ENGINEERING DESIGN PROTOCOLS FOR UNSATURATED SOILS

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Abstract: Guidance and recommendations need to be provided to engineers who desire to use unsaturated soil mechanics in engineering practice. Guidance should come in the form of engineering protocols that define acceptable, high-standard engineering practice. Practicing engineers can then have greater confidence in applying unsaturated soil mechanics. Engineering protocols for unsaturated soil mechanics can be divided into "preliminary design" situations and "final design" situations. Both design levels involve the use of a variety of estimation procedures that have been proposed for various classes of constitutive behavior. This paper attempts to provide guidance and recommendations with regard to the estimation procedures that are most appropriate for engineering practice. Many of the initial estimation procedures were initiated in agriculture-related disciplines but have required later refinement for usage in engineering. Guidance and recommendations are provided regarding the use of Soil-Water Characteristic Curves, (SWCC), and consequently to the generation of unsaturated soil property functions, (USPF).

1. INTRODUCTION

A natural and logical progression for the development of suitable engineering design protocols should involve: i.) the development of a fundamental science with suitable constitutive laws to describe the material behavior of concern, ii.) the verification of suitable laboratory testing procedures to measure appropriate soil properties, and iii.) the designation of design procedures (i.e., protocols) that should be followed in engineering practice (Fredlund and Rahardjo, 1993). While this is the preferred approach, it is not the way that most unsaturated soils protocols have been developed. It has been more common that solutions have been proposed and it is only later that the most appropriate engineering design procedures emerge. To a large extent, this has been the pattern associated with the design of "cover systems" that became a common solution for many situations in the 1980s. Many "cover systems" have now been designed over the years there still does not appear to be a clear engineering design protocol. Consequently, there is considerable uncertainty regarding the design procedures that should be followed to produce a satisfactory engineering design. The same difficulty appears to be associated with other application areas in unsaturated soil mechanics.

A variety of estimation techniques have become an acceptable part of engineering protocols when applying unsaturated soil mechanics. Little has been published on the suitability of each of the estimation procedures for various types of problems. The question can be asked, "Are all estimation procedures for unsaturated soil property functions, USPF, suitable for all engineering problems?" If not, "Under what conditions should various estimation and testing procedures be used?" These are questions for which practicing engineers desire answers in order to utilize sound engineering protocols with confidence. A lack of designated high-level engineering protocols certainly makes practicing engineers more vulnerable to litigation. It also makes it difficult for engineers to insist on adopting high-level engineering practices.

The objective of this paper is to set forth guidance on recommendations for sound engineering protocols for the implementation of unsaturated soil mechanics. It is outside of the scope of the paper to detail suitable engineering protocols for all applications of unsaturated soil mechanics. An attempt is made to provide general guidelines related to the estimation of the SWCC, and to a lesser extent to the USPF, since the SWCC, provides the basis for estimating all other unsaturated soil property functions.

2. COMPONENTS OF A "BOUNDARY VALUE PROBLEM"

The general framework associated with a "boundary value problem" provides a context for practical engineering formulations related to unsaturated soil behavior (Fredlund, 2000, 2006), (Figure 1).

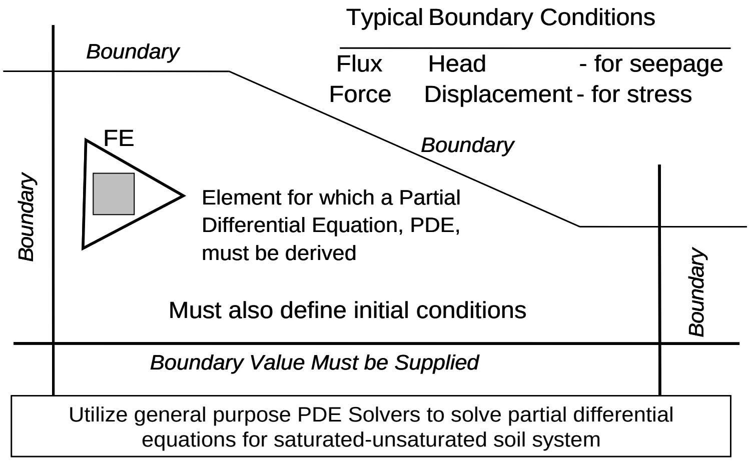


Figure 1. General contextual framework for solving a "boundary value problem".

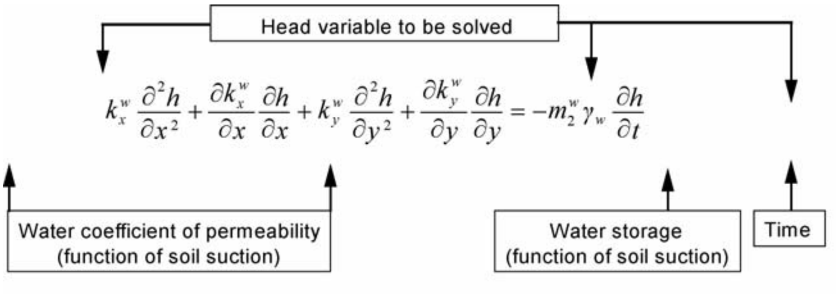


Figure 2. Example of a partial differential equation for two-dimensional saturated –unsaturated flow through a porous media.

2.1 Starting with an R.E.V.

At the "heart" of boundary value formulations is the description of fundamental physical soil behavior for a "Representative Elemental Volume" (i.e., a REV). The physical behavior of the REV of the continuum is derived within the restraints of the conservative laws of physics; namely, i.) conservation of mass, or ii.) conservation of energy. The description of the physical behavior of the REV results in the derivation of a partial differential equation, (i.e., a PDE). Figure 2 shows the type of partial differential equation that is derived for two-dimensional water flow through a saturated-unsaturated porous medium. Those aspects related to conservative physical restraints can be separated from the soil properties which may be constants or have nonlinear functional character.

The physics pertaining to the REV need to be extended to a "finite element" with a particular designated geometry (e.g., triangle). The finite elements can be reproduced over a region until the boundaries of the selected geometry are reached. A numerical technique such as the finite element method is then used to solve a series of equations that embrace the physics for the entire region under consideration.

2.2 Geometry, boundary conditions and soil properties

The primary components of a "boundary value problem" can be listed as follows:

1) Determination of the geometry and the stratigraphy that define the ground surface and the separation of soil strata (or other materials). The geometry and stratigraphy can be described for a one-, two- or three-dimensional representation of the engineering problem. In some situations a one-dimensional representation is satisfactory; however, with advanced computing capabilities, two- and three-dimensional representations become relatively easy, more realistic, and more representative of *in situ* conditions. The description of the geometry and stratigraphy for problems involving unsaturated soil is essentially the same as

for a saturated soils problem. There is no need to spend more time on this aspect of the problem.

2) Assessment of the boundary and initial conditions for analyzing the problem-at-hand. The primary boundary condition that generally needs to be assessed is the ground surface moisture flux. This means the precipitation (rainfall) as well as "actual evaporation" and evapo-transpiration needs to be quantified for a typical one-year period. Initial conditions also need to be determined because the problem being analyzed is nonlinear in character.

The assessment of the moisture flux conditions at the ground surface plays a major role in performing a reliable simulation of an unsaturated soils problem. In fact, it can be said that most unsaturated soils problems are moisture flux problems due to the proximity of unsaturated soils to the ground surface. Precipitation conditions needs to be assessed based on measured weather data. Where sufficient data exists, it might be necessary to determine a precipitation record that represents: i) a typical average year, ii) a precipitation record that represents a typical dry year, and, iii) a year that represents a typical wet year.

A coupled moisture flow and thermal model is necessary for the prediction of "actual evaporation" (Wilson et al, 1994; 1997). The solution takes the form of a "soil-atmosphere model". Evapo-transportation involves the effect of vegetation and requires further approximations to represent the effect of "leaf area index", growing season and other variables.

3) The soil properties for an unsaturated soil take on the character of unsaturated soil property functions, USPF. Estimation procedures have become generally acceptable for the assessment of unsaturated soil property functions (Fredlund, 1996; Fredlund et al, 1997). However, not all estimation procedures are equally reliable and the geotechnical engineer needs to understand the limitations and assumptions associated with each USPF (Fredlund et al, 2003). Some of the factors of importance will later be elaborated upon in this paper.

The assessment of the soil-water characteristic curve forms the basis for all other unsaturated soil property functions. This paper explains some of the factors that must be taken into consideration in order to obtain the most reliable possible solution for unsaturated soils problems. There is a linkage between the soil-water characteristic curves and the unsaturated soil property functions and this linkage must always be maintained in order to obtain reasonably accurate results from numerical analysis.

4) The above information forms the basis for performing a numerical modeling simulation of the problem-at-hand (Fredlund, 1998). The partial differential equation that describes the physics of the problem is nonlinear when unsaturated soils are involved and consequently there are often challenges in achieving convergence to the correct solution. It is important to meet two criteria in solving numerical nonlinear problems; namely, i) the computer software must produce convergence when solving the problem, and ii) there must be convergence to the correct solution. In other words, convergence alone is not a sufficient criterion since there must be assurance that the technique utilized produces convergence to the correct solution.

Unsaturated soils problems are distinctive in that the soil properties always take on a nonlinear soil property form. The unsaturated soil properties are a function of the primary variable for which a solution is being sought. Consequently, the partial differential equation must be solved through use of an iterative technique. It is common for the PDEs being solved to fall within the category of being highly nonlinear. As a result, convergence becomes of primary concern when solving unsaturated soil mechanics problems. The digital computer has become a necessary computational tool when solving engineering design problems within the "boundary value problem" context. Figure 3 shows a simple saturated-unsaturated seepage analysis for the dissipation of hydraulic head through a dam. In this particular case, the finite element mesh has been designed based on mathematical algorithms to ensure that the nonlinear solution converges to the correct values for "hydraulic head". As such, the finite element mesh becomes part of the mathematical solution and is quite irrelevant to the geotechnical engineer. Rather, the geotechnical engineer is simply concerned with obtaining the correct hydraulic head contours.

Not all computer software packages are equally capable of solving unsaturated soils problems. Researchers in the mathematics and computing science disciplines have made significant progress with respect to the solution of highly nonlinear partial differential equations. Software dedicated to solving unsaturated soils problems needs to take advantage of the latest findings in complimentary mathematics and computing disciplines.

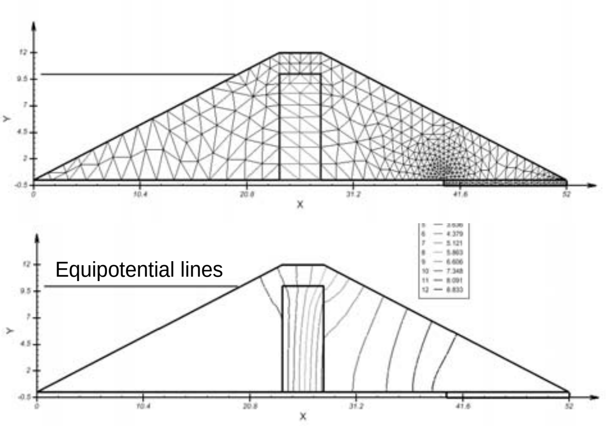


Figure 3. Finite element solution for dissipation of hydraulic head through a dam.

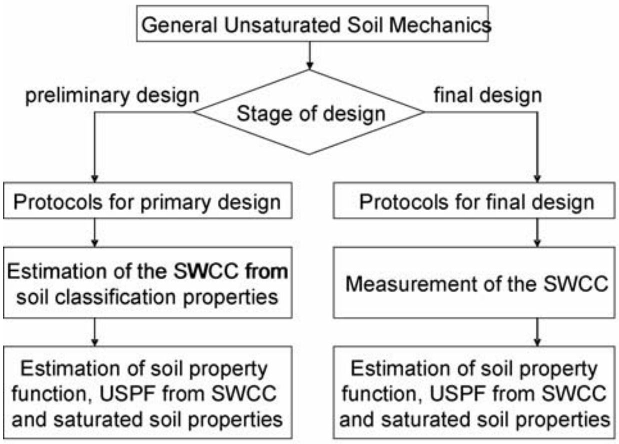


Figure 4. General classification of engineering protocols for unsaturated soils.

3. DEFINITIONS

When considering the practice of unsaturated soil mechanics, the word **PROTOCOL** can be defined as: "procedures and practices that are acceptable for prudent unsaturated soils engineering practice". The term, protocol, is commonly used in connection with computing procedures (e.g., telecommunications Protocols or Internet Protocols) or standards for global acceptance (e.g., the Kyoto Protocol). The definition has a sense of flexibility and is meant to set a temporary standard that could be changed with time. The term is meant to reflect a generally accepted, high standard for use in practice.

Engineering protocols for unsaturated soils applications should provide general guidelines for the implementation of engineering procedures. These procedures should reflect the most recent research published in journals and at research conferences. In other words, there needs to be a clear "connect" between research and engineering practice without turning engineering projects into research type projects.

A complete engineering Protocol needs to address all aspects of engineering design ranging from the geometric description of the problem to the assessment of suitable soil properties and boundary conditions. The Protocol also needs to address the type of analysis that should be performed when solving a particular problem. However, the focus of the Protocol in this paper is directed towards the determination of suitable unsaturated soil properties (i.e., SWCC and USPF).

4. CATEGORIZATION OF ENGINEERING DESIGN PROTOCOLS

Engineering design protocols can generally be placed within one of two primary categories; namely, i) **preliminary design protocols**, and ii) **final design protocols** as shown in Figure 4. It could be reasoned that there might be other intermediate protocol categories but there is also benefit in keeping the categorization of design protocols as straight forward as possible.

4.1 Preliminary design protocols

Preliminary engineering design protocols should be kept as simple as possible while still taking advantage of the many estimation procedures that have been proposed, tested and verified in the research literature. It is suggested that "preliminary design protocols" utilize estimation procedures for the assessment of the soil-water characteristic curve, SWCC. There are a variety of estimation procedures that have been proposed and some of the attributes of each procedure are later discussed.

4.2 Final design protocols

It is suggested that "final design protocols" be based on the measurement of soil-water characteristic curves, SWCC. In other words, soil samples representative of the material involved in the field should be tested in the laboratory under the range of soil suctions that are anticipated to occur during the life of the structure. Only in situations of extremely low risk is it not necessary to perform measurements of the SWCC.

It is the measurement or the estimation of the SWCC that becomes the primary determining factor separating a "preliminary design protocol" or a "final design protocol". Estimation procedures for the determination of unsaturated soil property functions are later discussed but do not form the basis for whether a "preliminary design" or a "final design" is being performed.

It is recommended that the most appropriate estimations techniques be used for the assessment of the SWCC of various materials when undertaking a "preliminary design". The SWCC results should later be confirmed through laboratory measurements for the "final design". It may also be necessary to confirm the performance of the design through *in situ* monitoring of water content and matric suction during the life of the structure.

4.3 Verification or monitoring category

There are engineering projects where verification through field monitoring should be made to ensure that the engineered structure is performing in accordance with the design. The author prefers to leave verification through field monitoring as an independent confirmation test on design rather than another design category.

Field monitoring requires the measurement of the same two variables that define the soil-water characteristic curve. In other words, it will be necessary to measure *in situ* water content and soil suction (i.e., generally matric suction). In measuring these two variables in the field it is possible to confirm that the *in situ* SWCCs are in reasonable agreement with the design curves.

5. OTHER FACTORS AFFECTING THE ENGINEERING DESIGN PROTOCOL

The classification of soil type associated with a particular engineering problem must be known to the engineer. Usually the saturated soil properties will be measured or estimated. The detail and reliability sought in assessing the soil properties depends upon a number of factors related to the engineering project. If the soil classification is known, as well as the saturated soil properties, it is always possible to obtain an estimation of the unsaturated soil properties.

All unsaturated soil property functions can be estimated from saturated soil properties and soil-water characteristic curves. The SWCC can also be estimated from the soil classification properties, in particular, from the grain-size distribution curve. Consequently, there is no reason for an engineer to say that he cannot take the unsaturated soil portion of the problem into consideration in the analysis. A more relevant question is, "How accurate must the SWCC be known for the problem at-hand?" To answer this question, the engineer must be aware of the factors that affect the assessment of the SWCC.

The risks associated with the engineering design need to be taken into consideration. If the risks associated with a hazard are low, an estimation of the SWCC and subsequent unsaturated soil properties is justifiable. If the risks are high, it is prudent to more accurately assess the SWCC (i.e., possibly the desorption and adsorption curves) and subsequent unsaturated soil properties.

Saturated-unsaturated soil problem analyses can be viewed as the solution of a partial differential equation, PDE, within the context of a boundary value problem. As such, there will be a "key" variable that is solved for in the PDE solution. In the case of a water flow seepage analysis, the "key" variable is "hydraulic head", *h*. The hydraulic head variable has a reduced sensitivity to the unsaturated soil properties. Therefore, it can be said that "hydraulic heads" can be computed with reasonable accuracy because it is quite insensitive to the input unsaturated soil property functions. This is generally the case when using the hydraulic heads as input to a slope stability analysis.

The reason for the insensitivity to the soil properties can be observed from the finite element form of the PDE being solved (Figure 5). The [B] transpose matrix is multiplied by the constitutive [C] matrix and then by the [B] matrix. Therefore, there is a canceling effect around the constitutive [C] matrix. The same insensitivity to the soil properties can be observed in a stress analysis (i.e., with stresses being insensitive), or any other engineering PDE formulation.

$$\int [B]^{\mathrm{T}} [k_w][B] \mathrm{d}A \{h_{wn}\} - \int [L]^{\mathrm{T}} \overline{v}_w \mathrm{d}S = 0$$

where:

$[B]$=matrix of the derivatives of the area coordinates

$$\begin{Bmatrix} v_{wx} \\ v_{wy} \end{Bmatrix} = [k_w][B]\{h_{wn}\}$$

where:

v_{wx}, v_{wy} = water flow rates within an element in the x- and -y direction, respectively.

Figure 5. Finite element formulation to solve for "hydraulic heads" and flux velocities.

On the other hand, if the amount of water flow (i.e., water flux) in a seepage analysis, (also shown in Figure 5), is of interest to the engineer, then the soil properties play a vital role. The water flux can be seen to be directly related to the constitutive [C] or permeability matrix. Water infiltration or water flux is of major concern with regard to the design of "cover" systems.

The engineer must be aware that there are drying and wetting branches associated with the soil-water characteristic curve. This phenomenon is referred to as "hysteresis". For engineering problems where the variable of interest from the solution is highly dependent upon the unsaturated soil property function, it is necessary to determine a suitable drying curve and a wetting curve. When performing the numerical modeling it is important to ensure that the correct bounding curve is used when solving the engineering problem as illustrated in Figure 5. The desorption and adsorption curves can be assumed to be the same (i.e., congruent), apart from a shift of the inflection point on the drying and wetting curves. Pham (2001) and Pham et al. (2002; 2003) suggested using the estimations shown in Table 1 for the amount of shift between the drying and wetting curves when measured data is not available with regard to the actual shift.

Table 1. Suggested shifts of the inflection poinbetween the drying and wetting curves for various soils (Pham, 2001)

Soil type	Range of typical shifts (% of a log cycle)	Average shift (% of a log cycle)
Sand	15% to 35%	25%
Silt and loam	35% to 60%	50%
Clay	···	Up to 100%

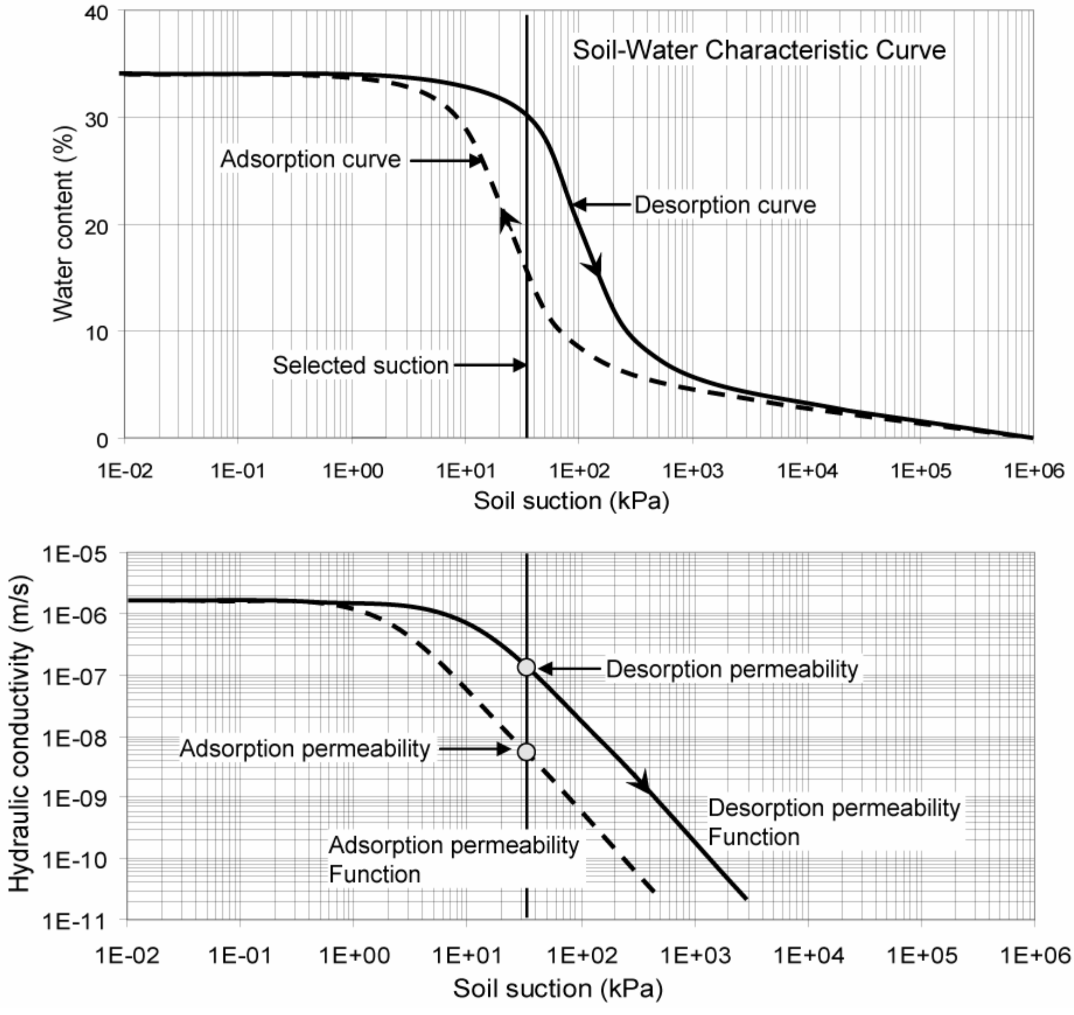
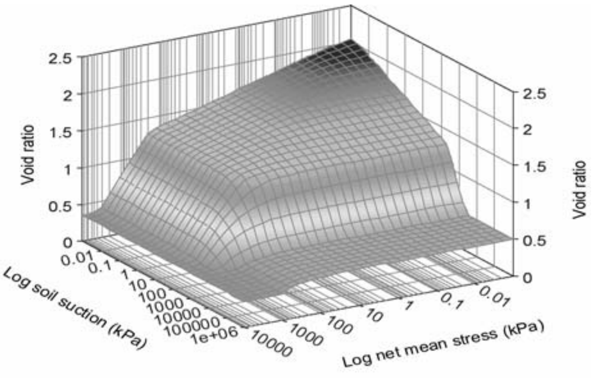


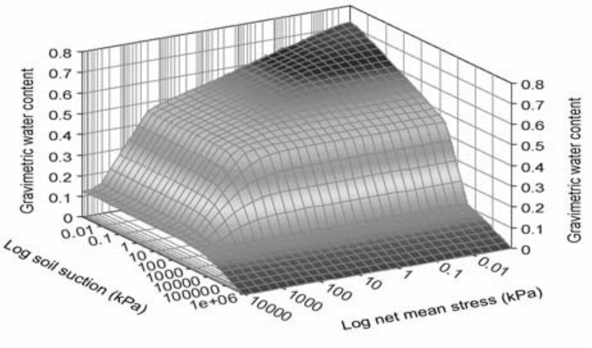
Figure 5. Hysteresis in the permeability function associated with the drying and wetting SWCC branches.

6. CENTRALITY OF SOIL-WATER CHARACTERISTIC CURVES, SWCC

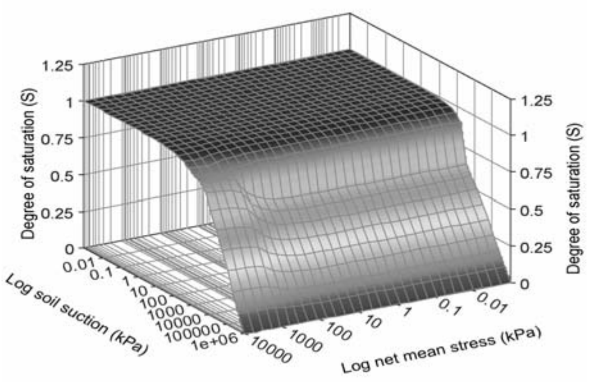
The soil-water characteristic curves, SWCCs, become pivotal to the solution of unsaturated soils problems. The SWCCs constitute one of the bounding relationships of the volume-mass constitutive properties for an unsaturated soil. Figure 6 shows the volume-mass constitutive surfaces for a clay soil. The degree of saturation versus soil suction relationship allows a clear discernment of the air entry value for a clay soil that may undergo considerable volume change prior to desaturation. If the volume change of the soil is small it is possible to use the gravimetric water content versus soil suction relationship to determine the air entry value and the residual water content. The volume-mass constitutive surfaces for a sand soil are illustrated in Figure 7. Gravimetric water content does not require a volume measurement in the laboratory and is considerably easier to measure than degree of saturation. It should also be noted that the volumetric water content is required in the seepage formulation for an unsaturated soil. Figure 8 shows a typical soil-water characteristic curve with its three distinct zones of desaturation.



(a) Void ratio

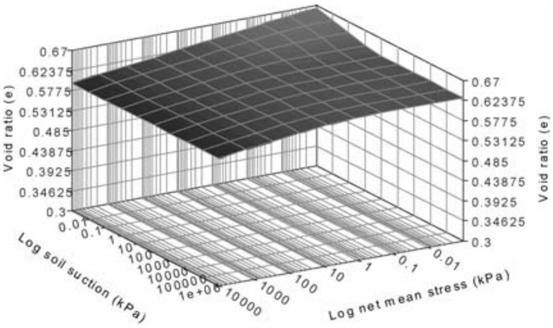


(b) Gravimetric water content

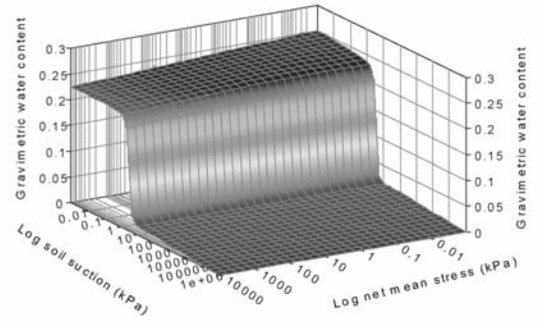


(c) Degree of saturation

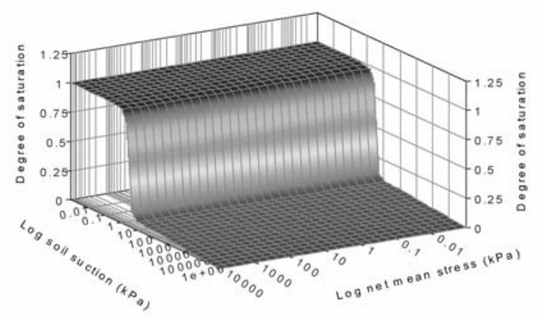
Figure 6. Volume-mass constitutive surfaces for a highly plastic clay preconsolidated to 200 kPa *(Pham, 2004).*



(a) Void ratio



(b) Gravimetric water content



(c) Degree of saturation

Figure 7. Volume-mass constitutive surfaces for a sand soil (Pham, 2004).

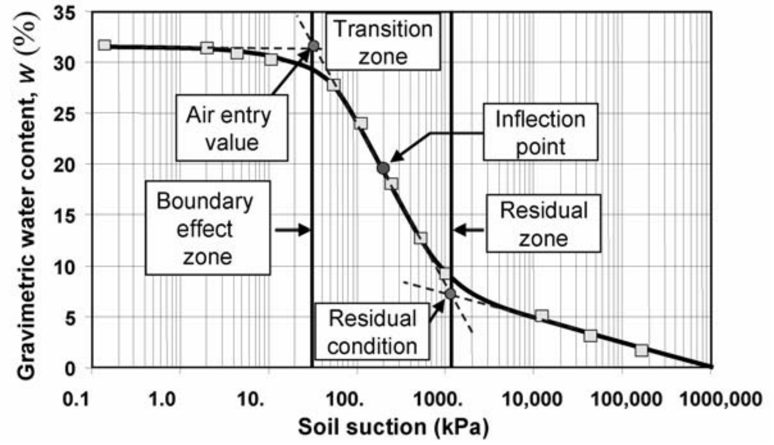


Figure 8. Zones of desaturation defined by the desorption branch of the soil-water characteristic curve, SWCC.

The SWCC is an approximation of the actual volume-mass constitutive surface that should be used in the estimation of unsaturated soil properties. The reliability of the estimated unsaturated soil property functions are related to the accuracy of the soil-water characteristic curve. The key transition points on the SWCC are the air entry value, and the residual value for suction and water content. The key transition points subdivide the SWCC into the "boundary effect" zone, the "transition" zone, and the "residual" zone (Vanapalli et al., 1998). The same three zones of desaturation can be defined for the drying (or desorption) branch and wetting (or adsorption) branch.

7. EQUATIONS FOR THE SOIL-WATER CHARACTERISTIC CURVE, SWCC

There are a large number of empirical equations that have been proposed to best-fit laboratory water content versus soil suction data. Generally desorption (or drying) curves are measured in the laboratory. Some of the SWCC equations that have been proposed are shown in Table 2. These equations appear in the research literature using one of three main variables to describe the amount of water in the soil; namely, gravimetric water content, volumetric water content or degree of saturation (Fredlund, 2002; 2006). The degree of saturation variable is the only term that clearly defines the air entry value for a soil that undergoes volume change as soil suction increases. On the other hand, volumetric water content is associated with the water storage term that appears in the derivation of the saturated-unsaturated transient seepage PDE. It is the derivative of the volumetric water content versus soil suction relationship that describes the water storage term for a transient seepage analysis. For purposes of comparison, each SWCC equation shown in Table 2 is written in terms of gravimetric water content.

Several of the most commonly used SWCC equations are sigmoidal in character and as such do not provide a reasonable description of water storage in the low suction range (i.e., below the air entry value). It should be noted that many of the SWCC equations have been derived for usage in disciplines other than geotechnical engineering. Consequently, the requirements for modeling geotechnical and geo-environmental engineering problems may not be fully satisfied.

Proposed SWCC equations can also be categorized according to the portion of the curve that is adequately described by the equation. Figure 10 shows a typical SWCC along with the manner in which a sigmoidal function can be best-fit to laboratory data. A sigmoidal curve is a continuous function that can adequately best-fit the SWCC from the air-entry value to the residual water content. A sigmoidal function does not perform well in the region below the air entry value and above residual conditions**.** Figure 11 provides a categorization of some SWCC equations that have been proposed in the research literature (Sillers, 1996; Sillers et al., 2001). Some equations have 2 fitting soil parameters, while others have 3 and 4 fitting soil parameters. Some equations are continuous while others are discontinuous. The larger the number of fitting parameters, the more likely the SWCC equation will closely fit the laboratory data. However, the SWCC function must be of the correct character if it is going to describe the entire range of soil suctions.

Table 2. Presentation of soil-water characteristic curve fitting equations along with average Akaike information from Sillers (1997)

Index	Name	Equation description	Fitting param eters	Average Aikake$^{(1)}$ information
1	Gardner(1958)	$w(\psi)=\dfrac{w_s}{1+a\psi^n}$	*a,n*	-739
2	Brooks and Corey (1964)	$w(\psi)=w_s \quad \text{for}\, \psi\le\psi_{ae}$ $w(\psi)=w_s\left(\dfrac{\psi}{a}\right)^{-n} \quad \text{for}\, \psi\le\psi_{ae}$	*a,n*	-764
3	King (1964)	$w(\psi)=w_s\left[\dfrac{\cosh(\psi/\psi_0)^b-\dfrac{w_s-w_r}{w_s+w_r}\cosh(a)}{\cosh(\psi/\psi_0)^b+\dfrac{w_s-w_r}{w_s+w_r}\cosh(a)}\right]$	ψ_o，*a*，*b*，w_r	N/A$^{(2)}$
4	Brutsaert(1966)	$w(\psi)=\dfrac{w_s}{1+\left(\dfrac{\psi}{a}\right)^n}$	*a*，*n*	-777
5	van Genuchten(1980)	$w(\psi)=\dfrac{w_s}{\left[1+(a\psi)^n\right]^m}$	*a*，*m*，*n*	-824
6	van Genuchten-Mualem (1980)	$w(\psi)=\dfrac{w_s}{\left[1+(a\psi)^n\right]^{(1-1/n)}}$	*a*，*n*	-769
7	Genuchten-Burdine (1980)	$w(\psi)=\dfrac{w_s}{\left[1+(a\psi)^n\right]^{1-2/n}}$	*a*，*n*	-762
8	Tani equation(1982)	$w(\psi)=w_s\left(1+\dfrac{a-\psi}{a-n}\right)\exp\left(-\dfrac{a-\psi}{a-n}\right)$	*a*，*n*	-475
9	Boltzman equation (1984)	$w(\psi)=w_s \quad \text{for}\, \psi\le\psi_{ae}$ $w(\psi)=w_s\exp\left(\dfrac{a-\psi}{n}\right) \quad \text{for}\, \psi\le\psi_{ae}$	*a*，*n*	-617
10	Fermi equation(1987)	$w(\psi)=\dfrac{w_s}{1+\exp\left(\dfrac{\psi-a}{n}\right)}$	*a*，*n*	-375
11	Fredlund and Xing (1994)	$w_w=C(\psi)\dfrac{w_s}{\left[\ln\left[e+\left(\dfrac{\psi}{a}\right)^n\right]\right]^m}$	*a*，*n*，*m*	-1007
12	Feng and Fredlund (1999)	$w(\psi)=\dfrac{w_s b+c\psi^d}{b+\psi^d}$	*b*，*c*，*d*	N/A$^{(2)}$

continued

Index	Name	Equation description	Fitting param eters	Average Aikake(1) information
13	Pereira and Fredlund(2000)	$w(\psi)=w_r+\frac{w_s-w_r}{\left[1+\left(\frac{\psi}{c}\right)^b\right]^a}$	a，b，c，w_r	N/A(2)
14	Gilson and Fredlund(2004)	$w(\psi)=\frac{S_1-S_2}{1+(\psi/\sqrt{\psi_b\psi_{res}})^d}+S_2$ $S_i=\frac{\tan\theta_i(1+r_i^2)\ln(\psi/\psi_i^a)}{(1-r_i^2\tan^2\theta_i)}+(-1)^i\cdot\frac{(1+\tan^2\theta_i)}{(1-r_i^2\tan^2\theta_i)}$ $\sqrt{r_i^2\ln^2(\psi/\psi^2)+\frac{a^2(1-r_i^2\tan^2\theta_i)}{(1+\tan^2\theta_i)}}+S_i^a$ i=1，2； $\theta_i=-(\lambda_{i-1}+\lambda_i)/2$ are hyperbolae rotation angles； $r_i=\tan(\lambda_{i-1}-\lambda_i/2)$ are the aperture angles tangents； $\lambda_0=0$ and $\lambda_i=$ $\arctan\left[(S_i^a-S_{i+1}^a)/(\ln(\psi_{i+1}^a/\psi_i^a))\right]$ are the eesaturation slopes ； $S_1^a=1$ ； $S_2^a=S_{res}$ ； $S_3^a=0$ ； $\psi_1^a=\psi_b;\psi_2^a=\psi_{res};\psi_3^a=10^6;$ and $d=2\cdot\exp(1/\ln(\psi_{res}/\psi_b))$ is a weight tactor for S_1 and S_2 that produces a continuous and smooth curve.	ψ_b，ψ_i， a，b，c，θ_r	N/A(2)

(1)Average Akaike information: The average Akaike (1974) information was obtained from fitting three different forms of the equations with more than 200 soil-water characteristic curves (Sillers, 1997). The more negative the average Akaike number the better is the numerical fit.

(2) N/A means “not available”.

The van Genuchten (1980) equation is typical of the proposed sigmoidal equations and can be written as follows:

$$w(\psi)=\frac{w_s}{\left[1+(a\psi)^n\right]^m} \tag{1}$$

where: a_v = soil parameter primarily related to the air entry value of the soil (1/kPa), n_v = soil parameter primarily related to the rate of water extraction from the soil once the air entry value has been exceeded, m = soil parameter primarily a function of residual water content.

While the van Genuchten (1980) equation might be the most commonly used continuous function, it primarily describes the SWCC from the air entry value to residual suction. Equations that relate the "m" and "n" soil parameters [i.e., Mualem (1976) and Burdine (1953)] do not improve the range for the best-fit of the equation.

The Fredlund and Xing (1994) equation is sigmoidal in character up to residual suction conditions after which it goes to a suction value of 1,000,000 kPa at zero suction. The equation is written as follows:

$$w(\psi)=C(\psi)\frac{w_s}{\left[\ln\left[e+\left(\frac{\psi}{a}\right)^n\right]\right]^m} \tag{2}$$

The Fredlund and Xing (1994) equation extends the SWCC as a straight line on a semi-log plot from residual conditions to a suction of 1,000,000 kPa at zero water content. The equation for the $C(\psi)$ "correction factor" takes the following form.

$$C(\psi) = 1 - \frac{\ln(1+\psi/\psi_r)}{\ln[1+(1000000/\psi_r)} \qquad (3)$$

where: a_f = soil parameter related mainly to the air entry value of the soil, n_f = soil parameter related mainly to the rate of water extraction from the soil, once the air entry value has been exceeded, m_f = soil parameter related mainly to residual water content, and $C(\psi)$ = correction factor applied from residual conditions to 1,000,000 kPa.

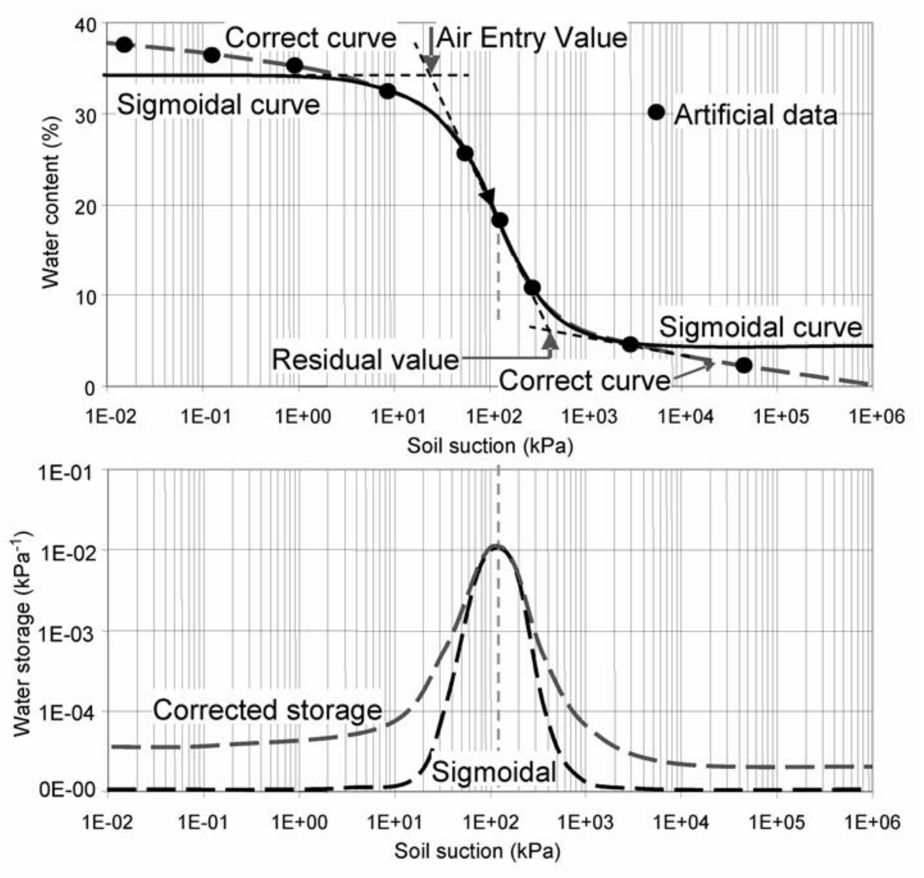


Figure 9. Illustration of the limitations associated with most soil-water characteristic curve, SWCC, equations.

The Gitirana and Fredlund (2004) SWCC equation has three independent parts that describe the SWCC equation from the air entry value to completely dry conditions. The initial portion of the SWCC curve is asymptotic below the air entry value.

$$w(\psi) = \frac{S_1 - S_2}{1 + (\psi / \sqrt{\psi_b \psi_{res}})^d} + S_2 \qquad (4)$$

The definition of all variables can be found in Gitirana and Fredlund (2003), as well as in Table 2. The Gitirana and Fredlund (2003) SWCC equation also has the advantage that all soil properties are completely independent of one another.

The Fredlund and Pham (2006) equation provides a description of the entire SWCC relationship from low suctions to completely dry conditions (Pham 2005).

$$w(\psi) = \left(\left[w_s - \frac{C_c}{G_s}\log(\psi) - w_r \right] \frac{a}{\psi^b + a} + w_r \right) \left(1 - \frac{\ln\left[1 + \frac{\psi}{\psi_r}\right]}{\ln\left[1 + \frac{10^6}{\psi_r}\right]} \right) \qquad (5)$$

where:
C_c = virgin compression index of the soil,
G_s = particles specific gravity,
w_r = curve-fitting parameter representing residual water content

w_s = curve-fitting parameter representing the water content of the slurry soil at an effective stress of 1kPa,

a, b = curve-fitting parameters, and

ψ_r = residual soil suction.

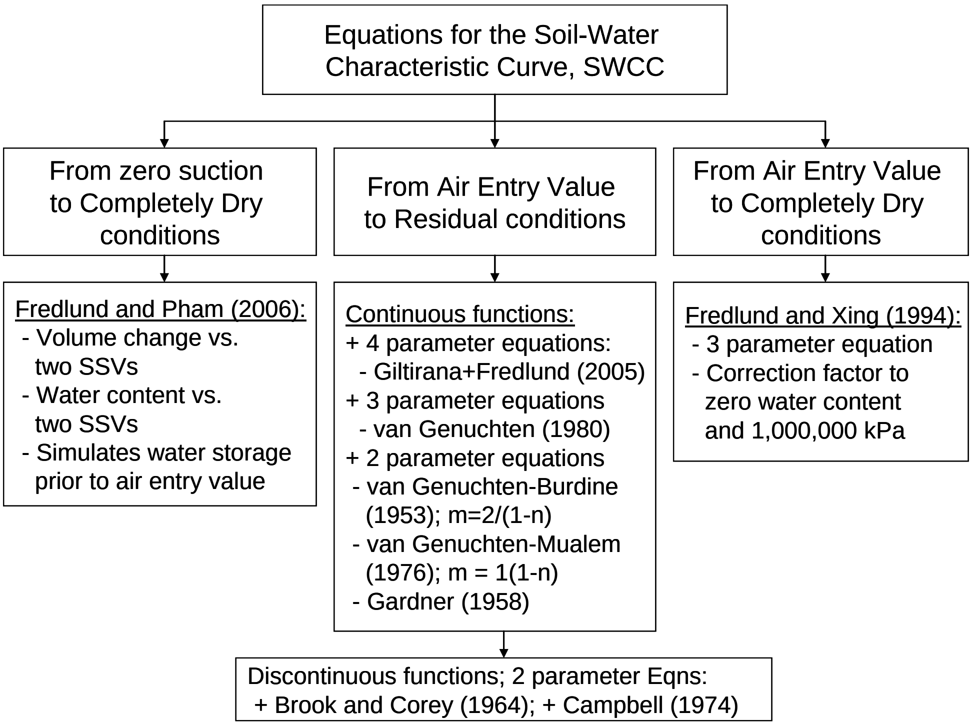


Figure 10. Categorization of soil-water characteristic curve equations according to the portion of the SWCC that is accurately defined for engineering purposes.

The Fredlund and Pham (2006) equation has also been extended to two independent stress state variables (i.e., net normal stress and soil suction). Corresponding equations have been derived for the void ratio and degree of saturation constitutive relationships. As a result, it is possible to define all volume-mass soil properties in terms of two independent stress state variables for an unsaturated soil. The volume-mass constitutive model is stress path dependent. The water storage character of the soil can be described for the entire net loading and soil suction range.

Each increase in rigor for the SWCC equation comes with an increase in the effort required to obtain the necessary soil parameters for the model. It must also be realized that many of the SWCC equations have been developed in disciplines other than engineering and the requirements for engineering practice may be somewhat more stringent.

8. COMPARATIVE STUDIES OF SOIL-WATER CHARACTERISTIC CURVES

There have been a number of statistical studies undertaken where a variety of SWCC equations have been best-fit to a database of laboratory test results. Tinjum et al. (1997) undertook a study of several empirical SWCC equations. The equations were applied to a dataset of measured soil-water characteristic curves. A conclusion from the study was that both two-parameter and three-parameter SWCC equations could be best-fit to laboratory data; however, the three-parameter equations provided a better fit due to the inherently greater flexibility of the equation. It was also concluded that it was appropriate to use estimated SWCCs for engineering applications. It was further noted that, "-- testing should be conducted in critical situations and final design --".

Leong and Rahardjo (1997) undertook a review on several SWCC empirical equations by applying each equation to a database of collected results. It was concluded that the SWCC empirical equations with the highest number of soil fitting parameters provided the closest fit to the datasets. The Fredlund and Xing (1994) SWCC equation was found to "perform marginally better" than the van Genuchten (1980) equation. The Fredlund and Xing (1994) equation is found to perform much better than other SWCC equations when the dataset contains measurements where the soil suctions was greater than residual suction.

Table 2 shows the results of a detailed study in which a total of 9 SWCC equations were best-fit to a database containing 231 soils (Sillers, 1996). The soil datasets correspond to 8 soil classification groups according to the USDA classification system. The Akaike Information Criterion statistical indicator was used to assess the "fit" of each of the 9 SWCC equations. The results showed that the Fredlund and Xing (1994) equation performed the best with an Akaike number of minus 1007. (Note: the more negative the Akaike number, the better the fit). The next best performing SWCC equation was the van Genuchten (1980) equation with the Fredlund and Xing (1994) "correction factor" applied beyond residual suction to give an Akaike number of -824. When the "correction factor" was not added, the Akaike number was poorer (i.e., it was less negative). The study also reported that the van Genuchten (1980) and the [van Genuchten (1980) – Mualem (1976)] equations appeared to be most commonly used equations according to the research literature.

The Siller (1996) study also showed that there was no advantages in fixing the relationship between the "m" and "n" variables in the van Genuchten (1980) equation as was proposed by Burdine (1953) and Mualem (1976). In fact, the best-fit analysis showed that the same quality of fit was obtained by leaving the "m" variable as 1.0 in the van Genuchten (1980) equation. On the other hand, the study showed that the Fredlund and Xing (1994) "correction factor" could be applied to other SWCC equation and in each case there was an improvement in the quality of fit.

The author is not aware of any independent studies that have been undertaken where the Fredlund and Pham (2006) equation and the Gitirana and Fredlund (2004) equation have been compared to other SWCC equations through use of a regression analysis on a database. However, the authors have in each case undertaken a best-fit regression analysis. The Gitirana and Fredlund (2004) equation performed similar to the Fredlund and Xing (1994) equation but has the advantage of having totally independent soil fitting parameters. The Fredlund and Pham (2006) SWCC equation would appear to be the most accurate equation available because of its ability to best-fit the entire range of soil suctions. The water storage function prior to the air entry value is accurately captured when using the Fredlund and Pham (2006) equation.

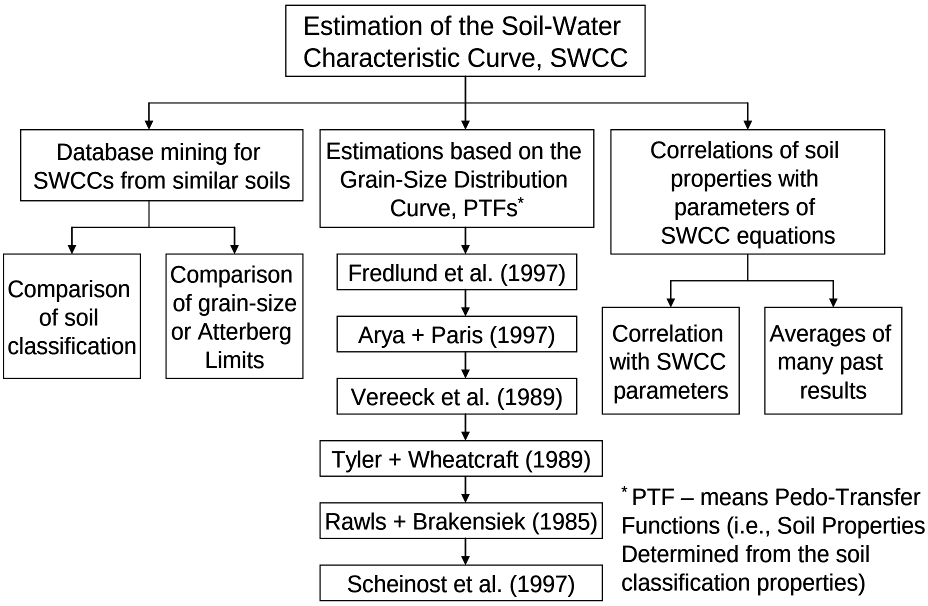
9. ESTIMATION OF THE SOIL-WATER CHARACTERISTIC CURVE, SWCC

There are three primary ways in which the drying curves (i.e., desorption branch), of the SWCC can be estimated. These are: i.) through database mining of previous test results, ii.) through estimation of the SWCC from the grain-size distribution, and, iii.) from the correlation of soil parameters for a specific SWCC equation with soil classification properties. These estimation procedures are shown in Figure 11.

Estimations of the soil-water characteristic curve from soil classification properties such as the grain-size distribution curve are referred to as pedo-transfer functions (Fredlund et al, 1999). Several pedo-transfer estimation procedures have been proposed in agriculture-related disciplines (Ayra and Paris, 1981). All of the proposed pedo-transfer functions are not of equal reliability. Neither can the most recently published methodologies be relied upon to be the most accurate. Table 3 provides statistical evidence on the quality of prediction for several pedo-transfer functions. All of the estimation techniques are programmed into the SoilVision software (1996). The results of the statistical study reveal that the Fredlund et al. (1999) technique is considerably superior in predicting the SWCC.

Estimating the SWCC from grain-size distribution curves (Fredlund et al., 2000), shows promise for use in engineering practice particularly at the "preliminary design" level. In fact, there already appears to be extensive usage of pedo-transfer functions in unsaturated soil engineering applications.

Pedo-transfer functions appear to provide reasonable SWCC estimations for sand and silt (or loam) soils. Obviously soils exhibiting macro structure, such as many clay soils, cannot have an adequate estimation of the SWCC from a grain-size distribution curve. The SWCC obtained from the pedo-transfer functions is the desorption curve.



* PTF – means Pedo-Transfer Functions (i.e., Soil Properties Determined from the soil classification properties)

Figure 11. Techniques available for the "estimation" of the soil-water characteristic curve, SWCC.

Table 3. Statistics on the comparison between estimated SWCCs using Pedo-transfer functions, (PTF), and measured SWCCs for a wide range of soil types (Fredlund et al. 2002)

Pedo-transfer function	Frequency Sum[1]	Air Entry Value[2]	Maximum Slope[3]
Fredlund et al.(1997)	96	0.59	0.49
Arya & Paris(1981)	55	0.86	0.59
Vereeck et al.(1989)	21	1.33	0.46
Tyler & Wheatcraft(1989)	16	3.44	0.99
Rawls & Brakensiek(1985)	15	0.79	7.85
Scheinost et al.(1997)	14	1.19	0.48

[1] The frequency sum consists of the summation of frequencies associated with R^2 values greater than 0.7. The higher the "Frequency Sum" the better the fit between the estimated SWCCs and the measured SWCCs.

[2] Numbers are R^2 differences between the estimated air entry value and the measured air entry values. The lowest R^2 value signifies the closest fit for the air entry value.

[3] Numbers are R^2 differences between the estimated maximum slope and the measured maximum slope for the SWCCs. The lowest R^2 value signifies the closest fit for the air entry value.

10. MEASUREMENT OF THE SOIL-WATER CHARACTERISTIC CURVE, SWCC

Soil-water characteristic curves can readily be measured in geotechnical laboratories through use of pressure plate type apparatuses. There are a variety of apparatuses that are in common use in agriculture-related disciplines. In addition, there are a number of apparatuses that have been designed to better meet the needs of geotechnical engineers.

The GCTS cell shown in Figure 12 allows the measurement of water content and volume change on a single soil specimen under various total stress conditions (Pham et al., 2002). Desorption SWCC can readily be measured but more time is required and greater experimental care is required when measuring the adsorption SWCC curve.

High soil suctions beyond 1500 kPa are most commonly determined through vapour pressure equilibrium in desiccators.

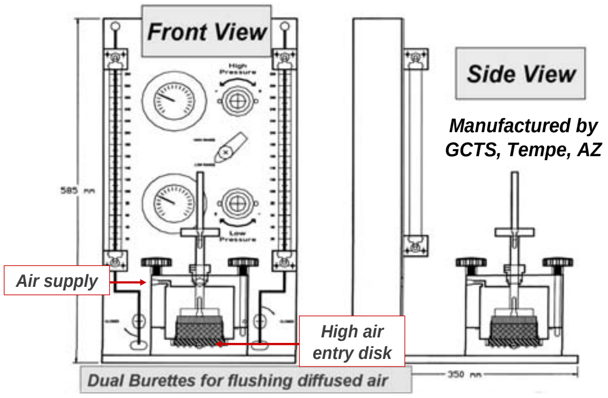


Figure 12. Schematic illustration of the GCTS soil-water characteristic curve cell.

11. INITIAL CONDITION OF THE SOIL SAMPLES TESTED

Figure 13 shows three initial conditions for a soil specimen that is to be tested in the laboratory (Fredlund, 2002). There will be a difference in the laboratory test results from specimens tested with each of the different initial conditions. The air entry value of the soil, as well as the rate of desaturation is affected by the initial density and amount of disturbance to the soil. The initial condition of the soil needs to be mentioned because much of the soil data contained in databases is obtained from agriculture-related disciplines where the disturbance of the soil is not of great concern. However, sample disturbance is of concern in geotechnical and geo-environmental engineering, particularly with soils with clay content.

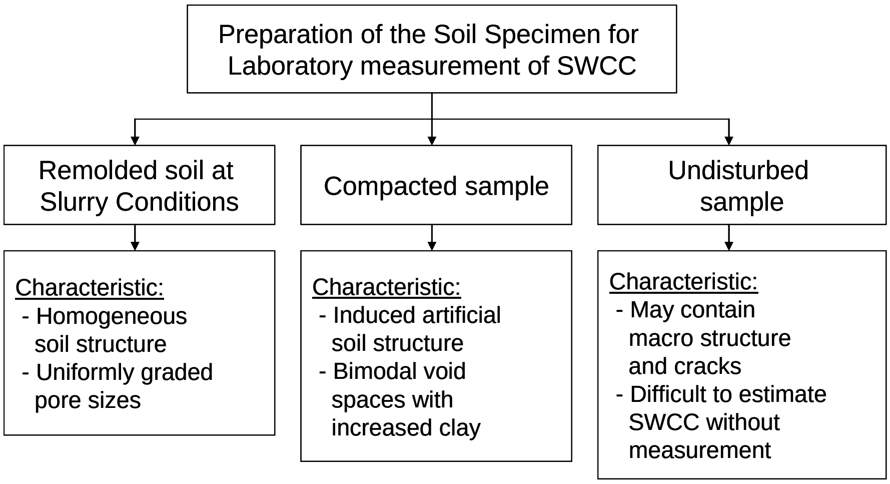


Figure 13. Effect of initial conditions of the soil specimen on the soil-water characteristic curve, SWCC.

12. LINKAGE BETWEEN THE SWCC AND THE UNSATURATED SOIL PROPERTIES

The emphasis of this paper has been on the measurement of the soil-water characteristic curve. The SWCC is subsequently used to undertake another set of estimations; namely, estimations for the hydraulic conductivity function, the shear strength function and the volume-mass change functions. Space does not allow all of theses estimation procedures to be considered within the context of "preliminary design" and "final design". It is important to emphasize that the accuracy of all subsequent estimations is highly dependent upon the accuracy with which the SWCC is characterized.

There is direct linkage between the SWCC and the estimated unsaturated soil property functions, USPFs (Fredlund et al., 1995; 1997). This linkage must be respected and maintained throughout all subsequent estimations and modeling analyses. In other words, it is not possible to use the SWCC to estimate an unsaturated soil property function and then subsequently change any aspect of the unsaturated soil property functions in order to alleviate nonlinearity problems associated with the solution of a partial differential equation. It must be realized that any change in the unsaturated soil property function results in a change in the soil being modeled. The change also produces an inconsistency between the unsaturated soil property functions.

Unsaturated soil property functions can produce highly nonlinear water storage and permeability functions, particularly for sand soils. The highly nonlinear functions can subsequently create convergence difficulties when solving partial differential equations such as the unsaturated seepage equation. When convergence difficulties are encountered, there is a temptation for the "modeler" to adjust one or more of the soil parameters to assist in obtaining a solution. This is "dangerous practice" and should NOT be undertaken. Rather, it is important to use partial differential equation solvers that can ensure convergence even under highly nonlinear conditions.

Misfeldt et al, (2006) reported on the "preliminary design" of a cover system where it was shown that a change in one of the soil parameters associated with the hydraulic conductivity function dramatically affect the outcome of the cover design. Changing any one soil parameter changes the soil that is being analyzed. It was suggested that estimation techniques can be used for "preliminary design" purposes; however, it is important that individual soil parameters associated with the unsaturated soil property functions NOT be changed when performing a seepage analysis.

13. UNSATURATED SOIL PROPERTY FUNCTIONS, USPF

The accuracy of the unsaturated soil property functions, USPF, used in the solution of partial differential equations for unsaturated soil systems depends upon: i.) the accuracy of the soil-water characteristic curve (i.e., SWCC equation), and ii.) the accuracy of the empirical estimation procedure for the USPFs (Fredlund et al, 1994; 1996; Vanapalli and Fredlund, 2000). In other words, it is important to achieve the closest representation of the physical behavior of the unsaturated soil as possible.

Let us consider the example problem of modeling a transient saturated-unsaturated seepage problem (Thieu et al, 2001). It is necessary to have the best possible representation of the: i) water storage function, and ii) hydraulic conductivity function. Figure 10 showed a typical SWCC along with the actual water storage function and the water storage function that might be generated from some of the empirical representations of the SWCC. It is also important to estimate the most accurate permeability function to represent the actual hydraulic conductivity of the unsaturated soil. It is beyond the scope of this paper to investigate the various permeability functions that can be estimated from SWCCs.

14. SELECTION OF ESTIMATION PROCEDURES

There are many estimation procedures associated with solving unsaturated soils problems. The estimation procedures can readily lead to confusion amongst practicing engineers. This paper has attempted to point out the importance of adequately estimating the SWCC for "preliminary design" purposes and the need for the measurement of the SWCC for "final designs".

It cannot be assumed that all estimation procedures are of equal accuracy when estimating the SWCC (Fredlund, 2002). It is incumbent upon a prudent engineer to utilize the estimation techniques that have been proven to be most accurate. When the best estimation techniques have been utilized for obtaining the SWCC, then the engineer needs to correspondingly utilize the most reliable estimation procedures for computing the necessary unsaturated soil property functions.

15. SUMMARY AND CONCLUSIONS

There are a number of general guidelines and recommendations with regard to engineering protocols. The following conclusions are made with regard to the assessment of unsaturated soil property functions.

1) The SWCC can be estimated for "preliminary design" purposes but must be measured for "final design" purposes. It is recommended that laboratory measurements be made on undisturbed soil samples whenever possible.

2) The estimation of unsaturated soil property functions is related to the drying (or desorption) branch of the SWCC. It is the responsibility of the geotechnical and geo-environmental engineer to determine whether the process being simulated in the field corresponds to the drying or wetting process. If the process corresponds to the wetting (or adsorption) process then the SWCC should be shifted in accordance with the average suggested values.

3) It is important to use the SWCC estimation procedures that have been shown to be the most

accurate when using the grain-size distribution curve.

4) It is important to utilize a SWCC equation that covers the entire range of soil suctions that are of interest for a particular engineering problem.

5) When estimating unsaturated soil property functions, USPF, it is important to use the estimations techniques that have been shown to provide the most accurate estimations for unsaturated soil properties.

6) There is a strong linkage between the SWCC and the unsaturated soil properties and this relationship must be maintained throughout the numerical modeling process. It is not considered acceptable engineering practice to adjust some of the soil parameters associated with unsaturated soil property functions in order to obtain convergence of the solution.

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