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A VOLUME-MASS CONSTITUTIVE MODEL FOR UNSATURATED SOILS

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ABSTRACT:
This paper presents a brief description of a new volume-mass constitutive model for unsaturated soils. The model requires relatively simple laboratory data for calibration. The volume and water content changes can be independently computed using the model. The model is also capable of taking into account: i) the hysteretic nature of the soil-water characteristic curve; and ii) both elastic and plastic deformations in the soil. The prediction results for 3 artificial soils (i.e., sand, silt and clay) are consistent with observed behavior.

RÉSUMÉ:
Cet article présente une courte description d'un modèle constitutif de la nouvelle volume-masse pour les sols insaturés. Le modèle exige des données relativement simples de laboratoire pour le calibrage. Les changements de volume et de teneur en eau peuvent être indépendamment calculés en utilisant le modèle. Le modèle est également capable de tenir compte: i) la nature par hystérésis de la courbe caractéristique de sol-eau; et ii) déformations élastiques et en plastique dans le sol. Les résultats de prévision pour 3 sols artificiels (c.-à-d., sable, vase et argile) sont conformés au comportement observé.

1 INTRODUCTION

Volume-mass constitutive relationships play an important role in modeling unsaturated soil behavior. The volume-mass constitutive relationships can be used in the assessment of other unsaturated soil properties such as shear strength and hydraulic conductivity. Many geotechnical problems should be solved as 'coupled' solutions of seepage, volume change and shear strength (Vu and Fredlund 2004). A rigorous volume-mass constitutive model will assist in archiving a seepage, volume change and shear strength solution with few simplifying assumptions.

There are a number of volume-mass constitutive models that have been proposed (Alonso et al., 1990; Wheeler and Sivakumar, 1995; Batz and Graham, 2003). Most models do not predict water content or assume that the degree of saturation is independent of net mean stress. A new volume-mass constitutive model has been proposed that is capable of: i) predicting water content; ii) taking into account both elastic and plastic strains, and iii) taking into account hysteretic nature of the soil-water characteristic curve. The volume-mass constitutive model was proposed for only isotropic loading/unloading conditions. A brief description of the model is presented in this paper.

2 TERMINOLOGY

There is no single, unique relationship between volume change and water content change for an unsaturated soil. Volume change and water content change in an unsaturated soil are controlled by two independent mechanisms: i) stress-strain (i.e., mechanical theory) behavior and ii) adsorption-drainage behavior (i.e., capillary theory). Therefore, water content change and volume change should be predicted separately in terms of a volume-mass constitutive model.

It is well accepted in the soil mechanics that the soil particles are incompressible; that is, deformations in the overall soil mass are directly related to changes in the volume of the pores. The soil-water appears only in pores; therefore, volume change and water content change are directly referenced to changes in the volume and shape of pores of the soil.

The "shape" of a pore can be referred to as "open pore diameter or neck pore diameter" and "body pore diameter" in the relation to the capillary theory (Haines, 1930). The "open pore diameter" is often referred to as the "drying soil suction" or the air entry value of the pore (Neél, 1942, 1943; Mualem, 1973, 1974). The "body pore diameter" is often referred to as the "wetting soil suction" or the water entry value of the pore.

Pores in a soil are comprised of various shapes and volumes. The pore-size distribution in a soil is a soil property that relates the volume and the shape of pores in the soil. The pore-size distribution of a soil at any stress state provides information regarding both the total volume and the volume of water in the soil; therefore, in order to predict volume and water content, it is necessary to predict the pore-size distribution of the soil. The pore-size distribution is changed when changing soil suction or net mean stress (Fig. 1). It is necessary to predict changes in the volume and in the drying/wetting suction of each group of pores along the pores-size distribution in order to predict the pore-size distribution of a soil at any stress states. In other words, the stress-strain relationship for the soil structure surrounding each pore group must be described.

Four major steps are proposed for the development of a volume-mass constitutive model; namely, i) propose basic assumptions for the response of a pore to changes in net mean stress and soil suction; ii) formulate stress-strain relationship for the soil structure surrounding a pore including: changes in the air entry value, water entry value, yield stress, volume and water content; iii) determine the compression and unloading-reloading indices for each group of pores (i.e., when pore are filled water); and 4) propose a model for the hysteretic nature of the soil-water characteristic curve in the context of pore-size distribution.

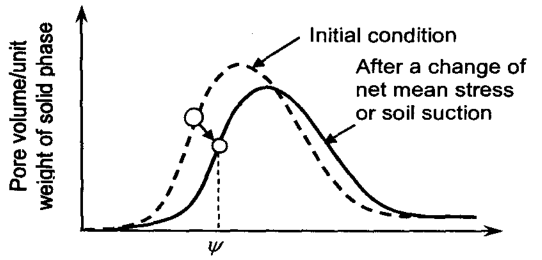


Figure 1. Schematic illustration of the change of the pore-size distribution due to a change in net mean stress or soil suction.

3 THEORY

3.1 Stress State Variables

In the case of isotropic loading condition (i.e., $\sigma_x = \sigma_y = \sigma_z$), the following stress state variables control the mechanical behavior of an unsaturated soil: i) net mean stress, $p = (\sigma_1 + \sigma_2 + \sigma_3)/3 - u_a$; ii) soil suction, $\psi = (u_a - u_w)$; and iii) mean effective stress, $p^* = (\sigma_1 + \sigma_2 + \sigma_3)/3 - u_w$. However, only two of the above three stress variables are independent (Fredlund and Morgenstern, 1977). Net mean stress, p, and soil suction, ψ, are two stress state variables that will be used for the proposed model. The void ratio, e, of the soil is the primarily variable used to represent the overall volume of the soil. The gravimetric water content, w, of the soil is the primary variable used to represent the amount of water in the soil.

3.2 Reference Pore-Size Distribution

The pore-size distribution curve of a soil is generally plotted on a semi-logarithmic soil suction graph (Fredlund, 1999; Simms and Yanful, 2001). There are two types of the pore-size distribution curves of a soil; namely, i) the plot of the ratio of pore volume per unit weight versus open pore diameter (i.e., drying suction or air entry value of the pore) referred to as the ***drying pore size distribution***, *DPD*, and ii) the plot of the ratio of pore volume per unit weight versus body pore diameter (i.e., wetting suction or water entry value of the pore) referred to as the ***wetting pore size distribution***, *WPD*. For an insignificant volume change soil, the *DPD* and *WPD* are directly related to the initial/boundary drying and the boundary wetting soil-water characteristic curve of the soil (Fig. 2).

For a significant volume change soil, the pore-size distribution of a soil changes with soil suction and net mean stress. Therefore, In order to predict the change in the pore-size distribution, it is necessary to select a reference pore-size distribution. The authors have observed that the drying pore-size distribution curve corresponding to an initially dry soil (i.e., at 10^6 kPa on the initial drying process of the slurry soil) can provide a meaningful and easy reference state. In this model, the *reference pore-size distribution curve* of a soil is defined as the pore-size distribution curve of a soil completely dried from the slurry.

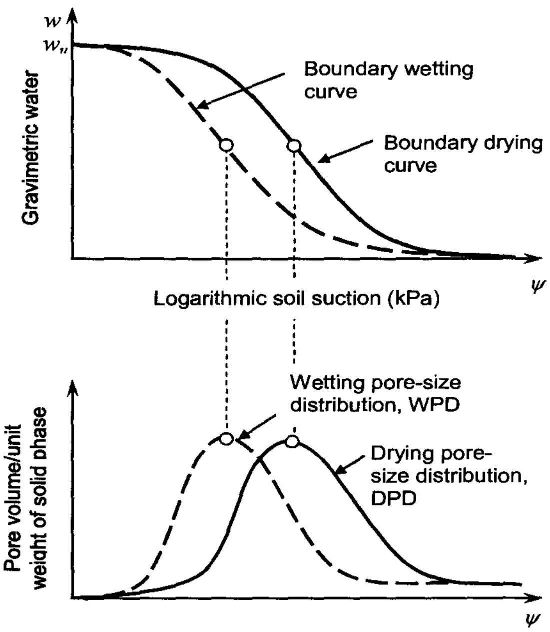


Figure 2. Schematic illustration of the change of the pore-size distribution due to a change in net mean stress or soil suction.

A soil has two reference pore-size distribution curves under completely dry conditions (i.e., with respect to drying and wetting suctions). The *reference DPD* of a soil provides information regarding the air entry value and the distribution in volume of pores in the soil (i.e., directly related to the initial/boundary drying soil-water characteristic curve). Similarly, the *reference WPD* of a soil provides information regarding the water entry value and the distribution in volume of pores in the soil (i.e., directly related to the boundary wetting soil-water characteristic curve).

3.3 Basic Assumptions

Six assumptions are made in the development of the proposed constitutive model,. These assumptions are based on the findings of previous studies. The six assumptions can be described as follows:

- <u>*Assumption 1:*</u>

In an unsaturated soil, a particular pore under consideration in the soil has only two states; namely, i) the pore is filled with water; or ii) the pore is dry.

- *Assumption 2:*

Each water-filled pore in the soil also has two indices, namely: i) virgin compression index, C_c^p and ii) unloading-reloading compression index, C_s^p.

- *Assumption 3:*

There are two types of pores: i) collapsible pores and ii) non-collapsible pores. The collapsible pores are relative big pores and the non-collapsible pores are relative small interconnected pores.

- *Assumption 4:*

Virgin and unloading-reloading compression indices for a pore in the soil are proportional to the volume of the pore at the reference stress state (i.e., 1 kPa net mean stress and zero soil suction).

- *Assumption 5:*

Pores are deformed, and water is absorbed and drained independently.

- *Assumption 6:*

Air-filled pores are incompressible (i.e., the compression indices of an air-filled pore is equal to zero).

3.4 Mathematical Formulation

The volume-mass constitutive relationships are stress path dependent (Alonso, 1993); therefore, the stress-strain relationship for the soil structure surrounding a pore was considered for four different stress paths (i.e., loading, unloading, drying, and wetting). Descriptions of the four stress paths are not presented in this paper due to limited space. In this section, compression indices and the effect of the net mean stress to the change in the air/water entry value of a pore is presented.

3.4.1 Compression indices of a pore when it is filled with water

The initial drying soil-water characteristic curve from the slurry of a significant volume change soil can be best-fit using the following equation:

[1]

$$w(\psi)=\left(\left[w_{sat}-\frac{C_c}{G_s}\log(\psi)-w_r\right]\frac{a}{\psi^b+a}+w_r\right)\left(1-\frac{\ln\left[1+\frac{\psi}{\psi_r}\right]}{\ln\left[1+\frac{10^6}{\psi_r}\right]}\right)$$

where: C_c = virgin compression index of the soil; G_s = Particles specific gravity; w_r = curve-fitting parameter represents the residual water content; w_{sat} = curve-fitting parameter represents the water content of the slurry soil at an effective stress of 1 kPa; and *a, b* = curve-fitting parameters.

The water content appears only in the collapsible pores can be calculated as follows:

$$[2]\quad w^c(\psi)=\left(\left[w_{sat}-\frac{C_c}{G_s}\log(\psi)-w_r\right]\frac{a}{\psi^b+a}+w_r\right)$$

where: a, b = curve-fitting parameters obtained from bet-fit equation [1] to the soil-water characteristic curve of the soil from slurry. Applying assumptions #2 and #4, the *reference DPD* of the soil can be calculated as follows:

$$[3]\quad f_d(\psi)=M_s\frac{d\left(w^c(\psi)\right)}{d\psi}-\frac{aC_c}{\psi^b+a}$$

where: $w^c(\psi)$ = water content appears in the collapsible pores; M_s = weight of the solid phase. The virgin compression index of the group of pores having air entry value of ψ on the reference pore-size distribution curve can be calculated as follows:

$$[4]\quad C_c^p(\psi)=-\frac{C_c ab\ln(10)\psi^b}{\left[\psi^b+a\right]^2}$$

Similarly, the unloading-reloading compression index of the group of pores having air entry value of ψ on the reference pore-size distribution curve can be calculated as follows:

$$[5]\quad C_s^p(\psi)=-\frac{C_s ab\ln(10)\psi^b}{\left[\psi^b+a\right]^2}$$

3.4.2 Effect of the net mean stress to the change in the air/water entry value of a pore

Two equation can be written for the strains of a pore when it is dried under zero net mean stress and when it is experienced a yield stress, p_y, and then dried under a constant net mean stress, *p*. The equation for the effect of the net mean stress to the air entry value of the pore can be written as follows:

$$[6]\quad \frac{\psi_{ae}}{\psi_{ae}(p,p_y)}=1-\eta\frac{\left[(C_c-C_s)\log(p_y)+C_s\log(\psi_{ae}+p)-C_c\log(\psi_{ae})\right]}{3\left[e_{sat}-C_c\log(\psi_{ae})-w_rG_s\right]}$$

where: p_y = yield stress of the pore; ψ_{ae} = water entry value of the pore having zero yield stress and wetting under zero net mean stress; $\psi_{ae}(p,p_y)$ = air entry value of the pore when yield stress is equal to p_y and drying under a net mean stress of *p*; p_y = yield stress; η = pore-shape parameter represents the effect of the net mean stress to the change in the diameter of a pore; and w_r = residual water content. Similarly, the equation for the effect of the net mean stress to the water entry value of the pore can be written as follows:

$$[7]\quad \frac{\psi_{we}}{\psi_{we}(p,p_y)}=1-\eta\frac{\left[(C_c-C_s)\log(p_y)+C_s\log(\psi_{we}+p)-C_c\log(\psi_{ae})\right]}{3\left[e_{sat}-C_c\log(\psi_{ae})-w_rG_s\right]}$$

where: ψ_{we} = water entry value of the pore having zero yield stress and wetting under zero net mean stress; and

$\psi_{we}(p, p_y)$ = water entry value of the pore when experienced an yield stress of p_y and wetting under a constant net mean stress of p.

3.4.3 *Yield stress of a pore*

Similar to the behavior of the soil at saturation, the volume of each group of pores in an unsaturated soil is controlled by: i) yield stress; ii) stress acting on the soil structure surrounding pores in the group; and iii) compression indices of the group of pores (i.e., when the pore group is filled with water).

At high soil suctions an unsaturated soil appears to be stiffer in response to the application of an external load under drained conditions (Alonso et al., 1990; Wheeler and Sivakumar, 1995; Blatz and Graham, 2003). The yield stress of the soil structure surrounding a water-filled pore is considered to be the maximum effective stress that ever acted on the pore. For example, if a pore is dried under zero net mean stress, yield stress of the pore is equal to the air entry value of the pore. If a pore is dried under a constant net mean stress of, p, yield stress of the pore that can be calculated as follows:

[8] $p_y = (p + \psi(p, p + \psi_{ae}))$

where: p = net mean stress, and $\psi(p, p+\psi_{ae})$ can be calculated using Eq. [6].

When a pore is filled with air, the soil structure surrounding the pore is incompressible (i.e., assumption #6); therefore, the yield stress of the soil structure surrounding the pore does not change with net mean stress and soil suction. The yield stress of the soil is represented by the yield stress of each group of pores along the pore-size distribution.

3.5 Hysteresis Model for Soil-Water Characteristic Curves

A simple model for hysteretic soil-water characteristic curves of a soil is applied in the volume-mass constitutive model. The volume-mass constitutive model is developed using the pore-size distribution curve. In order to incorporate the hysteretic nature of the SWCC into the proposed volume-mass constitutive model, the hysteresis model for the SWCC must be a physically-based model (i.e., using pore-size distribution).

Numerous physically-based hysteresis models for the SWCCs have been presented (Pham *et al.*, 2005). In a physically-based model, the pore-size distribution is commonly described as a function of two variables: air entry value and water entry value of the pore (i.e., two-dimensional pore-size distribution function). Once the pore-size distribution is calculated, the model can be calibrated. However, considerable measurement data are required to calibrate the model using a 2-dimensional pore-size distribution.

A hysteresis model is developed in this paper using two one-dimensional pore-size distribution functions (i.e., wetting and drying pore-size distributions). Scanning soil-water characteristic curves are horizontal on degree of saturation soil-water characteristic curve plots. This means that each group of pores has a unique relationship between the drying and wetting suction. Therefore, the relationship between the *reference DPD* and the *reference DPD* can be described as follows:

[9] $f_w(\xi) = f_d(\xi - \Delta\xi)$

where: $f_d(\xi)$ = reference drying pore-size distribution; $f_w(\xi)$ = reference wetting pore-size distribution, $\Delta\xi$ = difference between the air entry value and water entry value of the group of pores having air entry value of ψ (i.e., log-cycles); and $\xi = \log(\psi)$.

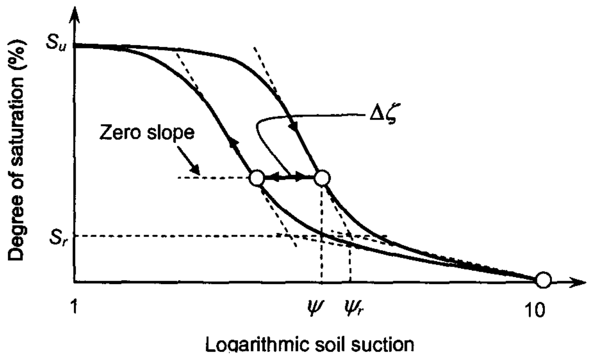


Figure 3. Schematic illustrations of the distance between the two boundary curves in log-cycle.

Pham et al. (2005) presented a model for the three key hysteretic soil-water characteristic curves of a soil (i.e., initial drying, boundary drying, and boundary wetting curves). If the distance and the slope ratio between the two boundary curves are known, the key hysteretic soil-water characteristic curves can be predicted from each others. The *reference WPD and DPD* can be calculated from the boundary soil-water characteristic curves of the soil (Eq. [3]). Therefore, two reference pore-size distribution curves can be calculated from: i) one of the three key soil-water characteristic curves; ii) the slope ratio and the distance between the two boundary curves.

The proposed model also takes into account the effect of the entrapped air. It is assumed that, the amount of air entrapped in a collapsible pore is proportional to the volume of the pore and denoted by entrapped air parameter, β. Once, a pore is empty a certain amount of air is entrapped in the pore.

3.6 Model Parameters

The proposed volume-mass constitutive model requires a simple laboratory test data for calibration. The required data for calibration can be described as follows:

i. The initial drying soil-water characteristic curve of the initially slurried soil.

ii. The pore-shape parameter (i.e., can be measured or chosen typical value for each type of soil).
iii. The parameters for the hysteretic nature of the soil water characteristic curve of the soil; namely, i) the distance between two boundary hysteretic curves, D_{SL}; and ii) the ratio between the slopes of the boundary drying and the boundary wetting curves, R_{SL}; and iii) the entrapped air parameter, β.

4 PRESENTATION OF THE MODEL PREDICTION

The application of the proposed volume-mass constitutive model is presented for three artificial soils in this section. The prediction results are presented for: i) several simple stress paths (i.e., 2D graphs) and ii) Several complete volume-mass constitutive surfaces (i.e., 3D graphs). Materials are described first, followed by a description of the stress paths, a presentation of prediction results and discussions.

4.1 Materials

Descriptions of the three artificial soils are presented in Table 1. The initial drying soil-water characteristic curves of the three soils starting from slurries are shown in Figure 4.

Table 1. Characteristics of the three artificial soils

Soil name		Sand	Silt	Clay
Soil-water characteristic curve from initially slurried	w_{sat}	0.28	0.38	0.780
	a	500	2.0E+5	5.0E+6
	b	3	2.5	2
	w_r	0.03	0.08	0.13
Characteristics of the hysteretic SWCCs	D_{SL}	0.2	0.35	0.5
	R_{SL}	2	1.5	1.5
	β	0.1	0.1	0.1
Compression indices	C_c	0.01	0.2	0.5
	C_s	0.003	0.04	0.1
Specific gravity	G_s	2.6	2.7	2.8
Pore-shape parameter	η	2	2	2

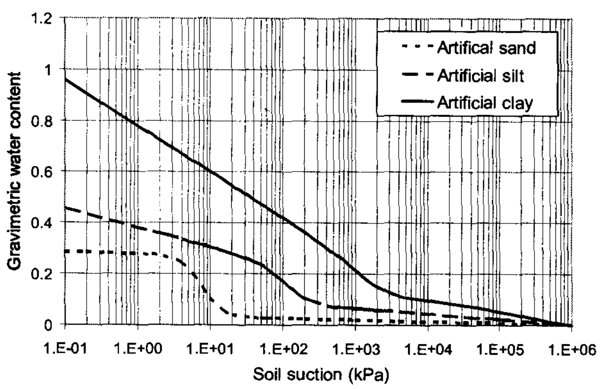


Figure 4. Initial drying soil-water characteristic curves for the artificial sand, silt and clay.

4.2 Prediction Results for Several Simple Stress Paths

The shrinkage curve of a soil can be predicted using the proposed constitutive model by measuring both the void ratio and the water content along the drying process from slurry to 10^6 kPa soil suction. The predicted shrinkage curves for the three artificial soils are shown in Figure 5. The shrinkage curve for the sand soil is almost a horizontal line since the sand does not change volume during a drying process. The shrinkage curve for silt and clay also appear reasonable. At high water contents, the shrinkage curves are 45 degree lines. The shrinkage curves are horizontal water contents below the shrinkage limit. The transitions of the shrinkage curve between the low and high water contents appear to be smooth and have reasonable curvature.

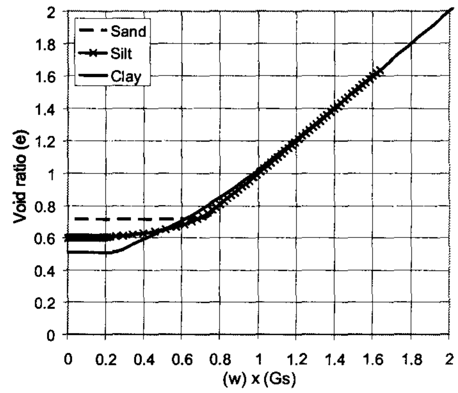


Figure 5. Shrinkage curves of the artificial sand, silt and clay.

For illustration purpose, let us predicted void ratio, gravimetric water content and degree of saturation for the artificial silt when following several simple stress paths. The stress paths are described as follows (Fig. 6): 1) the soil is initially slurried; 2) the soil is loaded to a net mean stress of 200 kPa; 3) the soil is then dried to 100 kPa and then wetted to 0.1 kPa soil suction; and 4) the soil is then dried to 10^6 kPa and then wetted to 0.1 kPa soil suction. The predicted void ratio, gravimetric water content and degree versus soil suction for the specified stress path are shown in Figures 7, 8 and 9.

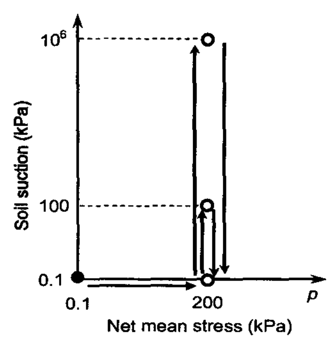


Figure 6. Stress path followed for the artificial silt.

Figures 7, 8 and 9 shows that the predicted results for the volume-mass constitutive relationships are reasonable. Hysteresis in the soil-water characteristic curve and plastic deformation have been taken into account in the simulation.

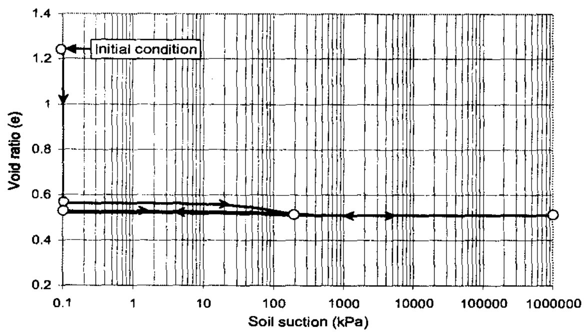


Figure 7. Void ratio for the artificial silt.

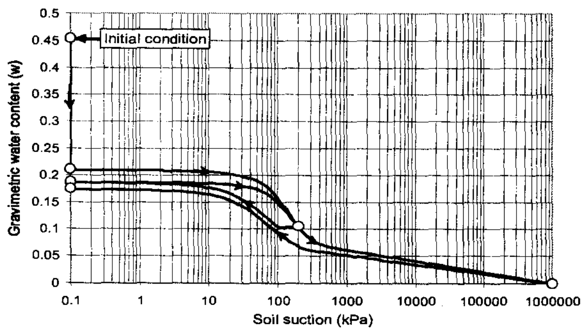


Figure 8. Gravimetric water content for the artificial silt.

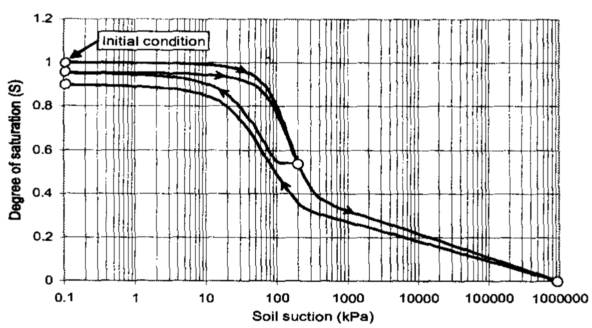
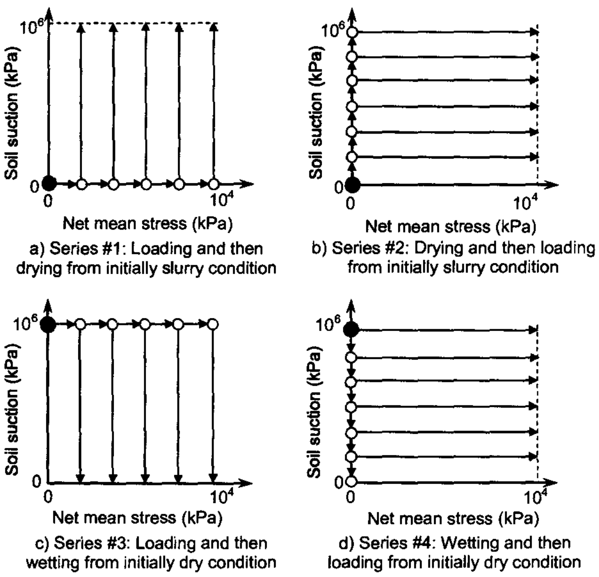


Figure 9. Degree of saturation for the artificial silt.

4.3 Prediction Results for the Volume-Mass Constitutive Surfaces

It is necessary to select several series of stress paths to present the volume-mass constitutive surfaces for the three artificial soils. The selected stress paths should be simple but capable of showing several important characteristics of an unsaturated soil (e.g., swelling and collapsible behaviors).

The four series of stress paths for studying the volume-mass constitutive surfaces of an unsaturated soil are described in Figure 10. In stress path #1 series, the soil is an initial slurry, loaded to a certain net mean stress and then dried under the constant net mean stress to a soil suction of 10^6 kPa. In stress path #2 series, the soil is an initial slurry, then dried to a certain soil suction and loaded under constant soil suction condition to a net mean stress of 10^4 kPa. In stress path #3 series, the soil is initially dried from a slurry, loaded to a certain net mean stress and then wetted under the constant net mean stress to a soil suction of 10^6 kPa. In stress path #4 series, the soil is initially dried from a slurry, wetted to a certain soil suction and then loaded under the constant soil suction to a net mean stress of 10^4 kPa.



a) Series #1: Loading and then drying from initially slurry condition

b) Series #2: Drying and then loading from initially slurry condition

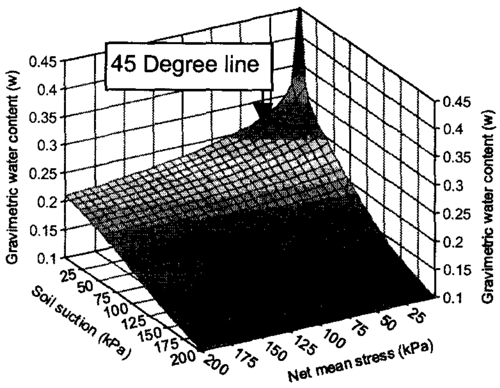
c) Series #3: Loading and then wetting from initially dry condition

d) Series #4: Wetting and then loading from initially dry condition

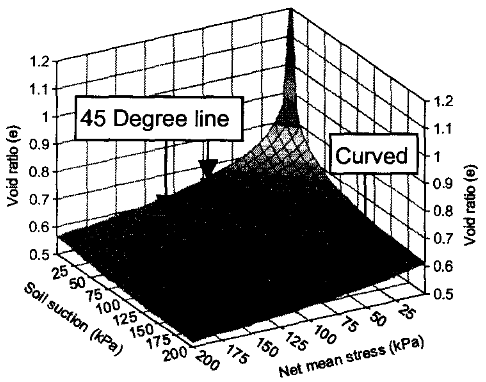
Figure 10. Four series of stress paths for studying volume-mass constitutive surfaces of an unsaturated soil.

The two stress path series #1 and #2 are used to study the mechanical behavior of various soil specimens with the same initial slurry condition and final stresses but followed by two different stress paths that involve a drying process. Similarly, the stress path series #3 and #4 are used to study mechanical behavior of various soil specimens with the same initial (i.e., dried from slurry) and final stresses but followed by two different stress paths that involve a wetting process.

Figure 11 shows that the calculated volume-mass constitutive surfaces agree well with all the postulates presented by Fredlund et al. (2000); namely, i) there is a one-to-one relationship between the effect of a change in net total stress and a change in soil suction, when the soil suction is less than air entry value of the soil (i.e., for both void ratio and water content surfaces); ii) there is a gradual curve forms from the air entry value to the primary water content reference condition, corresponding to a particular water content on the water content surface; and iii) there is a gradual curve forms from the air entry value to the second reference condition corresponding to a particular void ratio on the void ratio surface



a. gravimetric water content surface



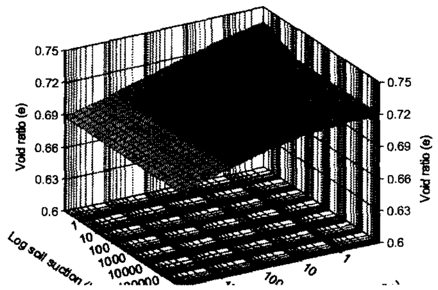
b. Void ratio surface

Figure 11. The constitutive surfaces for silt at low suctions and net mean stresses (stress path series #1 and #2).

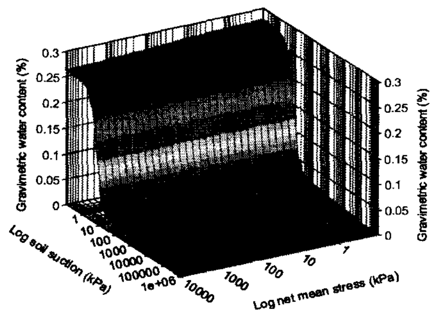
Figures 12 to 17 show several volume-mass constitutive surfaces predicted for the three artificial soils. The predicted constitutive relationships at zero soil suction and zero net mean stress planes appear to be reasonable. The volume-mass constitutive surfaces starting from initially slurry seem to have steeper slopes than that starting from air-dried condition (Figs 14 to 17). These prediction results appear reasonable.

The void ratio constitutive surfaces obtained by following stress paths #3 and #4 (Figs 15 to 17) seem to be unusual. At 10^6 kPa soil suction and 10^4 kPa net mean stress, the void ratio is higher than that at soil suction of 0.1 kPa and net mean stress of 10^4 kPa. The shape of the surfaces is similar to that measured by Matyas and Radhakrishna (1968). It is showed that the proposed volume-mass constitutive model can predict both collapsible and swelling behavior of a soil. Figure 13 shows that the air entry value of a soil is increased with each increment of net mean stress.

The void ratio surfaces measured for the artificial silt (Figs 14 to 17) show that the stress path related to a drying process is stress path independent and the stress path related to a wetting process is stress path independent.

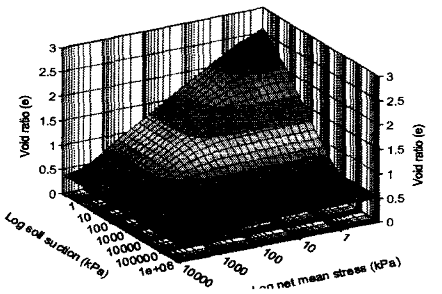


a. Void ratio

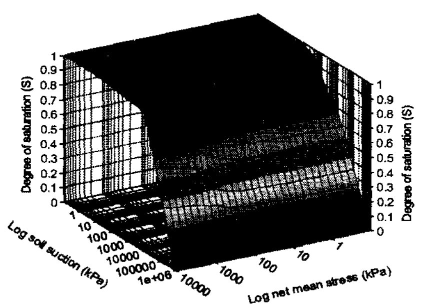


b. Gravimetric water content

Figure 12. The constitutive surfaces for sand (stress path series #1 and #2 in logarithmic scale).



a. Void ratio



b. Degree of saturation

Figure 13. The constitutive surfaces for sand (stress path series #1 and #2 in logarithmic scale).

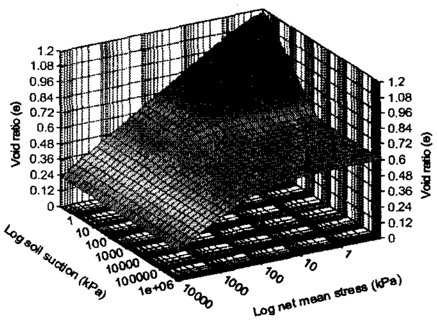


Figure 14. Void ratio surface for silt (stress path #1).

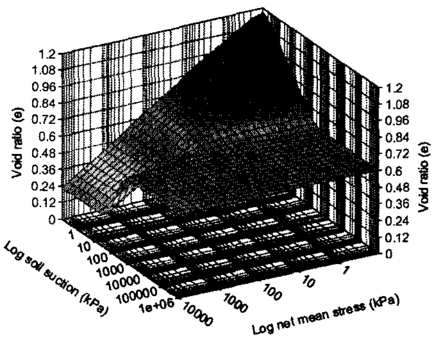


Figure 15. Void ratio surface for silt (stress path #2).

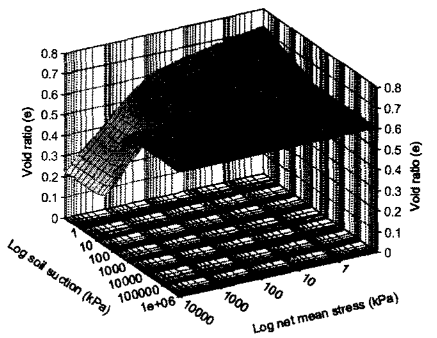


Figure 16. Void ratio surface for silt (stress path #3).

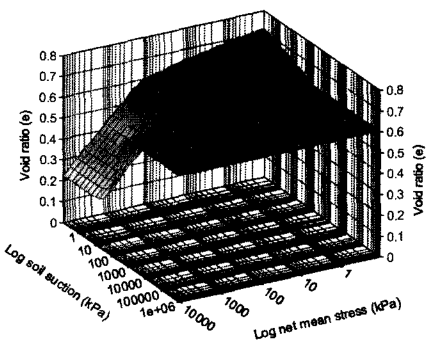


Figure 17. Void ratio surface for silt (stress path #4).

5 CONCLUSIONS AND RECOMENDATIONS

The proposed volume-mass constitutive model is capable of: 1.) predicting both volume and water content; 2.) taking into account the hysteretic nature of the soil-water characteristic curve; and 3.) predicting both swelling and collapsible behaviors of an unsaturated soil. The model can predict volume-mass constitutive relationships that are stress path dependent. The prediction results appear to be consistent with observed laboratory data.

In the theory of the proposed model, air-filled pores are assumed to be incompressible. It is recommended that for the next step in the model development, the air-filled pore compressibility be made a function of water content. The hysteresis model for the soil-water characteristic curve makes use of two one-dimensional pore-size distributions. It is recommended that a three-dimensional pore-size distribution be developed.

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