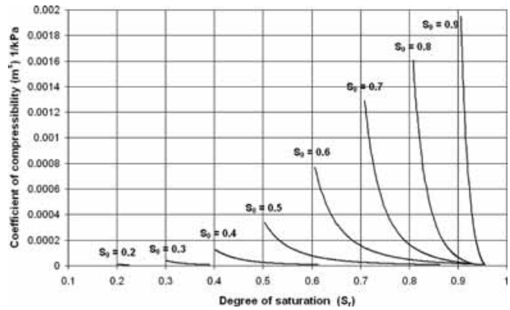

Figure 4. Variations of m^s_1 with initial degree of saturation (S_0) and during compaction for well graded sand with 5% bentonite.

development of air pressure on the dry side is not very significant, but will depend on m^s_1. It can be seen that the predicted and experimental density difference on Figure 1 is higher than that of Figure 2. In addition to the likely influence of the other assumptions made in the analysis, this difference seems to indicate that the drainage of air may lead to higher densities in the dry side. It is likely that the pore sizes in well graded sand with 5% bentonite content is higher than the same sand with 15% bentonite. These issues will be further examined through future targeted experiments.

Figure 4 shows the variation of m^s_1 with initial degree of saturation (S_0) and during compaction for the well graded sand with 5% bentonite content. It can be seen that coefficient of compressibility due to net stress (m^s_1) decreases with decreasing initial degree of saturation (S_0), as assumed previously by Kurucuk et al. (2007). However, during the compaction process, the degree of saturation increases from the initial value, but the coefficient of compressibility (m^s_1) decreases. This decrease of compressibility happens owing to the increase of net stress as the compaction progresses. It is also apparent that much of the compaction takes place in the early part of the process where the soil compressibility decreases rapidly.

5 CONCLUSION

This paper presents theoretical concepts to predict the compaction curve for soil during undrained K_0 or isotropic loading using unsaturated soil mechanics principles. It highlights the fact that the well-known inverted parabolic shape of the compaction curve may be theoretically predicted using unsaturated soil mechanics principles, arguably for the first time in literature. This was demonstrated using published experimental results, but it was necessary to make some assumptions. The controlling parameter governing the compaction process was identified as the coefficient of compressibility with respect to net stress or m_1^s. It was also identified that the variation in drainage conditions during compaction may influence the results. Future experiments will be targeted to develop a comprehensive set of data to examine the modelling assumptions and improve modelling capability.

ACKNOWLEDGEMENTS

Thanks are rendered to Monash University for providing a Monash Graduate Scholarship and financial assistance to the first author for her PhD candidature.

REFERENCES

Barden, L. & Sides, G.R. 1970. Engineering behaviour and structure of compacted clay. Journal Soil Mechanics and Foundations Division, ASCE, 96, No. SM4: 1171.

Fredlund, D.G. & Rahardjo, H. 1993. Soil mechanics for unsaturated soils. John Wiley & Sons, Inc.

Fredlund, D.G. & Morgenstern, N.R. 1976. Constitutive relations for volume change in unsaturated soils. Canadian Geotechnical Journal, 14, 3: 261–276.

Hilf, J.W. 1948. Estimating construction pore pressures in rolled earth dams. Proceedings of 2nd International Conference in Soil Mechanics and Foundation Engineering, 3: 234–240. Rotterdam, The Netherlands.

Hilf, J.W. 1956. An investigation of pore water pressures in compacted cohesive soils. Technical Memorandum 654, U.S. Department of the Interior, Bureau of Reclamation, Denver, Colorado.

Hogentogler, C.A. 1936. Essentials of soil compaction. Proceedings Highway Research Board, National Research Council, Washington, D.C., 309–316.

Kenai, S., Bahar, R. & Benazzoug, M. 2006. Experimental analysis of the effect of some compaction methods on mechanical properties and durability of cement stabilized soil. Journal of Material Science, 41: 6956–6964.

Kurucuk, N., Kodikara, J. & Fredlund, D.G. 2007. Prediction of compaction curves. 10th ANZ Conference on Geomechanics, 2: 115–119.

Lambe, T.W. 1960. Structure of compacted clay. Transactions, ASCE, 125: 682–705.

Lee, D.Y. & Suedkamp, R.J. 1972. Characteristics of irregularly shaped compaction curves of soil. Highway Research Board, 381: 1–9.

Li, Z.M. 1995. Compressibility and collapsibility of compacted unsaturated loessial soils. Unsaturated Soils. Proc. 1st Int. Conf. on Unsaturated Soils (UNSAT 95), Paris, France (ed. Alonzo, E.E. and Delage, P.), Rotterdam: Balkema, Vol. 1: 139–144.

Lloret, A., Villar, M.V., Sanchez, M., Gens, A., Pintado, X. & Alonso, E.E. 2003. Mechanical behaviour of heavily compacted bentonite under high suction changes. Géotechnique, 53(1): 27–40.

Montanez, J.E.C. 2002. Suction and volume changes of compacted sand-bentonite mixtures. PhD thesis, University of London, Imperial College of Science, London, England.

Olson, R.E. 1963. Effective stress theory of soil compaction. Journal Soil Mechanics and Foundations Division, ASCE, 89, No. SM2: 27–45.

Proctor, R.R. 1933. Fundamental Principles of Soil Compaction, Engineering News-Record, 111: 286.

Sheng, D., Fredlund, D.G. & Gens, A. 2007. A new modelling approach for unsaturated soils using independent stress state variables. Research Report No. 261.11.06, University of Newcastle, NSW 2308, Australia.