Proceeding of the 16th International Conference of Soil Mechanics and Geotechnical Engineering, Osaka, Japan, Sept. 12-16. 2005

Technical Session "4a: Slope Stability and Landslides" (tentative program)

September 13 Tuesday, 13:30-15:30 Rm 1001-1002 (10F)
Session Chair: *Hiroyuki. Nakamura (Japan)*
Session Secretary: *kazuyoshi Tateyama (Japan)*

General Report: Cam Tim Chau (*Hong Kong*)

Panelists Presentation: *Luciano Picarelli* (*Italy*) - The mobilised shear strength in first-time slides in OC clay

Willy Alvarenga Lacerda (*Brazil*) - Slow movements in saturated, long colluvial deposits of tropical regions

***D.G. Fredlund* (*Canada*)** - Where are we going in the analysis of landslides?

Atsushi Yashima (*Japan*) - Predicting method for large deformation during slope failure

Discussion Topics:
(a) First-time landslide and its slope movement before failure.
(b) Mechanism of slow movement landslides
(c) Factor of safety of sliding slopes

Where are we going in the Analysis of Landslides?

Delwyn G. Fredlund
Session 4a: Slope Stability and Landslides
September 13, 2005 Tuesday, 13:30-15:30 Room 1001-1002 (10F)

Introduction

There are two primary analyses commonly associated with landslides; namely, 1.) hydrological modeling of infiltration and groundwater flow, and 2.) slope stability analyses for the calculation of the factor of safety. Both deep-seated and shallow slides are commonly triggered by changes in pore-water pressures associated with infiltration and regional groundwater changes. The characterization of ground surface moisture flux conditions (from climatic conditions), is an important aspect that is often omitted in a slope stability study and needs increased attention; however, the focus of this presentation is on the methodologies emerging for the analysis of slopes.

The **objective** of this presentation is to illustrate the gradual changes that have taken place in the analysis of slopes and to comment on the advantages and disadvantages of the newer methodologies that have emerged.

Limit Equilibrium Methods of Slices

Limit equilibrium methods of analysis for landslides have been so widely used that it is hard for engineers to accept that there may need to be some changes in the methodology for landslide analysis. However, there have been some gradual changes in methodology that are emerging with regard to slope stability analyses. Some of the changes have been shown to have benefits and it is necessary to relate the results of new methodologies back to past experience.

There have been limitations associated with classical limit equilibrium methods of slices; namely, the shape of the slip surfaces that are analyzed must be assumed and the location of the critical slip surface must be found by trial and error. Consequently, the boundaries for the free body diagram to be analyzed are unknown and must be assumed.

Various limit equilibrium methods of slices differ primarily in the manner in which the normal force at the base of the slice is computed and the elements of statical equilibrium that are used when solving for the factor of safety (Fredlund and Krahn, 1977; Fredlund et al, 1981). Various methods of slices could be categorized in terms of a moment equilibrium factor of safety equation or a force equilibrium factor of safety equation, or both. It has been shown that the moment equilibrium factor of safety equation is relatively insensitive to the inter-slice force function as long as the shape of the slip surface is approximately circular. Alternatively, the force equilibrium factor of safety equation is relatively insensitive to the inter-slice force function as long as the slip surface is planar. Composite shaped slip surfaces produce intermediate conditions with both factor of safety being dependent upon the inter-slice force function.

The normal force on the base of the slice is generally computed by summing forces in a vertical direction on individual slices with an arbitrary assumption made regarding the inter-slice shear forces. Wilson and Fredlund (1983) used a linear elastic model in a finite element analysis, along with "switching on" gravity forces to provide a somewhat realistic shape for the inter-slice force function. There appears to have been a philosophical resistance to utilizing stress–strain modeling to assist in making a slope stability analysis determinate. This resistance was related primarily associated with attempting to analyze near failure conditions with a simple stress-strain model.

Enhanced Limit Methods

In 1969, Kulhawy suggested a "strength level" definition of the factor of safety equation and made use of a linear elastic stress analysis for the assessment of the normal force at the base of a slice. Several other finite element, stress analysis based approaches have been suggested for computing the factor of safety of a slope and these methods have became known as "enhanced limit methods". Studies by Scoular (1997) and Fredlund et al, (1997), showed a linear elastic, finite element stress analysis produced more realistic values for the normal forces along a selected slip surface than could be obtained through the conventional limit equilibrium methods of slices approach. In other words, "switching on" gravity for the entire mass of soil produced stress states that better reflected the overall ground surface geometry.

The enhanced limit method for the computation of the factor of safety was different from the conventional methods of slices in that: i.) the solution for the factor of safety became determinate, and ii.) the factor of safety equation was linear in form. The "enhanced limit" method was similar to the conventional methods of slices in that it was still necessary to assume a shape for the slip surface and it was necessary to perform a trial and error search for the critical slip surface. The analysis also made it possible to compute a "local" factor of safety along the slip surface in additions to the "global" factor of safety for the entire sliding mass. The computed factors of safety for the "enhanced limit" method and the conventional methods of slices that satisfied both moment and force equilibrium were shown to be similar (Scoular, 1997). It was revealed that the computed factor of safety was affected by the Poisson's ratio selected for the stress analysis. However, the effect was perceived to be of secondary significance. The "enhanced limit" method provided greater flexibility in analyzing slopes and opened the way for the simulation of more complex slope stability problems.

Optimization Methods (Dynamic Programming Method)

The application of optimization theories to the analysis for the stability of a slope essentially removes the assumptions related to the shape of the critical slip surface. In other words, the shape of the critical slip surface becomes a part of the analytical solution through optimization techniques such as Dynamic Programming. Baker (1980) applied the Dynamic Programming optimization technique while retaining Spencer's approach to the calculation of the factor of safety. Yamagami and Ueta (1988) combined the Dynamic Programming technique with the results of a finite element stress analysis. The analysis was thus similar to that of the "enhanced limit" method with the exception there was now no restriction on the shape of the slip surface. Pham and Fredlund (2003) continued research on the approach advanced by Yamagami and Ueta (1988) and performed a comparative parametric study involving a variety of slope geometries and soil conditions. Once again, the computed factors were similar in magnitude to those obtained from limit equilibrium methods of slices where moment and force equilibrium conditions were satisfied. Poisson's ratio had a secondary effect on the computed factors of safety.

Probably the greatest resistance to the use of the dynamic programming methodology is the use of a stress analysis for the calculation of the factor of safety of a slope. A recent study on the dynamic programming technique by Stianson et al. (2004) shows the results of a comparison of analyses performed using a linear elastic model and an elasto-plastic model for the calculation of the stress state. The results showed that the results from the linear elastic analysis and the elasto-plastic analysis were the same as long as the "global" factor of safety was greater than or equal to 1.0. The location of the critical factor of safety was also the same. When the computed factor of safety was less than 1.0, the results from the elasto-plastic analysis produced a deeper critical slip surface but the factor of safety tended towards 1.0. However, if the deformed shape of the slope was used in the elasto-plastic analysis, then the elasto-plastic analysis gave the same results as the linear elastic analysis (i.e., both in terms of the magnitude of the factor of safety and the location of the critical slip surface). These results help relieve concerns related to the use of a (linear) stress-strain analysis for the calculation of the factor of safety when using optimization techniques such as Dynamic Programming.

There are a number of benefits that emerge from the utilization of optimization techniques. First, the shape of the critical slip surface becomes a part of the solution. Second, the critical slip surface can be irregular in shape but must be kinematically admissible. Third, no assumption is required regarding the location of the critical slip surface. Fourth, force and moment equilibrium conditions are satisfied through the stress analysis. Fifth, the factor of safety equation is linear.

Summary of Trends in Slope Stability Analyses

Advances in the analysis for the factor of safety of a slope can be presented in the form of two "steps". The first "step" involves the use of stress analyses to compute normal stresses at the base of each slice. The computed factors of safety are referred to as the "enhanced limit" method for the calculation of the factor of safety. The second "step" involves the use of optimization theory to compute the shape, location and factor of safety for a soil mass. Advancements in computer technologies make the emerging changes for slope stability analysis attractive from the standpoint of engineering practice.

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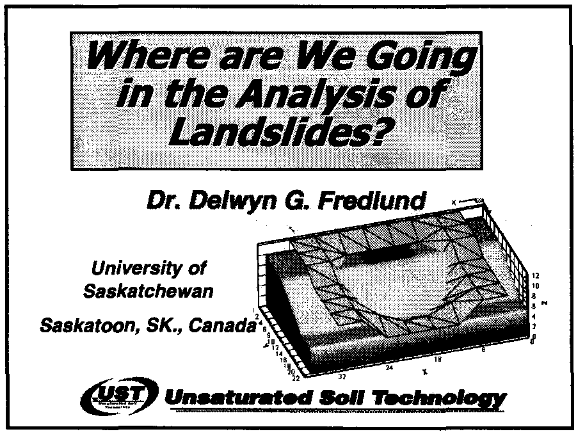

Where are We Going in the Analysis of Landslides?

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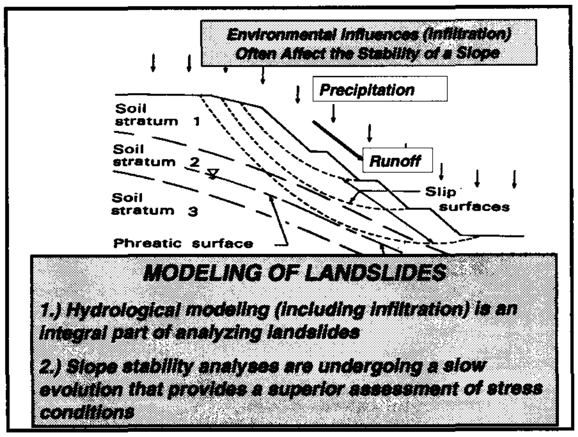
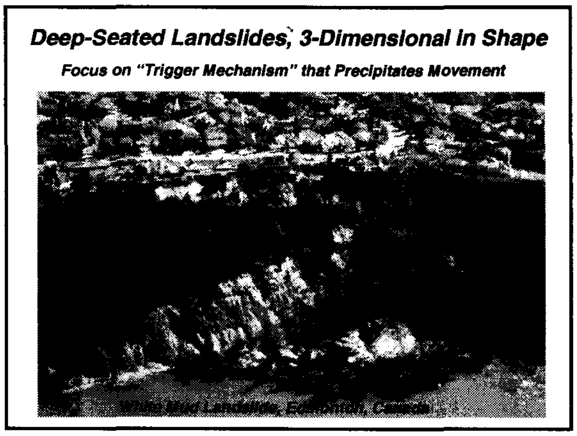


Environmental Influences (Infiltration) Often Affect the Stability of a Slope

MODELING OF LANDSLIDES

1.) Hydrological modeling (including infiltration) is an integral part of analyzing landslides

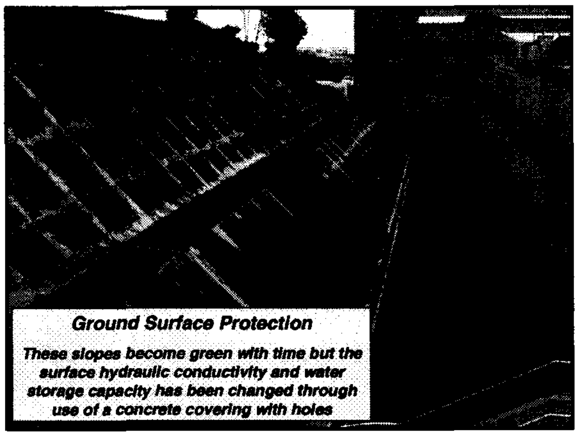
2.) Slope stability analyses are undergoing a slow evolution that provides a superior assessment of stress conditions



Deep-Seated Landslides, 3-Dimensional in Shape

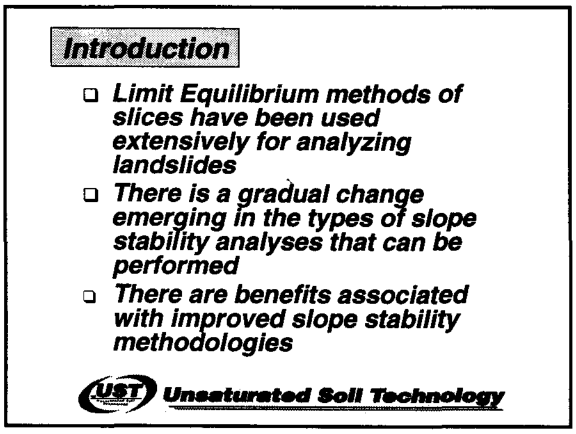
Focus on "Trigger Mechanism" that Precipitates Movement

White Mud Landslide, Edmonton, Canada



Ground Surface Protection

These slopes become green with time but the surface hydraulic conductivity and water storage capacity has been changed through use of a concrete covering with holes



Introduction

- Limit Equilibrium methods of slices have been used extensively for analyzing landslides
- There is a gradual change emerging in the types of slope stability analyses that can be performed
- There are benefits associated with improved slope stability methodologies

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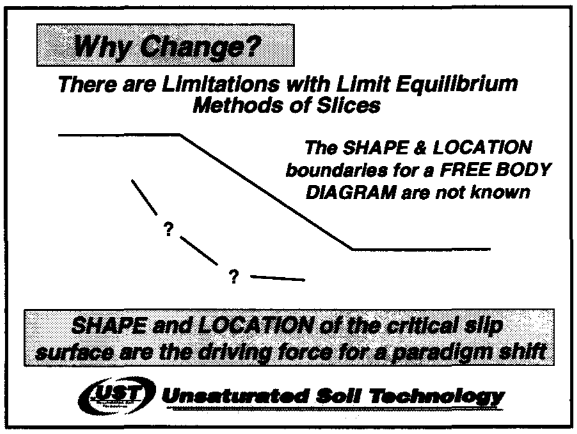

Why Change?

There are Limitations with Limit Equilibrium Methods of Slices

The SHAPE & LOCATION boundaries for a FREE BODY DIAGRAM are not known

SHAPE and LOCATION of the critical slip surface are the driving force for a paradigm shift

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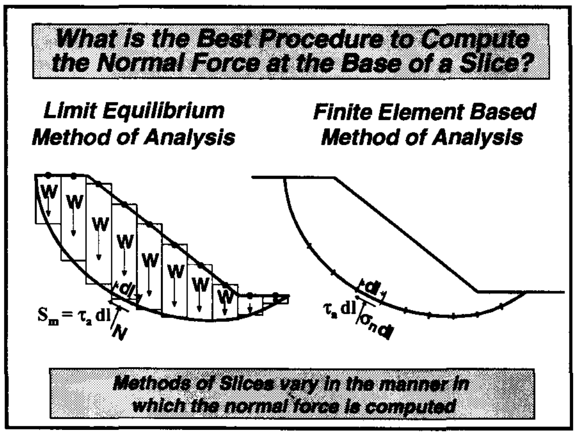
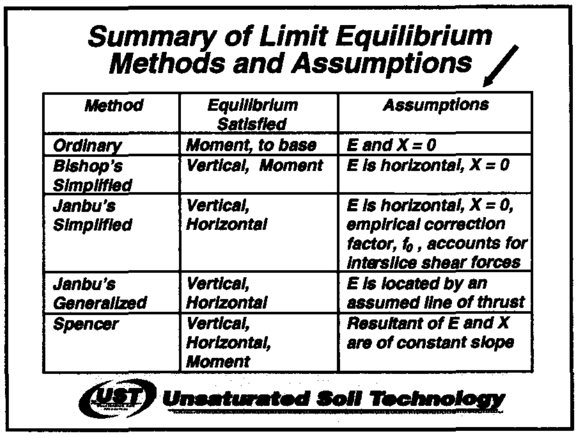


What is the Best Procedure to Compute the Normal Force at the Base of a Slice?

Limit Equilibrium Method of Analysis

Finite Element Based Method of Analysis

Methods of Slices vary in the manner in which the normal force is computed



Summary of Limit Equilibrium Methods and Assumptions

Method	Equilibrium Satisfied	Assumptions
Ordinary	Moment, to base	E and X = 0
Bishop's Simplified	Vertical, Moment	E is horizontal, X = 0
Janbu's Simplified	Vertical, Horizontal	E is horizontal, X = 0, empirical correction factor, f_0, accounts for interslice shear forces
Janbu's Generalized	Vertical, Horizontal	E is located by an assumed line of thrust
Spencer	Vertical, Horizontal, Moment	Resultant of E and X are of constant slope

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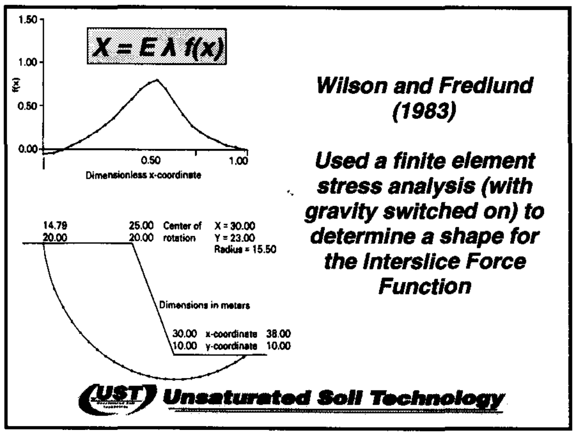

$$X = E \lambda f(x)$$

Wilson and Fredlund (1983)

Used a finite element stress analysis (with gravity switched on) to determine a shape for the Interslice Force Function

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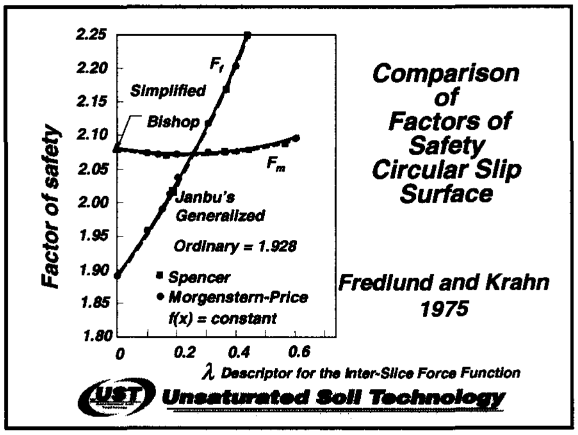

Comparison of Factors of Safety Circular Slip Surface

Fredlund and Krahn 1975

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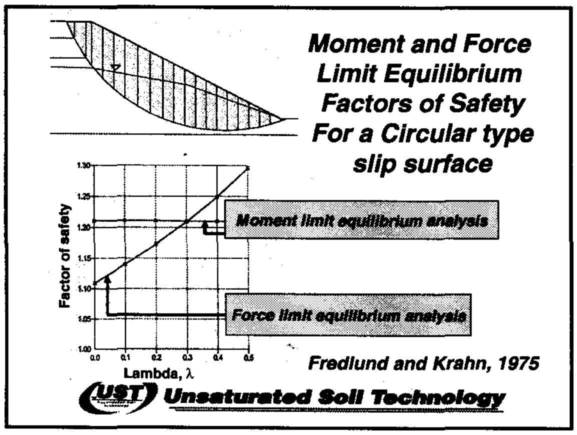

Moment and Force Limit Equilibrium Factors of Safety For a Circular type slip surface

Moment limit equilibrium analysis

Force limit equilibrium analysis

Fredlund and Krahn, 1975

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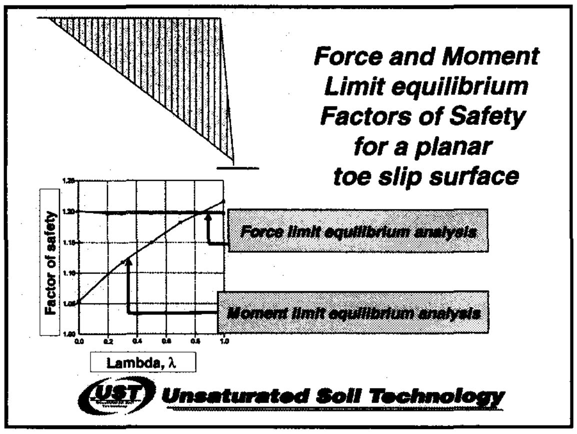

Force and Moment Limit equilibrium Factors of Safety for a planar toe slip surface

Force limit equilibrium analysis

Moment limit equilibrium analysis

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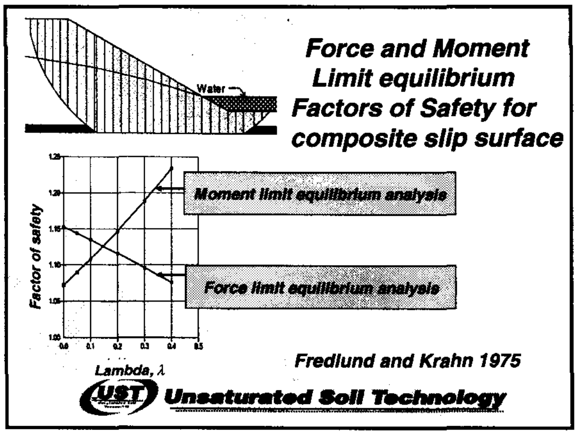

Force and Moment Limit equilibrium Factors of Safety for composite slip surface

Moment limit equilibrium analysis

Force limit equilibrium analysis

Fredlund and Krahn 1975

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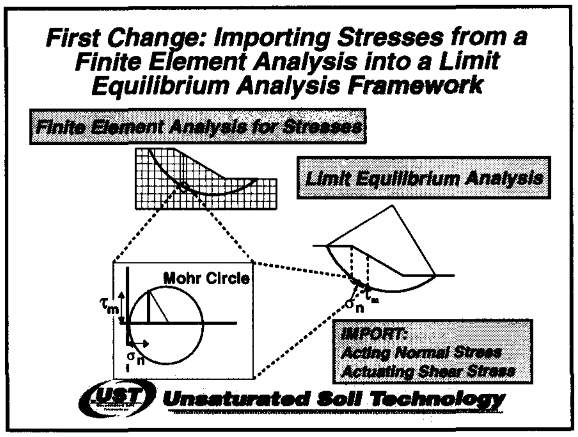

First Change: Importing Stresses from a Finite Element Analysis into a Limit Equilibrium Analysis Framework

Finite Element Analysis for Stresses

Limit Equilibrium Analysis

IMPORT:
Acting Normal Stress
Actuating Shear Stress

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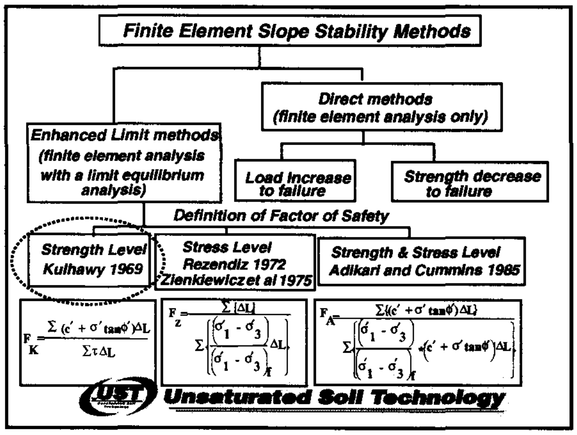

Finite Element Slope Stability Methods

- Enhanced Limit methods (finite element analysis with a limit equilibrium analysis)
  - Definition of Factor of Safety
    - Strength Level Kulhawy 1969
    - Stress Level Rezendiz 1972 Zienkiewicz et al 1975
    - Strength & Stress Level Adikari and Cummins 1985
- Direct methods (finite element analysis only)
  - Load increase to failure
  - Strength decrease to failure

$$F_K = \frac{\Sigma (c' + \sigma' \tan\phi')\Delta L}{\Sigma \tau \Delta L}$$

$$F_Z = \frac{\Sigma \{\Delta L\}}{\Sigma \left\{ \frac{(\sigma'_1 - \sigma'_3)}{(\sigma'_1 - \sigma'_3)_f} \Delta L \right\}}$$

$$F_A = \frac{\Sigma \{(c' + \sigma' \tan\phi')\Delta L\}}{\Sigma \left\{ \frac{(\sigma'_1 - \sigma'_3)}{(\sigma'_1 - \sigma'_3)_f} (c' + \sigma' \tan\phi')\Delta L \right\}}$$

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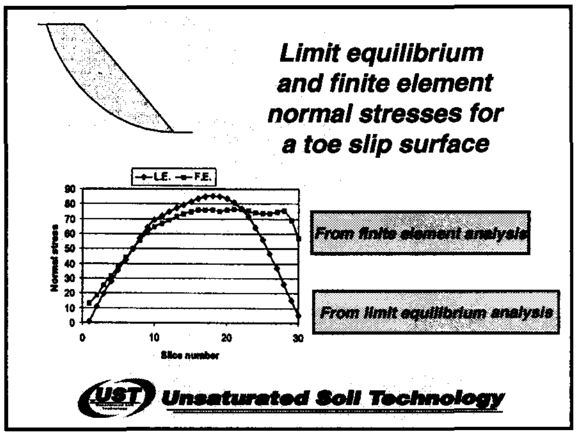

Limit equilibrium and finite element normal stresses for a toe slip surface

From finite element analysis

From limit equilibrium analysis

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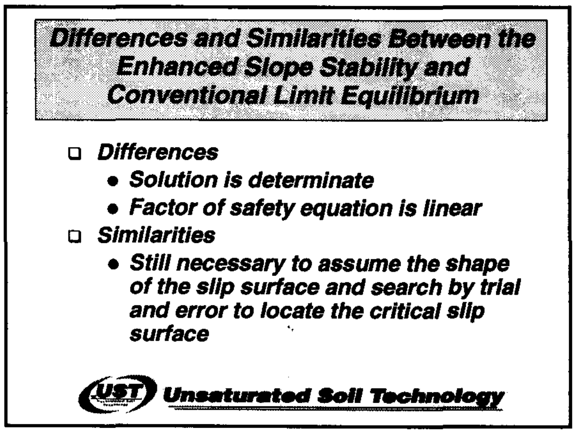

Differences and Similarities Between the Enhanced Slope Stability and Conventional Limit Equilibrium

- ☐ Differences
  - Solution is determinate
  - Factor of safety equation is linear
- ☐ Similarities
  - Still necessary to assume the shape of the slip surface and search by trial and error to locate the critical slip surface

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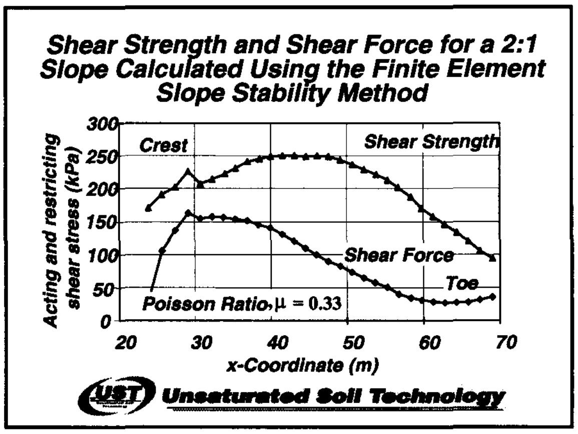

Shear Strength and Shear Force for a 2:1 Slope Calculated Using the Finite Element Slope Stability Method

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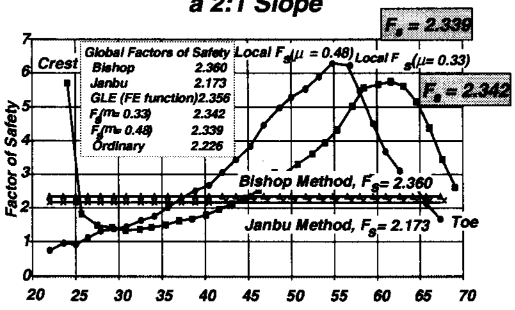
Local and Global Factors of Safety for a 2:1 Slope



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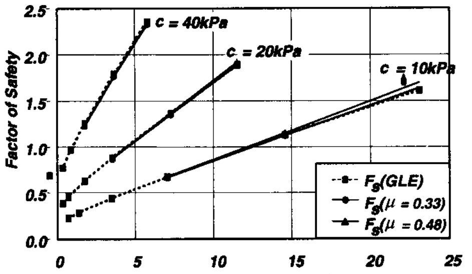
Factors of Safety Versus Stability Number for a 2:1 Slope as a Function of c'



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Next Question to Address

- ☐ Is it possible for the analysis to determine the Shape of the critical slip surface?
- ☐ Is it possible for the analysis to determine the Location of the Critical Slip Surface?
- ☐ Optimization Techniques (i.e., Dynamic Programming) can be used to find the pathway which minimizes a function of the shear strength available to the actuating shear stress within a soil mass

Slope Stability Analysis Using Dynamic Programming Combined with a Finite Element Stress Analysis

- ☐ Baker (1980) Dynamic Programming (DP) optimization techniques for slope stability analysis using Spencer's method
- ☐ Yamagami & Ueta (1988) and Zou et al. (1995) improved on the Baker (1980) solution by coupling Dynamic Programming with a Finite Element stress analysis
- ☐ Pham, H.T.V. (2002) Slope Stability Analysis Using Dynamic Programming Method Combined With a Finite Element Stress Analysis

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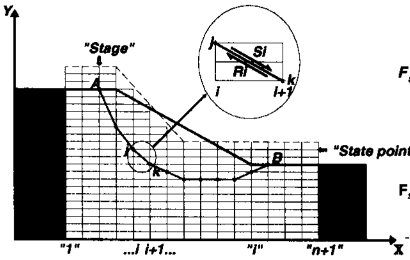
Definition of Factor of Safety

$F_s = \Sigma$ (Shear Strength) / Σ (Actuating Shear Stress)



Smooth curve

$$F_s = \frac{\int_A^B \tau_f \, dL}{\int_A^B \tau \, dL} \quad (1)$$

Discrete form

$$F_s = \frac{\sum_{i=1}^{n} \tau_{fi} \Delta L_i}{\sum_{i=1}^{n} \tau_i \Delta L_i} \quad (2)$$

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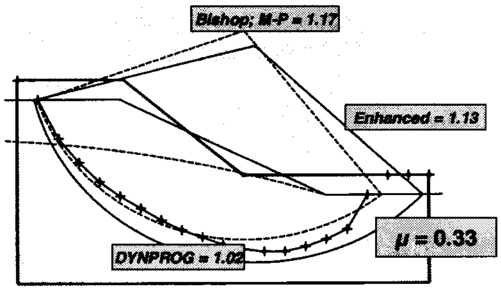
Example of a Homogeneous Slope



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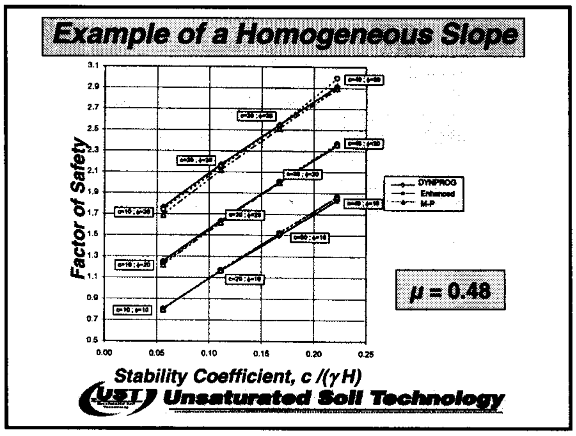

Example of a Homogeneous Slope

$\mu = 0.48$

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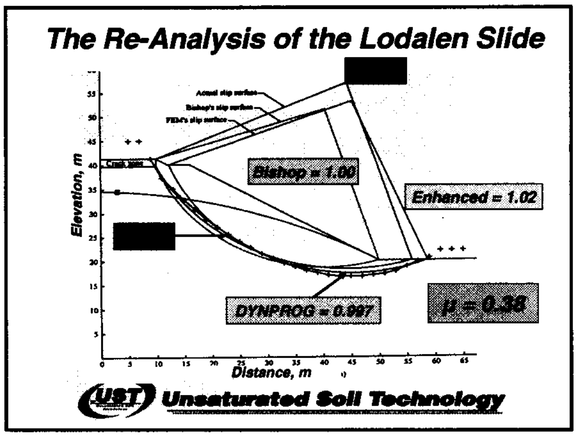

The Re-Analysis of the Lodalen Slide

Bishop = 1.00

Enhanced = 1.02

DYNPROG = 0.997

$\mu = 0.38$

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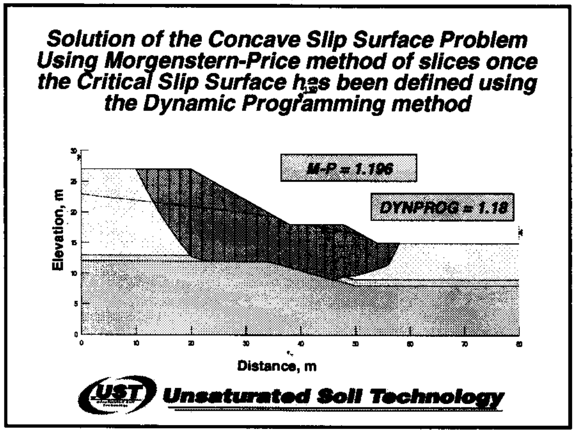

Solution of the Concave Slip Surface Problem Using Morgenstern-Price method of slices once the Critical Slip Surface has been defined using the Dynamic Programming method

M-P = 1.196

DYNPROG = 1.18

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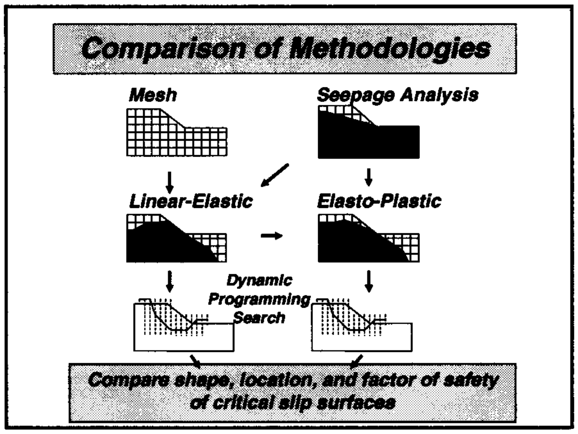


Comparison of Methodologies

Mesh

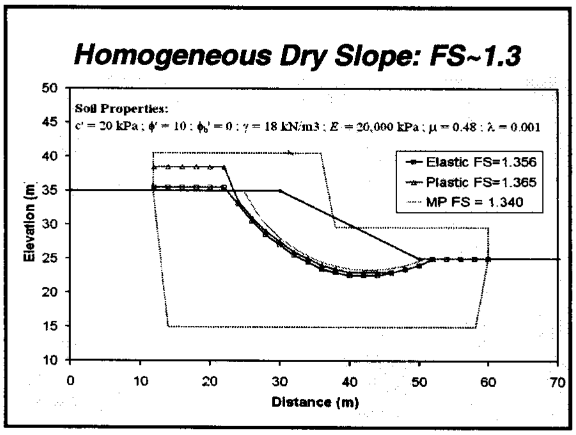
Seepage Analysis

Linear-Elastic

Elasto-Plastic

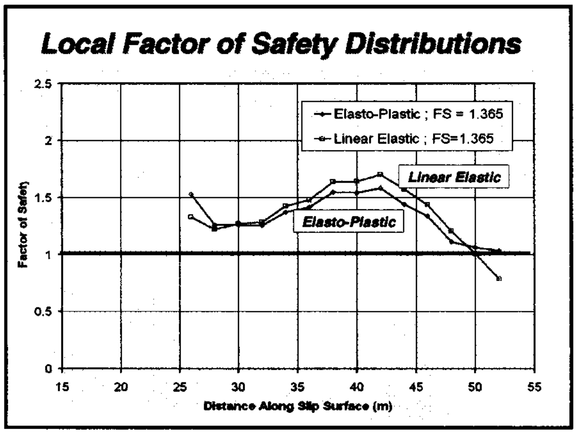
Dynamic Programming Search

Compare shape, location, and factor of safety of critical slip surfaces

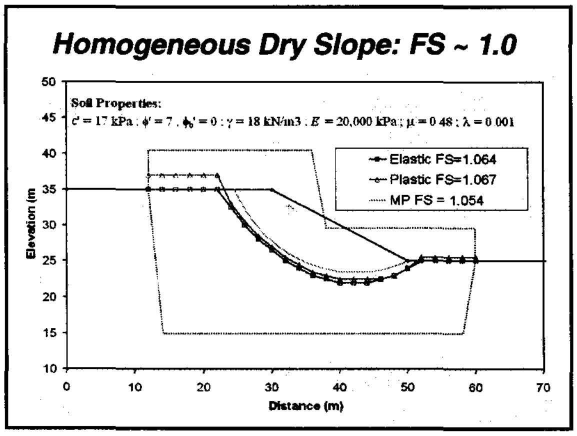


Homogeneous Dry Slope: FS~1.3

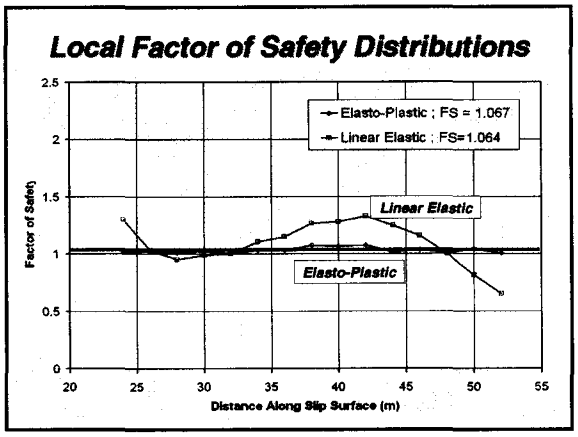
Soil Properties:
$c' = 20$ kPa ; $\phi' = 10$; $\phi_b' = 0$; $\gamma = 18$ kN/m3 ; $E = 20{,}000$ kPa ; $\mu = 0.48$; $\lambda = 0.001$



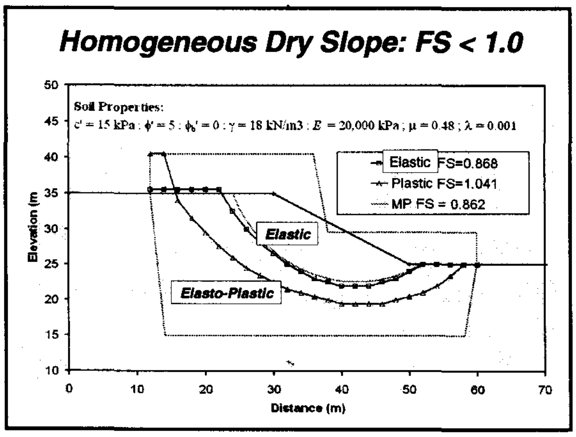
Local Factor of Safety Distributions



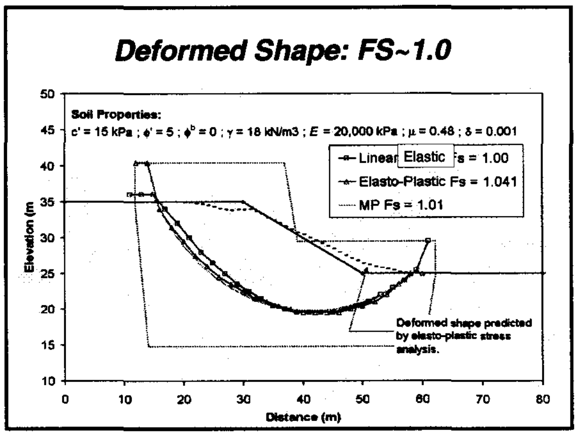
Homogeneous Dry Slope: FS ~ 1.0



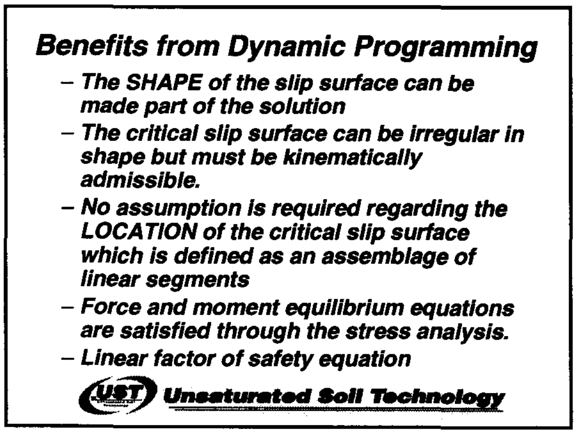
Local Factor of Safety Distributions



Homogeneous Dry Slope: FS < 1.0



Deformed Shape: FS~1.0



Benefits from Dynamic Programming

- ***The SHAPE of the slip surface can be made part of the solution***
- ***The critical slip surface can be irregular in shape but must be kinematically admissible.***
- ***No assumption is required regarding the LOCATION of the critical slip surface which is defined as an assemblage of linear segments***
- ***Force and moment equilibrium equations are satisfied through the stress analysis.***
- ***Linear factor of safety equation***

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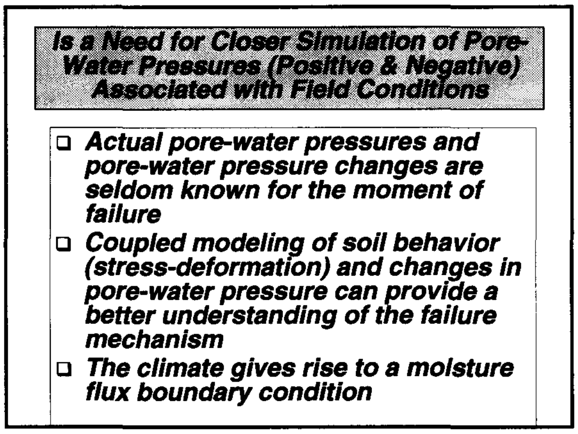
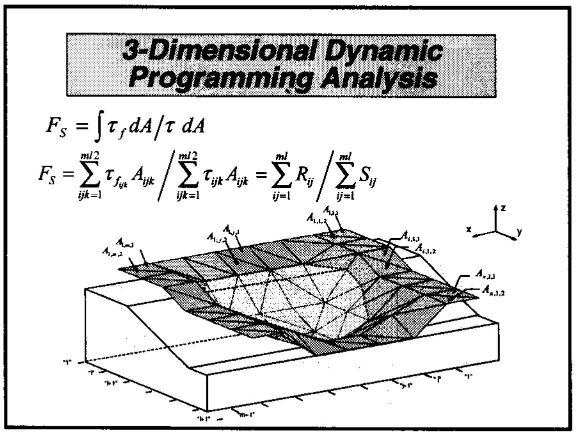


Is a Need for Closer Simulation of Pore-Water Pressures (Positive & Negative) Associated with Field Conditions

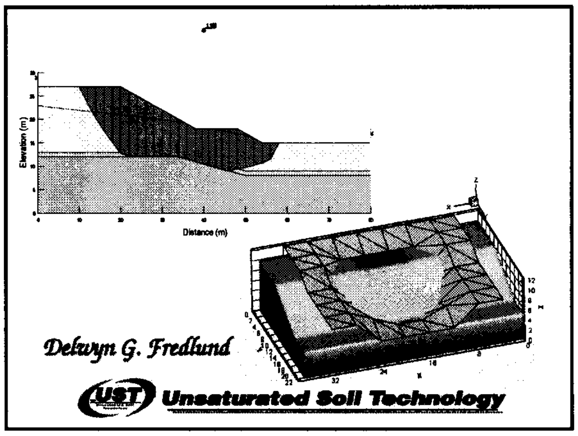
- ☐ ***Actual pore-water pressures and pore-water pressure changes are seldom known for the moment of failure***
- ☐ ***Coupled modeling of soil behavior (stress-deformation) and changes in pore-water pressure can provide a better understanding of the failure mechanism***
- ☐ ***The climate gives rise to a moisture flux boundary condition***



3-Dimensional Dynamic Programming Analysis

$$F_S = \int \tau_f \, dA \Big/ \tau \, dA$$

$$F_S = \sum_{ijk=1}^{ml2} \tau_{f_{ijk}} A_{ijk} \Big/ \sum_{ijk=1}^{ml2} \tau_{ijk} A_{ijk} = \sum_{ij=1}^{ml} R_{ij} \Big/ \sum_{ij=1}^{ml} S_{ij}$$



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