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EVALUATION OF THE VARIABILITY OF UNSATURATED SOIL PROPERTIES

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ABSTRACT

A statistical study was carried out with the objective of establishing typical measures of variability of soil-water characteristic curves (SWCCs) and hydraulic conductivity functions. A methodology was established based on appropriate fitting functions, parameters, and soil grouping criterion. A total of 186 dataset were sampled from a database of soils and analysed using the proposed methodology. Measures of variability were established for the parameters of each soil group, in terms of standard deviations and coefficients of variation. The information provided herein can be used as an approximation in geotechnical analyses.

RÉSUMÉ

Une étude statistique a été effectuée avec l'objectif d'établir des mesures typiques de variabilité des courbes caractéristiques sol-eau (SWCCs) et des fonctions hydrauliques de conductivité. Une méthodologie a été établie basée sur des fonctions convenables appropriées des paramètres et de critère groupant de sol. Un total de 186 courbes a été prélevé d'une base de données des sols et analysé en utilisant la méthodologie proposée. Des mesures de variabilité ont été établies pour les paramètres de chaque groupe de sol, en termes d'écarts type et coefficients de variation. Les informations fournies ici, peuvent être employées comme approximation dans des analyses géotechniques.

1. INTRODUCTION

Geotechnical analyses often require the quantification of the variability of the input variables. Generally, typical values of parameter variability can be established and used in engineering practice. Numerous studies can be found in the literature suggesting values for the typical variability of saturated soils parameters, usually presented in terms of the coefficient of variation. Conversely, a limited number of studies are available on the variability of unsaturated soil parameters. The lack of information on the variability for unsaturated soil parameters is due to limited available data and the lack of an appropriate methodology for the assessment of variability.

The primary objective of this paper is to present the results of a statistical study carried out using 186 datasets of soil-water characteristic curves and hydraulic conductivity functions. The variability information provided herein is intended as a first approximation and can be refined once additional and/or site-specific information becomes available.

2. GEOTECHNICAL VARIABILITY AND STATISTICAL ASSESSMENT APPROACHES

Geotechnical property variability results from several factors. Whitman (1984) identified four sources of parameter uncertainty; namely, (i) inherent spatial variability; (ii) random testing errors; (iii) "statistical" error due to a finite number of samples; and (iv) measurement bias. *Spatial variability* and *random testing errors* are characterised as *scatter* in the data. Data *scatter* averages over large soil volumes and its contribution to parameter uncertainty decreases as the volume of the problem increases. The other two sources of variability are *systematic errors* that do not average out over the soil volume and have a significant influence on the overall uncertainty. Phoon and Kulhawy (1999a) included estimation model uncertainty as a source of parameter uncertainty.

The components of soil property uncertainty can be evaluated using conventional descriptive statistics. A review of descriptive statistics applied in geotechnical engineering problems was presented by Christian et al. (1992). Some of the unbiased estimators commonly used for the mean, variance, coefficient of variation, correlation, and autocorrelation can be found in Christian et al. (1992).

The variance, $Var[x]$, and standard deviation, $\sigma[x]$, are commonly used measures of variability. The *variance* and *standard deviation* are strongly affected by the magnitude of the variable. As a result, the variance and standard deviation of different parameters cannot be compared.

The coefficient of variation, $CV[x]$, provides a useful measure of parameter variability. $CV[x]$ can often be assumed to be independent of the magnitude of the variable. One shortcoming is that the equation for $CV[x]$ breaks down if the mean of the variable approaches zero.

Christian et al. (1992) presented an example where field vane test data were used to characterise the variability of the undrained shear strength. The individual components of uncertainty, as described by Whitman (1984), where carefully estimated. The *scatter* component of variability was divided into *spatial variability* and *measurement noise* by means of an autocovariance function obtained from spatially distributed data. A large quantity of data is required in order to obtain an autocorrelation function.

Unfortunately, conventional descriptive statistics are rarely used in geotechnical engineering practice. The number of samples and tests required to obtain values of statistical significance is prohibitive for most projects, both in terms of costs and available time. As an alternative, an estimation of *CV* can be obtained by subjectively selecting the "highest and lowest conceivable values" of a given parameter and using the *three-sigma rule*. Dai and Wang (1992) and Duncan (2000) present examples of application of the *three-sigma rule*.

Another alternative to the above approaches is to adopt published values of typical variability and to neglect the separation of the distinct sources of uncertainty (Harr, 1987). Approximate values of *CV* for unsaturated soil parameters would provide useful information that could be used within this framework.

3 PROPOSED METHODOLOGY

The properties studied herein are the soil-water characteristic curve, SWCC, and the hydraulic conductivity function. Only the drying curves were used in the present study. A total of 186 data records were sampled from a soils database (SoilVision Systems, 2003). The number of records sampled from the SoilVision database was limited by the number of "complete" soil records. To be considered "complete", a soil record was required to have a grain-size distribution, the soil porosity, *n*, the drying soil-water characteristic curve, and the hydraulic conductivity function. The porosity was required for the total water storage estimation. The grain-size distribution was required to classify each soil record and form groups of soils with similar textures.

3.1 Fitting Equations and Corresponding Soil Parameters

Unsaturated soil properties are generally defined as nonlinear functions of the stress state variables. The nonlinear characteristics have resulted in more elaborate procedures for statistical assessment than that adopted for constant, saturated soil properties.

Confidence bands are often used in the statistical characterization of functions (Bates and Watts, 1988). This was the approach used in the statistical characterization of the SWCC by Mishra et al. (1989) and Zapata et al. (2000). Unfortunately, such an approach is not convenient for reliability analysis.

Reliability analyses can be more easily undertaken if property variability is characterised in terms of the variability of a finite and relatively small number of curve parameters. Parameters with physical significance and mathematically independent must be defined.

3.1.1 Soil-Water Characteristic Curve Equation

The soil-water characteristic curve data records can be fitted using the unimodal equations proposed by Gitirana Jr. and Fredlund (2004). Two equations were used herein; namely, the unimodal equation with two bending points and the unimodal equation with one bending point:

$$S = \frac{S_1 - S_2}{1 + \left(\psi / \sqrt{\psi_b \psi_{res}}\right)^d} + S_2 \qquad [1]$$

$$S_i = \frac{\tan\theta_i (1 + r_i^2) \ln(\psi/\psi_i^a)}{(1 - r_i^2 \tan^2\theta_i)} + (-1)^i \cdot \frac{(1 + \tan^2\theta_i)}{(1 - r_i^2 \tan^2\theta_i)} \sqrt{r_i^2 \ln^2(\psi/\psi_i^a) + \frac{a^2 (1 - r_i^2 \tan^2\theta_i)}{(1 + \tan^2\theta_i)}} + S_i^a \qquad [2]$$

where $i = 1, 2$; $\theta_i = -(\lambda_{i-1} + \lambda_i)/2$; $r_i = \tan((\lambda_{i-1} - \lambda_i)/2)$; $\lambda_0 = 0$; $\lambda_i = \arctan[(S^a_i - S^a_{i+1})/(ln(\psi^a_{i+1}/\psi^a_i))]$; $S_1^a = 1$; $S_2^a = S_{res}$; $S_3^a = 0$; $\psi_1^a = \psi_b$; $\psi_2^a = \psi_{res}$; $\psi_3^a = 10^6$; $d = 2 \cdot \exp(1/ln(\psi_{res}/\psi_b))$; and *a* is a parameter controlling the curve sharpness at the two bending points. The unimodal equation with one bending point can be obtained directly from Eq. 2, with $S = S_1$, $\theta_1 = -\lambda/2$; $r_1 = \tan(\lambda/2)$; and $\lambda = \arctan[1/(ln(10^6/\psi_b))]$;

According to Eq. 1 a SWCC with two bending points can be defined by four parameters; namely, ψ_b = air-entry value; ψ_{res} = residual suction; S_{res} = residual degree of saturation; and the sharpness parameter, *a*. These four parameters correspond to the number of shape features of a typical SWCC (Fig. 1). Figure 1 also shows that two shape features define a SWCC with one bending point; namely, ψ_b and *a*.

Other parameters can be defined; namely, λ_d, the primary drainage slope; and λ_{res}, the residual drainage slopes:

$$\lambda_d = \frac{1 - S_{res}}{\log_{10}\left(\psi_{res}/\psi_b\right)} \qquad [3]$$

$$\lambda_{res} = \frac{S_{res}}{\log_{10}\left(1{,}000{,}000/\psi_{res}\right)} \qquad [4]$$

The primary drainage slope, λ_d, indicates the distribution of the pore-sizes. The more uniform the pore-size distribution is (i.e., the less spread is the pore-size distribution) the steeper the primary drainage slope. The residual drainage slope, λ_{res}, does not bear as much physical meaning as λ_d.

Regardless of the parameters chosen to describe the SWCC, four parameters must be used because there are four distinct shape features. Difficulties could arise in probabilistic analyses using ψ_b and ψ_{res} due to the possibility of estimate points or random realisations of ψ_b and ψ_{res} where $\psi_b > \psi_{res}$. **The set of parameters□** ψ_b,□ λ_d, λ_{res}, and *a* was deemed more appropriate. The parameter *a* can be considered as a fixed value for each soil type.

Most sampled SWCC record data show curves with two bending points (i.e., curve 2 in Fig. 1). However, most of the clays have SWCCs with only one bending point and do not present a distinguishable residual point (i.e., curve 1 in Fig. 1). As a result, all clay data records were fitted using the unimodal equation with one bending point.

3.1.2 Hydraulic Conductivity Function

The hydraulic conductivity functions were fitted using a bi-linear function on a log versus log plot (see Fig. 2). The bi-linear shape was found to fit reasonably well for most experimental curves. The value of k^w_{sat} was not treated as a fitting parameter but as an independent fixed measurement corresponding to each sampled record:

$$k^w = k^w_{sat} \text{ for } (u_a - u_w) \square \psi_{bk}$$
$$k^w = k^w_{sat}\left[\psi_{bk}/(u_a - u_w)\right]^{\eta} \text{ for } (u_a - u_w) > \psi_{bk} \qquad [5]$$

where k^w = hydraulic conductivity; k^w_{sat} = saturated hydraulic conductivity; $(u_a - u_w)$ = matric suction; ψ_{bk} = break point in the hydraulic conductivity function; η = slope of the hydraulic conductivity function.

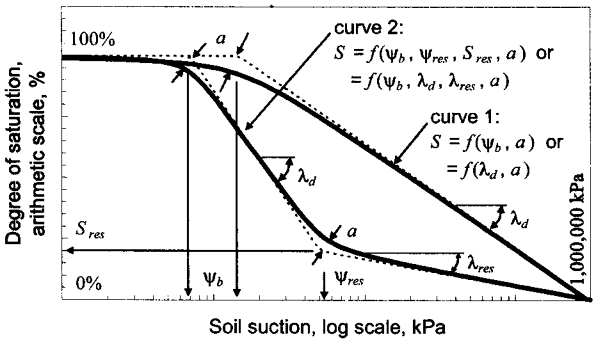

Figure 1 Idealization of unimodal soil-water characteristic curves with one and two bending points.

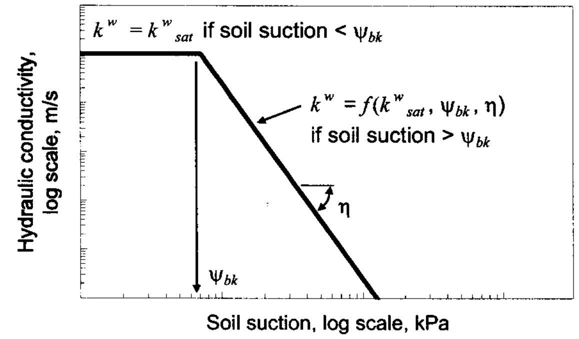

Figure 2 Idealization of a hydraulic conductivity function.

Equation 5 resembles the equation proposed by Brooks and Corey (1964), where η is defined by the pore-size distribution index obtained from the SWCC. In theory, the break point, ψ_{bk}, corresponds to the air-entry value, ψ_b. However, the air-entry value obtained from the SWCC fitting does not always match the break point observed in the hydraulic conductivity function.

In summary, the following soil parameters need to be statistically assessed in order to completely define the SWCC and hydraulic conductivity function: ψ_b, λ_d, λ_{res}, k^w_{sat}, ψ_{bk}, and η. The parameter "*a*" was assumed as a fixed value for each soil type. A value *a* equal to 0.075 was selected for Sands, *a* equal to 0.050 for Loams, and *a* equal to 0.025 for Clays.

The data records sampled from the SoilVision database was first imported into MS Excel 97^{tm}. Best-fit was performed by minimizing the sum of the squared residuals (SSR). The nonlinear minimization solver available in MS Excel 97^{tm} was utilized. Each curve parameter was considered as a random variable with a certain frequency distribution. The frequency distributions can be statistically characterized based on statistical descriptive measures. This approach is similar to that traditionally used for other soil property functions such as shear strength (Lumb, 1966, 1970, Whitman, 1984, Christian et al. 1994, Phoon and Kulhawy, 1999b).

3.2 Soil Grouping

The 186 soil records correspond to diverse soils with distinct characteristics. Ideally, the statistical assessment of individual "soil groups" is preferable even though the definition of the term "soil group" is not totally precise.

The number of "complete" records in the SoilVision database presenting the Atterberg limits is small, making the use of the USCS (ASTM, 1993) system unfeasible. As a result, the USDA (Soil Survey Staff, 1975) system has been adopted for soil grouping.

Figure 3 presents the textural diagram used by the USDA system for the classification of soils (Soil Survey Staff, 1975) along with the textural characteristics of the records sampled. The percentages of sand, silt and clay plotted in Fig. 3 were defined by the following grain-size intervals: Clay – x < 0.002 mm; Silt – 0.002 □ x < 0.05 mm; Sand – 0.05 □ x < 2 mm; and Coarse – 2 □ x < 300 mm

Three main groups are identified in Fig. 3; namely, sands (*Sa*), loams (L), and clays (*C*). The silt and silty fractions (*Si*) are placed within the loam fraction. The number of sampled records pertaining to each soil group is 62, making a total of 186 sampled records. Each of the three main soil groups is subdivided into a number of subgroups. Statistical analyses to each soil subgroup can be also performed, though an insufficient number of sampled records was found in certain subgroups, notably the silts group (*Si*), Silt Clay Loam group (*Si-C-L*), Clay Loam group (*C-L*), and the Sand Clay group (*Sa-C*).

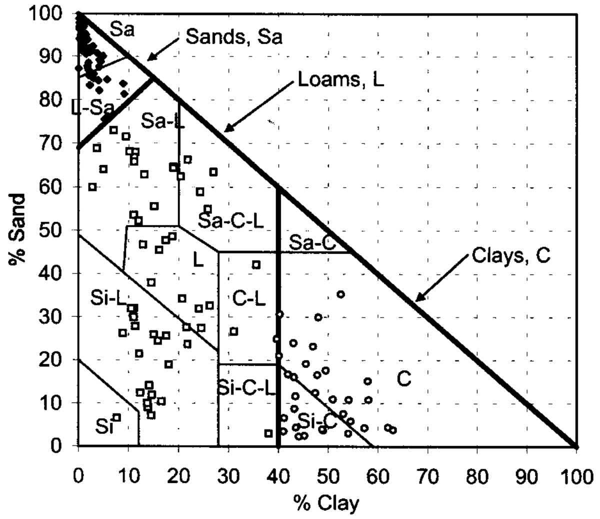

Figure 3 Sampled soil records classified according to the USDA system.

4. RESULTS OF THE STATISTICAL ASSESSMENT OF UNSATURATED SOIL PARAMETERS

The statistical characterization of the parameters defining the SWCC and hydraulic conductivity function involved several steps. First, 186 dataset were sampled from the SoilVision database. Figure 4 and 5 present the datasets for the sand soils as an example. Next, fitting analyses were performed for each of the 186 datasets using the equations presented in the previous sections. The fitting results obtained by considering all datasets of sand soils together are presented for illustrative purposes (see Figs. 4 and 5). Ultimately, the soil parameters n,□ ψ_b, λ_d, λ_{res}, k^w_{sat}, ψ_{bk}, and η were determined or collected for all sampled records. The parameters λ_d and λ_{res} were calculated based on Eqs. 3 and 4.

Normality tests presented by Gitirana Jr. (2005) have shown that the unsaturated soil parameters studied herein are log normally distributed. As a result, the natural logarithm of all parameters is considered herein, with exception of porosity.

4.1 Descriptive Statistics and Measures of Central Tendency of Unsaturated Soil Parameters

Table 1 present descriptive statistical parameters obtained for the three soil groups; namely, sands (*Sa*), loams (*L*), and clays (*C*). The minimum and maximum values are presented, along with measures of central tendency and a measure of data variability, the standard deviation. Three measures of central tendency were calculated; namely, the median, the mean, and the fitting parameters obtained by using all datasets together.

The mean value of porosity is 0.410 for sands, 0.501 for loams, and 0.534 for clays. The median values obtained do not differ considerably from the mean. The minimum, maximum, median, and mean porosities increased for finer grained soils, as expected. The standard deviations also increased for finer grained soils, indicating a possibly higher variability. The coefficients of variation presented in the next sections will give a more complete representation of parameter variability.

The mean and median values of $ln(\psi_b)$ are in reasonable agreement, corroborating the log normality of ψ_b. The maximum and mean values of air-entry value increased for finer grained soils, as expected. The overall trends regarding the value of ψ_{bk} and the soil texture was the same as that observed for ψ_b.

The mean and median values of $ln(\lambda_d)$ are in relatively better agreement, also in conformity with the log normality of λ_d. The values of λ_d for sands are higher than those of loams. Loam soils have broad pore-size distributions that reflect well-graded grain-size distributions.

The values of λ_{res} for sands were significantly lower than those of loams, as expected. The smaller pores found in loam soils are capable of holding water at higher soil suctions due to relatively larger capillary forces. Fractions of clay found in the loam soils also contribute to increase the values of λ_{res}.

The mean and median values of $ln(k^w_{sat})$ are in close agreement. This observation reflects the log normality of k^w_{sat}. The mean values of $ln(k^w_{sat})$ can be considered better measurements of central tendency. The range of variation of $ln(k^w_{sat})$ increased for finer grained soils. The relative variability of $ln(k^w_{sat})$ will be verified by better measures of variability, such as the coefficient of variation. The values of k^w_{sat} and $ln(k^w_{sat})$ decreased for finer grained soils, as expected.

The hydraulic conductivity function slope was also analysed in terms of natural logarithm. The mean and median values of $ln(\eta)$ are in relatively good agreement, reflecting the log normality of η. The observed changes of η according with the soil texture were as expected. The values of η for sands are higher than those of loams and clays. This trend was observed for the minimum, maximum, median, and mean values.

4.2 Variability of Unsaturated Soil Parameters

Table 2 presents a summary of the coefficients of variation, *CV*, obtained. As in the previous section, the parameters ψ_b, λ_d, λ_{res}, k^w_{sat}, ψ_{bk}, and η were analysed in terms of natural logarithm, because the use of a natural logarithmic transformation results in simpler and more meaningful results.

The results presented in Table 2 indicate that the values of *CV* tend to increase when soil records pertaining to distinct soil types are treated as a single group. This result was somehow expected, as the sampled soil records appear to form more distinct populations when analysed as textural groups.

The sand group appears to present lower values of *CV*, with a few exceptions. For instance, the *CV* of $ln(\lambda_{res})$ and of $ln(k^w_{sat})$ of sands was higher than that of loams; clays presented the lowest vales of *CV* of $ln(k^w_{sat})$. Other factors must be responsible for the lower *CV* of $ln(k^w_{sat})$ for loams and sands.

The coefficients of variation of porosity obtained were 13.5% for sands, 17.1% for loams, and 18.7% for clays. The *CV* of porosity increased for fine grained soils and none of the values appeared unrealistic. The values previously reported in the literature for porosity and void ratio vary from 10% to 20%. In most cases, the reports available do not indicate the soil type studied. The results obtained herein suggest that the variability of the soil records studied may be slightly higher than that of soil records obtained from a single location or soil formation. Nevertheless, the small difference indicates that the results presented herein are representative.

Figure 6 presents a plot of mean values of porosity versus the computed coefficients of variation. Each data point corresponds to a distinct soil group gathered from the sampled soil records. Two lines surrounding the data points were computed and plotted. These lines correspond to constant values of standard deviation of 0.05 and 0.10. One outlier was ignored. The data points plotted in Fig. 6 indicate that the measures of *CV* of porosity appear to show little variation with the mean value. The results obtained herein suggest that a value of *CV* between 13 and 19% should be adopted.

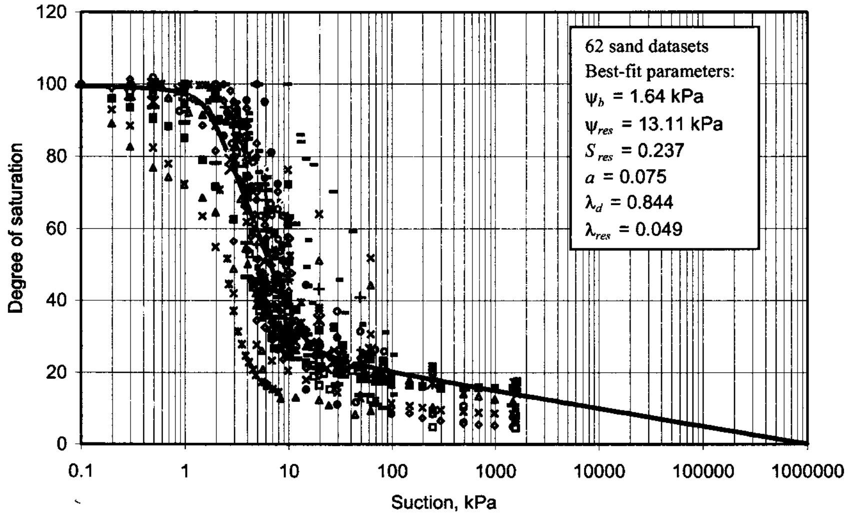

Figure 4 Soil-water characteristic curve for the sampled records of sand soils: experimental data and best-fit curve.

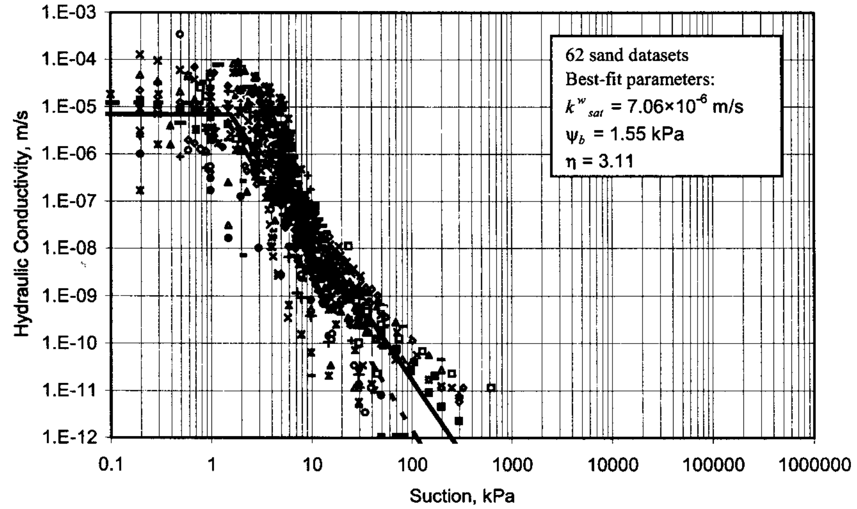

Figure 5 Hydraulic conductivity function for the sampled records of sand soils: experimental data and best-fit curve.

Table 1 Descriptive statistics for the hydraulic properties of unsaturated soils (a = 0.075 for sands, 0.050 for loams, and 0.025 for clays).

Group (1)	Measure (2)	n (3)	$ln(\psi_b)$ ln(kPa) (4)	$ln(\lambda_d)$ (5)	$ln(\lambda_{res})$ (6)	$ln(k^w_{sat})$ ln(m/s) (7)	$ln(\psi_{bk})$ ln(kPa) (8)	$ln(\eta)$ (9)
Sands	Min	0.297	-1.701	-0.904	-3.968	-14.78	-2.303	0.253
	Max	0.570	2.485	1.442	-2.068	-7.96	1.979	2.351
	Median	0.409	0.891	0.163	-3.119	-10.91	0.461	1.248
	Mean	0.410	0.856	0.198	-3.141	-11.34	0.119	1.268
	Std. Dev.	0.055	0.733	0.592	0.430	1.736	1.180	0.506
Loams	Min	0.378	-2.914	-1.734	-3.270	-17.17	-3.219	0.027
	Max	0.715	3.689	0.905	-1.940	-8.92	2.303	2.036
	Median	0.471	0.992	-0.991	-2.446	-12.64	0.000	0.721
	Mean	0.501	0.927	-0.737	-2.445	-12.58	-0.047	0.895
	Std. Dev.	0.085	1.090	0.689	0.291	1.870	1.391	0.492
Clays	Min	0.351	-3.347	--	--	-21.34	-3.507	0.089
	Max	0.790	8.504	--	--	-12.04	1.792	1.596
	Median	0.544	0.743	--	--	-15.96	0.731	0.693
	Mean	0.534	0.999	--	--	-16.03	0.527	0.682
	Std. Dev.	0.100	2.064	--	--	2.053	0.992	0.331

Table 2 Coefficients of variation for the hydraulic properties of unsaturated soils.

Group (1)	Sub-group (2)	Records (3)	n (4)	$ln(\psi_b)$ (5)	$ln(\lambda_d)$ (6)	$ln(\lambda_{res})$ (7)	$ln(k^w_{sat})$ (8)	$ln(\psi_{bk})$ (9)	$ln(\eta)$ (10)
All	---	186	20.2	151.6	294.4	18.1	20.6	608.8	53.7
Sands	All	62	13.5	85.6	299.3	13.7	15.3	988.7	39.9
	Sa	53	12.7	92.1	279.9	13.9	15.4	5439.9	43.0
	L-Sa	9	13.7	53.2	655.6	8.5	15.3	111.3	14.4
Loams	All	62	17.1	117.6	93.5	11.9	14.9	2962.5	55.0
	Sa-L	15	11.3	105.5	74.5	10.2	13.6	539.3	60.3
	Si-L	27	13.5	74.3	133.2	11.4	15.7	365.4	57.7
	L	12	18.0	684.0	49.6	8.9	13.5	177.2	33.3
	Sa-C-L & *C-L*	8	16.3	149.2	101.0	12.2	14.3	937.3	55.3
Clays	All	62	18.7	206.6	--	--	12.8	188.2	48.6
	Si-C	22	26.3	125.5	--	--	10.4	76.5	52.2
	Sa-C & *C*	40	13.4	295.2	--	--	13.3	369.2	44.7

The coefficient of variation of $ln(\psi_b)$ is 85.6% for sands, 117.6% for loams, and 206.6% for clays. The coefficients of variation of $ln(\psi_{bk})$ seem unreasonably high. The high values of CV of $ln(\psi_b)$ and remarkably high values of CV of $ln(\psi_{bk})$ are due to fact that some of the mean values of $ln(\psi_b)$ and $ln(\psi_{bk})$ are significantly close to 0 ln(kPa). The values of CV are exceptionally sensitive to small changes in the mean value when the mean value is between -1 and 1 ln(kPa) and close to 0 ln(kPa).

Figure 7 presents a plot of absolute values of the mean $ln(\psi_b)$ and $ln(\psi_{bk})$ versus the computed CV's. Each data point corresponds to a distinct soil group gathered from the sampled soil records. The two lines surrounding the data points correspond to constant values of SD equal to 0.8 ln(kPa) and 2.1 ln(kPa). Figure 7 shows that most data points are located between CV = 75 and 125%. However, when the mean values of $ln(\psi_b)$ and $ln(\psi_{bk})$ fall between -0.5 and 0.5 ln(kPa) and approach 0 ln(kPa), significantly higher values of CV are obtained.

The coefficients of variation for $ln(\psi_b)$ and $ln(\psi_{bk})$ show a clear trend with soil texture. The values of CV of $ln(\psi_b)$ and $ln(\psi_{bk})$ increase for fine grained soils. This result is expected, since it is generally known that sands have air entry values varying along a small range of soil suctions and loams and clays present widely varying values of air-entry value. In terms of comparison between the variability of $ln(\psi_b)$ and $ln(\psi_{bk})$, the results show that similar results are obtained in terms of standard deviation. Most data points are within a range of variation of standard deviation between 0.8 and 2.1 ln(kPa). However, the mean values of $ln(\psi_{bk})$ are generally smaller then the mean values of $ln(\psi_b)$. As a result, the values of coefficient of variation of $ln(\psi_b)$ are significantly affected because the mean values are close to 0 ln(kPa).

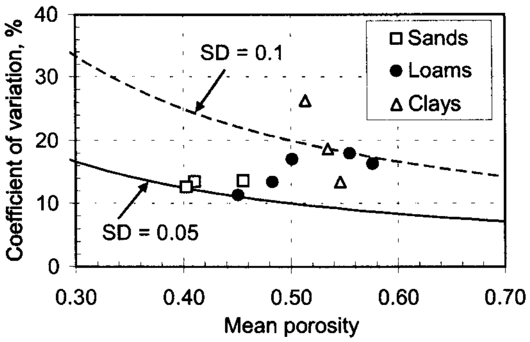

Figure 6 Mean versus the coefficient of variation of the soil porosity, n.

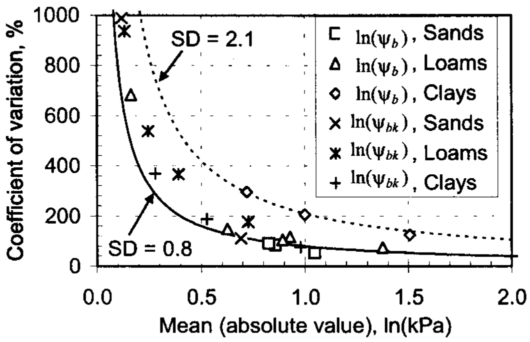

Figure 7 Mean versus the coefficient of variation of the natural logarithm of air-entry value obtained from the SWCC and from the k^w function, $ln(\psi_b)$ and $ln(\psi_{bk})$, ln(kPa).

The results obtained herein suggest that a value of *CV* between 75 and 125% should be adopted in cases where the mean values of $ln(\psi_b)$ or $ln(\psi_{bk})$ are not within the -0.5 to 0.5 *ln*(kPa) range. In cases where the mean values of $ln(\psi_b)$ or $ln(\psi_{bk})$ are within the -0.5 to 0.5 *ln*(kPa) range, the values of *CV* suggested sould be abandoned and the parameter variability should be defined respecting a range of standard deviations between 0.8 and 2.1 *ln*(kPa).

The variability of the remaining parameters was analysed in the same fashion. The coefficient of variation of $ln(\lambda_d)$ found is 299.3% for sands and 93.5% for loams. The *CV* of $ln(\lambda_d)$ of loams is significantly lower than that of sands. This result does not reflect the widely varying grain size distribution of the loams group and the knowingly narrower variation of grain size distribution of the sands group. This apparently "abnormal" result was due to the mean values of $ln(\lambda_d)$, that had a significant effect on the values of *CV*. The results suggest that a value of *CV* between 75 and 100% should be adopted in cases where the mean values of $ln(\lambda_d)$ are not within the -0.5 – 0.5 range. In cases where the mean values of $ln(\lambda_d)$ are within the -0.5 – 0.5 range, the values of *CV* suggested above should be abandoned and the parameter variability should be established respecting a range of standard deviations between 0.45 and 0.80.

The coefficient of variation of $ln(\lambda_{res})$ is 13.7% for sands and 11.9% for loams. The *CV* of $ln(\lambda_{res})$ of loams was slightly lower than that of sands. The mean values of $ln(\lambda_{res})$ did not have a significant effect on the values of *CV* as they did not fall within the -0.5 and 0.5 range. The results suggest that a value of *CV* between 8 and 12% should be adopted.

The coefficients of variation of $ln(k^w_{sat})$ found were 15.3% for sands, 14.9% for loams, and 12.8% for clays. The *CV* of $ln(k^w_{sat})$ appears to slightly decrease for fine grained soils. The measures of *CV* of $ln(k^w_{sat})$ appear to show little variation with the mean values. The results obtained herein suggest that a value of *CV* of $ln(k^w_{sat})$ between 13 and 16% should be adopted.

The coefficients of variation of $ln(\eta)$ found are 39.9% for sands, 55.0% for loams, and 48.6% for clays. The *CV* of $ln(\eta)$ do not show any clear relationship with soil texture, but appeared to slightly increase for fine grained soils. The results do not indicate any clear relationship between the measures of *CV* of $ln(\eta)$ and the mean values. The results obtained herein suggest that values of *CV* of $ln(\eta)$ between 40 and 55% should be adopted.

5. SUGGESTED VALUES FOR THE VARIABILITY OF UNSATURATED SOIL PARAMETERS

Table 3 presents a summary of the coefficients of variation for the unsaturated soil parameters n, $ln(\psi_b)$, $ln(\lambda_d)$, $ln(\lambda_{res})$, $ln(k^w_{sat})$, and $ln(\eta)$. The values of $ln(\psi_b)$ were established by combining the results obtained for $\ln(\psi_b)$ and $ln(\psi_{bk})$. General ranges considering all soil types are presented, along with specific information for each main soil group, *Sa*, *L*, and *C*.

Some ranges of coefficient of variation suggested in column 2 of Table 3 are not applicable when the mean value of the parameter is within the range -0.5 and 0.5. In that case, the values of standard deviation provided in column 2 of Table 3 must be adopted. The same criteria regarding the mean values between -0.5 and 0.5 must be extended for the variability of each soil group. Therefore, the values provided in columns 3, 4, and 5 of Table 3 must also be replaced by the standard values provided in column 2, when the mean value of the variable is within the range -0.5 and 0.5.

6. CONCLUDING REMARKS

This paper presented a comprehensive statistical study of unsaturated soil properties. The primary objective was to present approximate variability values that could be used in probabilistic geotechnical analyses. The property functions studied were the soil-water characteristic curve and the hydraulic conductivity function. The statistical study was based on a large database of soils (SoilVision Systems, 2003). The study was undertaken considering three soil groups established based on the USDA textural classification system; namely, sands, loams, and clays.

Table 3 Variability of unsaturated soil properties.

Soil parameter (1)	General ranges for all soils of the Coefficient of Variation (*CV*) and Standard Deviation (*SD*) (2)	Coefficient of Variation		
		Sands (3)	Loams (4)	Clays (5)
n	$CV = 13 - 19\%$	13%	17%	19%
$ln(\psi_b)$	$CV = 75 - 205\%$ for $-0.5 > E[ln(\psi_b)] > 0.5$ ln(kPa) otherwise, $SD = 0.8 - 2.1$ ln(kPa)	85%	115%	205%
$ln(\lambda_d)$	$CV = 75 - 100\%$ for $-0.5 > E[ln(\lambda_d)] > 0.5$ otherwise, $SD = 0.45 - 0.80$	100%	90%	--
$ln(\lambda_{res})$	$CV = 8 - 14\%$ for $-0.5 > E[ln(\lambda_{res})] > 0.5$ otherwise, $SD = 0.20 - 0.45$	14%	12%	--
$ln(k^w{}_{sat})$	$CV = 13 - 16\%$	16%	15%	13%
$ln(\eta)$	$CV = 40 - 55\%$	40%	55%	50%

Variability measures were presented in terms of standard deviations and coefficients of variation. Fairly constant coefficients of variations were determined for various soil properties and soil groups. It was found that the coefficient of variations may provide a poor measure of variability for some soil parameters. A combination of standard deviation and coefficient of variation values were proposed for these parameters.

The coefficients of variation and standard deviations presented herein include diverse sources of uncertainty that could not be assessed. Nevertheless, the presented standard deviations and coefficients of variations serve as a general indication and as a first approximation. The information presented herein can be refined in future studies, as more information is collected and organised using an appropriate database of soils.

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