AN ALTERNATIVE MODELLING APPROACH FOR UNSATURATED SOILS

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Abstract: Although a number of constitutive models for unsaturated soils exist in the literature, some fundamental questions have not been fully answered. There are questions related to (a) the change of the yielding stress with soil suction, (b) the plastic volume change associated with initial drying of slurry soils, and (c) the slope change of the normal compression lines at suctions larger than zero. This paper addresses these questions by proposing a new modelling approach for unsaturated soils.

1. INTRODUCTION

Since the pioneering work of Alonso et al. (1990), a number of elastoplastic constitutive models have been developed for unsaturated soils. Early models only deal with stress-suction-strain relationships of unsaturated soils (e.g., Kohgo et al. 1993; Wheeler and Sivakumar 1995; Cui and Delage 1996). These models are based on the same basic assumptions and largely fall in the same framework of Alonso et al. (1990), though different constitutive equations and different stress variables are used. The model by Alonso et al (1990), which is later referred to as the Barcelona Basic Model (BBM), remains as one of the fundamental models for unsaturated soils. More recent models have incorporated suction-saturation relationships with hysteresis into stress-strain relationships (Vaunat et al. 2000; Wheeler et al. 2003; Sheng et al. 2004; Sun et al. 2007).

Existing elastoplastic models for unsaturated soils usually use a loading-collapse yield surface that defines the variation of the apparent preconsolidation stress along the soil suction axis. The apparent preconsolidation stress is usually assumed to increase with increasing suction. Under such a framework, these models are able to reproduce some basic features of unsaturated soil behaviour (e.g., wetting-induced volume collapse). However, some fundamental questions have not yet been fully answered.

One such question is: "How does the yielding stress change with soil suction?" For a fully saturated soil, the effective stress principle prevails. The elastic zone on the plane of mean net stress versus suction should travel along the 45° line. Once the suction is sufficiently high so that the soil becomes unsaturated, increasing suction is likely less effective in causing plastic volume change and the yield surface should therefore drift away from the 45° line. However, the drift is likely to follow a smooth and continuous curve (the dashed line in Figure 1). This rationale leads to a somewhat different outcome than the commonly adopted loading-collapse yield surface that assumes the yielding stress increases with increasing suction (the dotted line in Figure 1).

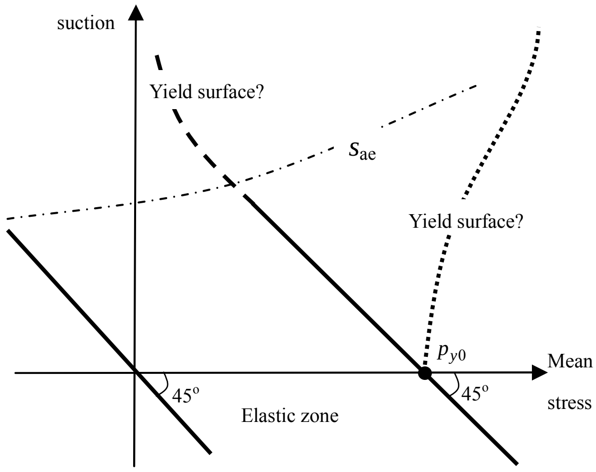


Figure 1. Elastic zone for saturated states.

The second question that arises is "Does drying cause plastic volume change of a slurry soil?" Indeed, if the soil is fully saturated, increasing suction has a similar effect on the soil volume to increasing mean stress. Therefore, drying a slurry soil upto the air entry suction is similar to consolidating the soil to an equivalent mean stress. This rationale also leads to the conclusion that the initial yield surface of a slurry soil must intercept with the suction axis.

Another question that needs to be addressed is: "What is the source and nature of the smooth curvature of the normal compression lines obtained at constant suction levels?" Experimental data show that the normal compression lines at constant suctions generally exhibit an apparent over-consolidation effect, even for a soil that has never been over-consolidated. The initial portions of these lines usually become increasingly flatter as suction increases (Lloret et al. 2003; Futail and Almeida 2005). In existing models, these curves are usually approximated by two asymptotic lines, with one representing the unloading-reloading line and the other the normal compression line. The slope of the normal compression lines is then considered to be a function of suction, with some data supporting a decreasing slope with increasing suction (Alonso et al.1990) and some supporting an increasing slope (Wheeler and Sivakumar 1995). While these approximations seem to be effective, seeking for an alternative approach that can lead to a more unified explanation of the slope change of these compression lines is a worthy effort.

The objective of this paper is to address the above-mentioned questions by presenting a different approach to model the basic features of unsaturated soil behaviour.

2. INDEPENDENT STRESS VARIABLES

Fredlund and Morgenstern (1977) presented the experimental and theoretical justifications of using two independent stress state tensors for constitutive modelling of unsaturated soils. Most existing elasto-plastic models have used two sets of stress variables to define the behaviour of unsaturated soils. This is particularly true for the more recent models that accommodate both stress-strain and suction-saturation behaviour. In this paper, the constitutive relationships are defined in terms of the two independent stress variables and their work-conjugate strains. The use of the independent stress variables facilitates the study of various stress paths commonly adopted in laboratory testing of unsaturated soils. The two independent stress variables and their work-conjugate strain variables are respectively:

$$\begin{pmatrix} \boldsymbol{\sigma} - \mathbf{m} u_a \\ u_a - u_w \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} \boldsymbol{\varepsilon} \\ \theta \end{pmatrix}$$

where $\boldsymbol{\sigma}$ is the total stress vector, u_a is the pore air pressure, u_w is the pore water pressure, $\boldsymbol{\varepsilon}$ is the soil skeleton strain vector, θ is the volumetric water content, and $\mathbf{m}^T=(1, 1, 1, 0, 0, 0)$.

3. CONSTITUTIVE EQUATIONS

3.1 Volumetric behaviour

For saturated soils, it is usually assumed that the specific volume, v, varies with the mean effective stress, $p' = p - u_w$:

$$\mathrm{d}v = -\lambda_{vp} \frac{\mathrm{d}p'}{p'} = -\lambda_{vp} \frac{\mathrm{d}p}{(p-u_w)} - \lambda_{vp} \frac{-\mathrm{d}u_w}{(p-u_w)} \tag{1}$$

where p' is the mean effective stress, λ_{vp} is the slope of the normal compression line (NCL) for normally consolidated soils, and is replaced by κ_{vp} for the slope of the unloading-reloading line (URL) for over-consolidated soils.

Instead of using equation (1), it has become more common to use a linear relationship between the logarithmic specific volume and the logarithmic mean effective stress lnp':

$$\mathrm{d}\varepsilon_v = -\frac{\mathrm{d}v}{v} = \lambda_{vp} \frac{\mathrm{d}p}{(p-u_w)} + \lambda_{vp} \frac{-\mathrm{d}u_w}{(p-u_w)} \tag{2}$$

where $\mathrm{d}\varepsilon_v$ is the rate of the volumetric strain. Equation (2) is supported by the experimental data of Butterfield (1979) and Hashiguchi (1995). The use of a double logarithmic relationship instead of the semi-logarithmic relationship is also motivated by the fact that the former leads to a decoupled model where the instantaneous elastic modulus is independent of the plastic strain (Collins and Kelley 2002). Otherwise, equations (1) and (2) are similar and the use of one or the other does not lead to a significant difference.

For unsaturated soils, the volume change due to a suction change may not necessarily be the same as that due to a change in the mean net stress. The equivalent equation then takes the form:

$$\mathrm{d}\varepsilon_v = \lambda_{vp} \frac{\mathrm{d}\bar{p}}{\bar{p}+s} + \lambda_{vs} \frac{\mathrm{d}s}{\bar{p}+s} \tag{3}$$

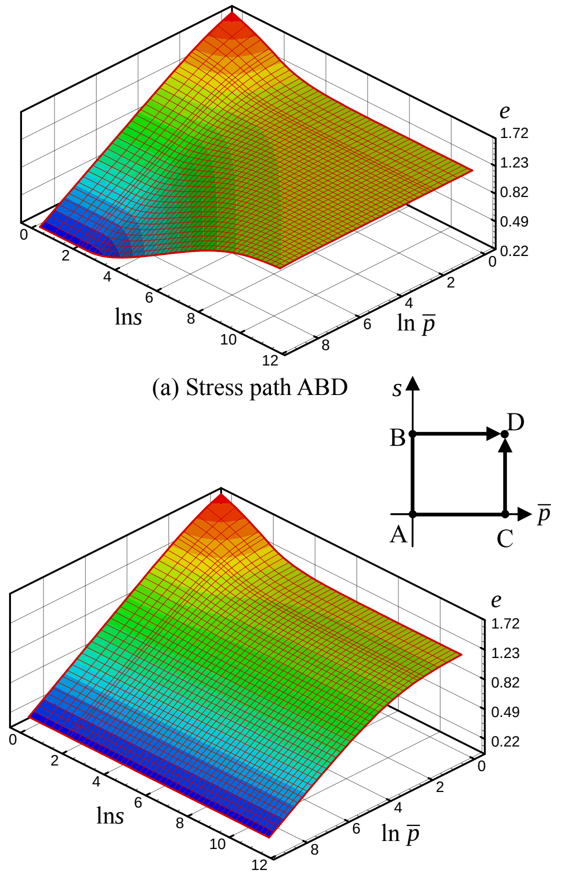
where $\bar{p} = p - u_a$ is the mean net stress, u_a is the pore air pressure, $s = u_a - u_w$ is the soil suction. The slope λ_{vs} is identical to the slope λ_{vp} when the soil is fully saturated, because decreasing pore water pressure has a similar effect to increasing mean stress on a saturated soil. Once the soil becomes unsaturated, experimental results show that the slope λ_{vs} gradually decreases to zero at high soil suctions. A simple, but not unique approximation for λ_{es} takes the form

$$\lambda_{vs} = \begin{cases} \lambda_{vp} & s < s_{sa} \\ \lambda_{vp} \dfrac{s_{sa}+1}{s+1} & s \geqslant s_{sa} \end{cases} \tag{4}$$

where s_{sa} is the saturation suction, which is the air entry value for a desaturating soil and the suction causing full saturation for a saturating soil. The equation defines a continuous function of suction.

Equation (3) is integrable for suction changes under a constant mean net stress or for stress changes under a constant suction. The integrated equation can be used to generate 3-dimensional plots which show the variation of the specific volume versus soil suction and net mean stress change. In Figure 2, two alternative stress paths ABD and ACD are considered, both starting from a saturated slurry soil. Along path ABD, the slurry soil is first dried to suction at point B and then loaded to point D. Along path ACD, the slurry soil is first loaded to point C and then dried to point D. The predicted volume response for both stress paths are shown in Figure 2.

Figure 2a shows that a soil that is first dried to a high suction becomes almost incompressible. The 3D surface in Figure 2a is projected onto the *e*-log $\bar{p}$ space in Figure 3, to give a better view of the respective volume changes. Figure 3 shows that when the slurry soil was first dried to a specified suction and then isotropically compressed, the normal compression lines (NCL) are no longer straight lines. The initial portion of a NCL looks like an unloading-reloading line (URL) for an over-consolidated soil, even though the soil has never been unloaded or over-consolidated! As the suction level increases, the initial portion of the NCL becomes increasingly flatter. If the soil were wetted at point D under a constant mean net stress to full saturation, volume collapse would occur, since the void ratio at this point is higher than the void ratio on the NCL for zero suction.



(a) Stress path ABD

(b) Stress path ACD

Figure 2. Specific volume versus suction and mean net stress (slurry soil, $\bar{p}_0 = 1$kPa, s_0=0 kPa, *N*=3, λ_{vp}=0.1, s_{sa}=10kPa*).*

The volumetric behaviour shown in Figure 3 is well documented for unsaturated soils and is supported by a large amount of experimental data in the literature. For example, the $e - \ln \bar{p}$ curves under constant suctions presented by Lloret *et al.* (2003) and Futai and Almeida (2005) are all similar to those shown in Figure 3. Figure 4 illustrates that equation (3) can well fit the experimental data of Lloret et al. (2003). The only parameters used in producing the curves in Figure 4 are the slope λ_{vp} and the initial void ratios at $\bar{p}$ =0.1MPa.

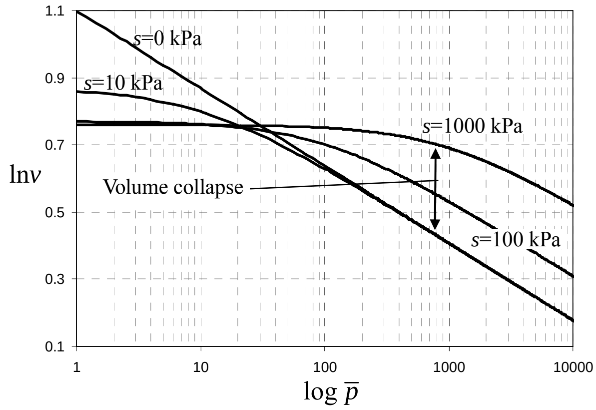


Figure 3. Normal compression lines at different suctions (slurry soil, N=3, λ_{vp}=0.1, s_{sa}=10kPa*).*

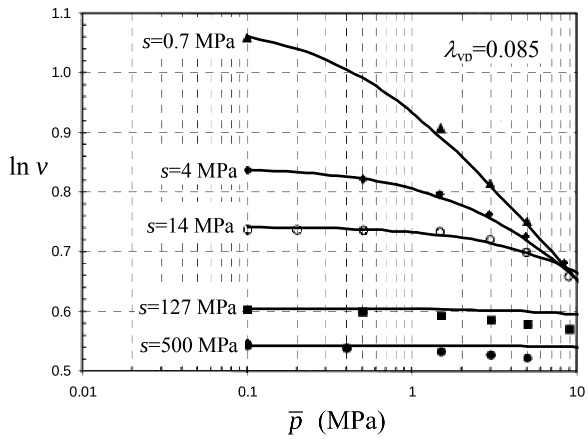


Figure 4. Comparison of equation (3) with experimental data of Lloret et al. (2003). The curve of s=0.7MPa *corresponds to the data of s*=0 MPa, *but with a precosolidation stress of* 0.7MPa *in Lloret et al.*

Figure 2b shows the volume change along stress path ACD where the saturated soil is first compressed to a mean net stress before it is dried. The 2D projection of Figure 2b is shown in Figure 5. The results show that there is little volume change during drying when the soil is first loaded to a high mean net stress (e.g. 1000 kPa). The curves in Figure 5 are of the same pattern as the experimental data of Richards *et al.* (1984), which is also shown in the figure. Since the void ratio at point D is almost the same as that at point C, no collapse would occur if the soil were wetted at point D under a constant mean net stress.

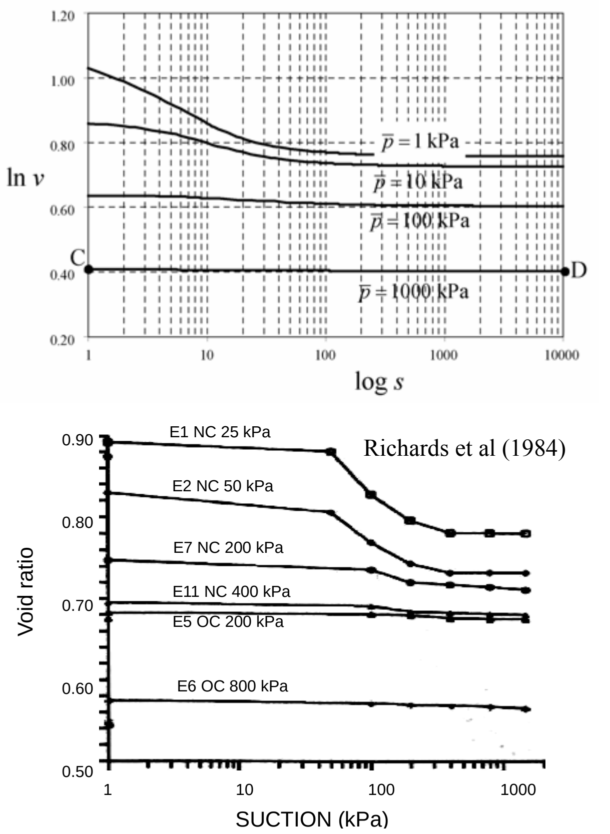


Figure 5. Void ratio versus suction along stress path ACD (compared with Richards et al. 1984).

3.2 Yielding stress and hardening law

In order to understand the projection of the yield surface onto the $\overline{p}-s$ plane for an unsaturated soil, let us start with a saturated soil. For simplicity, the Modified Cam Clay (MCC) model (Roscoe and Burland, 1968) can be used as a starting point, while the generalisation to other models of saturated soils follows in a similar manner. The pore water pressure can again be separated from the effective stress in the MCC model:

$$f = q^2 - M^2\left(p-u_{\mathrm{w}}\right)\left(p_{\mathrm{y}0}+u_{\mathrm{w}}-p\right) \equiv 0 \qquad (5)$$

where f is the yield function, q is the deviator stress, M is the slope of the critical state line, and $p_{\mathrm{y}0}$ is the yielding mean stress when u_{w}=0. If the yield surface is projected onto the $p-u_{\mathrm{w}}$ plane, the elastic zone is bounded by the two 45$^{\mathrm{o}}$ lines (Figure 1). Equation (5) can be rewritten as:

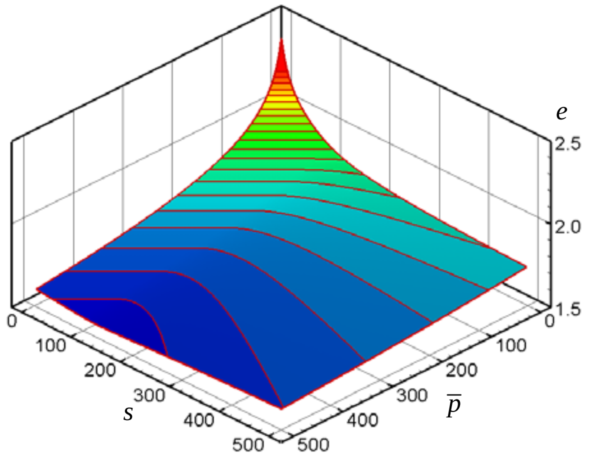
$$f = q^2 - M^2\left(p-p_0\right)\left(p_{\mathrm{y}}-p\right) \equiv 0 \qquad (6)$$

where $p_{\mathrm{y}} = p_{\mathrm{y}0}+u_{\mathrm{w}}$ is the yielding mean stress, and $p_0 = u_{\mathrm{w}}$ is the 45$^{\mathrm{o}}$ line that goes through zero.

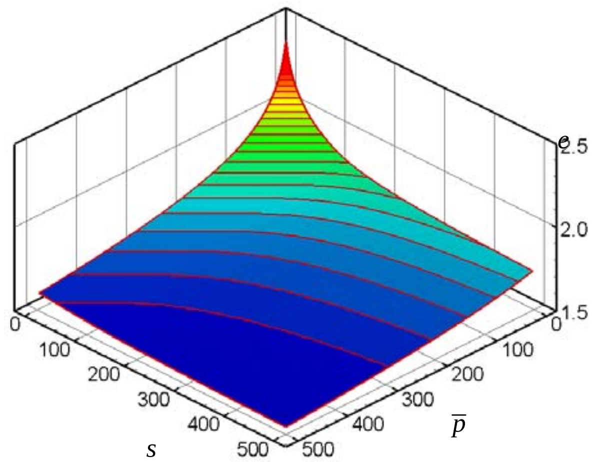
Once the soil suction is above the saturation suction, increasing suction is likely less effective as increasing stress in causing plastic volume change. Therefore, the bounding lines are expected to drift away from the 45$^{\mathrm{o}}$ lines. The yield function for unsaturated soils then takes the general form:

$$f = q^2 - M^2\left(\overline{p}-\overline{p}_0(s)\right)\left(\overline{p}_{\mathrm{y}}(s)-\overline{p}\right) \equiv 0 \qquad (7)$$

To understand the evolution of the yield surface for suctions above the saturation suction, we replot the 3D surfaces in Figure 2 in arithmetic scales in Figure 6. The saturation suction is changed to 100 kPa, to make it more visible on the arithmetic plots. The contours of the void ratio can be seen to follow the 45$^{\mathrm{o}}$ line for suctions below the saturation suction, but then drift away differently depending on the stress path. For stress path ABD, the void ratio at point D can be higher or lower than the void ratio point C, depending on the suction level at point D. For stress path ACD, the void ratio at point D is always lower than that at point C.



(a) Slurry soil dried to different suctions and then isotropically compressed



(b) Slurry isotropically compressed to different mean stresses and then dried

Figure 6. Contours of void ratio on the $\overline{p}-s$ plane, (a) following stress path ABD, (b) following stress path ACD (Slurry soil, N=3, λ_{vp}=0.1, s_{sa}=100 kPa).

The contours of the void ratio do not exactly represent the yield surface. Rather, the yield surfaces are represented by the contours of the plastic volumetric strain for isotropically hardening materials. Therefore, the elastic volume change has to be considered. Following critical state soil mechanics and prior discussions, we assume:

$$\mathrm{d}\varepsilon_{\mathrm{v}}^{\mathrm{e}} = -\frac{\mathrm{d}v^{\mathrm{e}}}{v} = \kappa_{\mathrm{vp}}\frac{\mathrm{d}\bar{p}}{\bar{p}+s} + \kappa_{\mathrm{vs}}\frac{\mathrm{d}s}{\bar{p}+s} \qquad (8)$$

where the slope of κ_{vs} is identical to the slope κ_{vp} when the suction is lower than the saturation suction and gradually decreases to zero as the suction increases above the saturation suction. A simple, but not unique, approximation is:

$$\kappa_{\mathrm{vs}} = \begin{cases} \kappa_{\mathrm{vp}} & s < s_{\mathrm{sa}} \\ \kappa_{\mathrm{vp}}\dfrac{s_{\mathrm{sa}}+1}{s+1} & s \geqslant s_{\mathrm{sa}} \end{cases} \qquad (9)$$

With equations (3) and (8), it is then possible to derive the projection of the yield surface onto the $\bar{p}-s$ plane. Let us commence with a soil that has been consolidated to $\bar{p}_{\mathrm{y0}}$ under zero suction. The plastic volumetric strain should be zero along the initial yield surface, leading to:

$$\frac{\mathrm{d}\bar{p}_{\mathrm{y}}}{\mathrm{d}s} = -\frac{\lambda_{\mathrm{vs}}-\kappa_{\mathrm{vs}}}{\lambda_{\mathrm{vp}}-\kappa_{\mathrm{vp}}} \qquad (10)$$

Equation (10) implicitly defines the initial yielding stress $\bar{p}_{\mathrm{y}}$ at an arbitrary suction s for such a consolidated soil. This equation can be integrated along the yield surface, leading to:

$$\bar{p}_{\mathrm{y}} = \begin{cases} \bar{p}_{\mathrm{y0}} - s & s < s_{\mathrm{sa}} \\ \bar{p}_{\mathrm{y0}} - s_{\mathrm{sa}} - (s_{\mathrm{sa}}+1)\ln\dfrac{s+1}{s_{\mathrm{sa}}+1} & s \geqslant s_{\mathrm{sa}} \end{cases} \qquad (11)$$

$$\bar{p}_0 = \begin{cases} -s & s < s_{\mathrm{sa}} \\ -s_{\mathrm{sa}} - (s_{\mathrm{sa}}+1)\ln\dfrac{s+1}{s_{\mathrm{sa}}+1} & s \geqslant s_{\mathrm{sa}} \end{cases} \qquad (12)$$

The yield functions so found define the initial elastic zone for a soil that was consolidated at zero suction. Figure 7 illustrates this initial elastic zone. In the figure, the soil has a saturation suction of 100kPa and was consolidated to 300kPa at zero suction. If such a soil is dried at zero net mean stress, plastic volume change will not occur until the suction reaches 730kPa.

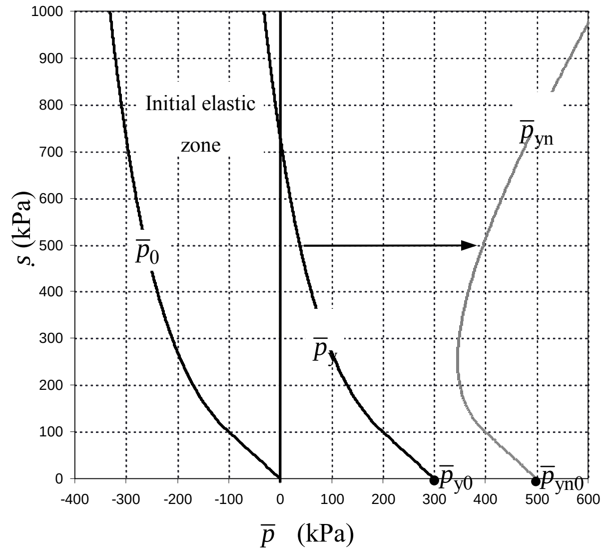


*Figure 7. Initial yield surface for a soil that was consolidated to 300kPa at zero suction and its evolution when the soil is then loaded at different suction levels (s_{sa}=100*kPa*).*

The yield surface $\bar{p}_{\mathrm{y}}$ shown in Figure 7 is only valid for a soil that was initially consolidated to 300 kPa at zero suction. A slurry soil that has never been consolidated has an initial yield surface that is a single point at the origin. If a soil is compressed to plastic yielding at suctions above the saturation suction, the yield surface will evolve to a quite different shape. The evolution of the yield surface is governed by a hardening law.

For isotropically hardening material, $\bar{p}_0$ never changes, but $\bar{p}_{\mathrm{y}}$ changes according to the plastic volumetric strain:

$$\mathrm{d}\varepsilon_{\mathrm{v}}^{\mathrm{p}} = \frac{\lambda_{\mathrm{vp}}-\kappa_{\mathrm{vp}}}{\bar{p}+s}\mathrm{d}\bar{p} + \frac{\lambda_{\mathrm{vs}}-\kappa_{\mathrm{vs}}}{\bar{p}+s}\mathrm{d}s \qquad (13)$$

Equation (13) is the hardening law that governs the evolution of the yield surface. Therefore, the evolution of $\bar{p}_{\mathrm{y}}$ under a constant suction s is governed by the following equation:

$$\mathrm{d}\varepsilon_{\mathrm{v}}^{\mathrm{p}} = \frac{\lambda_{\mathrm{vp}}-\kappa_{\mathrm{vp}}}{(\bar{p}_{\mathrm{y}}+s)}\mathrm{d}\bar{p}_{\mathrm{y}} \qquad (14)$$

Equation (14) shows that the evolution of $\bar{p}_{\mathrm{y}}$ depends on the suction level. If the soil in Figure 7 is compressed to plastic yielding at different suctions, a new yield surface then represents a contour of the total plastic volumetric strain. It means that the total plastic volumetric strain will be the same when every point on the current yield surface $\bar{p}_\text{y}$ is loaded to a new yield surface $\bar{p}_\text{yn}$ under a constant suction. Therefore, we have:

$$\int_{\bar{p}_\text{y}}^{\bar{p}_\text{yn}} \frac{\left(\lambda_\text{vp} - \kappa_\text{vp}\right)}{\left(\bar{p}_\text{y} + s\right)} \mathrm{d}\bar{p}_\text{y} = \int_{\bar{p}_\text{y0}}^{\bar{p}_\text{yn0}} \frac{\left(\lambda_\text{vp} - \kappa_\text{vp}\right)}{\left(\bar{p}_\text{y} + 0\right)} \mathrm{d}\bar{p}_\text{y} \qquad (15)$$

The above equation can be integrated, leading to:

$$\bar{p}_\text{yn} = \begin{cases} \bar{p}_\text{yn0} - s & s < s_\text{sa} \\ \bar{p}_\text{yn0}\left(1 + \dfrac{s - s_\text{sa}}{\bar{p}_\text{y0}} - \dfrac{\left(s_\text{sa}+1\right)}{\bar{p}_\text{y0}} \ln \dfrac{s+1}{s_\text{sa}+1}\right) - s & s \geqslant s_\text{sa} \end{cases} \qquad (16)$$

where $\bar{p}_\text{yn0}$ is the new yielding stress at zero suction. Equation (16) is plotted in Figure 7 for $\bar{p}_\text{yn0} = 500\ \text{kPa}$, which shows that the new yield surface $\bar{p}_\text{yn}$ takes a rather different shape than the old yield surface $\bar{p}_\text{y}$. The yielding stress along the new yield surface does no longer monotonically decrease with increasing suction. It first decreases (following the 45 line to a minimum value) and then increases. The new yield surface shown in Figure 7 confirms that the void ratio contours shown in Figure 2a. Note that the soil in Figure 2a is a slurry soil that was never consolidated.

Alternatively, wetting from the current yield surface under a constant mean net stress will invoke the hardening law:

$$\mathrm{d}\varepsilon_\text{v}^{p} = \frac{\lambda_\text{vs} - \kappa_\text{vs}}{\left(\bar{p} + s_\text{y}\right)} \mathrm{d}s_\text{y} \qquad (17)$$

where s_y is the suction value along the yield surface $\bar{p}_\text{y}$. Equation (11) can be rewritten in terms of s_y. Similarly, the total plastic volumetric strain will be the same if every point on the current yield surface s_y is dried to a new yield surface s_yn under a constant mean net stress, leading to:

$$s_\text{yn} = \begin{cases} \dfrac{A\bar{p} - 1}{1 - A} & \bar{p} \leqslant \bar{p}_\text{y0} - s_\text{sa} \\ \dfrac{B\bar{p} - 1}{1 - B} & \bar{p}_\text{yn0} - s_\text{sa} \geqslant \bar{p} > \bar{p}_\text{y0} - s_\text{sa} \\ \bar{p}_\text{yn0} - \bar{p} & \bar{p} > \bar{p}_\text{yn0} - s_\text{sa} \end{cases} \qquad (18)$$

where

$$A = \left(\frac{\bar{p}_\text{yn0}}{\bar{p}_\text{y0}}\right)^{\frac{\bar{p}-1}{s_\text{sa}+1}} \frac{s_\text{y}+1}{s_\text{y}+\bar{p}} \qquad B = \left(\frac{\bar{p}_\text{yn0}}{\bar{p}+s_\text{sa}}\right)^{\frac{\bar{p}-1}{s_\text{sa}+1}} \frac{s_\text{sa}+1}{s_\text{sa}+\bar{p}}$$

The new yield surface defined by equation (18) is plotted in Figure 8 for $\bar{p}_\text{yn0} = 500\ \text{kPa}$ kPa. Its shape is almost the same as the old yield surface s_y, indicating that drying does not significantly change the shape of the yield surface. The shape of the curve in Figure 8 confirms the void ratio contour shown in Figure 2b. It becomes clear that no collapse would occur in this case if the soil is wetted under a constant mean net stress.

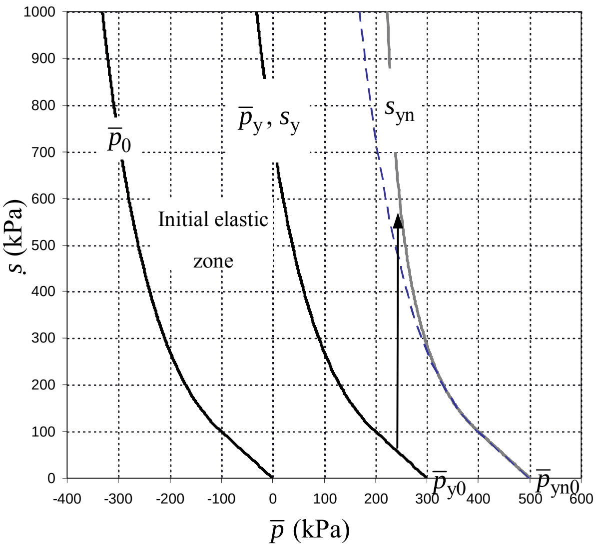


Figure 8. Initial yield surface for a soil that was consolidated to 300kPa *at zero suction and its evolution when the soil is then dried at different mean net stresses (the thin dashed line represents the initial yield surface for a soil that was consolidated to* 500kPa *at zero suction).*

3.3 Soil-water characteristics

The relationship between the volumetric water content and the stress state variables has to be defined. Extensive research has been done on the soil-water characteristics of a soil (see, e.g., Hillel 1982; Fredlund and Rahardjo 1993). As a first approximation, let us assume a piece-wise linear relationship between the degree of saturation S_r and logarithmic soil suction:

$$\mathrm{d}S_\text{r} = -\lambda_\text{ws} \frac{\mathrm{d}s}{s} \qquad (19)$$

where the slope λ_ws may change with suction. For soil suctions below the saturation suction, the soil is saturated and the degree of saturation remains essentially constant. For soil suctions above the residual suction, the degree of saturation gradually decreases to zero at a suction of 10^6 kPa. The slope λ_ws is assumed to be constant between the air entry and the residual suction:

$$\lambda_{ws} = \begin{cases} 0 & s < s_{sa} \\ \kappa_{ws} & s_{sa} \leqslant s < s_{ae} \\ \lambda_{ws} & s_{ae} \leqslant s < s_r \\ \kappa_{ws} & s \geqslant s_r \end{cases} \quad (20)$$

Hysteresis in soil-water characteristics is usually considered to be too important to ignore. Therefore, a wetting curve must be added and this curve is controlled by the water entry value s_{we} and has a similar slope, λ_{ws} (see Figure 9). A series of parallel lines having a slope κ_{ws}, are used to represent recoverable changes in S_r between the drying and the wetting curves. These curves are called "scanning curves". For the purpose of this study, the slope of the scanning curve is assumed to be identical to the slope of the drying curve for suctions beyond the residual value. The simplifications adopted here are similar to those in Wheeler et al. (2003). In the simplified model, the maximum suction that corresponds to full saturation is s_{sa}, not the air entry value, s_{ae}.

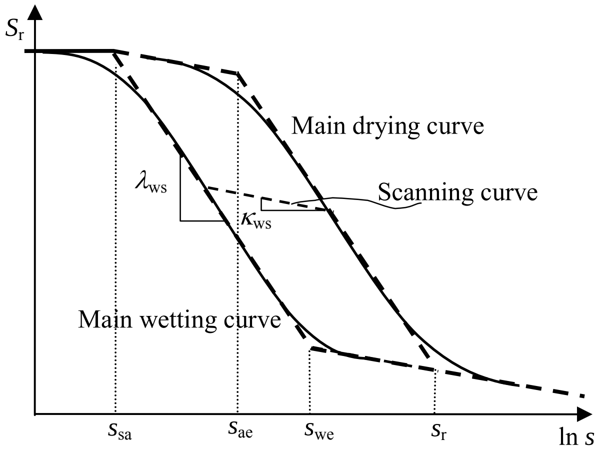


Figure 9. Degree of saturation versus suction (dashed lines represent simplification).

Hysteresis of soil-water characteristics can also be explained in the same framework of elasto-plasticity (Sheng et al. 2004). Under such a framework, an unsaturated state always lies within the main drying and wetting curves. Drying or wetting from within the hysteresis loops will only cause recoverable water content changes until the suction reaches the main drying or wetting curve. Once soil suction reaches the main drying or wetting curve, further drying or wetting will cause irrecoverable water content changes. Therefore, the drying and wetting curves define the boundaries of recoverable water content change and are similar to the normal compression line. The scanning curves define the recoverable water content change and are similar to the unloading-reloading line. On the $\bar{p} - s$ plane, two additional boundaries can be added, representing the main drying and wetting curves, respectively.

3.4 Incremental stress-strain relations

An incremental stress-strain relation can be derived for the proposed model. The plastic potential functions for yield surfaces are needed and, as a starting point, associativity of the flow rule can be assumed. The final incremental stress-strain relationship can be written in a form of:

$$\begin{pmatrix} \mathrm{d}\bar{\boldsymbol{\sigma}} \\ \mathrm{d}\theta \end{pmatrix} = \begin{pmatrix} \mathbf{D}_{ep} & \mathbf{W}_{ep} \\ \mathbf{R} & G \end{pmatrix} \begin{pmatrix} \mathrm{d}\boldsymbol{\varepsilon} \\ \mathrm{d}s \end{pmatrix} \quad (21)$$

where $\mathbf{D}_{ep}$ is a 6×6 matrix, $\mathbf{R}$ is a row vector of 6 elements, $\mathbf{W}_{ep}$ is a column vector of 6 elements, and G is a scalar. The derivation of equation (21) can be found in Sheng *et al.* (2006). In equation (21), the soil suction is treated as a strain variable, in consistent with the displacement finite element method. The incremental stress-strain relationship defined by equation (21) can readily be implemented into the finite element method to solve boundary value problems.

The model presented in this paper is referred to as the SFG model and contains the following material parameters:

- λ_{vp}: slope of the NCL for saturated soil
- κ_{vp}: slope of the URL for saturated soil
- M: slope of the CSL on the $q - \bar{p}$ plane
- λ_{ws}: slope of the main drying curve
- κ_{ws}: slope of the scanning curve
- s_{sa}: saturation suction
- s_{we}: water entry value
- s_{re}: residual suction,
- μ: Poisson's ratio

and the following initial conditions:

- $\bar{p}_{y0}$: initial preconsolidation stress at s=0
- e_0: initial void ratio or the specific volume

Compared to the Modified Cam Clay model, the mechanical part of the SFG model has only one new parameter, i.e. the saturation suction. Some of the parameters can also be functions of the suction or mean net stress. For example, the saturation suction can be a function of the mean net stress.

Due to the space limit, no numerical examples will be presented here. More detailed information refers to Sheng et al. (2006).

4. CONCLUSIONS

This paper presents a new elastoplastic model for unsaturated soils using independent stress state variables. In so doing, it addresses some previously unanswered questions. These questions relate to the change of the yielding stress with suction and the smooth curvature of the normal compression lines at constant soil suctions. A new volume-stress-suction relationship is proposed to model the volume changes caused by independent changes of stresses and soil suction. The Modified Cam Clay model is used as the base model for saturated states and it is generalised to unsaturated states through a smooth transition. The projection of the yield surface on the plane of mean net stress versus suction is derived. The evolution of the yield surface under different stress paths is illustrated. The presented model also accommodates hysteresis associated with wetting and drying.

It is shown that the yielding stress initially decreases with increasing suction below the saturation suction, but may increase or decrease for suctions above the saturation suction, depending on the stress path. The yielding stress for a soil that has never been over-consolidated at suction above zero always decreases with increasing suction. However, compaction or consolidation of the soil at suction above zero can change the shape of the yield surface. It also is shown that the smooth curvature of the normal compression lines at constant suctions is a natural result of the proposed volume-stress-suction relationship. The variation of the soil compressibility with suction is well captured by the model.

Compared to existing models in the literature, the proposed SFG model seems to be more flexible in modelling different types of unsaturated soils. The model works well for soils that are dried or loaded from initially slurry conditions and for soils that have low to high air entry values. It works for compacted soils in a similar way as existing models in the literature.

The SFG model provides a fundamental framework for modelling the basic features of unsaturated soil behaviour. As with its counterparts in the literature, it does not address more complex issues such as plastic volume expansion during wetting for expansive clays and volume collapse at zero mean stress for loess soils. Some functions used in the proposed model may also have to be elaborated upon for quantitative prediction of unsaturated soil behaviour.

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