Proceedings of the Fourth International Conference on Unsaturated Soils, ASCE, Carefree, Arizona, April 2-6, Vol. 2, pp. 2121-2132. 2006

Numerical Modeling of Slab-On-Grade Foundations

M. D. Fredlund[1], J. R. Stianson[2], D. G. Fredlund[3], H. Vu[4], and R. C. Thode[5]

[1]SoilVision Systems Ltd., 2109 McKinnon Ave S., Saskatoon, SK S7J 1N3; PH (306) 477-3324; FAX (306) 955-4575; email: murray@soilvision.com
[2]University of Alberta, Department of Civil and Environmental Engineering, Edmonton Alberta, T6G 2W2
[4]Clifton Associates Ltd., 340 Maxwell Crecent, Regina, SK S4N 5Y5
[5]SoilVision Systems Ltd., 2109 McKinnon Ave S., Saskatoon, SK S7J 1N3

Abstract

Residential foundations such as slabs-on-ground are often constructed on unsaturated, expansive soils and have to resist deformation associated with external loads and changes in matric suction in the soil. Residential foundations are lightly loaded, and therefore the displacements are mostly due to changes in matric suction. Changes in matric suction can occur as a result of variation in climatic conditions, change in depth of water table, water uptake by vegetation, removal of vegetation or the excessive watering of a lawn.

The theoretical context for the analysis of slabs-on-ground placed on expansive soils involves the numerical modeling of moisture flow through the soil and the associated deformations associated with changes in the stress state. The theory associated with moisture movement and volume change has been formulated and widely accepted as part of unsaturated soil mechanics.

This paper presents the use of the finite element method to address this complex problem. Various aspects of the separate water flow and deformation aspects will be presented. The combination of stress analysis and seepage analysis is presented. An analysis of the final estimated deformations provides insight into this complex process.

1. Introduction

Residential foundations such as slabs-on-ground are often constructed on unsaturated, expansive soils and have to resist deformation associated with external loads and changes in matric suction in the soil. Residential foundations are lightly loaded, and therefore the displacements are mostly due to changes in matric suction. Changes in matric suction can occur as a result of variation in climatic conditions, change in depth of water table, water uptake by vegetation, removal of vegetation or the excessive watering of a lawn.

The soil beneath an impervious cover can deform into either an Edge Drop mode or an Edge Lift mode (Figure 1). Edge Drop occurs when the matric suction in the soil surrounding the slab gradually increases due to evaporation and/or transpiration. Soil volume decreases more around the perimeter of the slab than under the interior of the slab, because the soil around the slab becomes dryer than the soil beneath the cover. In reverse, Edge Lift occurs when the matric suction in the soil decreases due to the infiltration of water in to the soil mass around the slab.

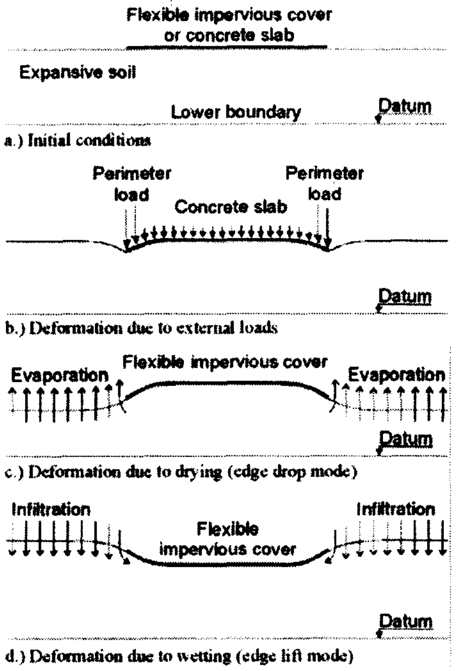

Figure 1 Illustration of the soil response to external loads and changes in matric suctions (after Post-tensioning Institute, 1996)

This report presents a methodology that can be used to simulate the Edge Lift (i.e., swelling) and Edge Drop (i.e., shrinkage) problems around an impervious concrete cover (or a uniform thickness slab) placed on an unsaturated, expansive soil. The focus of the report is to investigate the unique soil slab interaction that takes place in the Edge Drop condition; namely, the separation of the soil from the slab. A two-dimensional example problem involving a flexible and stiff impervious cover (or a slab-on-ground) is used as the primary example to illustrate the suggested methodology for predicting the expected separation between the soil and the slab. Two scenarios are analyzed. The first scenario compares the predicted vertical displacements along the ground surface for the Edge Drop case using a flexible cover to the response predicted using a concrete slab with various levels of stiffness. The second scenario presents the methodology developed to predict the separation distance between the soil and the stiff concrete slab for the Edge Drop condition. Information is also provided regarding the interpretation of the modeling results to obtain the parameters necessary to design a slab-on-grade foundation.

2. Solving the Saturated/Unsaturated Seepage Analysis and the Stress Deformation Analysis Required to Model the Edge Drop and Edge Lift Scenarios

An "uncoupled" approach is adopted to solve the required seepage and stress deformation analyses. In the "uncoupled" approach, the water phase continuity (i.e., seepage) equation is solved separately from the equilibrium (i.e., stress-deformation) equations. The interdependence of the equations is accommodated in an iterative manner where the flow portion of the formulation is solved for a given time period and the resultant pore-water pressure changes are used as input in a stress/deformation analysis. In turn, volume changes and induced stresses from the deformation analysis are used in the computation of the soil properties for the next time period in the seepage analysis. The "uncoupled" solutions are obtained through the use of the general partial differential equation solver in SVFLUX (SoilVision, 2001[a]) and SVSOLID (SoilVision, 2002[b]).

2.1 Governing Partial Differential Equation for Seepage in Unsaturated Swelling Soil

Volume change in an unsaturated expansive soil is primarily related to a change in pore-water pressures. The seepage analysis becomes central to determining the overall change in volume. There are two primary soil properties required for a transient unsaturated seepage analysis:

1) Soil-water characteristic curve
2) Hydraulic conductivity relationship (sometimes referred to as the permeability function)

Slab-on-ground problems are typically transient, unsaturated soil problems. A fully defined soil-water characteristic curve (SWCC) and an unsaturated hydraulic conductivity curve are therefore required in a typical analysis. These functions are required by the governing seepage, partial differential equation, shown in equation [1] (Fredlund and Rahardjo 1993).

[1] $$\frac{\partial}{\partial x}\left(kx(\psi)\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(ky(\psi)\frac{\partial h}{\partial y}\right)=m^2{}_w\gamma_w\frac{\partial h}{\partial t}$$

where:

h = total head,
$k_x(\psi)$ = hydraulic conductivity of the soil in the x direction,
$k_y(\psi)$ = hydraulic conductivity of the soil in the y direction,
γ_w = the unit weight of water (9.81 kN/m^3),
$m^2{}_w$ = the slope of the soil-water characteristic curve. This coefficient is a function of soil suction.

Two things can be noted from the above governing partial differential equation. Firstly, hydraulic conductivity is a function of soil suction. Secondly the $m^2{}_w$ coefficient is the slope of the soil-water characteristic curve and is also a function of soil suction. These two functions are the most critical in determining the solution of the partial differential equation.

2.2 Governing Partial Differential Equation for Soil Structure Equilibrium

The two-dimensional plane strain equation governing overall static equilibrium for an unsaturated soil can be written as follows (Fredlund and Vu, 2003):

$$\frac{\partial}{\partial x}\left(c_{11}\frac{\partial u}{\partial x}+c_{12}\frac{\partial v}{\partial y}\right)+c_{33}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)-d_s\frac{\partial(u_a-u_w)}{\partial x}+b_x=0 \tag*{[2]}$$

$$c_{33}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)+\frac{\partial}{\partial y}\left(c_{12}\frac{\partial u}{\partial x}+c_{22}\frac{\partial v}{\partial y}\right)-d_s\frac{\partial(u_a-u_w)}{\partial y}+b_y=0 \tag*{[3]}$$

where:

$$c_{11}=c_{22}=\frac{(1-\mu)E}{(1+\mu)(1-2\mu)},\ c_{12}=\frac{\mu E}{(1+\mu)(1-2\mu)},\ c_{33}=\frac{E}{2(1+\mu)},\ d_s=\frac{E}{(1-2\mu)H},$$

E = elasticity parameter for the soil structure with respect to a change in the net normal stress,
H = elasticity parameter for the soil structure with respect to a change in matric suction, and
μ = Poisson's Ratio.

Equations [2] and [3] can be used to compute the displacements in horizontal and vertical directions under an applied load and/or due to changes in matric suction.

The solution of the stress/deformation equations under specified boundary conditions requires the definition of the initial matric suction, initial stress conditions, the elasticity parameter functions associated with the volume change of the soil, and the results from a seepage analysis (i.e., for changes in soil suction).

3. Modeling Scenarios to Illustrate the Soil Slab Interaction under Edge Drop Conditions

The following problem presents the results showing the shrinkage of the soil around the edge of a flexible cover as a result of evaporation from the surrounding soil surface. The first step involves creation of the seepage problem in the SVFLUX software. The analysis performed in SVFLUX will yield the initial and final pore-water pressure distributions. These distributions will then be used as input into the SVSOLID stress/deformation model.

The geometry consists of a single soil region 12m wide x 3m deep as shown in Figure 2 (Fredlund and Vu, 2003). The flexible or concrete cover extends 6m from the left boundary and symmetry is assumed with respect to the central axis. The seepage boundary conditions used to establish the initial suction profile included a constant suction of 400 kPa at the bottom boundary, no flow permitted at the edge boundaries, and a constant suction of 20 kPa applied to the uncovered ground surface. Therefore, a linear suction profile was established. For the transient analysis an evaporation rate equal to 10mm/day is applied to the uncovered top boundary surrounding the flexible slab. The soil properties used in the seepage analysis are shown in Table 1.

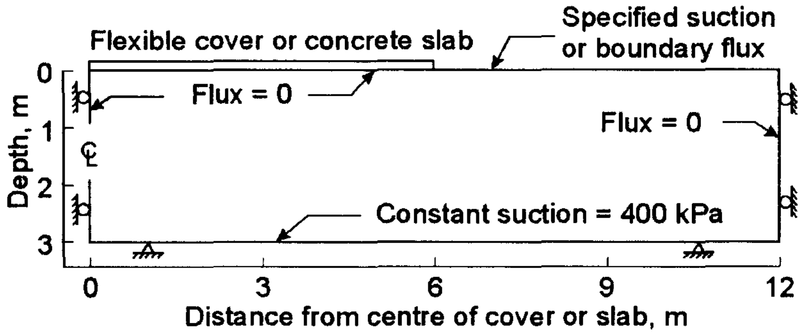

Figure 2 Geometry and boundary conditions for the Edge Drop scenario

Table 1 Assumed soil properties for the 2D Edge Drop seepage analysis.

Soil Properties	Values
Coefficient of permeability at saturation, k_s	$1x10^{-8}$ m/s
Volumetric water content at saturation, θ_s	0.45
Parameters for SWCC (Fredlund & Xing, 1994) and permeability function (Leong and Rahardjo, 1997)	a = 300 kPa n = 1.5 m = 1 p = 1

The stress deformation boundary conditions include a roller condition applied to the left and right boundaries, a pinned condition applied to the bottom boundary, and the top boundary (ground surface) is free to move. The initial vertical stress state in the soil is determined from the total unit weight multiplied by the elevation (the depth below the ground surface datum). The horizontal stress is determined from the vertical stress by applying the coefficient of earth pressure at-rest, K_o. The soil properties used in the stress deformation analysis are shown in Table 2

Table 2 Assumed soil properties for the Edge Drop stress analysis.

Soil Properties	Edge Drop Values
Total Unit Weight, γ_t	17.2 kN/m^2
Initial Void Ratio, e_o	1
Swelling Index, C_s	0.15
Swelling Index, C_m	0.13
Poisson's Ratio, μ_s	0.4
Coefficient of earth pressure at rest, K_o	0.33

3.1 Scenario #1: Comparing ground displacements for the edge drop example using a flexible or stiff concrete slab

The movement of the soil under the flexible slab is reported after 1, 3, and 5 days of evaporation has taken place. The effect of the slab stiffness is investigated by comparing the differential vertical deflection, □ (Figure 3), calculated using a flexible slab to the results calculated using a stiff slab with elastic modulus equal to 20 MPa, 10 GPa, and 100 GPa. The soil is forced to remain in contact with the slab.

Figure 3 presents a comparison of the vertical displacement along the ground surface considering a flexible slab or a concrete slab with an elastic modulus equal to 100 GPa. The vertical displacements along the ground surface were used to compute the differential deflection at the base of the concrete slab as shown in Figure 3.

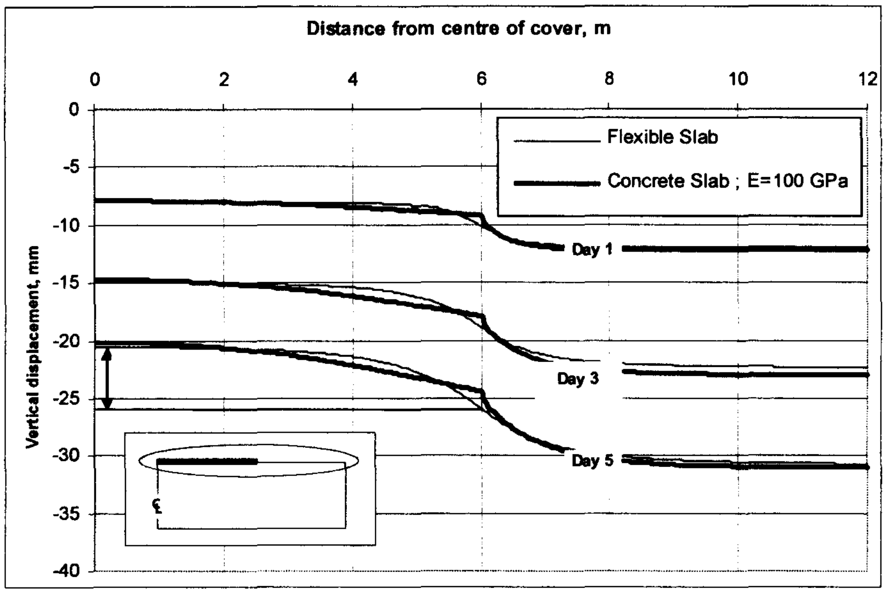

Figure 3 Comparison of the vertical displacement at the ground surface for the edge drop case considering a flexible slab and a stiff concrete slab with an elastic modulus of 100GPa.

The calculated differential deflections are presented in Table 3. A differential deflection of 5.33 mm is predicted after five days of evaporation when a flexible cover is considered. Including a concrete slab with a modulus of 20 MPa decreases the expected differential deflection by less than 1 mm to 4.94 mm. Increasing the elastic modulus of the concrete slab to 10 GPa resulted in a small increase when compared to the 20 MPa concrete slab resulting in a differential deflection of 5.10 mm. These results indicate that while values of differential movement can be computed to the fraction of a millimeter, the reliable resolution of the results is less (i.e., probably to the nearest millimeter). Considering a 100 GPa concrete slab decreases the expected differential deflection by approximately 1 mm to 4.27 mm, when compared to the flexible slab cover.

Table 3 Values of differential deflection for the Edge Drop example considering a flexible slab cover and a concrete slab with elastic modulus equal to 20 MPa, 10 GPa, or 100 GPa.

Elastic Modulus	Day 1 Δ (mm)	Day 3 Δ (mm)	Day 5 Δ (mm)
Flexible Cover	2.09	3.87	5.33
20 MPa	1.80	3.55	4.94
10 GPa	1.50	3.48	5.10

100 GPa	1.29	3.16	4.27

The modeling procedure followed for all cases considered for the edge drop example where the concrete slab was included in the analysis did not allow for any separation between the soil and the concrete slab. In reality, the soil can separate at a particular locations under the slab corresponding to the moisture variation distance, e_m. The next scenario is included to present a trial and error methodology that was developed to investigate the effect of separation of the concrete slab from the underlying soil.

3.2 Scenario #2: Methodology used to determine the separation distance between the slab and the underlying soil for the edge drop example

The purpose of this example problem is to illustrate a procedure that was used to model the separation of the concrete slab and the underlying soil. The main components of the procedure can be described in three steps.

1) A stress analysis model (i.e., SVSOLID) was prepared to analyze the edge drop scenario using the computed initial and final pore water pressure conditions from SVFLUX. For the first stress analysis model the slab was considered to be flexible. A plot of vertical displacement versus horizontal distance along the ground surface was computed.

2) A second stress analysis model was prepared using the same initial and final pore water pressure conditions as used in the first step but the effect of the concrete slab was taken into consideration. The assumption was made that the concrete slab remained in contact with the soil at all points. In other words, the soil was not allowed to separate from the concrete slab. The vertical displacements and vertical stresses were computed along the ground surface. The vertical displacements profile was added to the plot prepared from the first step. A separate plot of the vertical stress profile along the ground surface was also prepared and the location along the bottom of the concrete slab where significant tensile stresses develop was identified.

3) A series of stress analysis models were then completed using the estimation of the soil separation distance that would correspond to a condition of no tensile stresses along the bottom of the slab. The first soil separation distance was estimated from the vertical stress profile prepared in the second step. Subsequent soil separation distances were estimated from the vertical displacements and vertical stresses computed along the ground surface as each consecutive soil separation model was solved. Three additional stress analysis trials were required to predict the actual soil separation distance for the studied Edge Drop conditions.

The results from the soil separation analyses were compared to the results from the previous case where the slab was considered to be either flexible or extremely stiff with no separation allowed between the slab and the soil.

The vertical displacements for the flexible slab scenario and the rigid slab scenario with no separation have already been presented in Figure 3. These computational runs satisfy step 1.), and the first part of step 2.), of the procedure proposed to model the separation of the soil from the concrete slab. To complete step 2.), the vertical stress profile along the ground surface for the rigid concrete slab scenario with no separation between the soil and the slab was prepared. It was observed that tension existed over a distance of approximately 0.6m from the edge of the slab. Therefore, the soil will separate from the bottom of the slab over a distance of 0.6m during the first attempt to model the slab separation. The model was modified by separating the soil from the concrete slab by leaving a 0.01 m gap between the soil and the slab as shown in Figure 4.

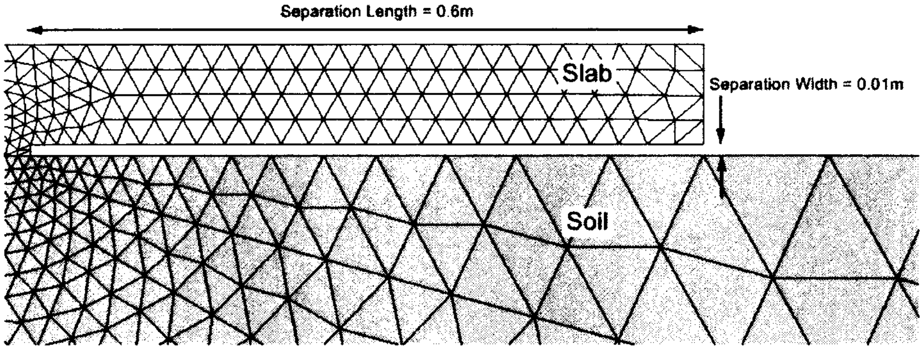

Figure 4 Illustration of the separation provided between the expansive soil and the concrete slab.

The vertical displacement profile resulting from a 0.6m separation between the soil and the concrete slab allowed the soil beneath the slab that would otherwise be in tension to compress in a manner approaching the behavior of the flexible slab scenario. There was; however, a small distance at the contact where the soil was being pulled above the displacement profile predicted using a flexible slab, similar to the rigid slab case with no gap shown in Figure 5. This means that a slightly larger gap would be required to completely eliminate the slab from applying tension to the soil.

A second separation model was run including a separation distance between the soil and the slab of 0.75m. The resulting vertical displacement profile is shown in Figure 5. Increasing the separation to 0.75m has allowed the remainder of the soil that was in tension when a gap of 0.6m was used to align with the displacement profile predicted using a flexible slab from the point of separation (5.25m) to the edge of the problem domain (12m).

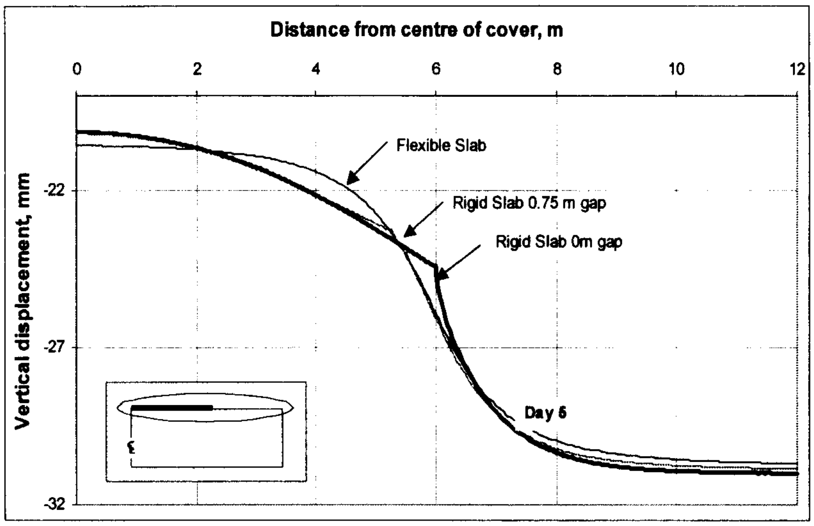

Figure 5 Comparison of the vertical displacement profiles using a flexible slab, a rigid slab with no gap, and a rigid slab with a gap of 0.75m

This example has illustrated how finite element software (SVSOLID) can be used to study the amount of soil separation that can be expected if the surrounding soil shrinks due to evaporation from the surface.

4. Determining the Bending Moments and Shear Force in the Concrete Slab

The shear and bending moments in the concrete slab can be computed using Equation [4] and Equation [5], respectively (Timoshenko and Woinowsky-Krieger, 1959).

$$V = \tau_{xy} * t \tag{4}$$

$$M = \frac{E(t)^3 \left(d^2v / dx^2 \right)}{(12)(1-\mu^2)} \tag{5}$$

The above equations have been implemented in the SVSOLID software proving the ability to plot the shear force and bending moment distributions along the length of the concrete slab. The most critical values required in the design of the concrete slab can be easily obtained from the plotted distributions.

The calculation of moment and force in the slab is based on the assumption that slab is thin and the neutral plane is in the middle of the slab. The accuracy of the calculated bending moments and shear forces will decrease with increasing thickness of the slab (i.e., considering the ratio of the slab thickness and the edge lift or edge drop distance).

5. Interpretation of Computer Analysis

There are two variables that play a primary role in the design of slabs-on-ground. The variables are the maximum anticipated values for the i.) horizontal distance of moisture movement, e_m, (also referred to as the edge penetration distance), and ii.) vertical differential soil movement, y_m. These variables are used for the design of the concrete slab-on-ground.

5.1 Edge Moisture Variation Distance, e_m

The "edge moisture variation distance", e_m, is defined as the distance measured inward from the edge of the slab of a shallow foundation where the moisture content of the soil varies due to drying and wetting influences around the perimeter. The selection of the e_m requires an assessment of the point where the change in matric suction becomes insignificant, or more correctly, the point where the volume change of the soil becomes insignificant. This point is difficult to determine because the volume change behavior is related to matric suction changes that take place on a logarithmic basis.

A tentative procedure is suggested for the assessment of the edge moisture distance using a plot of the change in matric suction versus horizontal distance below the concrete slab for scenario #1, shown in Figure 6. The change in matric suction immediately below the concrete slab is plotted on a logarithmic scale. This makes it possible to select a threshold value at which the change in matric suction can be used to estimate the distance e_m. For example, if the threshold change in matric suction were selected to be 0.01 kPa then the edge moisture distance, e_m, for an evaporation rate of 10mm/day would be equal to 1.9m as shown in Figure 6. However, it is probably more realistic to select a threshold suction change of about 1 kPa, in which case the edge moisture distance would be closer to 1.0 m.

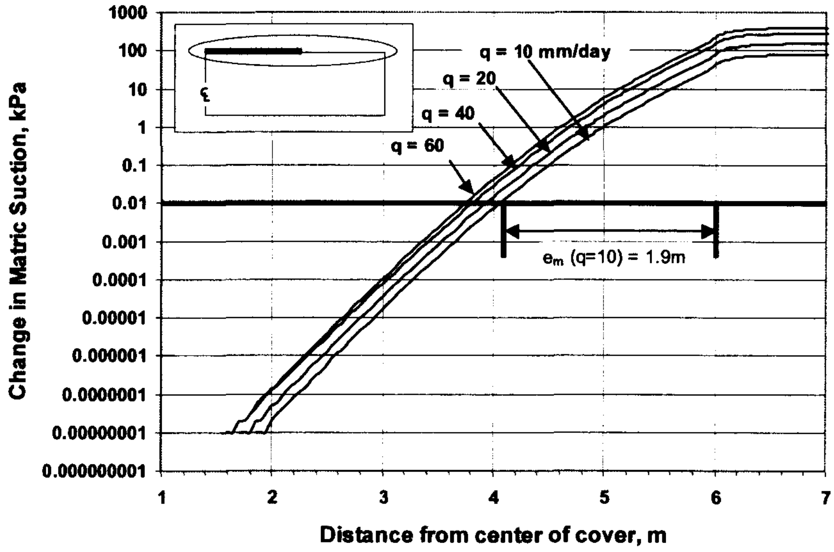

Figure 6 Proposed method for estimating the edge moisture distance, e_m.

Another possible criterion that could be used for the selection of the edge moisture variation distance is one related to the point where the tensile stress between the concrete slab and the soil goes to zero. In other words, it would be the computed distance where to where the slab and the soil separate. In this case, the procedure for the determination of the edge moisture distance would follow the procedure described Scenario #2.

5.2 Differential Soil Movement, y_m

The "differential soil movement", y_m, is defined as the maximum vertical differential soil movement that is likely to occur. It is easy to determine y_m from a plot of the vertical displacement along the ground surface.

6. Conclusions

During the process of numerical modeling a number of conclusions were reached. The conclusions obtained during the course of this study are as follows.

1) The differential vertical deflections calculated considering a flexible slab were compared to the results calculated using a concrete slab with elastic modulus equal to 20 MPa, 10 GPa, and 100 GPa. Elastic modulus values equal to 20 MPa and 10 GPa were shown to reduce the differential deflection by less than 1mm while using an elastic modulus equal to 100 GPa was shown to reduce the differential deflection by approximately 1mm.
2) A trial and error procedure developed to model the separation of the concrete slab from the underlying soil was presented.
3) Methodologies were presented for determining several important design parameters for the concrete slab from the output provided from SVSOLID. The design parameters included the shear force (V), bending moments (M), edge moisture distance (e_m), and the differential soil movement (y_m).
4) It is possible to determine the edge moisture distance (e_m) from a seepage analysis alone. The distance of possible pore-water pressure change under the slab is a function of amount of precipitation, duration of precipitation, and unsaturated soil properties.

7. References

Fredlund, D.G and Rahardjo, H. 1993. Soil Mechanics for Unsaturated Soils. John Wiley & Sons, New York, 560 p.

Fredlund, D.G and Vu, H.Q. 2003. Numerical Modelling of Swelling and Shrinking soils around slabs-on-ground. Proceeding, Post-Tensioning Institute Annual Technical Conference, Huntington Beach, CA, USA.

Post-Tensioning Institute. 1996. Design and construction of post-tensioned slabs-on-ground, Second edition, 101 p.

SoilVision 2001^a SVFLUX Users Manual

SoilVision 2002^b SVSOLID Users Manual

SoilVision 2004^c MODELING SOIL-STRUCTURE INTERACTON OF SLABS ON EXPANSIVE SOILS, Post Tensioning Institute

Timoshenko, S., and Woinowsky-Krieger, S. 1959. Theory of Plates and Shells, McGraw-Hill, 580 p.

Vu, H.Q. and Fredlund, D.G 2004. Prediction of one-, two-, and three-dimensional heave in expansive soils. Canadian Geotechnical Journal (41):713-737.

12