Correlation between unsaturated soil parameters

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Abstract: A statistical study was carried out with the objective of establishing typical values of statistical measures of soil-water characteristic curve parameters (SWCCs) and hydraulic conductivity function parameters. This paper presents the scatter plots obtained and the coefficients of correlation between the unsaturated soil properties. A methodology was established based on appropriate fitting functions, adequate parameters, and soil grouping criterion. A total of 186 datasets were sampled from a database of soils and analyzed using the proposed methodology. Typical coefficients of correlation were established for the parameters of each soil group. The information provided herein gives important insight into the relationship between unsaturated soil properties and is useful in reliability-based analysis and design.

Resumo: Um estudo estatístico foi desenvolvido com o objetivo de estabelecer valores típicos para as medidas estatísticas de parâmetros de curvas características e parâmetros de funções de condutividade hidráulica. Este artigo apresenta os gráficos de dispersão obtidos e os coeficientes de correlação entre as propriedades de solos não saturados. Uma metodologia foi estabelecida com base em funções de ajuste apropriadas, parâmetros adequados e um critério de agrupamento de solos. Um total de 186 conjuntos de dados foi amostrado de um banco de dados de solos e analisados utilizando a metodologia proposta. Coeficientes de correlação típicos foram estabelecidos para os parâmetros de cada grupo de solo. As informações apresentadas neste artigo oferecem importantes dados sobre a relação entre os parâmetros de solos não saturados e é útil em análises e dimensionamento probabilístico.

Keywords: statistics; soil-water characteristic curve; hydraulic conductivity; coefficient of correlation.
Palavras-chave: estatística; curva característica; condutividade hidráulica; coeficiente de correlação.

1 INTRODUCTION

Typical values of statistical measures of soil parameters are often established and used in engineering practice. Numerous studies can be found in the literature suggesting values for the typical statistical measures of saturated soils parameters. A limited number of studies are available on the statistical measures of unsaturated soil parameters, due to limited available data and the lack of an appropriate methodology for the assessment of variability.

The primary objective of this paper is to present part of the results of a statistical study carried out using 186 datasets of soil-water characteristic curves and hydraulic conductivity functions. This paper focuses on the correlation analyses. Scatter plots have been drawn and analyzed in order to determine if the unsaturated soil properties are significantly correlated.

The statistical measures presented herein are intended as a first approximation and can be refined for site-specific information.

2 METHODOLOGY AND DATA

A total of 186 data records were sampled from the SoilVision database (SoilVision Systems, 2003). The number of sampled records was limited by the number of “complete” soil records. To be considered “complete”, a soil record was required to have a grain-size distribution, soil porosity, *n*, the drying soil-water characteristic curve, and the hydraulic conductivity function. Only the drying curves were used in the present study. The porosity indicates the total water storage. The grain-size distribution was required to classify each soil record and form groups of soils with similar textures.

2.1 Fitting equations and soil parameters

Unsaturated soil properties are generally defined as nonlinear functions of the stress state variables. The soil-water characteristic curve data records can be fitted using the unimodal equations proposed by Gitirana Jr. and Fredlund (2004). Two equations were used herein; namely, the unimodal equation with two bending points and the unimodal equation with one bending point:

$$S = \frac{S_1 - S_2}{1 + \left(\psi / \sqrt{\psi_b \psi_{res}}\right)^d} + S_2 \tag{1}$$

$$S_i = \frac{\tan\theta_i (1 + r_i^2)\ln(\psi/\psi_i^a)}{(1 - r_i^2 \tan^2\theta_i)} + (-1)^i \times \frac{(1 + \tan^2\theta_i)}{(1 - r_i^2 \tan^2\theta_i)} \times \sqrt{r_i^2 \ln^2(\psi/\psi_i^a) + \frac{a^2(1 - r_i^2 \tan^2\theta_i)}{(1 + \tan^2\theta_i)}} + S_i^a \tag{2}$$

where $i = 1, 2$; $\theta_i = -(\lambda_{i-1} + \lambda_i)/2$; $r_i = \tan((\lambda_{i-1} - \lambda_i)/2)$; $\lambda_0 = 0$; $\lambda_i = \arctan[(S^a_i - S^a_{i+1})/(\ln(\psi^a_{i+1}/\psi^a_i))]$; $S_1^a = 1$; $S_2^a = S_{res}$; $S_3^a = 0$; $\psi_1^a = \psi_b$; $\psi_2^a = \psi_{res}$; $\psi_3^a = 10^6$; $d = 2 \times \exp(1/\ln(\psi_{res}/\psi_b))$; and "$a$" is a parameter controlling the curve sharpness at the two bending points. The unimodal equation with one bending point can be obtained directly from Eq. 2, with $i = 1$, $\theta_1 = -\lambda/2$; $r_1 = \tan(\lambda/2)$; and $\lambda = \arctan[1/(\ln(10^6/\psi_b))]$;

According to Eq. 1 a SWCC with two bending points can be defined by four parameters; namely, ψ_b = air-entry value; ψ_{res} = residual suction; S_{res} = residual degree of saturation; and the sharpness parameter, a. These four parameters correspond to the number of shape features of a typical SWCC, as presented in Fig. 1. Figure 1 also shows that two shape features define a SWCC with one bending point; namely, ψ_b and a.

Other parameters can be defined; namely, λ_d, the primary drainage slope; and λ_{res}, the residual drainage slopes:

$$\lambda_d = \frac{1 - S_{res}}{\log_{10}\left(\psi_{res}/\psi_b\right)} \tag{3}$$

$$\lambda_{res} = \frac{S_{res}}{\log_{10}\left(1{,}000{,}000/\psi_{res}\right)} \tag{4}$$

The primary drainage slope, λ_d, indicates the distribution of the pore-sizes. The more uniform the pore-size distribution is the steeper the primary drainage slope. The residual drainage slope, λ_{res}, does not bear as much physical meaning as λ_d.

Regardless of the parameters chosen to describe the SWCC, four parameters must be used because there are four distinct shape features. Difficulties could arise in probabilistic analyses using ψ_b and ψ_{res} due to the possibility of estimate points or random realizations where $\psi_b > \psi_{res}$, which is physically impossible. Therefore, the set of parameters ψ_b, λ_d, λ_{res}, and a was deemed more appropriate.

The parameter "a" was considered as a fixed value for each soil type. A value of a equal to 0.075 was selected for Sands, a equal to 0.050 for Loams, and a equal to 0.025 for Clays.

The hydraulic conductivity functions were fitted using a bi-linear function on a log versus log plot (see Fig. 2):

$$\begin{aligned} k^w &= k^w_{sat} \text{ for } (u_a - u_w) \le \psi_{bk} \\ k^w &= k^w_{sat}\left[\psi_{bk}/(u_a - u_w)\right]^{\eta} \text{ for } (u_a - u_w) > \psi_{bk} \end{aligned} \tag{1}$$

where k^w = hydraulic conductivity; k^w_{sat} = saturated hydraulic conductivity; $(u_a - u_w)$ = matric suction; ψ_{bk} = break point in the hydraulic conductivity function; η = slope of the hydraulic conductivity function.

Best-fit for SWCCs and hydraulic conductivity data was performed by minimizing the sum of the squared residuals (SSR). Each curve parameter was considered as a random variable with a certain frequency distribution. The frequency distributions were statistically characterized based on statistical descriptive measures.

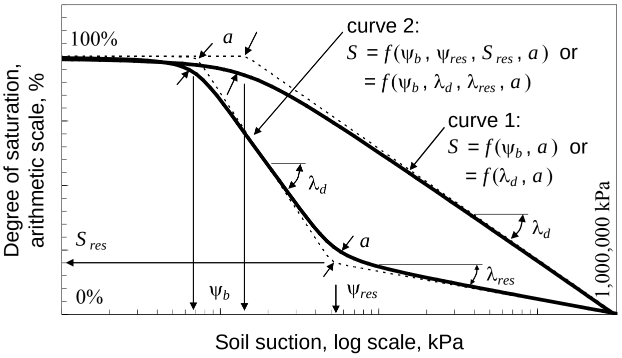

Figure 1. Idealization of unimodal soil-water characteristic curves with one and two bending points.

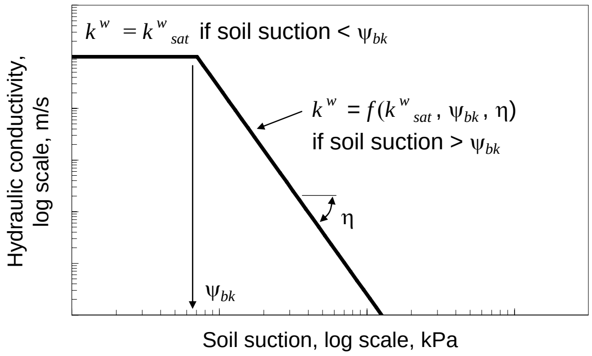

Figure 2. Idealization of hydraulic conductivity functions.

2.2 Sample Grouping

The 186 soil records sampled from the database correspond to diverse soils with distinct characteristics. Ideally, the statistical assessment of individual "soil groups" is preferable even though the definition of the term "soil group" is not totally precise.

The USDA (Soil Survey Staff, 1975) system has been adopted as a soil grouping criterion. The number of "complete" records presenting Atterberg limits data is small, making the use of the USCS (ASTM, 1993) system unfeasible.

Figure 3 presents the textural diagram used by the USDA system for the classification of soils (Soil Survey Staff, 1975) along with the data points for the records sampled. The percentages of sand, silt and clay plotted in Fig. 3 were defined by the following grain-size intervals:

Clay	$x < 0.002$ mm;
Silt	$0.002 \leq x < 0.05$ mm;
Sand	$0.05 \leq x < 2$ mm; and
Coarse	$2 \leq x < 300$ mm

Three main groups are identified in Fig. 3; namely, sands (Sa), loams (L), and clays (C). The silt and silty fractions (Si) are placed within the loam fraction. The number of sampled records pertaining to each soil group is 62, making a total of 186 sampled records.

3 RESULTS AND DISCUSSION

The correlation coefficient, ρ, between each pair of unsaturated soil parameters was determined with the aid of Minitab 13 (Minitab Inc., 2000). The parameters studied were n, $\ln(\psi_b)$, $\ln(\lambda_d)$, $\ln(\lambda_{res})$, $\ln(k^w_{sat})$, $\ln(\psi_{bk})$, and $\ln(\eta)$. The correlation coefficient is a measure of the degree of linear relationship between two variables. The correlation coefficient assumes a value between – 1 and +1. If one variable tends to increase as the other decreases, the correlation coefficient is negative. Conversely, if the two variables tend to increase together the correlation coefficient is positive.

A two-tailed correlation test was applied. The null hypothesis of the test performed was H0: coefficient of correlation is zero. P-values were computed, representing the probability of making a type 1 error, which is "*rejecting the null hypothesis when it is true.*" The smaller the P-value, the smaller is the probability that a mistake would be made by rejecting the null hypothesis. The cut-off value used was 5%, that is, the null hypothesis was rejected when the P-value was less than 5%.

Table 1 presents the results of the correlation analyses. The coefficients of correlation are presented for all pairs of variables, along with the symbol (*), indicating those tests for which the P-value obtained is less than 5% (i.e., the probability of rejecting the hypothesis that $\rho = 0$ when it is true is less than 5%). Further tests, using soil subgroups can be found in Gitirana Jr. (2005).

Several scatter plots are presented in Figs. 4 to 10, for the pairs of variables that presented significant correlation, considering the parameters n, $\ln(\psi_b)$, $\ln(\lambda_d)$, $\ln(\lambda_{res})$, $\ln(k^w_{sat})$, $\ln(\psi_{bk})$, and $\ln(\eta)$. These scatter plots were used in the analysis of the data and as a qualitative verification of the analysis results.

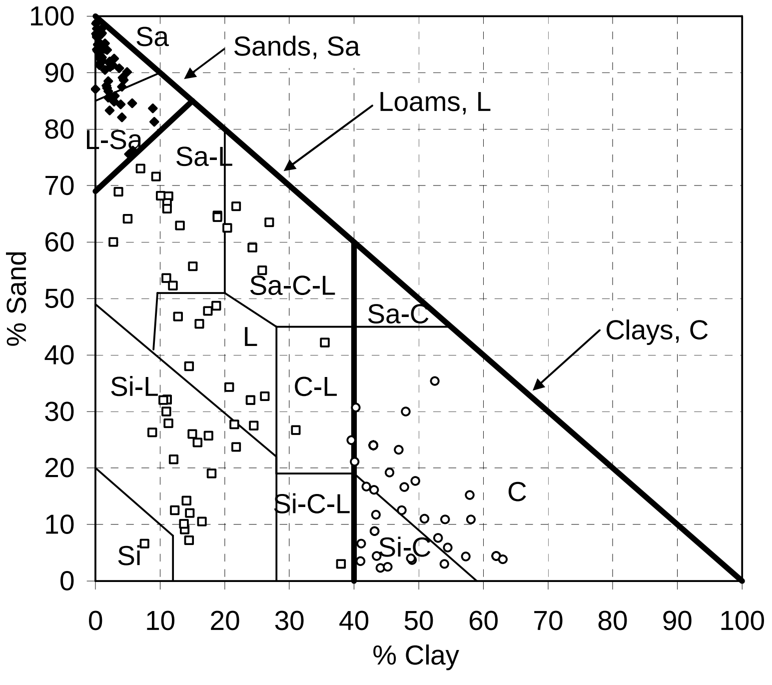

Figure 3. Sampled soil records classified according to the USDA system.

Table 1. Correlation matrix for unsaturated soil properties.

Parameter	Group	n	$ln(\psi_b)$	$ln(\lambda_d)$	$ln(\lambda_{res})$	$ln(k^w_{sat})$	$ln(\psi_{bk})$	$ln(\eta)$
n	All soils	1						
	Sands	1						
	Loams	1						
	Clays	1						
$ln(\psi_b)$	All soils	-0.147*	1					
	Sands	-0.079	1					
	Loams	-0.096	1					
	Clays	-0.282*	1					
$ln(\lambda_d)$	All soils	-0.311*	0.374*	1				
	Sands	0.084	0.587*	1		*symmetric*		
	Loams	-0.032	0.443*	1				
	Clays	--	--	--				
$ln(\lambda_{res})$	All soils	0.534*	0.211*	-0.399*	1			
	Sands	0.354*	0.270*	0.096	1			
	Loams	0.227	0.281*	-0.080	1			
	Clays	--	--	--	--			
$ln(k^w_{sat})$	All soils	-0.210*	-0.216*	0.354*	-0.233*	1		
	Sands	0.193	-0.276*	0.234	-0.086	1		
	Loams	0.249	-0.174	0.192	0.095	1		
	Clays	0.228	-0.347*	--	--	1		
$ln(\psi_{bk})$	All soils	0.078	0.140	0.373*	-0.041	-0.408*	1	
	Sands	0.073	0.187	0.127	0.141	-0.427*	1	
	Loams	0.012	0.320*	0.626*	-0.167	-0.275*	1	
	Clays	0.032	0.023	--	--	-0.630*	1	
$ln(\eta)$	All soils	-0.151*	0.025	0.675*	-0.227*	0.393*	0.500*	1
	Sands	0.092	0.083	0.365*	0.105	0.057	0.755*	1
	Loams	0.079	0.228	0.847*	-0.095	0.280*	0.700*	1
	Clays	0.300*	-0.084	--	--	0.023	0.338*	1

Note: (*) indicates the correlation coefficients for which the *P*-value is less than 5%.

The correlation coefficients presented in Table 1 were calculated considering all data records as a single group and also considering the three soil groups (Sa, L, and C) individually. Some correlation coefficients decreased when the data records where considered as a single group and others increased. Therefore, no clear trend was observed.

Nevertheless, the P-values tended to decrease when the data records where considered as a single group. This is expected since an increase in the data points number results in a decrease in the uncertainty associated with a type 1 error.

Porosity did not present substantial correlation with any parameter, with exception of the parameter ln(λ_{res}), as presented in Fig. 4. Still, the coefficient of correlation between n and ln(λ_{res}) does not have a P-value lower than 5% for all soil groups. Similarly, the variable ln(λ_{res}) does not present substantial correlation with any other parameter, with exception of the aforementioned correlation with porosity and some mild correlation with ln(ψ_b), as presented in Fig. 6.

Noteworthy correlations involving the air-entry value were found considering both ln(ψ_b) and ln(ψ_{bk}). While ln(ψ_b) presented considerable correlations with the SWCC-related parameters, ln(ψ_{bk}) showed significant correlations with the parameters related to the k^w function. The parameters ln(ψ_b) and ln(λ_d) presented correlation coefficients that varied from +0.374 to +0.587.

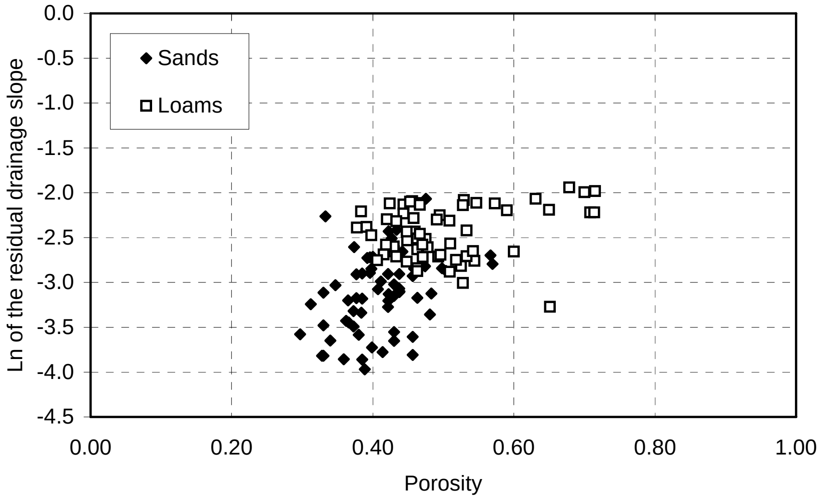

Figure 4. Scatter plot: porosity versus natural logarithm of the residual drainage slope, λ_{res}.

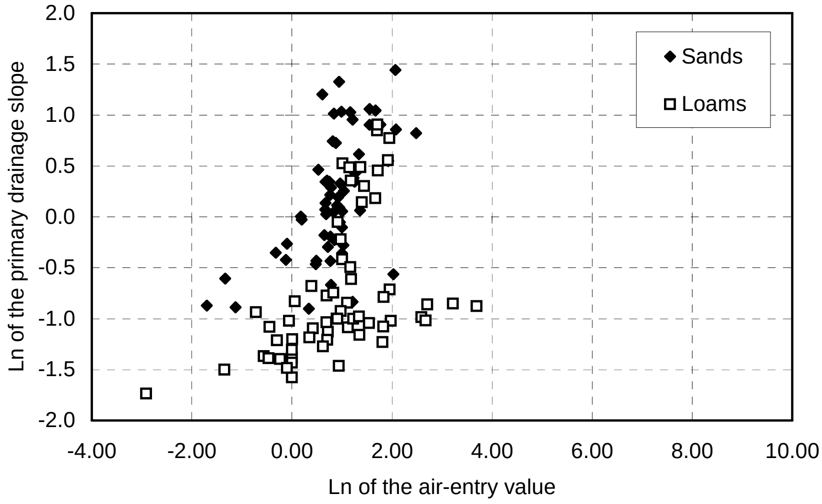

Figure 5. Scatter plot: natural logarithm of the air-entry value (kPa) versus natural logarithm of the primary drainage slope, λ_d.

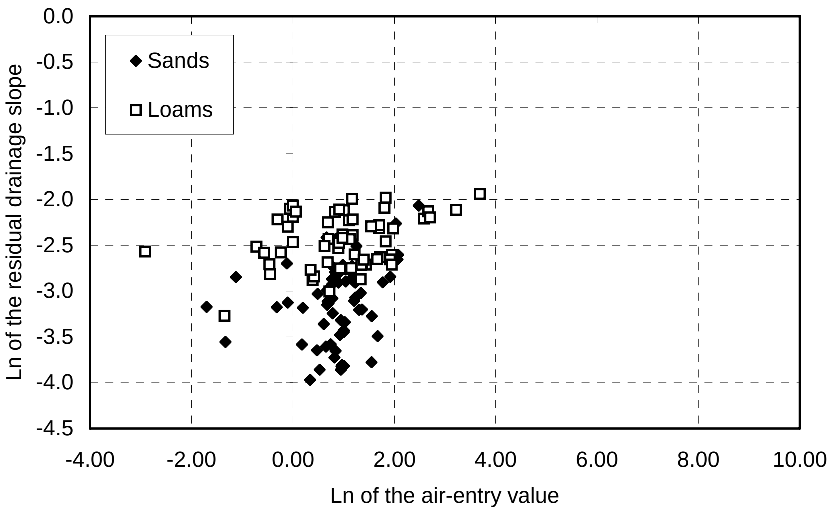

Figure 6. Scatter plot: natural logarithm of the air-entry value (kPa) versus natural logarithm of the residual drainage slope, λ_{res}.

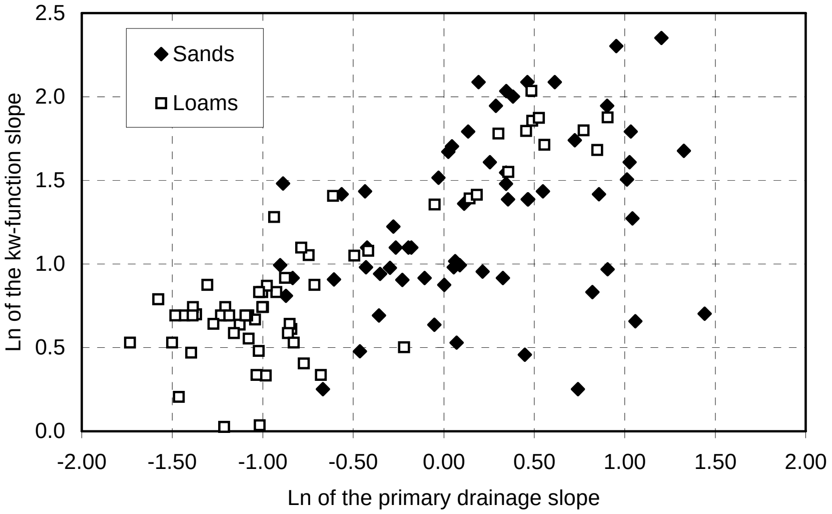

Figure 7. Scatter plot: natural logarithm of the primary drainage slope, λ_d, versus the natural logarithm of the slope of the hydraulic conductivity function.

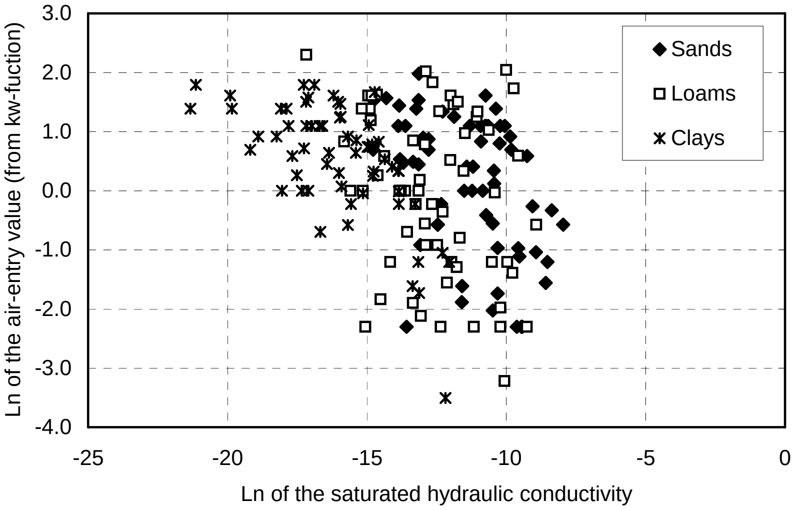

Figure 8. Scatter plot: natural logarithm of the saturated hydraulic conductivity (m/s) versus the natural logarithm of the air-entry value (kPa) obtained from the hydraulic conductivity function.

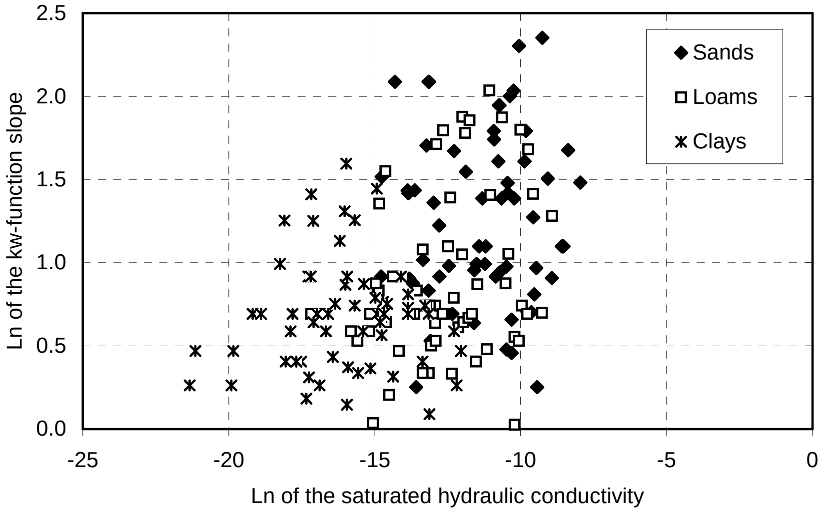

Figure 9. Scatter plot: natural logarithm of the saturated hydraulic conductivity (m/s) versus the natural logarithm of the slope of the hydraulic conductivity function.

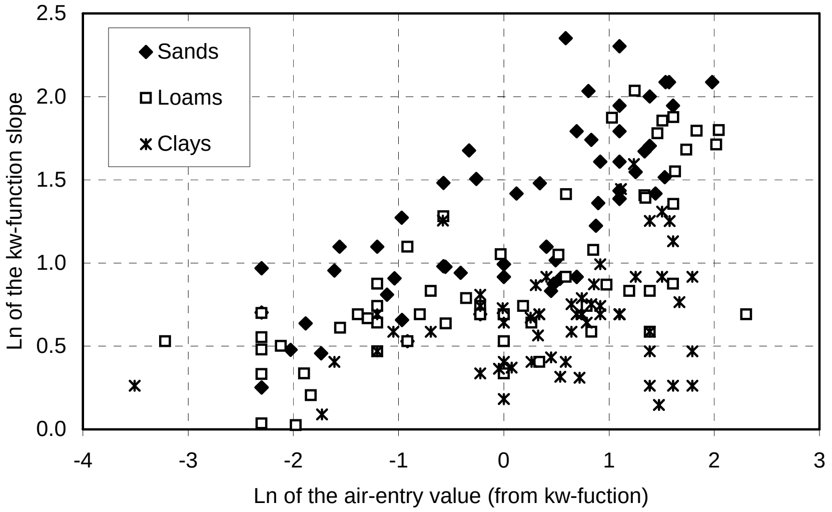

Figure 10. Scatter plot: natural logarithm of the air-entry value (kPa) obtained from the hydraulic conductivity function versus the natural logarithm of the slope of the hydraulic conductivity function.

This significant positive correlation, shown in Fig. 5, indicates that soils with lower air-entry value tend to be the same soils that have “widely graded” grain and/or pore-size distributions. It’s important to mention that these values of coefficient of correlation may sometimes not bear any physical meaning and simply explain a trend in the natural occurrence of data.

The parameters ln(ψ_b) and ln(λ_{res}) presented correlation coefficients that varied from +0.211 to +0.281 (Fig 6). This correlation was not expected, since in principle there is little physical meaning for the relationship between the air-entry value and the residual drainage slope. Nevertheless, the coefficient of correlation is relatively low.

High degrees of correlation were found between the parameters ln(λ_d) and ln(η), varying from +0.365 to +0.847. The scatter plot for these variables is presented in Fig. 7. The high degree of positive correlation was expected. Several mechanistic models of prediction of the hydraulic conductivity function indicate that the slope of the k^w function increases with increasing values of λ_d (e.g., Brooks and Corey, 1964).

Figure 8 presents the scatter plot for ln(k^w_{sat}) and ln(ψ_{bk}). The parameters ln(k^w_{sat}) and ln(ψ_{bk}) presented moderate to high correlation coefficients that varied from -0.275 to -0.630. The negative correlation between ln(k^w_{sat}) and ln(ψ_{bk}) was also expected since the same factors that cause higher air-entry values are responsible for lower values of k^w_{sat}, such as larger percentage of fines.

The parameters ln(k^w_{sat}) and ln(η) presented correlation coefficients that varied from +0.280 to +0.393. The loam group indicated statistically insignificant correlation. The mild positive correlation found between ln(k^w_{sat}) and ln(η) was expected, since soils with higher saturated hydraulic conductivities tend to have steeper hydraulic conductivity functions (see Fig. 9). However, other factors may interfere and make the relationship between both variables certainly not unique.

The parameters ln(ψ_{bk}) and ln(η) presented correlation coefficients that varied from +0.338 to +0.755. This highly significant degree of correlation indicates that soils with larger air-entry value tend to be the same soils that have steep grain and/or pore-size distributions. It appears that this correlation is merely an indication of a trend in the natural occurrence of these soil properties.

4 CONCLUSIONS

This paper presented a statistical study that was carried out with the objective of establishing typical coefficients of correlation between unsaturated soil parameters. The parameters studied herein are those associated with the soil-water characteristic curve parameters and the hydraulic conductivity function.

Scatter plots were presented and the coefficients of correlation between the unsaturated soil parameters calculated accordingly. The analyses were carried out following a methodology established based on appropriate fitting functions, parameters, and soil grouping criterion. A total of 186 datasets were sampled from a database of soils and analyzed using the proposed methodology.

The typical coefficients of correlation provided herein give important insight into the relationship between unsaturated soil properties. The coefficients of correlation must be combined with typical coefficients of variation and mean values in order to allow a reliability-based analysis and design.

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