A Practical Volume-Mass Constitutive Model for Unsaturated Expansive Soils

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Abstract: This paper presents a brief description of a new volume-mass constitutive model. The model requires relatively simple data for calibration. The volume and water content changes can be independently computed using the model. Closed-form equations for the volume-mass constitutive surfaces corresponding to any set of stress paths can be predicted. The model is capable of taking into account: i) hysteresis in the soil-water characteristic curve, SWCC; and ii) elastic and plastic deformations in the soil. Model prediction results for an artificial silt are presented in this paper.

INTRODUCTION

Volume-mass constitutive relationships are of value in modeling unsaturated soil behavior, and can be used in the assessment of other unsaturated soil properties such as shear strength and hydraulic conductivity. A number of volume-mass constitutive models have been proposed (Alonso et al. 1990; Wheeler and Sivakumar 1995; Batz and Graham 2003). Most models cannot predict water content changes or assume that the degree of saturation is independent of net mean stress. A new volume-mass constitutive model is proposed herein for only isotropic loading condition (i.e., ignoring the existence of the shear stress).

TERMINOLOGY

Volume and water content changes in a soil are controlled by two different mechanisms: i) stress-strain (i.e., mechanical) behavior and ii) adsorption-drainage behavior (i.e., capillary theory). Consequently, the water content change and volume change of an unsaturated soil must be independently predicted in a volume-mass constitutive model.

Deformation of a soil mass is directly related to a change in the volume of pores in the soil. The pore-size distribution curve (PDC) of a soil at any stress state provides information regarding both the total volume and the volume of water in the soil, and can be used to predict the PDC of the soil. The PCD is a function of soil suction and net mean stress. In the proposed model, a reference pore-size distribution reference state is first selected and the stress-strain relationship for the soil structure surrounding a pore group is then calculated.

THEORY

Stress state variables and reference PDC

Net mean stress, $p = (\sigma_1 + \sigma_2 + \sigma_3)/3 - u_a$, and soil suction, $\psi = (u_a - u_w)$, are two stress state variables used in the proposed model. The void ratio, *e,* and the gravimetric water content, *w*, of the soil are the two primarily variables that can be used to represent the overall volume and water content of the soil.

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The PDC corresponding to the completely dry condition obtained from an initial slurry soil (i.e., at 10^6 kPa on the initial drying process of the slurry soil) provides a meaningful reference state. In this model, the *reference pore-size distribution curve* of a soil is defined as the PDC of a soil. Another part of the reference stress state was arbitrarily chosen as p = 1kPa and ψ = 0 kPa.

A soil has two reference pore-size distribution curves at completely dry conditions (i.e., with respect to drying and wetting suctions). The *reference drying DPC* of a soil provides information regarding the air entry value and the distribution in volume of pores in the soil. Similarly, the *reference wetting PDC* of a soil provides information regarding the water entry value and the distribution in volume of pores in the soil.

Assumptions

Six assumptions based on the results of previous studies were adopted for the proposed constitutive model.

- *Assumption 1:* A particular pore under consideration in the soil has only two states; namely, i) the pore is filled with water; or ii) the pore is dry.
- *Assumption 2:* Each water-filled pore in the soil also has two indices, namely: i) virgin compression index, C_c^p and ii) unloading-reloading compression index, C_s^p.
- *Assumption 3:* There are two types of pores: i) compressible pores and ii) non-compressible pores. The compressible pores are relatively large pores and the non-compressible pores are relatively small-interconnected pores.
- *Assumption 4:* Virgin and unloading-reloading compression indices for a pore in the soil are proportional to the volume of the pore at the reference stress state.
- *Assumption 5:* Pores are deformed, and water is absorbed and drained independently.
- *Assumption 6:* Air-filled pores are incompressible.

Mathematical formulation

The initial drying SWCC from the slurry of a significant volume change soil can be best-fitted using the following empirical equation:

$$w(\psi)=\left(\left[w_{sat}-\frac{C_c}{G_s}\log(\psi)-w_r\right]\frac{a}{\psi^b+a}+w_r\right)\left(1-\frac{\ln\left[1+\frac{\psi}{\psi_r}\right]}{\ln\left[1+\frac{10^6}{\psi_r}\right]}\right) \tag{1}$$

where C_c = virgin compression index of the soil; G_s = Particles specific gravity; w_r = curve-fitting parameter represents the residual water content; w_{sat} = curve-fitting parameter represents the water content at reference stress state; and a, b = curve-fitting parameters. The virgin compression index of the group of pores having air entry value of ψ on the reference pore-size distribution curve can be calculated as follows:

$$C_c^p(\psi) = -\frac{C_c ab \ln(10)\psi^b}{\left[\psi^b + a\right]^2} \tag{2}$$

Similarly, the unloading-reloading compression index of the group of pores having air entry value of ψ on the reference pore-size distribution curve can be calculated as follows:

$$C_s^p(\psi) = -\frac{C_s ab \ln(10)\psi^b}{\left[\psi^b + a\right]^2} \tag{3}$$

Comparing the strain of a pore that is dried under zero net mean stress and a pore that is dried under a yield stress, p_y, and then dried under a constant net mean stress, p, the equation for the effect of the net mean stress on the air entry value can be written:

$$\frac{\psi_{ae}}{\psi_{ae}(p, p_y)} = 1 - \eta \frac{\left[(C_c - C_s)\log(p_y) + C_s \log(\psi_{ae} + p) - C_c \log(\psi_{ae})\right]}{3\left[e_{sat} - C_c \log(\psi_{ae}) - w_r G_s\right]} \tag{4}$$

where ψ_{ae} = water entry value of the pore having zero yield stress and wetting under zero net mean stress; $\psi_{ae}(p, p_y)$ = air entry value of the pore when yield stress is equal to p_y and drying under a net mean stress of p; η = pore-shape parameter represents the effect of the net mean stress to the change in the diameter of a pore. Similarly, the equation for the effect of the net mean stress to the water-entry value of the pore can be obtained.

The yield stress of the soil structure surrounding a water-filled pore is considered to be maximum effective stress that ever acted on the pore. If a pore is dried under zero net mean stress, the yield stress of the pore is equal to the air entry value of the pore. If a pore is dried under a constant net mean stress of, p, the yield stress of the pore can be calculated:

$$p_y = (p + \psi(p, p + \psi_{ae})) \tag{5}$$

where $\psi(p, p + \psi_{ae})$ can be calculated using Eq. (4). When a pore is filled with air, the pore is incompressible (i.e., assumption #6); therefore, yield stress of the soil structure surrounding the pore does not change with net mean stress and soil suction. The two constitutive equations for the volume-mass constitutive surfaces follow the stress paths shown in Figure 1 (i.e., Eqs. (6) and (7)). The constitutive surfaces correspond to a slurry soil that is initially loaded to a net mean stress, p_0 at zero soil suction and then dried.

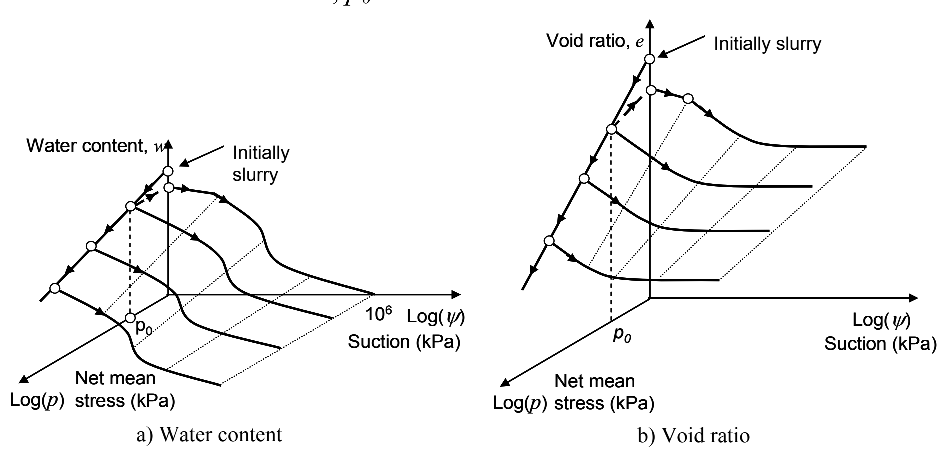


Figure 1. Schematic illustration of the volume-mass constitutive surfaces of an initially slurried specimen that is dried under various constant net mean stresses.

Equations have been written for the water content and the void ratio constitutive surfaces (See Pham, 2005). The equation for the degree of saturation surface can be derived from the equations for the gravimetric water content surface and void ratio surface.

Hysteresis model for SWCC

A hysteresis model is developed using two one-dimensional pore-size distribution functions (i.e., wetting and drying pore-size distributions). Scanning curves are horizontal on degree of saturation SWCC plots (similar to the Wheeler et al. (2003) model). This means that each group of pores has a unique relationship between the drying and wetting suction. Therefore there is a relationship between the *reference DPD* and the *reference DPD*

Model parameters

The required data for calibration can be described as follows:

- the initial drying SWCC of the initially slurry soil specimen.
- pore-shape parameter, η,
- the parameters for the hysteretic nature of the SWCC of the soil (Pham et al. (2005)).
- the compression indices of the soil.

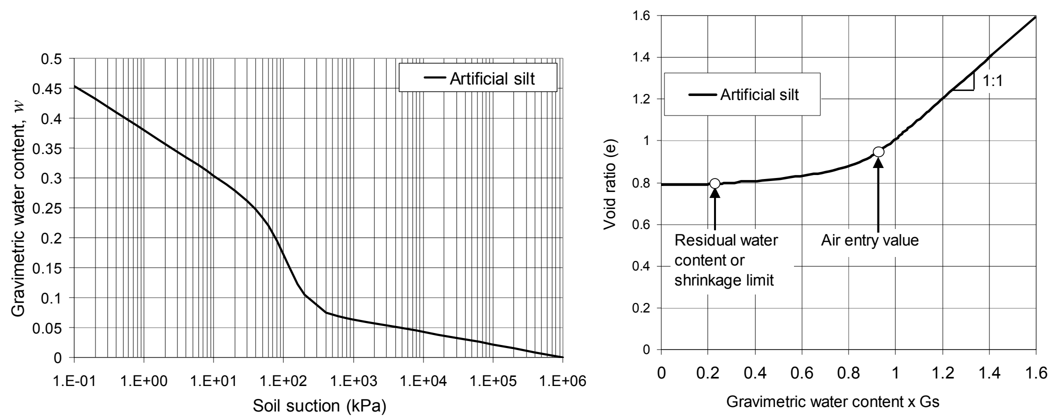
PRESENTATION OF THE MODEL PREDICTION

The volume-mass constitutive equation for an artificial silt is presented that has soil properties as shown in Table 1. The initial drying SWCC of the silt is shown in Figure 2.

Table 1. Soil properties for the artificial silt.

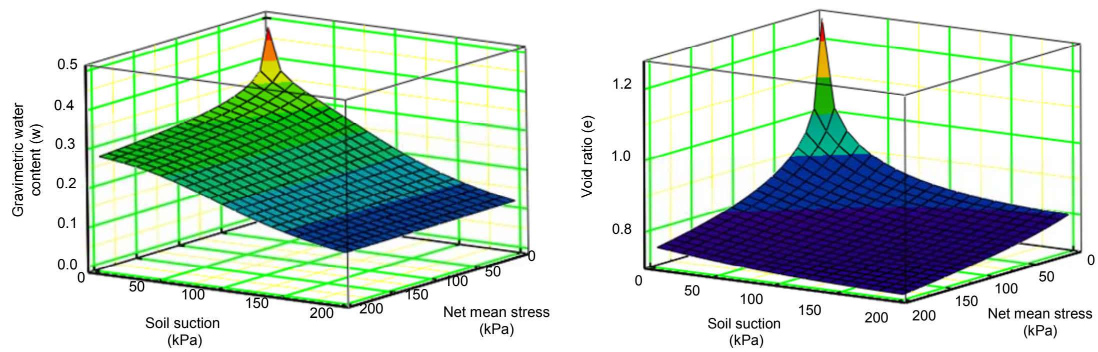
Soil-water characteristic curve				Hysteresis parameters			Compression indices		
w_{sat}	a	b	w_r	D_{SL}	R_{SL}	β	C_c	C_s	G_s
0.45	200000	2.5	0.08	0.35	1.5	0.1	0.2	0.04	2.7

The predicted shrinkage curve for the artificial silt is shown in Fig. 2 has a reasonable shape. At high water contents, the shrinkage curves are 45 degree lines. The shrinkage curves are horizontal at low water contents. Plots of the volume-mass constitutive surfaces along with the physical characteristics for the artificial silt are presented in Figure 3. The results show that the calculated volume-mass constitutive surfaces.

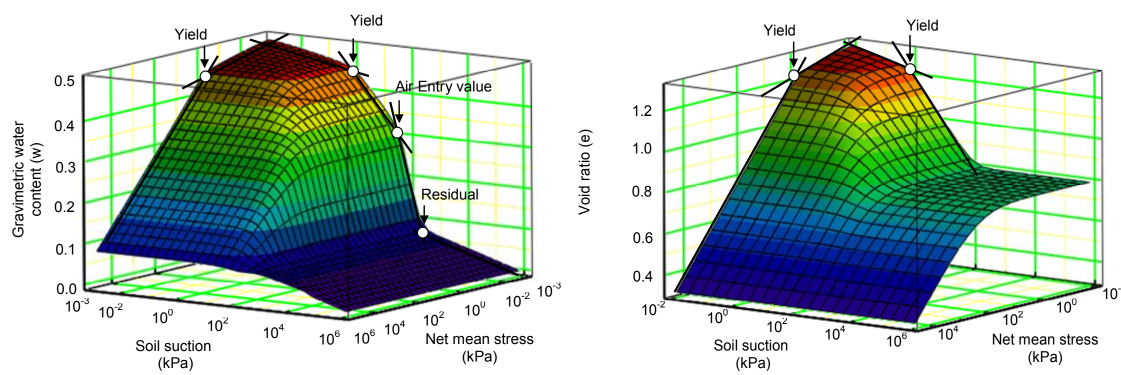


a) Soil-water characteristic curve b) Shrinkage curve

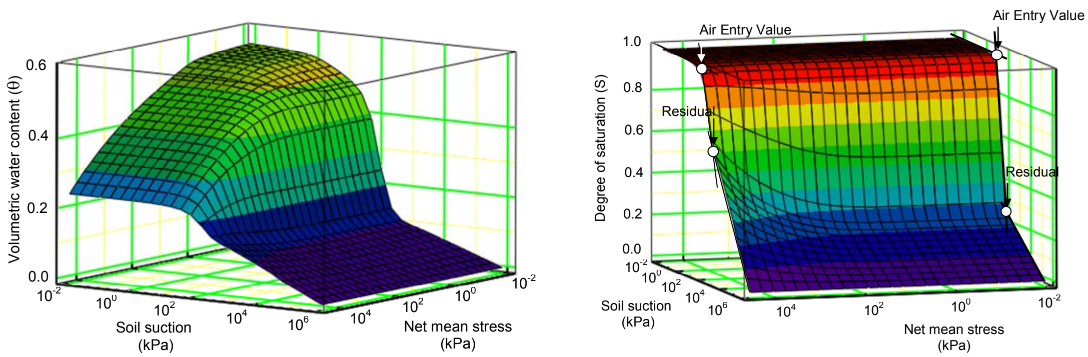
Figure 2. Initial drying SWCC curve and predicted shrinkage curve for the artificial silt.



a) Gravimetric water content (low stresses) b) Void ratio (low stresses)



c) Gravimetric water content (wide ranges) d) Void ratio (wide ranges)



e) Volumetric water content f) Degree of saturation

Figure 3. Predicted volume-mass constitutive surfaces for the artificial silt.

CONCLUSIONS

The proposed volume-mass constitutive model is capable of: 1.) Predicting both volume and water content; 2.) Take into account hysteretic nature of the SWCC; and 3.) Predicting both swelling and collapsible behaviors of an unsaturated soil. The model can predict volume-mass constitutive relationships that are stress path dependent. The prediction results appear to be reasonable.

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