A VOLUME-MASS CONSTITUTIVE MODEL FOR UNSATURATED SOILS IN TERMS OF TWO INDEPENDENT STRESS STATE VARIABLES

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Abstract: This paper presents a description of a new volume-mass constitutive model for unsaturated soils. The model requires relatively simple laboratory tests to obtain the calibration information. Volume and water content changes can be independently computed using the model. The model is also capable of taking into account: i) the hysteretic nature of the soil-water characteristic curve; and ii) both elastic and plastic deformations in the soil. The predicted results for several artificial soils (i.e., sand, silt and clay) are consistent with observed behavior. A presentation of the model predictions for the volume-mass constitutive surfaces is also presented.

INTRODUCTION

Volume-mass constitutive relationships play an important role in modeling unsaturated soil behavior. The volume-mass relationships allow calculation of all volume-mass soil properties corresponding to any stress state. In addition, the volume-mass constitutive relationships can be used in the assessment of unsaturated soil property function for shear strength and hydraulic conductivity. Many geotechnical problems should be solved as ‘coupled’ solutions of seepage, volume change and shear strength (Pereira 1996; Vu and Fredlund 2004). A rigorous volume-mass constitutive model will assist in achieving an integrated seepage, volume change and shear strength solution.

There are a number of volume-mass constitutive models that have been proposed for unsaturated soils (Pham 2005). Most models do not predict water content in terms of the stress state variables or assume that the degree of saturation is independent of net mean stress. Several models make use of two independent stress state variables (i.e., Alonso et al. 1990; Wheeler and Sivakumar 1995; Blatz and Graham 2003), while others make use of stress state variables incorporating soil properties (i.e., Kohgo et al. 1993; Jommi 2000; Wheeler et al. 2003; Tamagnini
In this paper, a new more rigorous volume-mass constitutive model has been proposed that is capable of: i) independently predicting water content, void ratio, and degree of saturation; ii) taking into account both elastic and plastic strains, and iii) taking into account hysteretic nature of the soil-water characteristic curve. The volume-mass constitutive model is proposed for isotropic loading/unloading conditions but is also applicable for $K_0$ loading/unloading conditions. A description for the volume-mass model and model predictions are presented in this paper.

**TERMINOLOGY**

There is no single, unique relationship between volume change and water content change for an unsaturated soil. Volume change and water content change in an unsaturated soil are controlled by two independent mechanisms: i) stress-strain (i.e., mechanical theory) behavior and ii) adsorption-drainage behavior (i.e., capillary theory). Therefore, water content change and volume change should be predicted separately within the context of a volume-mass constitutive model.

The soil particles are assumed to be incompressible; that is, deformations in the overall soil mass are directly related to changes in the volume of the pores. The soil-water appears only in pores; therefore, volume change and water content change are directly referenced to changes in the volume and shape of pores of the soil.

The “shape” of a pore can be defined by an “open pore diameter or neck pore diameter” and “body pore diameter” in relation to the capillary theory (Haines 1930). The “open pore diameter” is generally referred to as the “drying soil suction” or the air entry value of the pore (Neél 1942, 1943; Mualem 1973, 1974). The “body pore diameter” is generally referred to as the “wetting soil suction” or the water entry value of the pore.

The pores in a soil are comprised of various shapes and volumes. The pore-size distribution of a soil at any stress state provides information regarding both the total volume and the volume of water in the soil; therefore, in order to predict volume and water content, it is necessary to utilize the concept of the pore-size distribution of the soil. The pore-size distribution is changed when changing soil suction or net mean stress (Figure 1). It is required to predict changes in the volume and in the drying/wetting suction of each group of pores along the pores-size distribution at any stress states. In this paper, a reference pore-size distribution at a reference stress state is selected. The stress-strain relationship for the soil structure surrounding each pore group is then described.

The development of a volume-mass constitutive model includes: i) the proposal of basic assumptions for the response of a pore to changes in net mean stress and soil suction; ii) the formulation of the stress-strain relationship for the soil structure surrounding a pore including: changes in the air entry value, water entry value, yield stress, volume and water content; iii) the determination of the compression and unloading-reloading indices for each group of pores; and iv) the proposal of a model for the hysteretic nature of the soil-water characteristic curve within the context of a pore-size distribution.
THEORY AND ASSUMPTIONS FOR THE MODEL

Stress state variables

The proposed model makes use of two stress state variables (Fredlund and Morgenstern 1977); namely, net mean stress, \( p = (\sigma_1 + \sigma_2 + \sigma_3)/3 - u_a \), and soil suction, \( \psi = (u_a - u_w) \). The reference stress state is chosen to be a net mean stress = 1 kPa and soil suction = 0 kPa. This reference stress state is equivalent to the stress state where net mean stress = 0 kPa and soil suction = 1 kPa (i.e., assuming the air entry value of soil is greater than 1 kPa). The void ratio, \( e \), of the soil is the primarily variable used to represent the overall volume of the soil. The gravimetric water content, \( w \), of the soil is the primary variable used to represent the amount of water in the soil.

Reference Pore-Size Distributions

The pore-size distribution curve of a soil is generally plotted on a semi-logarithmic soil suction graph (Fredlund 1999; Simms and Yanful 2001). There are two types of the pore-size distribution curves for a soil; namely, i) the plot of the ratio of pore volume per unit weight versus open pore diameter (i.e., drying suction or air entry value of the pore) referred to as the drying pore size distribution, \( DPD \), and ii) the plot of the ratio of pore volume per unit weight versus body pore diameter (i.e., wetting suction or water entry value of the pore) referred to as the wetting pore size distribution, \( WPD \). For a soil that exhibits an insignificant volume change, the \( DPD \) and \( WPD \) are directly related to the initial/boundary drying and the boundary wetting soil-water characteristic curve of the soil (Figure 2).

The pore-size distribution of a soil with a significant volume change changes with soil suction and net mean stress. In order to predict changes in the pore-size distribution, it is necessary to select a reference pore-size distribution. The authors have observed that the drying pore-size distribution curve corresponding to an initially dry soil (i.e., at 10^6 kPa on the initial drying of a slurry soil) can provide a meaningful reference state. In the proposed model, reference pore-size distributions of a soil are chosen at completely dry conditions from the slurry. A soil has two reference pore-size distribution curves under completely dry conditions; namely, the reference Drying Pore-Size Distribution (DPD), denoted as \( f_d(\psi) \), and the reference Wetting Pore-Size Distribution (WPD), denoted as \( f_w(\psi) \).
Basic Assumptions

Seven assumptions are made in the development of the proposed volume-mass constitutive model. These assumptions are based on the findings of previous studies. The seven assumptions can be described as follows:

- **Assumption No. 1**: In an unsaturated soil, a particular pore under consideration in the soil has only two states; namely, i) the pore is filled with water; or ii) the pore is empty.

Assumption No. 1 has been widely accepted in the development of a number hysteresis models for the soil-water characteristic curve (Paulovassilis 1962; Topp, 1971; Mualem 1973, 1974).

- **Assumption No. 2**: Soil suction affects only the water-filled pores and does not affect the empty pores, while net mean stress has an effect on all pores in the soil.

Assumption No. 2 is similar to the explanation given for the mechanical

- **Assumption No. 3**: Each water-filled pore in the soil has two indices; namely, i) virgin compression index, \( C_v \) and ii) unloading-reloading compression index, \( C'_v \).

The soil particles are assumed to be incompressible but it is reasonable to assume that a pore in the soil has two compression indices that are related to the elastic and plastic volume change of the pore. It can be shown that the summation of the compression indices of all pores in the soil is equal to that of the overall soil mass.

- **Assumption No. 4**: There are two types of pore; namely, i) collapsible pores and ii) non-collapsible pores. The collapsible pores are relative large pores and the non-collapsible pores are relative small interconnected pores. The interconnected pores are assumed to be incompressible.

The assumption regarding collapsible and interconnected pores is similar to the explanation related to macro and micro structures that has been presented by several researchers (Alonso et al. 1994; Wheeler and Sivakumar 1995). Measured pore-size distribution curves for numerous soils under various loading and compaction conditions show that the micro pores do not seem to change with the change in volume of the soil (Sridharan et al. 1971; Ahmed et al. 1974; Delage and Graham 1995; Al-Mukhtar 1995; Alonso et al. 1995; Wan et al. 1995; Lloret et al. 2003). Therefore, interconnected pores are assumed to be incompressible.

- **Assumption No. 5**: The virgin compression index and unloading-reloading compression index for a pore in the soil is proportional to the volume of the pore at the reference stress state (i.e., 1 kPa net mean stress and zero soil suction).

- **Assumption No. 6**: Pores are deformed and water is absorbed and drained through independently mechanisms.

The non-collapsible pores are assumed to be incompressible (assumption No. 4), and it is reasonable to assume that the collapsible pores deform independently.

- **Assumption No. 7**: The unloading-reloading index of an air-filled pore is essentially equal to zero.

Silva et al. (2002) assumed that the unloading-reloading index of an unsaturated soil at a constant soil suction decreases with each increment of soil suction. The experimental data by Pham (2005) showed that the unloading-
reloading index of an air-dried soil can be considered to be zero. Therefore, assumption No. 7 appears to be reasonable.

**Volume change of a pore under different stress paths**

The volume-mass constitutive relationships are stress path dependent (Alonso 1993; Pham 2005). The stress-strain relationship for the soil structure surrounding a pore needs to be considered for four different stress paths (i.e., loading, unloading, drying, and wetting). Let us consider a pore in a representative soil element with a volume of the solid phase equal to $V_s$. A description of four stress paths that the pore in the soil element could be subjected to is presented in this section.

**Drying-wetting processes under zero net mean stress**

Let us consider a drying process where the soil suction is less than the air entry value and the pore is filled with water. The stress state acting on the soil structure surrounding the pore is equal to the soil suction ($u_a - u_w$). When the soil suction is higher than the air entry value, the pore becomes filled with air and soil suction does not affect the soil structure surrounding the pore. The yield stress of the pore is equal to the air entry value of the pore. The volume of the air-filled pore is the same as that of the pore at a soil suction equal to the air entry value while the water content in the pore is equal to zero. A schematic illustration of the volume and water content changes of a pore along the initial drying process from an initially slurry condition are shown in Figure 3. The volume of a pore, $v^p(\psi,0)$, at a soil suction of $\psi$ along a drying process can be calculated as follows:

$$
v^p(\psi,0) = \begin{cases} 
  v^p(1,0) - V_s C^p_c \log(\psi) & \text{for } \psi \leq \psi_{ae} \\
  v^p(1,0) - V_s C^p_c \log(\psi_{ae}) & \text{for } \psi > \psi_{ae}
\end{cases}
$$

where $\psi_{ae} =$ air entry value of the pore (i.e., reference air entry value = $\psi_{ae}$), $C^p_c =$ virgin compression index of the pore, $V_s =$ volume of the solid phase of the representative soil element, and $v^p(\psi,0) =$ volume of the pore at a soil suction of $\psi$ and zero net mean stress. The volume of water in a pore, $v^w(\psi,0)$, along a drying process can be written as follows:

$$
v^w(\psi,0) = \begin{cases} 
  v^w(1,0) - V_s C^w_c \log(\psi) & \text{for } \psi \leq \psi_{ae} \\
  0 & \text{for } \psi > \psi_{ae}
\end{cases}
$$

where $C^w_c =$ virgin compression index of the pore, and $v^w(\psi,0) =$ volume of water at a soil suction of $\psi$ and zero net mean stress.

The volume of the pore along the wetting process of an air-filled pore does not change until the soil suction is equal to the water entry value (Figure 3). When soil suction is less than the water entry value, the pore is filled with water and the soil continues to swell as soil suction decreases. The volume of the pore at any soil suction, $\psi$, along a wetting process (i.e., from initially air-filled) can be calculated as follows:
where $C'_p$ = unloading-reloading index of the pore, and $\psi_{we}$ = water entry value of the pore (i.e., reference water entry value = $\psi_{ae}$). An equation for the volume of water in the pore along a wetting process (i.e., initially air-filled) can be written as follows:

$$v^w(\psi,0) = \begin{cases} v^p(\psi_{ae},0) + V_s C'_s \log(\psi_{ae}/\psi) & \text{for } \psi \leq \psi_{we} \\ v^p(\psi_{ae},0) & \text{for } \psi > \psi_{we} \end{cases}$$

(3)

for $\psi_{ae} < 0$, $(\psi)$

$$v^p(\psi_{ae},0) = \begin{cases} v^p(\psi_{ae},0) + V_s C'_s \log(\psi_{ae}/\psi) & \text{for } \psi \leq \psi_{we} \\ 0 & \text{for } \psi > \psi_{we} \end{cases}$$

(4)

$\psi_{we}$ = water entry value of the pore (i.e., reference water entry value = $\psi_{ae}$).

An equation for the volume of water in the pore along a wetting process (i.e., initially air-filled) can be written as follows:

$$v^w(\psi,0) = \begin{cases} v^p(\psi_{ae},0) + V_s C'_s \log(\psi_{ae}/\psi) & \text{for } \psi \leq \psi_{we} \\ 0 & \text{for } \psi > \psi_{we} \end{cases}$$

(4)

$\psi_{we}$ = water entry value of the pore (i.e., reference water entry value = $\psi_{ae}$).

**Figure 3.** Schematic illustration of volume and water content changes of a pore during the initial drying process.

**Drying process under a constant net mean stress**

The air entry value of a pore is a function of net mean stress and can be expressed as $\psi_{ae}(p)$. The volume of a pore can be presented as a function of both net mean stress, $p$, and soil suction, $\psi$. The volume of the pore under dry conditions (i.e., dry under zero net mean stress) can be calculated as follows:

$$v^p(\psi_{ae},0) = v^p(1,0) - C'_c V_s \log(\psi_{ae})$$

(5)
where $\psi_{ae}$ = air entry value of the pore when drying under zero net mean stress, $v^p(1,0)$ = volume of the pore at the reference stress state (i.e., 1 kPa soil suction and zero net mean stress). When the soil is dried under a constant net mean stress, the air entry value of the pore is higher than that of the pore that is dried under zero net mean stress (Figure 4). When the pore is filled with water, net mean stress and soil suction plays the same role; Therefore, the volume of a pore at a soil suction equal to the air entry value, $\psi_{ae} (p)$, and a net mean stress of $p$, can be presented as follows:

$$v^p(\psi_{ae} (p), p) = v^p(1,0) - C_v p \log(\psi_{ae} (p) + p)$$

(6)

where $v^p(\psi_{ae} (p), p)$ = volume of the pore at net mean stress, $p$, and soil suction equal to air entry value of the pore if dried under a net mean stress, $p$, $\psi_{ae}(p)$ = air entry value of the pore if the soil is dried under a constant net mean stress of $p$.

An equation for the relationship between a change in soil suction and volume change of the pore can be derived by applying the capillary equation for the relationship between matric suction and the diameter of the capillary tube. The following equation can be obtained when assuming that deformation of the pore is isotropic (i.e., $\varepsilon_x = \varepsilon_y = \varepsilon_z$):

$$\frac{\psi_a}{\psi_b} \frac{3V_0}{3V_0 - \Delta V}$$

(7)

where $\psi_b$ = air entry value (or water entry value) of the pore before deformation (i.e., volume of $V_0$), and $\psi_a$ = air entry value (or water entry value) of the pore after deformation.

The air entry/water entry value of a pore depends on the smallest/largest open pore diameter that is connected to the pore. The above equation may not be able to describe the change in the air entry value (or water entry value) under anisotropic loading conditions. For simplicity, a parameter called the pore shape parameter, $\eta$, is added to represent the relationship between the air entry value of a pore that is dried under zero net mean stress in relation to that of the pore when it is dried under a constant net mean stress, $p$:

$$\frac{\psi_a}{\psi_b} \frac{3V_0}{3V_0 - \eta \Delta V}$$

(8)

where $\eta$ = pore-shape parameter ($\geq 1$) depends on the soil and the stress history. An equation can be derived for the relationship between the air entry value of a pore when drying from slurry condition under zero net mean stress and that of the pore when subjected to a yield stress, $p_y$, and drying under a constant net mean stress, $p$, as follows:

$$\frac{\psi_{ae}}{\psi_{ae}(p, p_y)} = 1 - \frac{[(C + C_y) \log(p_y) + C_v \log(\psi_{ae} + p) - C_v \log(\psi_{ae})]}{3(\varepsilon_{sat} - C_v \log(\psi_{ae}))}$$

(9)

where $\psi_{ae}(p, p_y)$ = air entry value of the pore when yield stress is equal to $p_y$. 
and drying under a net mean stress of $p$, $p_y = \text{yield stress of the soil and can be calculated as follows:}\

$$p_y = p_y(\psi_{ae}) = P(\psi_{ae}, p, p_0) = \begin{cases} 
  P_0 & \text{for } (\psi_{ae} + p) \leq p_0 \\
  (p + \psi_{ae}) & \text{for } (\psi_{ae} + p) > p_0
\end{cases}$$

\[ (10) \]

where $p_0 = \text{yield stress of the pore prior to the drying process.}$

\[ \frac{(\tan^{-1}(\psi_{ae} + p - p_0) + 1.571)(\psi_{ae} + p - p_0) - p_0}{3.142} \]

**Figure 4.** Schematic illustration of the effects of net mean stress to the air entry value, volume and water content of a pore along a drying process.

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**Wetting process under a constant net mean stress**

The water entry value of a pore having a yield stress of $p_y$ and wetting under a constant net mean stress, $p$, (i.e., $\psi_{we}(p, p_0)$) is higher than that of the pore when it is dried and wetted under zero net mean stress (i.e., reference water entry value, $\psi_{ae}$).
Similar to the procedure developed for the drying process, the relationship between the two values of the water entry values of the pore can be calculated as follows:

\[
\frac{\psi_{\text{we}}}{\psi_{\text{we}}(p, \psi_y)} = 1 - \frac{(C_e - C_s) \log(p_y) + C_e \log(\psi_{\text{we}} + p) - C_e \log(\psi_{\text{we}})}}{3(e_{\text{sat}} - C_e \log(\psi_{\text{we}}))} \tag{11}
\]

where \(\psi_{\text{ae}}\) = air entry value of the pore at zero net mean stress (i.e., reference air entry value), \(\psi_{\text{we}}\) = water entry value of the pore having zero yield stress and wetting under zero net mean stress (i.e., reference water entry value), \(\psi_{\text{we}}(p, \psi_y)\) = water entry value of the pore when it has experienced a yield stress of \(p_y\) and wetted under a constant net mean stress of \(p\), where \(p_y\), equal to the yield stress of the pore with a pore-shape parameter, \(\eta\).

Alonso (1993) observed that any stress paths related only to the wetting processes are stress path independent. Therefore, it is reasonable to assume that the volume of a pore that is wetted to a selected soil suction, \(\psi\), and then subjected to a loading process by a net mean stress, \(p\), is equal to that of the pore that is first loaded to a net mean stress, \(p\), and then wetted to a soil suction, \(\psi\). There are two types of pores after a wetting process; namely, i) water-filled pores and ii) air-filled pores. A pore that is filled with air after the wetting process is called the continually air-filled pore. For a water-filled pore, the volume of the pore at a soil suction of, \(\psi\), and a net mean stress of, \(p\), can be calculated as follows:

\[
v^p(\psi, p, \psi_y) = v^p(1,0) - V_s C_e^p \log(p_y) + V_s C_s^p \log(p_y / (p + \psi)) \tag{12}
\]

The yield stress of a pore that is filled with water after the wetting process can be calculated as follows:

\[
p_y = p_y(\psi_{\text{we}}) = \begin{cases} p_0 & \text{for } (\psi + p) \leq p_0 \\ p + \psi_{\text{we}} & \text{for } (\psi + p) > p_0 \\ \frac{(\tan^{-1} \left( \frac{\psi_{\text{we}} + p - p_0}{1.571(\psi_{\text{we}} + p - p_0)} \right) + 3.142}{p_0} & \text{for } (\psi + p) > p_0 \end{cases} \tag{13}
\]

where \(\psi_{\text{we}}\) = reference water entry value of the pore, \(p_0\) = yield stress prior to the wetting process. In order to be consistent with the elasto-plastic theory for saturated soils, the volume-mass constitutive model for unsaturated soils must have a yielding of the soil structure surrounding continually air-filled pores. Three alternative assumptions for the stress-strain relationship of the soil structure surrounding a continually air-filled pore are as follows:

- **Alternative 1:**

The simplest alternative solution is to assume that continually air-filled pores are incompressible. In this case, the yield stress of a continually air-filled pore does not change when changing soil suction and net mean stress. The equation for the yield stress, \(p_y\), can be written as follows:

\[
p_y = p_y(\psi_{\text{ae}}) = p_0 \tag{14}
\]
where \( p_y \) = yield stress of the *continually air-filled pore* (i.e., having a reference air entry value of \( \psi_{ae} \)) after the wetting process, and \( p_0 \) = yield stress of the pore prior to the wetting process.

• *Alternative 2:*

Let us assume that the compression curve of a *continually air-filled pore* at a constant soil suction on a logarithmic net mean stress scale can be expressed as a combination of two straight lines; namely, the virgin compression and unloading-reloading lines. The unloading-reloading index of an air-filled pore is equal to zero (i.e., assumption #7). Therefore, *volume change of an air-filled pore depends only on the yield stress of the pore*. The yield stress of a *continually air-filled pore* at a specific volume is a function of the virgin compression index of the pore (i.e., when it is filled with air).

The virgin compression index of a *continually air-filled pore* should be between the virgin compression indices of the pore when the soil is completely dry and that of the pore when soil suction is equal to zero. Figure 5 illustrates compression curves of a pore at three different soil suctions: i) at saturation (zero soil suction); ii) when soil suction equal to a certain soil suction, \( \psi \), at which the pore is filled with air; and iii) at 10⁶ kPa (i.e., soil is completely dry). The compression curve of the pore at soil suction of 10⁶ kPa can be described by two straight lines; namely, i) from zero net mean stress equal to soil suction (i.e., horizontal line because the unloading-reloading index of the air-filled pore is equal to zero) and ii) at net mean stresses higher than the air entry value of the pore with a slope of \( p_{cd} \). The compression curve of the pore at a soil suction of \( \psi \), can also be described using two straight lines; namely, i) at net mean stresses less than air entry value of the pore (i.e., horizontal line because the loading-unloading of a air-filled pore is equal to zero), and ii) at net mean stresses higher the air entry value with a slope of \( C_{vp} \). The volume of an air-filled pore at a net mean stress less than the air entry value of the pore is equal to the volume of the pore at dry conditions (i.e., dried from slurry under zero net mean stress).

The virgin compression index of the *continually air-filled pore* is assumed to be a function of the following variables; namely, i) water content in the soil (i.e., as observed from the data measured by Sharma (1998)); ii) virgin compression index of the pore when soil suction is equal to zero (i.e., saturation); and iii) virgin compression index of the pore when soil suction is equal to 10⁶ kPa (i.e., oven-dry condition). For simplicity, let us propose an equation for the virgin compression index of the *continually air-filled pore* (i.e., having a reference air entry value of \( \psi_{ae} \)) at a soil suction of \( \psi \) as follows:

\[
C_{vp}^p = (S(\psi))^m (C_c^p - C_{cd}^p) + C_{cd}^p
\]  

(15)

where \( m \) = soil parameter (i.e., for the compressibility of dry pores), \( C_{vp}^p \) = virgin compression index of the air-filled pore at soil suction of \( \psi \), \( C_c^p \) and \( C_{cd}^p \) = virgin compression indices of the air-filled pore at soil suctions of zero and 10⁶ kPa, respectively, and \( S(\psi) \) = degree of saturation at soil suction of \( \psi \) on the
corresponding soil-water characteristic curve of an initially slurried soil (i.e., when net mean stress is equal to zero).

Figure 5. Schematic illustration of the compression curves of a pore at three different soil suctions (i.e., 0, \( \psi \), and \( 10^6 \) kPa).

At a specific volume, the equivalent yield stress of a continually air-filled pore is assumed to be equal to the stress state variable that brings volume of the pore from the initial stress state to the same volume of the pore (i.e., when the pore is filled with water). The yield stress of the continually air-filled pore (i.e., reference air entry value = \( \psi_{ae} \)), can be obtained:

\[
\frac{\log(P^* - \psi_{ae})}{(10^6)^{\psi_{ae}}} = \frac{C_e^p |C_o^p|}{c^p} 
\]

(16)

- **Alternative 3:**
  
  At completely dry conditions there is a certain amount of relative small particles and cementing material (i.e., chemicals) that act to bond the larger particles together. Along the wetting process, water gradually fills the pores and eliminates the bonding strength that keeps soil particles in position (Lawton et al. 1991a, 1991b; Pereira 1996). Let us assume that all bonds are removed when all interconnected pores (i.e., non-collapsible pore) are filled with water. This means that the continually air-filled pores reach maximum collapse and the equivalent yield stress on the soil structure surrounding a continually air-filled pore is equal to the magnitude of the net mean stress. It is reasonable to assume that the equivalent yield stress of the soil structure surrounding continually air-filled pores is a function of the amount of water in the interconnected pores and the net mean stress. The equivalent yield stress of the soil structure surrounding a continually air-filled pore is independent of the reference air entry value of the pore, \( \psi_{ae} \), and can be calculated as follows:
\[
p_y^*(\psi_{ae}) = \begin{cases} 
p \left( \frac{w(\psi, p)}{w_r} \right)^n & \text{for } w(\psi, p) < w_r \\
p & \text{for } w(\psi, p) \geq w_r 
\end{cases}
\] (17)

where \( w(\psi, p) \) = water content in the soil at soil suction of \( \psi \), and net mean stress of \( p \), \( w_r \) = residual water content of the soil, and \( n \) = soil parameter.

- **Summary:**
  The yield stress calculated using equation (16) or (17) is the yield stress for a continually air-filled pore inducing from only the wetting process at a constant net mean stress, \( p \), of an air-dried specimen from initially slurry. The actual yield stress of a continually air-filled pore at any stress state can be calculated as follow:

\[
p_y = p_y^*(\psi_{ae}) = \begin{cases} 
p_y^*(\psi_{ae}) & \text{for } p_0 \leq p_y^* \\
p_0 & \text{for } p_0 > p_y^* 
\end{cases}
\] (18)

where \( p_y^* \) = yield stress calculated using equation (16) or (17), and \( p_0 \) = yield stress of the pore prior to the wetting process.

The volume of an air-filled pore is only a function of the yield stress of the pore (i.e., unloading-reloading index of the pore is equal to zero). The volume of an air-filled pore can be calculated as follows:

\[
v^p(\psi, p, p_y) = v^p(1,0) - V_y C_v \log(p_y)
\] (19)

where \( p_y \) = yield stress of the pore.

**Loading-unloading processes at a constant soil suction**

Two types of pores are considered during loading-unloading processes of an unsaturated soil; namely, i) air-filled pores; and ii) water-filled pores. When the soil structure surrounding an air-filled pore is loaded, the pore will collapse and the water entry value of the pore is decreased. If the soil suction is less than the water entry value, the pore absorbs water and becomes a water-filled pore. Loading-unloading processes of the soil structure surrounding a water-filled pore are similar to that of a saturated soil and have been described in the above sections. If soil suction is higher than the water entry value, the pore is still filled with air after the loading process and called the continually air-filled pore. Three alternative solutions for the volume have been described in the above section.

**Mathematical formulations for the drying process of an initially slurry soil**

Equations are presented in this section for the compression indices associated with each group of pores along the pore-size distribution. The equation for the reference pore-size distribution function is also described. Details of the mathematical derivation of each equation can be found in Pham (2005).

**A curve-fitting model for the soil-water characteristic curve**

The initial drying soil-water characteristic curve from the slurry of a volume
change soil can be best-fit using the following equation:

\[
\begin{align*}
 w(\psi) &= \left( w_{sat} - \frac{C_c}{G_s} \log(\psi - w_r) \right) \frac{a}{\psi^b + w_r} + w_r \left\{ \ln \left[ \frac{1 + \frac{\psi}{\psi_{r}}} \right] \right. \\
&\left. \left. - \ln \left[ \frac{1 + \frac{10^6}{\psi_{r}}} \right] \right\} \right. \\
&= r
\end{align*}
\]  

where: \( C_c = \) virgin compression index of the soil; \( G_s = \) particles specific gravity; \( w_r = \) curve-fitting parameter represents the residual water content; \( w_{sat} = \) curve-fitting parameter represents the water content of the slurry soil at an effective stress of 1 kPa; \( a, b = \) curve-fitting parameters; and \( \psi_r = \) residual soil suction.

**Equation for the virgin compression index of pores and the reference drying pore-size distribution**

An equation for the virgin compression index of pores along the reference pore-size distribution is presented in this section. A soil is completely dry at a soil suction of \( 10^6 \) (Fredlund and Xing 1994). Some soils may be dry at somewhat lower soil suctions. Let us denote the lowest soil suction at which a soil is completely dry as \( \psi_{max} \) (i.e., \( \xi_{max} = \log(\psi_{max}) \) on a logarithmic soil suction scale). Consider a representative soil element of a slurry dried soil where the volume of solids is equal to \( V_s \). The total volume of the pores in the soil element under dry conditions (i.e., at a soil suction \( \geq \psi_{max} \)) can be calculated as follows:

\[
V_p = V_a = \int_0^{\xi_{max}} f_d(x) dx
\]  

where \( f_d(x) = \) reference drying pore-size distribution function (DPD), and \( V_p, V_a = \) total volume of pores and total volume of air phase of the soil element at a soil suction of \( \psi_{max} \), respectively. The water content and total volume of the soil at any stress states can be obtained in a similar manner. Taking into account assumptions No. 3 and 5, an equation for the virgin compression index of a pore group can be derived:

\[
C_c^p(\xi) = \frac{\frac{d}{d^2\xi} \left[ \frac{G_s w(10^6)C_c}{e_{sat} - C_c^p(\xi)} \right]}{\left[ e_{sat} - C_c^p(\xi) \right]}
\]

where: \( e_{sat} = \) void ratio of the soil at reference stress state, \( w(\psi) = \) water content along the initial drying soil-water characteristic curve. Similarly, an equation for the reference DPD can also be obtained as follow:

\[
\int_0^{\xi_{max}} f_d(x) dx = V_{dry} - G_s V_s \left[ w(\psi) - \int_0^{\xi_{max}} \frac{w(10^6)C_c}{e_{sat} - C_c^p(\xi)} dx \right]
\]

**Volume change of collapsible and non-collapsible pores**

An equation for the prediction of the volume change along the drying process
has been presented in the previous section. It was previously noted that there was no
distinction between collapsible pores and interconnected pores. Non-collapsible
pores are interconnected pores that exhibit no significant hysteresis between the
wetting and drying processes. When soil suction exceeds the residual soil suction,
water exists only in the non-collapsible pores. The total volume of the non-
collapsible pores at residual soil suction is assumed to be equal to the volume of
water at residual soil suction.

Figure 6. Schematic illustration of the gravimetric water content in collapsible
and interconnected pores along the initial drying process.

The water content in a soil at a particular soil suction can be divided into two
components; namely, the i) water content in collapsible pores and ii) water content
in the non-collapsible pores. At soil suctions higher than residual soil suction, the
water content in the soil is equal to the water content in collapsible pores (Figure 6).
At soil suctions less than residual soil suction, the water content in the soil is equal
to the water content in the collapsible pores and the water content at residual soil
suction. Let us denote a function that presents gravimetric water content in the
collapsible pores as $w_c(\psi)$. The function $w_c(\psi)$ can be calculated as follows:

$$
 w_c(\psi) = \begin{cases} 
 w(\psi) - w_r & \text{for } \psi \leq \psi_r \\
 0 & \text{for } \psi > \psi_r 
\end{cases}
$$

where $w(\psi) =$ gravimetric water content along the initial drying curve for an
initially slurried soil, and $\psi_r =$ residual water content of the soil.

Summary of the compression indices of pores

Let us consider the existence of the non-collapsible pores. The virgin
compression index of the group of pores having an air entry value, $\psi_v$, can be
calculated as follows (i.e., by substituting Eq. (24) into Eq. (22)):
\[
C_c^p(\xi) = - \frac{d\left(C_c \frac{a}{(10^i)^b + a}\right)}{d\xi} = - \frac{C_c a b \ln(10)(10^i)^b}{\left[(10^i)^b + a\right]^2}
\]  
(25)

where \(\xi = \log(\psi)\), \(C_c\) = virgin compression index of the soil at saturation, \(a, b\) = curve fitting parameters obtained from a best-fitting of the SWCC of the soil at initially slurry conditions (Eq. (20)). Similarly, the unloading-reloading compression index of the group of pores having logarithmic air entry value of \(\xi\), can be calculated as follows:

\[
C_s^p(\xi) = - \frac{d\left(C_s \frac{a}{(10^i)^b + a}\right)}{d\xi} = - \frac{C_s a b \ln(10)(10^i)^b}{\left[(10^i)^b + a\right]^2}
\]  
(26)

where \(C_s\) = unloading-reloading index of the soil at saturation. An equation for the reference DPD of the soil can be obtained by differentiating both sides of equation (23) and then simplifying.

\[
f_d(\psi) = - \frac{d[w(\psi)]}{dy} G_y \ln(10) - \frac{a C_c}{\psi^b + a}
\]  
(27)

where \(w(\psi)\) = gravimetric water content along the initial drying curve of an initially slurried soil. Similar to the compression indices of water-filled pores, it is possible to derive an equation for the virgin compression index of air-filled pores. The dried virgin compression index of the group of pore having logarithmic air entry value of \(\xi\), can be calculated as follows:

\[
C_{cd}^p(\xi) = - \frac{C_{cd} a b \ln(10)(10^i)^b}{\left[(10^i)^b + a\right]^2}
\]  
(28)

where \(C_{cd}\) = virgin compression index of the soil under dried condition.

**Hysteresis model for soil-water characteristic curves**

A new model for hysteretic soil-water characteristic curves based on the pore-size distribution curve is applied in the volume-mass constitutive model. A hysteresis model is developed in this paper using two one-dimensional pore-size distribution functions (i.e., wetting and drying pore-size distributions). Scanning soil-water characteristic curves are assumed to be horizontal with respect to the degree of saturation of the soil-water characteristic curve (Wheeler et al. 2003). This means that each group of pores has a unique relationship between the drying and wetting suction. The relationship between the reference DPD and the reference DPD can be described as follows:

\[
f_w(\xi) = f_d(\xi - \Delta \xi) \quad (29)
\]

where: \(f_d(\xi)\) = reference drying pore-size distribution; \(f_w(\xi)\) = reference wetting pore-size distribution, \(\Delta \xi\) = difference between the air entry value and water entry value of the group of pores having an air entry value of \(\psi\) (i.e., log-cycles);
and $\xi = \log(\psi)$.

Figure 7. Schematic illustration of the distance between the two boundary curves in log-cycle.

Figure 8. Volume of the water-filled pores and the air-filled pores during the initial drying process.

Pham et al. (2005) presented a model for the three key hysteretic soil-water characteristic curves of a soil (i.e., initial drying, boundary drying, and boundary wetting curves). If the distance between curves and the slope ratio between the two boundary curves are known, the key hysteretic soil-water characteristic curves can be predicted. The reference WPD and DPD can be calculated from the boundary soil-water characteristic curves of the soil (Eq. (27)). Therefore, two reference pore-size distribution curves can be calculated from: i) one of the three key soil-water characteristic curves; ii) the slope ratio and the distance between the two boundary curves.

The proposed model also takes into consideration the effect of the entrapped air. It is assumed that the amount of air entrapped in a collapsible pore is proportional to the volume of the pore and denoted by entrapped an air parameter, $\beta$ (Fig. 8). Once a pore is empty, and a certain amount of air is entrapped in the pore.

A complete volume-mass constitutive model for soils
In summary, the following equations can be used to calculate the volume of a water-filled pore:

\[
v^p(\psi) = v^p(1,0) - V_s C^p \log(p_y) + V_s C^p \log(p_y / (\psi + p))
\]  \hspace{1cm} (30)

The volume of an air-filled pore is stress path dependent. The volume of an air-filled pore after a drying process can be calculated as follows:

\[
v^p(\psi) = v^p(1,0) - V_s C^p \log(p_y) + V_s C^p \log(p_y / (\psi_{ae}(p, p_y) + p))
\]  \hspace{1cm} (31)

The volume of an air-filled pore does not change after an unloading process. After a loading process or a wetting process, the volume of an air-filled pore can be calculated using the following equation:

\[
v^p(\psi) = v^p(1,0) - V_s C^p \log(p_y)
\]  \hspace{1cm} (32)

where \(p_y\) = yield stress of the pore (i.e., can be calculated using one of the following Eqs. (14), (16) and (17)). When taking into consideration collapsible and non-collapsible pores, equations for the change in the air entry value of a pore can be obtained by modifying equations (15) as follow:

\[
\psi_{ae}(p, p_y) = 1 - \eta \frac{(C_c - C_s) \log(p_y) + C_s \log(\psi_{ae} + p) - C_c \log(\psi_{ae})}{3(e_{sat} - C_c \log(\psi_{ae}) - w_r G_s)}
\]  \hspace{1cm} (33)

Similarly, an equation for the change in the water entry value of a pore can be obtained by modifying equation (16) as follow:

\[
\psi_{we}(p, p_y) = 1 - \eta \frac{(C_c - C_s) \log(p_y) + C_s \log(\psi_{we} + p) - C_c \log(\psi_{we})}{3(e_{sat} - C_c \log(\psi_{we}) - w_r G_s)}
\]  \hspace{1cm} (34)

If soil suction is less than the water entry value of the pore along a wetting path then the pore is filled with water, otherwise, it is filled with air. Along a drying process, the pore is filled with water if soil suction is less than the air entry value; otherwise, the pore is filled with air. The air entry value and water entry value of a pore can be calculated using equations (33) and (34), respectively.

At a particular stress state, the pore-size distribution of the soil can be calculated. The volume of water in a representative soil element can be calculated as a summation (i.e., integration) of volume of pores along the pore-size distribution from the current soil suction to \(10^6\) kPa. The volume of the representative soil element can be calculated as a summation (i.e., integration) from the zero soil suction to \(10^6\) kPa. The equations for the volume-mass constitutive surfaces are presented in the following section.

**Equations for volume-mass constitutive surfaces**

The volume-mass constitutive surface have experimentally shown to be stress path dependent (Karube 1986; Alonso 1993; Pham 2005). Each volume-mass constitutive surface pertains to a particular testing stress path and an equation is required to describe the surface. The objective of this section is to show that it is possible to derive closed-form equations for the volume-mass constitutive surfaces.
(i.e., both overall volume and water content) corresponding to any stress paths. Two equations are shown for the volume-mass constitutive surfaces followed by simple stress paths (Figure 9). The stress paths correspond to a slurry soil initially loaded to a net mean stress of \( p_0 \) at zero soil suction and then dried under various constant net mean stresses. For this stress path, the soil parameter, \( m \), is not required since there is no change in net mean stress after pores filling with air.

![Figure 9. Schematic illustration of the volume-mass constitutive surfaces obtained from loading-unloading processes of an initially slurried specimen that is dried under various constant net mean stresses.](image)

Equations for the water content and the void ratio constitutive surfaces can be written as follows:

\[
w(\psi, p, p_0) = w(\psi, 0, 0) - \\
\frac{w^c(\psi_s)(C_e - C_s)\log(P(\psi, p, p_0)) - \xi_s C_e + C_s \log(\psi + p))}{(e_{sat} - C_\xi s - w_r G_s)}
\]

\[
e(\psi, p, p_0) = e_{sat} - G_s w(\psi, 0, 0) - \left[ \int_0^{\xi_s} \frac{G_s w^c(10^x)C_e}{e_{sat} - C_\xi x - w_r G_s} dx \right]
\]

\[
\frac{-w(\psi, p, p_0)G_s + \left( \log(P(\psi, p, p_0)) - \xi_s \right) \left( \frac{G_s w^c(\psi_s)(C_e - C_s)}{(e_{sat} - C_\xi s - w_r G_s)} \right)}{(e_{sat} - C_\xi x - w_r G_s)}
\]

\[
\left[ \int_0^{\xi_s} \frac{G_s w^c(10^x)(C_e - C_s)}{(e_{sat} - C_\xi x - w_r G_s)} dx \right]
\]

\[
\left[ \int_0^{\xi_s} \frac{G_s w^c(10^x)C_e}{(e_{sat} - C_\xi x - w_r G_s)} dx \right] - C_s \log(1 + p)
\]

\[
\left[ \int_0^{\xi_s} \frac{G_s w^c(10^x)C_e}{(e_{sat} - C_\xi x - w_r G_s)} dx \right] - (C_e - C_s) \log(P(1, p, p_0))
\]

where \( w(\psi, 0, 0) \) = water content at soil suction, \( \psi_s \), on the initial drying curve.
of the slurry soil, \( P(\psi, p, p_0) = (\tan^{-1}(\psi + p - p_0) + 1.571)(\psi + p - p_0) + p_0 \), \( \xi_s = \frac{3.142}{\log(\psi_s)} \),

\[
\psi_s = \left(1 - \frac{\eta [(C_c - C_s) \log(P(\psi, p, p_0)) + C_s \log(\psi_{ae} + p) + C_c \log(\psi_{ae})]}{3[e_{sat} - C_c \log(\psi_{ae}) - w_r G_s]} \right)^\psi,
\]

\( e_{sat} = \) void ratio at the reference stress state, \( \eta = \) parameter for representing stress history of the soil, \( w^\psi(\psi) = \left( w_{sat} - \frac{C_s}{G_s} \log(\psi) - \frac{a}{\psi^b + a} \right) \), \( a, b = \) curve fitting parameters for the initial drying curve of the slurry specimen, \( G_s = \) Specific gravity of the soil particles, \( w_{sat} = \) gravimetric water content at reference stress state, \( C_c, C_s = \) compression indices of the soil at saturated condition (i.e., zero soil suction), \( p = \) net mean stress, \( p_0 = \) uniform yield stress, and \( w_r = \) residual water content of the soil. The equation for the degree of saturation surface can be derived from the equations for gravimetric water content surface (Eq. (35)) and void ratio surface (Eq. (36)) as follows:

\[
S(\psi, p, p_0) = \frac{w(\psi, p, p_0) G_s}{e(\psi, p, p_0)} \tag{37}
\]

**Determination of the model parameters**

The proposed volume-mass constitutive model requires relatively simple and conventional laboratory test data for calibration. The data required for calibration can be described as follows:

i. the initial drying soil-water characteristic curve of the initially slurry soil specimen.

ii. pore-shape parameter, \( \eta \), represents the effect of net mean stress and stress history on the change in the air entry values of pores in the soil. It is suggested that this parameter be selected as equal to 2 (Pham 2005) if another soil-water characteristic curve at a different pre-consolidation pressure has not been tested.

iii. the parameters for the hysteretic nature of the soil-water characteristic curve of the soil; namely, i) the distance between the two inflection points of the two boundary hysteretic curves, \( D_{SL} \); and ii) the ratio between the slopes of the boundary drying and the boundary wetting curves, \( R_{SL} \). These values can be estimated as suggested by Pham et al. (2005).

iv. the virgin compression index of the air-dried from an initially slurry specimen (i.e., completely dry specimen).

v. the soil parameter, \( m \), is required to determine the compressibility of the soil structure surrounding a continually air-filled pores. In the case of no measured data, the soil parameter can be assumed to be equal to 1.

The compression curve of an air-dried specimen becomes straight at the value of net mean stress beyond the residual soil suction (Figure 10). The dry virgin compression index, \( C_{vd} \), of an air-dried can then be estimated as the same manner
for saturated soils. For most soils, value of $C_{cd}$ can be assumed to be equal to zero (Pham 2005).

**Figure 10.** Compression curve of an air-dried from the slurry soil specimen and the method for the estimation of the virgin compression index of a dry specimen, $C_{cd}$.

**PRESENTATION OF MODEL PREDICTIONS**

The application of the proposed volume-mass constitutive model is presented for three artificial soils. The prediction results are presented for: i) several simple stress paths (i.e., 2D graphs) and ii) Several complete volume-mass constitutive surfaces (i.e., 3D graphs). Materials are described first, followed by a description of the stress paths and a presentation of prediction results.

**Materials**

Descriptions of the three artificial soils are presented in Table 1. The initial drying soil-water characteristic curves of the three soils starting from slurry conditions are shown in Figure 11. The virgin compression index of the three soils under completely dry conditions, $C_{cd}$, equal to zero.

**Table 1. Characteristics of the three artificial soils**
Figure 11. Initial drying soil-water characteristic curves for the artificial sand, silt and clay.
**Prediction results for several simple stress paths**

The shrinkage curve of a soil can be predicted using the proposed constitutive model by measuring both the void ratio and the water content along the drying process from slurry to $10^6$ kPa soil suction. The predicted shrinkage curves for the three artificial soils are shown in Figure 12. The shrinkage curve for the sand soil is essentially a horizontal line since the sand does not change volume during a drying process. The computed shrinkage curve for silt and clay also appear reasonable. At high water contents, the shrinkage curves form 45 degree lines. The shrinkage curves are horizontal water contents below the shrinkage limit. The transitions of the shrinkage curve between the low and high water contents appear to be smooth and have reasonable curvature.

For illustration purpose, let us predict void ratio, gravimetric water content and degree of saturation for the artificial silt when following several simple stress paths. The stress paths are described as follows (Figure 13). The soil is initially a slurry and subjected to a drying-wetting cycle to a soil suction of 100 kPa. The soil is then dried to $10^6$ kPa and then subjected to a net mean stress of 2000 kPa. The soil is then subjected to a wetting process to 0.1 kPa soil and followed by an unloading process to a net mean stress of 1 kPa. The predicted void ratio, gravimetric water content and degree versus soil suction for the specified stress path are shown in Figure 14.

![Figure 12. Shrinkage curves of the artificial sand, silt and clay.](image.png)
Figures 13. Stress path followed for the artificial silt.

Figures 14 shows the prediction results for the artificial silt followed stress path described in Figure 13. It is shown that the predicted results for the volume-mass constitutive relationships are reasonable. Hysteresis in the soil-water characteristic curve and plastic deformation have been taken into account in the simulation. The soil parameter $m$ controls the collapse behaviour of the soil. The smaller the value for the soil parameter, $m$, the earlier the collapse occurs in the soil during the wetting process.

a) Void ratio
Prediction results for the volume-mass constitutive surfaces

It is necessary to select several series of stress paths to present the volume-mass constitutive surfaces for the three artificial soils. The selected stress paths should be simple but capable of showing several important characteristics of an unsaturated soil (e.g., swelling and collapsible behaviors).

The four series of stress paths for studying the volume-mass constitutive surfaces of an unsaturated soil are described in Figure 15. In stress path series #1, the soil is initially slurry, loaded to a certain net mean stress and then dried under the constant net mean stress to a soil suction of $10^6$ kPa. In stress path series #2, the soil is initially slurry, then dried to a certain soil suction and loaded under constant soil suction condition to a net mean stress of $10^4$ kPa. In stress path series #3, the soil is initially dried from a slurry, loaded to a certain net mean stress and then wetted under the constant net mean stress to a soil suction of $10^6$ kPa. In stress path series #4, the soil is initially dried from a slurry, wetted to a certain soil suction and then loaded under the constant soil suction to a net mean stress of $10^4$ kPa.
Figure 15. Four series of stress paths for studying volume-mass constitutive surfaces of an unsaturated soil.

The two stress path series #1 and #2 are used to study the mechanical behavior of various soil specimens when starting with the same initial slurry condition and ending at the same final stress state, but followed by two different stress paths during the drying process. Similarly, stress path series #3 and #4 are used to study mechanical behavior of various soil specimens with the same initial (i.e., dried from slurry) and final stress states but following two different stress paths that involve a wetting process.

Figure 16 shows that the calculated volume-mass constitutive surfaces agree well with all the postulates presented by Fredlund et al. (2000); namely, i) there is a one-to-one relationship between the effect of a change in net total stress and a change in soil suction, when the soil suction is less than air entry value of the soil (i.e., for both void ratio and water content surfaces); ii) there is a gradual curve that forms from the air entry value to the primary water content reference condition, corresponding to a particular water content on the water content surface; and iii) there is a gradual curve that forms from the air entry value to the second reference condition corresponding to a particular void ratio on the void ratio constitutive surface.
Figures 16 to 19 show several volume-mass constitutive surfaces predicted for the three artificial soils. The predicted constitutive relationships at zero soil suction and zero net mean stress planes appear to be reasonable. The volume-mass constitutive surfaces starting from initially slurry seem to have steeper slopes than that starting from an air-dried condition. The shapes of the volume-mass constitutive surfaces predicted using the proposed model appear to be reasonable.
Figure 18. The constitutive surfaces for clay (stress path series #1 and #2 in logarithmic scale).

Figure 19. The constitutive surfaces for the artificial silt (stress path series #1, #2, #3 and #4 in logarithmic scale).

Figure 18b shows that the air entry value of a soil is increased with each increment of net mean stress. The void ratio constitutive surfaces obtained by
following stress paths #2, #3 and #4 in Figure 19 seem to be somewhat unusual (i.e., at $10^6$ kPa soil suction and $10^4$ kPa net mean stress, the void ratio is higher than that at soil suction of 0.1 kPa and net mean stress of $10^4$ kPa). However, the shape of the surfaces is similar to that measured by Matyas and Radhakrishna (1968). It shows that the proposed volume-mass constitutive model can predict both collapsible and swelling behavior in soils.

The void ratio surfaces measured for the artificial silt in Figure 19 show that the stress path related to a drying process is stress path dependent and the stress path related to only wetting processes is stress path independent. The proposed volume-mass constitutive model is capable of predicting volume-mass constitutive relationships that are stress path dependent.

CONCLUSIONS AND RECOMENDATIONS

The proposed volume-mass constitutive model is capable of: 1) predicting both volume and water content at all stress states corresponding to a wide variety of stress paths; 2) taking into account the hysteretic nature of the soil-water characteristic curve; and 3) predicting both swelling and collapsible behavior of an unsaturated soil. The model can predict volume-mass constitutive relationships that are stress path dependent. The prediction results appear to be consistent with observed laboratory data.

The hysteresis model for the soil-water characteristic curve makes use of two one-dimensional pore-size distributions. The two new parameters are: the pore-shape parameter, $\eta$, and compressibility of dry pore parameter, $m$. It is recommended that suggested ranges for the two parameters be the focus of future research programs.

REFERENCES


