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A General Air-phase Permeability Function for Airflow through Unsaturated Soils

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Abstract

The study of multiphase flow in porous media, (e.g., air and water flow in unsaturated soils), is relevant to many areas such as contaminant transport and remediation, oil extraction, concrete materials behavior, geothermal reservoir engineering, and landslide prevention. Air-phase flow is the primary concern of this paper. The relevance and concepts of soil-air characteristic curve are introduced. A procedure is proposed to establish the relationship between the air-phase permeability and matric suction. The Klinkenberg effect, which involves an enhancement of air-phase permeability through slippage of air molecules along boundaries of air-filled pores, is also considered in the air permeability function. Two case studies are used to validate the proposed procedure based on experimental data reported in the literature. Reasonable agreement is observed between the derived air permeability equations and test results in both cases.

Introduction

The study of multiphase flow in porous media has application in areas such as contaminant transport, remediation, oil extraction, concrete material behavior, geothermal reservoir engineering, and landslide prevention. In response to an increased pore-air pressure in a soil, the net normal stress will decrease (Fredlund and Morgenstern 1977). Accordingly, the shear strength and stiffness of the soil can be significantly reduced (Wheeler et al. 1991). This means that if there is a change in the pore-air pressure in the soil mass, the short-term stability of slopes will be decreased and deformations might occur. The flow of air through soils is the primary concern of this paper.

Many experiments have been conducted to investigate the air coefficient of permeability of soils at different degrees of saturation (Irmay 1954, Wyllie 1962, Falta et al. 1989, Delage et al. 1998, Brooks and Corey 1966, Parker et al. 1987, Cary et al. 1989). Brooks and Corey (1964) proposed an empirical relationship between the air-phase permeability and matric suction. A relationship between the air permeability and the water content was introduced. The water content was expressed in terms of matric suction resulting in a relationship between the air permeability and matric suction. However, few attempts have been made to establish a systematic procedure for incorporating the soil-water characteristic curve, SWCC, with the relationship between the air permeability and the volumetric air content (or degree of saturation) and to derive the relationship between the air coefficient of permeability and matric suction. The objective of this paper is to develop a procedure for establishing the relationship between the air-phase coefficient of permeability and matric suction. The dependency of air permeability on the absolute air pressure is also considered in the air permeability function.

Air Flow Law

Fick's law can be used to describe air flow through unsaturated soils (Blight, 1971). Fick's law can be expressed as follows: (Fredlund and Rahardjo 1993)

$$v_a = -k_a \frac{\partial h_a}{\partial y} \tag{1}$$

where v_a is the volume rate of the air flow across a unit area of the soil at the exit point of flow; k_a is the air coefficient of permeability; and h_a is the pore-air pressure head (i.e., $u_a / \rho_{ma} g$ where u_a is pore-air pressure; ρ_{ma} is a constant air density corresponding to the pressure used in the measurement of the mass rate; and g is the gravitational constant).

The coefficient of air permeability, k_d, can be considered as a constant for simplicity. However, k_d will vary with absolute pore-air pressure and this is called the Klinkenberg effect (Klinkenberg 1941). The Klinkenberg effect is an enhancement of the air-phase permeability through the slippage of air molecules along the boundaries of air-filled pores. This occurs when the mean free path of air molecules approaches the dimensions of the pores. The Klinkenberg effect is significant in fine-grained porous media and increases with decreasing air-phase pressure. The effect can be approximated using the following relationship:

$$k_{\text{int}} = k_{\text{int}\infty}\left(1 + \frac{b}{u_a}\right) \tag{2}$$

where k_{int} is the intrinsic permeability with regard to the air-phase; $k_{\text{int}\infty}$ is the air-phase permeability at high air-phase pressure; and b is a parameter of the porous medium. Heid et al. (1950) proposed an empirical relationship between $k_{\text{int}\infty}$ and b based on experimental data

$$b = \left(3.98\times10^{-5}\right)k_{\mathrm{int}\infty}^{-0.39} \quad (3)$$

where b is in atmospheres and $k_{\mathrm{int}\infty}$ is in centimeters squared. This equation has been adopted by the American Petroleum Institute for correcting the Klinkenberg effect (Baehr and Hult 1991).

For a hypothetical homogeneous and isotropic porous medium, the maximum percent error in estimating the air permeability by ignoring the Klinkenberg effect, was studied by Baehr and Hult (1991). It was illustrated through pneumatic tests that the Klinkenberg effect can be of practical significance (>10%) in estimating the air permeability of porous media when the intrinsic permeability is $10^{-7}\,mm^2$ or less.

Air Flow in Unsaturated Soils

When a soil becomes unsaturated, the air coefficient of permeability of the soil is largely influenced by the degree of water saturation. When the degree of water saturation is extremely low, the air coefficient of permeability approaches its maximum value and as the degree of water saturation increases, the air coefficient of permeability decreases until the suction decreases to the air entry value. The air-entry suction of a soil (i.e., bubbling pressure) is matric suction where the air starts to enter the largest pores in the soil. Below this value, the airflow takes the form of diffusion in soil-water and the air coefficient of permeability becomes very low.

Soil-air characteristic curve

The concept of soil-air characteristic curve, SACC, is used to describe the relationship between the degree of air saturation and matric suction. The SACC is later used to construct the air permeability function. Denoting $S_w(\psi)$ as the degree of saturation with respect to the water phase, $S_a(\psi)$ as the degree of saturation with respect to the air phase, $\theta(\Psi)$ as the volumetric water content, and $\theta_a(\Psi)$ as the volumetric air content, it is possible to write

$$S_a(\Psi) = 1 - S_w(\Psi) \quad (4)$$

$$\theta_a(\Psi) = \frac{e}{1+e} - \theta(\Psi) \quad (5)$$

where e is void ratio, ψ is matric suction. Equations 4 and 5 are soil-air characteristic curve equations (SACC equations). The relationship between the degree of water saturation and matric suction can best be represented by a soil-water characteristic curve, SWCC. The water content defines the amount of water contained within the pores of the soil and the suction can be either matric suction of the soil (i.e., $u_a - u_w$, where u_a is the pore-air pressure and u_w is the pore-water pressure) or the total suction (i.e., matric suction plus osmotic suction). At high suctions (i.e., greater than about 1500 kPa), matric suction and total suction are generally assumed to be equivalent.

Numerous empirical equations have been proposed to describe the SWCC (Burdine 1953, Brooks and Corey 1964, van Genuchten 1980, Fredlund and Xing 1994). Fredlund and Xing (1994) proposed the most comprehensive three-parameter model for the SWCC,

$$\theta = \theta_s \left(1 - \frac{\ln\left(1 + \frac{\Psi}{\Psi_r}\right)}{\ln\left(1 + \frac{10^6}{\Psi_r}\right)} \right) \frac{1}{\left(\ln\left(e + \left(\frac{\Psi}{a}\right)^n \right) \right)^m} \tag{6}$$

where θ_s is saturated volumetric water content, Ψ_r is the soil suction in residual condition. Residual suction will generally be in the range of 1500 kPa to 3000 kPa (Fredlund and Xing 1994). The equation uses three fitting parameters, namely, *a, n* and *m*. The parameter *a* is related to, but greater than, the air entry value of the soil and has the units of suction. The *a* value corresponds to the suction at the inflection point on the curve. The parameter *n* is related to the pore size distribution of the soil or the slope defining desaturation. The more uniform the pore sizes in the soil, the larger the value of *n*. The parameter *m* is related to the asymmetry of the model. Small values of *m* result in a moderate slope in the low suction range and a steeper slope in the high suction range. Combining Eqs. 4 and 6 yields an expression for the soil-air characteristic curve.

Relationships between air coefficient of permeability and air saturation

A number of empirical relationships between the air coefficient of permeability and the air saturation have been proposed. Published air permeability relations fall into two main categories, namely, those that assume power functions for the air saturation and those based on soil-water retention curves (Stylianou and DeVantier, 1995). Power relations include equations proposed by Irmay (1954), Wyllie (1962), Falta et al. (1989), and Delage et al. (1998). Relations based on the soil-water retention curve approach include Brooks and Corey (1966), Parker et al. (1987), and Cary et al. (1989).

Brooks and Corey (1964) proposed an empirical relationship between the air-phase permeability and the degree of water saturation,

$$k_a = k_d (1 - S_e)^2 \left(1 - S_e^{(2+\lambda)/\lambda}\right) \tag{7}$$

where S_e is the effective degree of saturation with respect to the water phase, $S_e = \frac{S_w - S_r}{1 - S_r}$; S_r is the residual degree of saturation; λ is a coefficient.

Van Genuchten (1980) introduced a simple relationship between pressure head and water content to describe the soil-water retention curve. The equation allows the derivation of closed-form expressions for the relative hydraulic conductivity based on the model of Mualem (1976). Parker et al. (1987) extended the van Genuchten method for the prediction of a three-phase system of relative permeabilities. The relative air coefficient of permeability is assumed to be a function only of the degree of air saturation regardless of the presence of other fluid phases. The extension of Mualem's model to the gas-phase case can be expressed as follows:

$$K_r = \left(1 - S_e\right)^{1/2}\left(1 - S_e^{1/q}\right)^{2q} \tag{8}$$

where q is a parameter depending on the pore-size distribution. Van Genuchten (1980) limited q within a range from 0 to 1.0.

Equation 8 can be extended using the Fredlind and Xing (1994) equation to read as follows:

$$K_r' = \left(1 - S\right)^{1/2}\left(1 - S^{1/q}\right)^{2q} \tag{9}$$

where S is any degree of saturation from zero to 100%. Equation 9 extends the range for the degree of saturation below the residual suction of the soil. The extension is important since it is the air coefficient of permeability under completely dry conditions that becomes a reference condition for the air permeability function.

Relationships between air coefficient of permeability and matric suction

Numerous empirical equations have been proposed to simulate the soil-water characteristic curve as reviewed in previous section. The Fredlund and Xing (1994) model is used in this paper because it applies for the entire range of suctions. When the Fredlund and Xing (1994) equation is used in conjunction with Eq. 8, it is necessary to modify the original equation. Using the relation $S_w = \frac{1+e}{e}\theta$, the effective degree of saturation with respect to water can be expressed as follows:

$$S_e = \frac{\frac{1+e}{e}\theta - S_r}{1 - S_r} \tag{10}$$

Substituting Eq. 6 into Eq. 10 allow the Fredlund and Xing (1994) equation to be used with Maulem's model (1976). The effective degree of saturation is

$$S_e = \left[\frac{1+e}{e}\left(1 - \frac{\ln\left(1 + \frac{\Psi}{\Psi_r}\right)}{\ln\left(1 + \frac{10^6}{\Psi_r}\right)}\right)\frac{1}{\left[\ln\left(e + \left(\frac{\Psi}{a}\right)^n\right)\right]^m} - S_r\right]\frac{1}{1 - S_r} \tag{11}$$

Further substituting Eq. 11 into Eq. 8 yields an air permeability function.

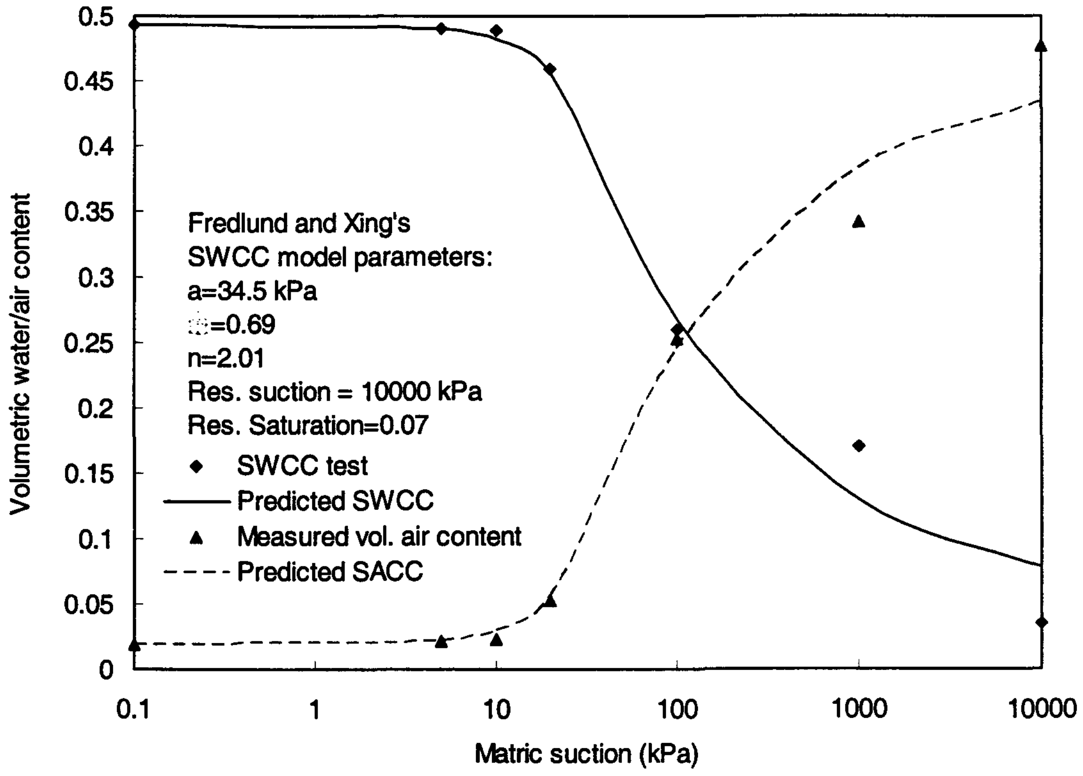
Some other empirical relationships between the air-phase permeability and the effective degree of saturation have been suggested. Also, many SWCCs have been proposed. By using different combinations of these relationships between the air-phase permeability and the effective degree of saturation and the SWCC functions, numerous air permeability functions can be obtained. The effectiveness of various combined relationships should be validated using experimental data of air coefficient of permeability.

No changes are required to the Fredlund and Xing (1994) equation when it is used in conjuction with Eq. 9.

Validation of Air Permeability Equations

The derived air permeability equation can be used to fit a series of air permeability test results found in the literature. First, test results for SWCCs of different soils are best-fit using the Fredlund and Xing (1994) equation. Then the relationship between the degree of air content and matric suction is established based on the SWCC results. Substituting these relationships into an air permeability function, a relationship between the air coefficient of permeability and the soil suction can be established. The relationships are then compared with experimental data.

Samingan et al. (2003) conducted a series of air permeability tests as well as soil-water characteristic tests on four Singapore residual soil samples. The results of specimen UP-2 has been extracted for the present study. The specimen consists of 43% sand, 45% silt, and 12% clay. The air permeability test results show hysteresis. For simplicity, the drying curve was chosen for study. Initial void ratio, and saturated volumetric water content of the specimen are 1.05 and 0.49, respectively. The Fredlund and Xing (1994) model was adopted to simulate the experimental results using a least-square method. The complicated soil parameters *a*, *n*, and *m* were computed for the Fredlund and Xing model assuming a residual soil suction is 10,000 kPa. The simulation results are shown in Fig. 1.



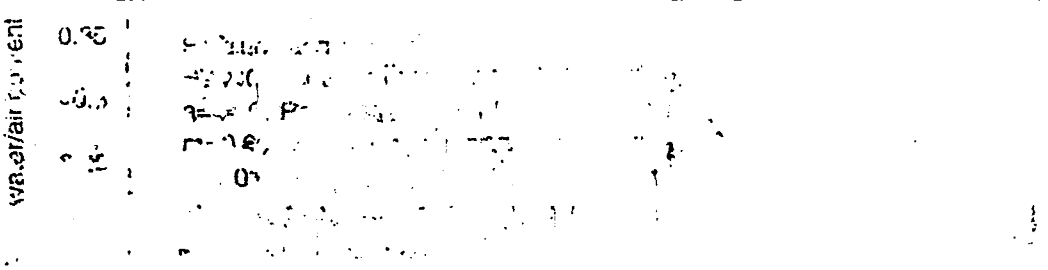


Figure 1. Predicted SWCC and SACC versus tests by Samingan et al. (2003)

According to Eqs. 5 and 6, the soil-air characteristic curve is plotted in Fig. 1. It can be seen that a reasonable agreement exists between the experimental results and the best-fit curves. Substituting Eq. 11 into Eq. 8 yields a function between the relative air coefficient of permeability and soil suction. Assuming that the air coefficient of permeability under dry condition is $k_d = 1\times10^{-7} m/s$, an equation between the air coefficient of permeability and soil suction can be achieved. In this equation, q is variable that can be determined using a least squared technique. The generated air permeability equation is plotted together with the experimental data in Fig. 2. The parameters for the air permeability equation are also shown in Fig.2. Reasonable agreement was found in the range of test points.

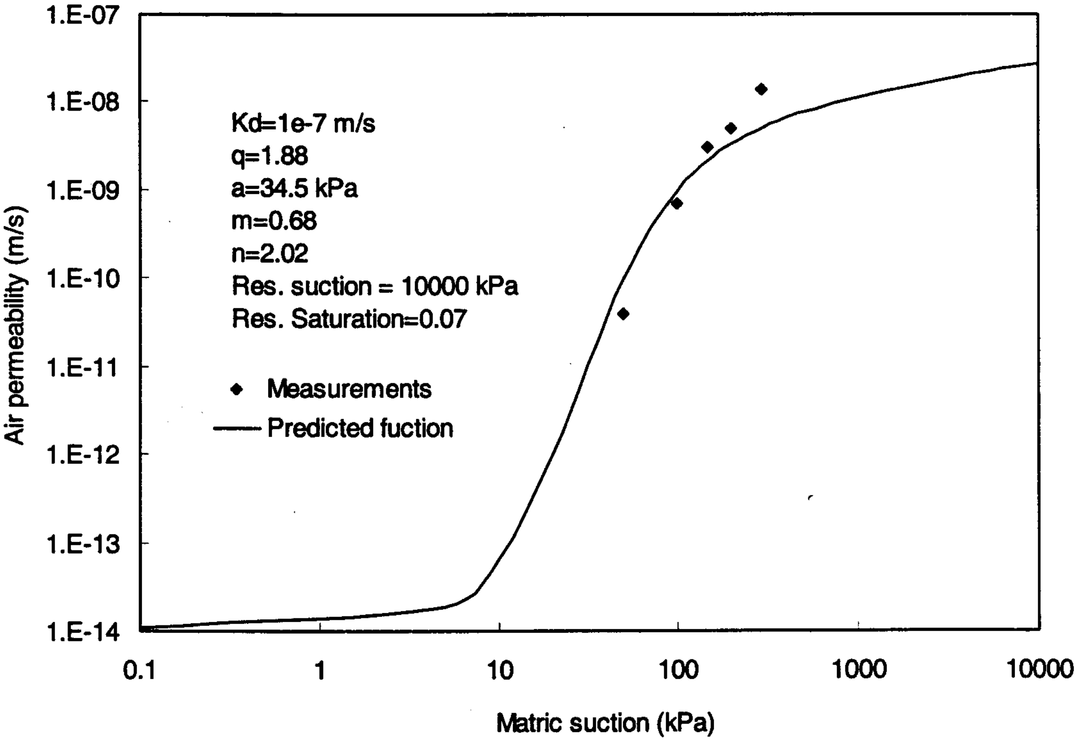


Figure 2. Predicted air coefficient of permeability versus tests by Samingan et al. (2003)

Moldrup et al. (2003) presented data on soil-water retention and gas diffusion coefficient as a function of water content for 22 Japanese soils. The measurements were originally conducted to evaluate factors that influence plant diseases and crop yield. Besides the water retention curves, gas diffusivity, and total porosity, there is limited information about the soils and the physical characteristics. Results from test Miura 4 Andisol is analyzed in this paper. The Fredlund and Xing (1994) model was used to best-fit the SWCC experimental results. The least square method was adopted to obtain the parameter *a*, *n*, and *m* of the SWCC equation, and the residual matric suction was assumed to be 3000 kPa. The simulation results are shown in Fig. 3. In view of Eqs. 5 and 6, the corresponding soil-air characteristic curve are plotted in Fig. 3. According to Eq. 8 and 11 the generated air permeability equation is plotted together with experimental data in Fig. 4. It can be seen in Figs. 3 and 4 that reasonable agreement between the experimental results and the fitted curves is achieved.

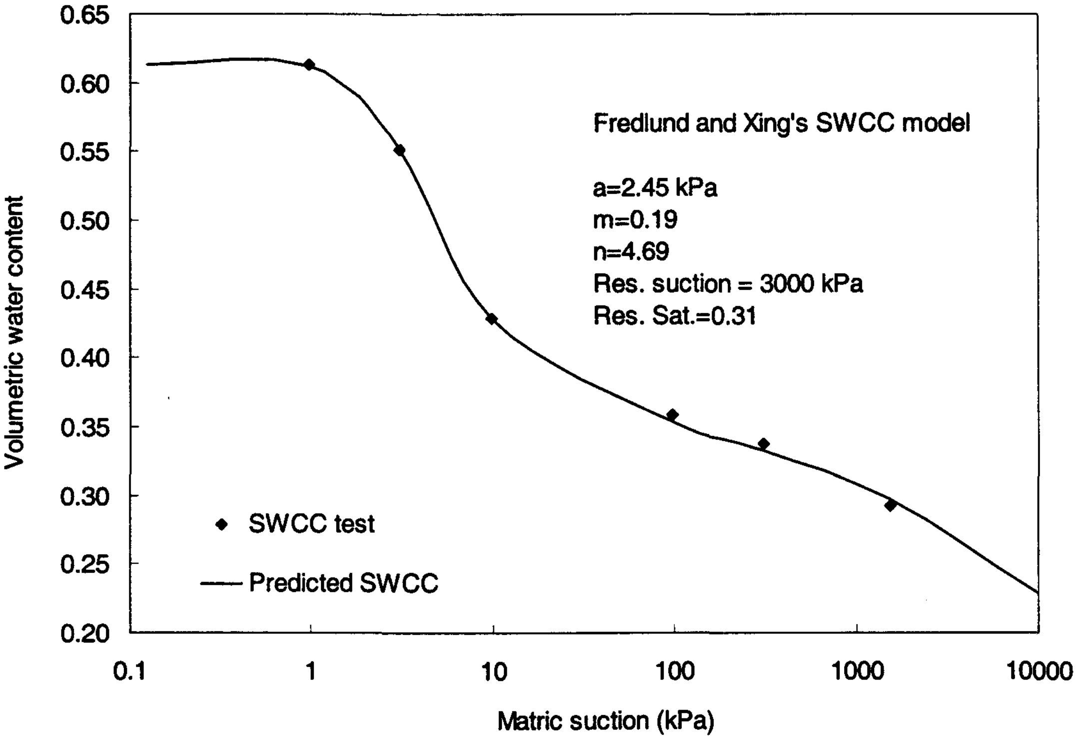


Figure 3. Predicted SWCC and SACC versus tests by Moldrup et al. (2003)

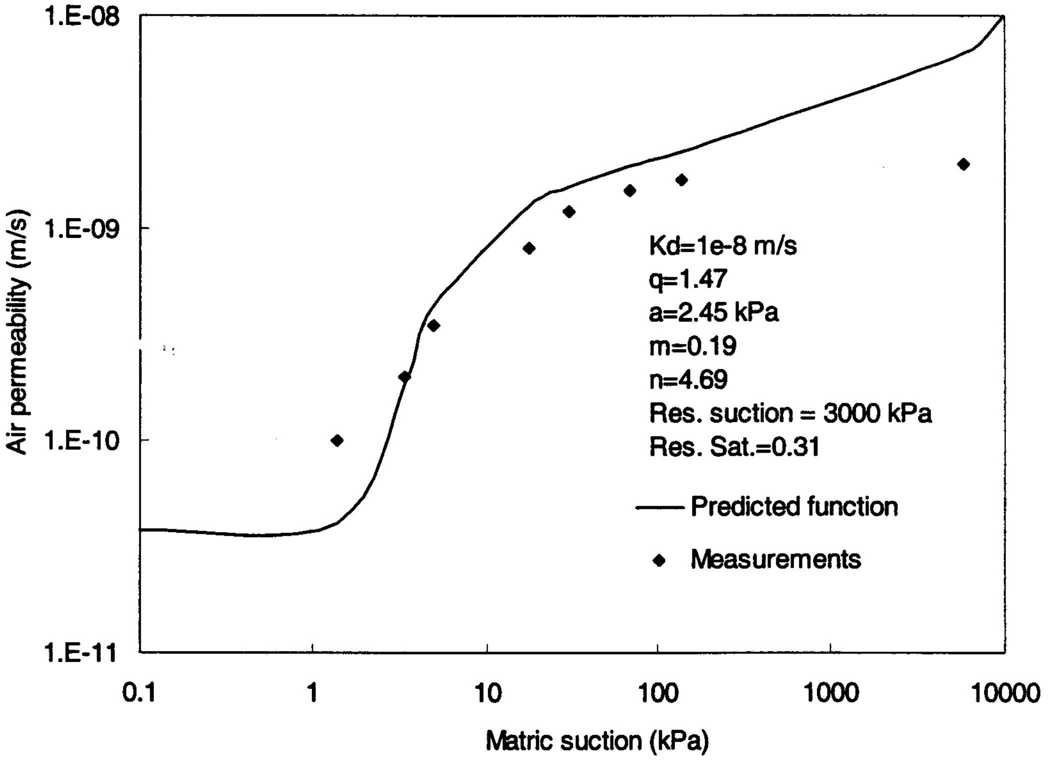


Figure 4. Predicted air coefficient of permeability versus tests by Moldrup et al. (2003)

Summary

The soil-air characteristic curve is established based on a soil-water characteristic curve for the derivation of air permeability equation. A systematic procedure for establishing the air-phase permeability function in terms of matric suction is proposed. The air permeability function incorporates simultaneously the effects of water content and the Klinkenberg effect. The latter is an enhancement of air-phase permeability through slippage of air molecules along the boundaries of air-filled pores.

Two case studies were conducted. The Fredlund and Xing (1994) model was used to best-fit the SWCCs against the measured data. After obtaining the SWCCs, the SACCs can be readily constructed. The air permeability equations are then obtained based on the SACCs obtained. Reasonable agreement was found between the experimental and predicted air coefficient of permeability values at different degrees of saturation.

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