Why Did Unsaturated Soil Mechanics Not Emergence Along With Saturated Soil Mechanics?

Professor Emeritus Delwyn G. Fredlund
University of Saskatchewan
Saskatoon, SK, Canada

The Tien H. Wu Lecture
Ohio State University
Columbus, Ohio, U.S.A.

May 12, 2006
Dr. Tien H. Wu

Education & Degrees
B.S. St. John's University, 1947, Shanghai, China
M.S. University of Illinois, Champaign, IL, 1948
Ph.D. University of Illinois, Champaign, IL, 1951

Research and Teaching Interests
Strength properties of soil and rock
Stability of embankments and natural slopes
Groundwater and seepage
Soil-reinforcement and soil-bio engineering
Risk and reliability assessment for foundations and slopes
Lifelong Dedication to Research and Teaching of Soil Mechanics


Tien H. Wu Lecture
3-D representation of the air-water interface (Contractile Skin)

Tortuosity
Why Did Unsaturated Soil Mechanics Not Emergence Along With Saturated Soil Mechanics?

The Tien H. Wu Lecture
Ohio State University
Columbus, Ohio, U.S.A.

May 12, 2006
Why Did Unsaturated Soil Mechanics Not Emergence Along With Saturated Soil Mechanics?

- Challenges (or Hindrances) to the development of Unsaturated Soil Mechanics
- Description of the Stress State
- Fundamental Constitutive Relations
- Role of the Soil-Water Characteristic Curve
- Use of SWCC in the Constitutive Relations
- Solution of a Series of PDEs
- Modeling Unsaturated Soils Problems
Objective

• To describe the **Challenges Faced** and the **Solutions Generated** in moving towards the **Implementation of Unsaturated Soil Mechanics**

**QUESTIONS TO KEEP IN MIND:**

1.) Was it necessary for **Unsaturated Soil Mechanics** to lag behind **Saturated Soil Mechanics**?

2.) Will **Unsaturated Soil Mechanics** ever become routine in geotechnical engineering practice?
Gradual Emergence of Unsaturated Soil Mechanics

- **1950s:** Independent measurement of pore-air and pore-water pressure through use of high air entry ceramic disks
- **1960s:** Laboratory testing of unsaturated soils
- **1970s:** Constitutive relations proposed and tested for uniqueness for unsaturated soils
- **1980s:** Solving formulations for classic Boundary Value Problems
- **1990s:** Establishing procedures for determination of unsaturated soil property functions
- **2000+:** Implementation into routine engineering practice
Challenges to the Implementation of Unsaturated Soil Mechanics

• Challenge #1:
  – To discover appropriate Stress State Variables for describing the physical behavior of unsaturated soils

• Solution #1:

How have we done?
Challenges to the Implementation of Unsaturated Soil Mechanics

• **Challenge #2:**
  To develop devices that could measure a wide range of negative pore-water pressures (i.e., high matric suctions)

• **Solution #2:**

*How have we done?*
Challenges to the Implementation of Unsaturated Soil Mechanics

- **Challenge #3:**
  - To develop (and test for uniqueness) constitutive relations suitable for describing unsaturated soil behavior

- **Solution #3:**

**How have we done?**
Challenges to the Implementation of Unsaturated Soil Mechanics

• **Challenge #4:**
  - To overcome the **excessive costs** associated with the determination (i.e., **measurement**) of unsaturated soil properties (i.e., nonlinear functions)

• **Solution #4:**

*How have we done?*
Challenges to the Implementation of Unsaturated Soil Mechanics

• **Challenge #5:**
  - To solve nonlinear partial differential equations for unsaturated soils without having convergence difficulties during the iterative solution process

• **Solution #5:**

  How have we done?
Challenges to the Implementation of Unsaturated Soil Mechanics

• **Challenge #6:**
  - To *promote and teach* unsaturated soil mechanics at universities and in engineering practice

• **Solution #6:**

*How have we done?*
Local vertical zones of unsaturated soils

Regional distribution of unsaturated soils

- Groundwater table
  - Water filling the voids
  - Air in a dissolved state

- DRY ZONE
  - Discontinuous water phase
  - Air filling most voids

- TWO PHASE ZONE
  - Continuous water
  - Continuous air

- CAPILLARY ZONE
  - Water filling most voids
  - Air phase discontinuous

- Saturated soil

Tien H. Wu Lecture
Zones of Desaturation Defined by a Soil-Water Characteristic Curve, SWCC

Gravimetric water content, $w$ (%) vs. Soil suction (kPa)

- **Air entry value**
- **Boundary effect zone**
- **Transition zone**
- **Inflection point**
- **Residual zone**
- **Residual condition**
Challenges to the Implementation of Unsaturated Soil Mechanics

• **Challenge #1:**
  - To discover appropriate Stress State Variables for describing the physical behavior of unsaturated soils

• **Solution #1:**
  - Designation of independent Stress State Variables based on multiphase continuum mechanics principles

![Diagram of stress state variables](image)

(u_a - u_w) (σ_z - u_a)

τ_{xz} τ_{xy} τ_{zx} τ_{yz} τ_{yx} (u_a - u_w) (σ_a - u_a)
Definition of stress state at a point in an unsaturated soil

- Defines the stress state at a point in a continuum
- State variables are independent of soil properties

Derivation of the Stress State is based on the superposition of equilibrium stress fields for a multiphase continuum
State Variable Stage (Unsaturated Soils)

- Stress Tensors form the basis for a Science because we live in a 3-D Cartesian coordinate world

**Net Total Stress Tensor**

\[
\begin{bmatrix}
(c_x - u_a) & \tau_{yx} & \tau_{zx} \\
\tau_{xy} & (\sigma_y - u_a) & \tau_{zy} \\
\tau_{xz} & \tau_{yz} & (\sigma_z - u_a)
\end{bmatrix}
\]

**Matric Suction Stress Tensor**

\[
\begin{bmatrix}
(u_a - u_w) & 0 & 0 \\
0 & (u_a - u_w) & 0 \\
0 & 0 & (u_a - u_w)
\end{bmatrix}
\]
Variations in Stress State Description

\[ \sigma' = (\sigma - u_a) + \chi (u_a - u_w) \]
\[ \sigma' = \text{effective stress} \]
\[ \chi = \text{parameter related to saturation} \]

\[ \sigma_{ij}^* = \sigma_{ij} - [S u_w + (1 - S) u_a] \delta_{ij} \]

\[ \sigma_{ij} = \text{total stress tensor}, \]
\[ \delta_{ij} = \text{Kronecker delta or substitution tensor}, \]
\[ \sigma_{ij}^* = \text{Bishop’s soil skeleton stress (Jommi 2000)} \]

Above proposed equations are constitutive relations.
• **Challenge #2:**
  - To develop *devices* that can measure a wide range of negative pore-water pressures (i.e., *high matric suctions*)

• **Solution #2:**
  - New instrumentation such as the *high suction tensiometers* and indirect *thermal conductivity suction sensors* provide viable techniques for the laboratory and the field
Monitoring for Verification Purposes

- Measurement of matric suction:
  - Direct measurement techniques
    - Low range tensiometers (< 90 kPa)
    - High range direct tensiometers (< 1200 kPa)
      - Presently primarily for laboratory use
  - Indirect measurement techniques
    - Thermal conductivity sensors
- Measurements of water content: TDR Technology
- Measurement of movement: same as for saturated soils
Monitoring of Water Content

Measures the dielectric constant for the soil around the rods. Dielectric constant varies with the water content of the soil.

TDR ThetaProbe, ML2x manufactured by AT Delta Devices, U.K.
Monitoring of Matric Suction

*Measures the thermal conductivity of a standard ceramic that varies in water content with the applied matric suction*

**Wetting**

\[
\begin{align*}
a &= 8.546 \\
b &= 20.110 \\
c &= 12.672 \\
d &= 1.530
\end{align*}
\]
In Situ Matric Suction measurements using Thermal Conductivity sensors at 1.0 to 1.3 m below roadway.
Direct, high suction sensor used to measure suctions greater than one atmosphere on the side of a triaxial specimen (Meilani, 2004)

- Silicone rubber grommet
- Rubber membrane
- Latex rubber, to seal the rubber membrane and grommet
- Mini suction probe
- O-ring
- 5-bar high air-entry ceramic disk

Water in the compartment is pre-pressurized to destroy cavitation nuclei

Tien H. Wu Lecture
Challenges to the Implementation of Unsaturated Soil Mechanics

• **Challenge #3:**
  - To develop (and test for uniqueness) **constitutive relations** suitable for describing unsaturated soil behavior

• **Solution #3:**
  - Constitutive relations for **saturated soils** needed to be extended to embrace the effect of **changing degrees of saturation**
Fundamental Constitutive Relations for Unsaturated Soils

• Constitutive Behaviors in Classic Soil Mechanics:
  – Seepage
  – Shear strength
  – Volume-mass changes: Void ratio, water content changes

• Other topics in soil mechanics:
  – Heat flow (Freeze-Thaw and Evaporation)
  – Air flow
  – Contaminant transport

Each constitutive relationship requires a nonlinear soil property function; therefore, Unsaturated Soil Mechanics might be referred to as NONLINEAR SOIL MECHANICS!

Tien H. Wu Lecture
Water Seepage Constitutive Relations

\[ h = \frac{u_w}{\rho_w g} + Y \]

Driving potential for water flow is hydraulic head, \( h \)

\[ v_x = -k_{wx} \frac{dh}{dx} \]

Darcy’s law (1856) for flow in the \( x \)-, \( y \)-, and \( z \)-direction

\[ v_y = -k_{wy} \frac{dh}{dy} \]

Coefficient of permeability, \( k_w \) is a function of matric suction; therefore, the flow law is nonlinear and subject to hysteresis

\[ v_z = -k_{wz} \frac{dh}{dz} \]
Shape of the water permeability function for glass beads tested by Mualem (1976)

Coefficient of permeability (m/s)

Drying

Wetting

Soil suction (kPa)

Tien H. Wu Lecture
The SWCC for the glass beads showing hysteresis during drying and wetting.
Water Storage in an Unsaturated Soil

Soil-Water Characteristic Curve, SWCC

Also has a hysteretic effect

Water storage function is the slope of the SWCC; Required for transient seepage analyses
Air Flow Constitutive Relations

\[ v_{ax} = -k_{ax} \frac{du_a}{dx} \]

**Driving potential for air flow is Pore-air pressure,** \( u_a \)

\[ v_{ay} = -k_{ay} \frac{du_a}{dy} \]

**Fick’s law for flow in the x-, y-, and z-direction**

\[ v_{az} = -k_{az} \frac{du_a}{dz} \]

**Coefficient of permeability,** \( k_a \) **is a function of matric suction; therefore, the flow law is nonlinear and subject to hysteresis**

**Observation:** Soil properties for unsaturated soils become **nonlinear functions** and are **hysteretic** in character.

Tien H. Wu Lecture
Shear Strength Constitutive Relations

\[ \tau = c' + (\sigma_n - u_a) \tan \phi' + (u_a - u_w) \tan \phi^b \]

\text{Linear form of the extended Mohr-Coulomb shear strength equation}

Fredlund, Morgenstern and Widger, 1978

\[ \tau = c' + (\sigma_n - u_a) \tan \phi' + (u_a - u_w) f_1 \]

\( f_1 = \text{function showing the rate of increase in shear strength with matric suction} \)
Extended Mohr-Coulomb failure surface (Fredlund, Morgenstern and Widger, 1978)

Shear strength versus suction is nonlinear and affected by hysteresis

Net normal stress, \((\sigma - u_a)\)

Shear strength, \(\tau\)

Air entry value

\(\phi'\)

\((u_a - u_w)\)
Multistage direct shear test results on compacted glacial till (Gan et al., 1988)

Soil-Water Characteristic Curve for glacial till

Shear strength, \( \tau \) (kPa)

\[
(\sigma_f - u_a)_f = 72.6 \text{ kPa}
\]

\[
(\sigma_f - u_a)_f \tan \phi' = 34.6 \text{ kPa}
\]

\[
c' = 10 \text{ kPa}
\]

Matric suction, \((u_a - u_w)\) (kPa)

Optimum Initial Water Content Specimen

AEV = 60 kPa

0 kPa, \( e = 0.517 \)

25 kPa, \( e = 0.514 \)

Tien H. Wu Lecture
Reference compression curves for a Saturated Soil

\[ \nu = (1 + e) \]

Elastic-plastic form

\[ \lambda = \frac{C_c}{\ln(\nu)} \approx \kappa \approx C_c \]

\[ \kappa = \frac{C_s}{\ln(\nu)} \approx \lambda \approx C_s \]

Yield stress

Preconsolidation pressure

Effective mean stress

Effective vertical stress

Tien H. Wu Lecture
Limiting or Bounding relationships for $K_o$ loading of a typical clayey silt soil

Volume-mass constitutive relations for an unsaturated soil take the form of two, 3-dimensional surfaces with distinct breaks that identify changes in soil behavior.

New points:
- Air entry value
- Residual point
Volume–Mass Constitutive Surfaces for Regina Clay – Preconsolidated at 200 kPa (Pham, 2004)

Basic volume-mass equation

\[ S \ e = w \ G_s \]

Void ratio, \( e \)

Water content, \( w \)

Degree of saturation, \( S \)

Tien H. Wu Lecture
Volume–Mass Constitutive Surfaces for Regina Clay Preconsolidated at 200 kPa (Pham, 2004)

Water content, w

SWCC

Yield

Residual value

Air entry value

Soil suction

Net total stress

Log net mean stress (kPa)

Log soil suction (kPa)

Gravimetric water content

Tien H. Wu Lecture
Volume–Mass Constitutive Surfaces for Beaver Creek Sand (Pham, 2004)

Basic volume-mass equation

$$S e = w G_s$$

Void ratio, e

Degree of saturation, S

Water content, w

Gravimetric water content

AEV

SWCC

Basic volume-mass equation

$$S e = w G_s$$
Challenges to the Implementation of Unsaturated Soil Mechanics

• **Challenge #4:**
  - To overcome the **excessive costs** associated with the determination (i.e., measurement) of unsaturated soil properties (i.e., nonlinear functions)

• **Solution #4:**
  - Indirect, estimation procedures have been developed to obtain unsaturated soil property functions based on **Soil-Water Characteristic Curves**

![Graph showing predicted vs. experimental water content versus suction (kPa)]
Role and Measurement of the Soil-Water Characteristic Curve, SWCC

- Soil-Water Characteristic Curves, SWCC define the relationship between the amount of water in a soil and soil suction (i.e., matric suction and total suction)

- SWCC has been successfully used to estimate all unsaturated soil property functions

- Water permeability
- Air permeability
- Shear strength
- Thermal flow
- Incremental elasticity

• ASTM Standard
• D6836-02 (2003)
Measured drying and wetting curves on processed silt (Pham, 2002)

- AEV = 10 kPa
- WEV = 4.5 kPa
- Residual = 120 kPa
- Residual = 62 kPa

Soil suction (kPa)
Gravimetric water content (%)
Pressure Plate Apparatus to Measure Void Ratio and Water Content While Applying Total Stress and Matric Suction

Manufactured by GCTS, Tempe, AZ

Front View

Side View

Air supply

High air entry disk

Dual Burettes for flushing diffused air

Tien H. Wu Lecture
Equations to Best-Fit SWCC Data

Numerous equations have been proposed:
- Brooks & Corey (1964)
- van Genuchten (1980)

Fredlund and Xing (1994)

\[ w(\psi) = C(\psi) \times \frac{W_s}{\{\ln(\psi + \frac{\psi}{\psi_e})^n f\}^m f} \]

Gravimetric water content

Soil suction (kPa)

Air entry value

Rate of desaturation

Asymmetry Variable

Correction Factor

Fredlund and Xing (1994)

Tien H. Wu Lecture
Hysteresis in the Soil-Water Characteristic Curve

- **Hysteretic SWCC Models** will eventually be available for geotechnical usage
- *Presently, the Geotechnical Engineer must decide which curve to use:*
  - Select *wetting curve* or *drying curve* based on process being simulated
- **Hysteresis loop shift at point of inflection:**
  - **Sands:** 0.15 to 0.35 Log cycle
    - *Average:* 0.25 Log cycle
  - **Loam soils:** 0.35 to 0.60 Log cycle
    - *Average:* 0.50 Log cycle

Estimation Values
Model measurements of water content and matric suction showing the SWCC relationship from water contents and matric suctions during wetting and drying simulations (Tami et al, 2004)
Approaches that can be used to obtain the Soil–Water Characteristic curves

Determination of Soil-Water Characteristic Curves, SWCC

Laboratory measurement of water content versus suction

- Pressure plate < 1500 kPa
- Vacuum desiccators > 1500 kPa

SWCC predictions from grain size distribution

- Numerous models

SWCC predictions from grain size & Atterberg limits

- Parameters for numerous models

Dataset “mining” for typical SWCC

- Soils with similar grain size or soil classification

Varying accuracy depending on the methodology used

Tien H. Wu Lecture
Soil-Water Characteristic Curve computed from a Grain Size Distribution Curve

Fredlund et al, 1997

Tien H. Wu Lecture
Incorporation of SWCC into the Constitutive Relations for Unsaturated Soils

- **Unsaturated soil property functions** rely on the **saturated soil properties** PLUS the **soil-water characteristic curve, SWCC**

- **SWCC** (i.e., Air entry value and Residual value) can be incorporated through:
  - Direct insertion of the **SWCC equation**
  - Various forms of **Integration** along the SWCC
  - **Differentiation** of the SWCC

- **Unsaturated soil property functions** render the solution of a problem **nonlinear**

Tien H. Wu Lecture
### Seepage Constitutive Relations in Terms of SWCC

<table>
<thead>
<tr>
<th>Permeability Models</th>
<th>References for the Soil-Water Characteristic Curve</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_r = k_w / k_{sat} )</td>
<td><strong>van Genuchten (1980)</strong></td>
</tr>
<tr>
<td><strong>Burdine (1953)</strong></td>
<td>( k_r(\psi) = \frac{{1 - (\alpha \psi)^{n-r}}[1 + (\alpha \psi)^n]^{-m}}{[1 + (\alpha \psi)^n]^{-n}} )</td>
</tr>
<tr>
<td>( m = 1 - \frac{\gamma}{n} )</td>
<td>( k_r(\psi) = \frac{{1 - (\alpha \psi)^{n-r}}[1 + (\alpha \psi)^n]^{-m}}{[1 + (\alpha \psi)^n]^{-n}} )</td>
</tr>
<tr>
<td><strong>Mualem (1976)</strong></td>
<td>( k_r(\psi) = \frac{{1 - (\alpha \psi)^{n-r}}[1 + (\alpha \psi)^n]^{-m}}{[1 + (\alpha \psi)^n]^{-n}} )</td>
</tr>
<tr>
<td>( m = 1 - \frac{\gamma}{n} )</td>
<td>( k_r(\psi) = (\alpha \psi)^{-\gamma - \gamma \lambda} )</td>
</tr>
</tbody>
</table>

Tien H. Wu Lecture
Seepage Constitutive Relations in Terms of SWCC

<table>
<thead>
<tr>
<th>Permeability Models</th>
<th>References for the Soil-Water Characteristic Curve</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_r = k_w/k_{sat}$</td>
<td>Fredlund, Xing &amp; Huang (1994)</td>
</tr>
</tbody>
</table>

Fredlund, Xing & Huang (1994):

$$k_r = \frac{\int_{\ln(\psi_{aev})}^{b} \frac{\theta(e^y) - \theta(\psi)\theta'(e^y)}{e^y} dy}{\int_{\ln(\psi_{aev})}^{b} \frac{\theta(e^y) - \theta_s(e^y)}{e^y} dy}$$

Campbell (1974):

$$k_r = \left(\frac{\psi}{\psi_{aev}}\right)^{-\frac{\gamma - \frac{\theta}{b}}{b}}$$

$b = \ln(1000000)$

$\theta(\psi) =$ Soil water content

$y =$ Dummy variable of integration representing the logarithm of integration

Tien H. Wu Lecture
Usage of several functions to predict permeability functions from the SWCC for a particular soil and a suggested lower limit for the permeability function.

Experimental data

Overall $K_w + K_v$

Van Genuchten - Mualem

Fredlund, Xing & Huang

Brooks and Corey

van Genuchten - Burdine

Vapor $K_v$

Soil suction (kPa)

Tien H. Wu Lecture
Shear strength Constitutive Equation Written in Terms of SWCC

Vanapalli et al. (1996)

\[ \tau = c' + (\sigma_n - u_a) \tan \phi' + (u_a - u_w) \Theta_n \tan \phi' \]

**SWCC**

\[ \Theta_n = \frac{\theta - \theta_r}{\theta_s - \theta_r} \]

**Shear strength**

**Effective cohesion**

**Net normal stress on the failure plane**

**Angle of internal friction**

**Matric suction**

Tien H. Wu Lecture
Challenges to the Implementation of Unsaturated Soil Mechanics

• **Challenge #5:**
  – *To solve nonlinear partial differential equations for unsaturated soils without having convergence difficulties during the iterative solution process*

• **Solution #5:**
  – *Adaptive mesh (grid) generation techniques in computer technology facilitates convergence*
Problem Solving Environments, PSEs, for Soil Mechanics Partial Differential Equations, PDEs

- All classic areas of soil mechanics can be viewed in terms of the solution of a Partial Differential Equation
- Water flow through porous soils (Saturated or Unsaturated)
- Air flow through unsaturated soils
- Stress analysis for slope stability, bearing capacity and earth pressure
- Stress-Deformation volume change and distortion
  - Incremental elasticity
  - Elasto-plastic models
Solving a Boundary Value Problem

Element for which a Partial Differential Equation, PDE, must be derived

Boundary Value Must be Supplied

Boundary

Must also define initial conditions

Boundary

Boundary

Typical Boundary Conditions

Flux    Head      - for seepage
Force   Displacement - for stress

Utilize general purpose PDE Solvers to solve partial differential equations for saturated-unsaturated soil system
Partial Differential Equation for Saturated-Unsaturated Water Flow Analysis

\[ k_x^w \left( \frac{\partial}{\partial x} \right)^2 h + k_y^w \left( \frac{\partial}{\partial y} \right)^2 h + k_x^w \frac{\partial}{\partial x} h + k_y^w \frac{\partial}{\partial y} h = -m_y^w \gamma_w \frac{\partial}{\partial t} h \]

- **Head variable to be solved**
- **Water coefficient of permeability** (function of soil suction)
- **Water storage** (function of soil suction)
- **Time**

Tien H. Wu Lecture
Partial Differential Equation for Unsaturated Air Flow Analysis

\[ k_a \frac{\partial u_a}{\partial x} + k_a \frac{\partial u_a}{\partial y} + \frac{\partial k_a}{\partial x} \left( \frac{\partial u_a}{\partial x} \right) + \frac{\partial k_a}{\partial y} \left( \frac{\partial u_a}{\partial y} \right) = -\left( \frac{e}{1 + e} S_a - u_a m^w \right) \frac{\omega_a g}{RT} \frac{\partial u_a}{\partial t} \]

- **Pore-air pressure** (primary variable to be solved)
- **Air coefficient of permeability** (function of soil suction)
- **Air storage and compressibility** (function of soil suction)
- **Time**
Partial Differential Equation for Saturated-Unsaturated Stress-Deformation Analysis

\[
\frac{\partial}{\partial x} \left[ \begin{array}{c} D_{\xi\xi} \frac{\partial u}{\partial x} + D_{\eta\eta} \frac{\partial v}{\partial y} \\ \frac{\partial}{\partial y} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \end{array} \right] + \frac{\partial}{\partial y} \left[ \begin{array}{c} D_{\xi\xi} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \\ \frac{\partial u}{\partial x} + D_{\eta\eta} \frac{\partial v}{\partial y} \end{array} \right] = \gamma_t.
\]

\[D_{11}, D_{12}, D_{44} = \text{Combination of } E \text{ and } \mu \text{ which are function of soil suction and net total stresses}\]

Stress-deformation analyses have a degrees of freedom in each of the Cartesian coordinate directions

Tien H. Wu Lecture
Convergence of Nonlinear Partial Differential Equations

- **Convergence** is the single most pressing problem facing modelers
- Most successful solutions have involved Adaptive Grid Refinement methods, AGR (Oden, 1989; Yeh, 2000)
- **Mesh is dynamically upgraded** during the solution based on error estimates
- **AGR becomes extremely important** when solving the nonlinear PDEs associated with Unsaturated Soil Mechanics
Two-dimensional seepage analysis through an earthfill dam with a clay core.

Optimized mesh for saturated-unsaturated seepage analysis

Thieu and Fredlund, 1998

Equipotential lines
Problem illustrating the solution of a 3-dimensional, saturated-unsaturated seepage PDE

Optimized, automatically generated finite element mesh

Modeling of a waste tailings pond

Tien H. Wu Lecture
Stress analysis PDE combined with the **Dynamic Programming** procedure to compute the factor of safety.

**Shape and location of the slip surface are a part of the solution.**

Elevation

Distance

Finite ElementS hear Stress

DP Generated Critical Slip Surface

FOS= 1.3

Tien H. Wu Lecture
Prediction of **Heave or Collapse** of a Soil

- Requires the solution of a saturated-unsaturated seepage model and a stress-deformation model

Must consider effects of:

- Nonlinearity
- Coupling

**Saturated-Unsaturated Seepage Model**

Computes changes in matric suction

**Saturated-Unsaturated Stress-Deformation Model**

Computes deformations
Consider Edge Lift for a Flexible Impervious Cover

Boundary conditions and initial conditions must be specified both seepage and stress-deformation

Flexible cover

Depth, m

Distance from centre of cover or slab, m

Flux = 0

Constant suction = 400 kPa

Infiltration, q

Flux = 0
Can have one optimized Adaptive Mesh generated for seepage model and another for the stress-deformation model.
Matric Suction at Ground Surface after One Day of Infiltration for Various Infiltration Rates

Distance from centre of cover, m

Distance under slab

Matric suction, kPa

Initial

Specified zero suction

Distance under slab

Tien H. Wu Lecture
Vertical Displacements at Ground Surface
after One Day of Infiltration

Distance under slab

specified zero suction

Distance from centre of cover, m

Amount of edge heave

Heave, (mm)

0 2 4 6 8 10 12 14 16 18 20 22 24 26

Distance under slab

q = 60 mm/day

q = 50

q = 40

q = 30

q = 20

q = 10

Tien H. Wu Lecture
Challenges to the Implementation of Unsaturated Soil Mechanics

• Challenge #6:
  – *To promote implementation of unsaturated soil mechanics into engineering practice*

• Solution #6:

  - Educational materials and visualization tools have been produced to better teach and understand unsaturated soil mechanics
Concluding Remarks

• **Unsaturated Soil Mechanics** needs to be first understood from the standpoint of the Constitutive equations describing soil behavior.

• **Constitutive Equations** can be written in terms of the SWCC and are referred to as **Unsaturated Soil Property Functions, USPF**.

• **Direct** and **Indirect** procedures are available for the assessment of the SWCC.

• It is **always** possible to obtain an estimate of the required **Unsaturated Soil Property Functions** for geotechnical engineering applications.
There are many soil mechanics researchers who deserve credit not only for researching saturated soil behavior but also for improving our understanding of unsaturated soil behavior.

Columbus, Ohio
May 12, 2006

Thank You

Tien H. Wu Lecture