The prediction of one-, two-, and three-dimensional heave in expansive soils

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Abstract: The prediction of heave in unsaturated, expansive soils has historically been studied primarily as a one-dimensional type analysis. This paper proposes a methodology that can be used for the prediction of one-, two-, or three-dimensional heave. It is suggested that negative pore-water pressures (i.e., soil suctions) can be estimated through a saturated–unsaturated seepage analysis. The results of the seepage analysis are then used as input for the prediction of displacements in a stress–deformation analysis. The formulation of the governing partial differential equations for both seepage and stress–deformation is based on the general theory of unsaturated soils using two independent stress state variables. The elasticity parameter functions required for a stress–deformation analysis can be calculated from various tests, including conventional one-dimensional oedometer tests. The proposed method is studied and tested against data collected on a case history involving a slab-on-ground floor on Regina clay. The predicted results from the two-dimensional analysis agree well with the measured data in terms of both total vertical displacements and final water contents in the soil. The results of a parametric study are also presented to show the effect of measured soil parameters (i.e., swelling index and initial void ratio) and assumed parameters (i.e., Poisson’s ratio and coefficient of earth pressure at rest) on the predicted displacements.

Key words: heave prediction, unsaturated soil, expansive soil, seepage analysis, stress analysis.

Introduction

Lightly loaded structures constructed on expansive soils are often subjected to severe distress subsequent to construction as a result of changes in the pore-water pressures in the soil. The structures most commonly damaged are roadways, airport runways, small buildings, irrigation canals, spillway structures, and all near ground surface structures associated with infrastructure development. Changes in the pore-water pressure can occur as a result of variations in climate, change in the depth to water table, water uptake by vegetation, removal of vegetation, or the excessive watering of a lawn.

The problems associated with expansive soils have been addressed in many international and regional conferences. There have been three symposiums on expansive clays (from 1957 to 1960), seven international conferences on expansive soils (from 1965 to 1992), three international conferences on unsaturated soils (from 1995 to 2002), and many other regional conferences. The research literature shows that the
prediction of heave associated with the wetting of an expansive soil has received more attention than any other problem involving unsaturated soils (Fredlund 2000).

The worldwide interest in research on expansive soils in the last four decades has resulted in numerous methods being proposed for the prediction of heave. The heave prediction methods are based either on one-dimensional oedometer test results or on direct matric suction measurements (Fredlund and Rahardjo 1993). Although an analytical tool for the prediction of heave is extremely important, there has been little advancement in the development of an analytical method for solving engineering problems. There does not appear to be a computer program that has been written and widely accepted for multidimensional heave predictions in expansive soils. It is important that such an analytical tool be developed for one-, two-, and three-dimensional problems.

Review of the prediction of heave in expansive soils

The prediction of heave in unsaturated, expansive soils has been studied primarily as a one-dimensional type analysis. The available methods for the prediction of heave generally make use of the linear relationship between void ratio (or vertical strain) and the logarithms of net normal stress or soil suction. Figures 1 and 2 show typical plots of vertical strain versus the logarithm of net normal stress and matric suction. These approaches can be divided into methods based on the stress path followed in the analysis (i.e., actual stress path or total stress path; Fig. 3). The methods that are based on matric suction measurements, and that follow the actual (or in situ) stress path (i.e., suction change path), include those proposed by Richards (1966), Aitchison and Woodburn (1969), Aitchison and Martin (1973), Lytton (1977), Snethen (1980), McKeen (1992), and Perko et al. (2000). These methods require the determination of soil suction and the measurement (or estimation) of the soil property relating a change in soil suction to volume change. The primary difficulties associated with these methods are related to the accuracy of soil suction measurements and appropriate soil property (i.e., volume change index with respect to changes in matric suction, \( C_m \)). The methods that follow a total stress path make use of the results from one-dimensional oedometer tests (i.e., the swelling index with respect to changes in net normal stress, \( C_s \), and swelling pressure). Common methods for the prediction of heave based on oedometer test results include the double oedometer method (Jennings and Knight 1957); the Sullivan and McClelland method (Sullivan and McClelland 1969); and the Fredlund, Hasan, and Filson method (Fredlund et al. 1980). The oedometer methods are used in many countries because soil suction measurements are not required and the soil properties (i.e., swelling index, \( C_s \)) can be more readily and accurately measured (Fredlund and Rahardjo 1993). One disadvantage associated with the oedometer-based methods is the difficulty in measuring a unique swelling pressure, since it is sensitive to the testing procedure. The ability to accurately measure volume changes in a highly fissured and fractured soil is a limitation affecting all methods (or stress paths).

This paper proposes a theoretically based method that can be used for the prediction of heave in one, two, and three dimensions using the input soil data from a one-dimensional oedometer test (i.e., swelling index, \( C_s \), with swelling pressure; and (or) swelling index, \( C_m \), with matric suction conditions).

Theory for the swelling process in an expansive soil

The historical development of a general volume change theory for unsaturated soils has been presented in Fredlund and Rahardjo (1993). The mechanics of volume change in unsaturated, expansive soils under isothermal conditions involves two processes (i.e., water flow and stress–deformation). The governing partial differential equations for both saturated–unsaturated seepage and stress–deformation are derived based on the following assumptions: (i) the air phase

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is continuous and remains at atmospheric pressure; (ii) soil is isotropic, nonlinear, and elastic; (iii) strains are small; (iv) pore water is incompressible; and (v) the effects of air diffusing through water, air dissolving in the water, and the movement of water vapor are negligible.

Strain–displacement relations

Let us consider a three-dimensional field with \( x, y, \) and \( z \) as the rectangular Cartesian coordinates (i.e., \( x \) for the horizontal direction and \( y \) for the vertical direction) and with \( u_i \) being components of the displacement vector (i.e., \( u, v, \) and \( w \) for the \( x, y, \) and \( z \) directions, respectively). The components of the strain tensor for the soil structure, \( \varepsilon_{ij} \), are written in terms of displacements as follows:

\[
\varepsilon_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)
\]

The normal strains can be designated as \( \varepsilon_x, \varepsilon_y, \) and \( \varepsilon_z \) for the \( x, y, \) and \( z \) directions, respectively. For infinitesimal deformations, volumetric strain, \( \varepsilon_v \), is the sum of the normal strain components:

\[
\varepsilon_v = \frac{\partial u_i}{\partial x_i} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \varepsilon_x + \varepsilon_y + \varepsilon_z
\]

Fredlund and Morgenstern (1977) proposed that the constitutive behaviour of unsaturated soils be described using two independent stress state variables, namely, net normal stress \( (\sigma - u_a) \) and matric suction \( (u_a - u_w) \). The use of independent stress state variables has formed the basis for the formulations of shear strength and volume change problems for unsaturated soils (Fredlund and Rahardjo 1993).

An unsaturated soil is considered as a four-phase mixture (Fredlund 1979), with two phases that come to equilibrium under applied stress (i.e., soil particle and contractile skin) and two phases that flow under applied pressure (i.e., the air and the water). The total volume change of an unsaturated soil element must be equal to the sum of volume changes associated with each phase. If the soil particles are assumed incompressible and the volume change of the contractile skin assumed internal to the element, the continuity requirement for an element of unsaturated soil reduces to

\[
\frac{\Delta V_e}{V_0} = \frac{\Delta V_s}{V_0} + \frac{\Delta V_a}{V_0}
\]

where \( V_0 \) is the initial overall volume of an unsaturated soil element, \( V_s \) is the volume of soil voids, \( V_w \) is the volume of water in the element, and \( V_a \) is the volume of air in the element.

Two constitutive relationships are required to describe the volume change associated with an unsaturated soil, one for the soil structure (in terms of void ratio or volumetric strain) and one for the water phase (in terms of degree of saturation or water content). Constitutive relationships for soil structure and water phase are presented in indicial notation for the formulation of the partial differential equations for the water flow and stress–deformation processes. The stress state of an
unsaturated soil can be written in terms of two independent stress tensors, namely, the net normal stress tensor, \( \sigma_{ij} - u \delta_{ij} \), and the matric suction tensor, \( (u_a - u) \delta_{ij} \). If the air phase is assumed to be continuous and remain at atmospheric pressure, a flow law is required only for the water phase.

**Soil structure constitutive relationship**

The constitutive relationship for the soil structure can be written in an incremental elasticity form using elasticity parameters (Fredlund and Rahardjo 1993):

\[
\frac{d\varepsilon_{ij}}{d\sigma_{ij}} = \frac{1 + \mu}{E} d(\sigma_{ij} - \delta_{ij}u_a) - \frac{\mu}{E} d(\sigma_{kk} - 3u_a)\delta_{ij} + \frac{d(u_a - u_w)}{H} \delta_{ij}
\]

where \( \sigma_{ij} \) are components of the total stress tensor for the soil structure, \( \sigma_{kk} = (\sigma_x + \sigma_y + \sigma_z) \), \( \delta_{ij} \) is the Kronecker delta, \( E \) is the elasticity parameter for the soil structure with respect to a change in the net normal stress, \( H \) is the elasticity parameter for the soil structure with respect to a change in matric suction, and \( \mu \) is Poisson’s ratio.

Equation [4] can be used to write the equation for an incremental volumetric strain using the coefficients of volume change in a compressibility form as follows:

\[
\frac{d\varepsilon_v}{d\sigma_{mean} - u_a} = m_1^s d(\sigma_{mean} - u_a) + m_2^s d(u_a - u_w)
\]

where \( \sigma_{mean} = \sigma_{kk}/3 = (\sigma_x + \sigma_y + \sigma_z)/3 \) is the mean net total stress, \( m_1^s = 3 (1 - 2\mu)/E \) is the coefficient of volume change with respect to a change in net normal stress, and \( m_2^s = 3/H \) is the coefficient of volume change with respect to a change in matric suction.

The unloading constitutive relationship for the soil structure is presented graphically in the form of a constitutive surface in Fig. 4a. The coefficients of volume change, \( m_1^s \) and \( m_2^s \), are slopes on the soil structure constitutive surface and can be obtained by differentiating the surface with respect to net normal stress and matric suction, respectively. The coefficients of volume change for the soil structure can be written as a function of void ratio as follows:

\[
\frac{d\varepsilon_v}{d\sigma_{mean} - u_a} = \frac{\partial \varepsilon_v}{\partial (\sigma_{mean} - u_a)} = \frac{1}{1 + e_0} \frac{\partial e}{\partial (\sigma_{mean} - u_a)}
\]

\[
\frac{d\varepsilon_v}{d(u_a - u_w)} = \frac{1}{1 + e_0} \frac{\partial e}{\partial (u_a - u_w)}
\]

where \( d\varepsilon_v = de/(1 + e_0) \), \( e_0 \) is the initial (i.e., referential) void ratio of the soil, and \( e \) is the void ratio of the soil.

**Water phase constitutive relationship**

The water phase constitutive relationship can be presented in an incremental compressibility form as follows (Fredlund and Rahardjo 1993):

\[
\frac{dV_w}{V_0} = m_1^w d(\sigma_{mean} - u_a) + m_2^w d(u_a - u_w)
\]

where \( m_1^w \) is the coefficient of water volume change with respect to a change in net normal stress, and \( m_2^w \) is the coefficient of water volume change with respect to a change in matric suction.


\[
\frac{dV_w}{V_0} = \beta_{w1} d\varepsilon_v + \beta_{w2} d(u_a - u_w)
\]

where

\[
\beta_{w1} = \frac{m_1^w}{m_1^s} \text{ or } \frac{m_1^w E}{3(1 - 2\mu)}
\]

\[
\beta_{w2} = \frac{m_2^w - m_1^w m_2^s}{m_1^s} \text{ or } \frac{m_2^w - m_1^w E}{(1 - 2\mu)H}
\]

The unloading constitutive relationship for the water phase is presented graphically as the constitutive surface in Fig. 4b. The coefficients of water volume change, \( m_1^w \) and \( m_2^w \), indicate the amount of water taken on or released by the soil because of a change in the net normal stress and matric suction. Therefore, the slopes on the water phase constitutive surface can be obtained by differentiating the surface with respect to net normal stress and matric suction, respectively:
\[ m_1^w = \frac{\partial \theta_w}{\partial (\sigma_{\text{mean}} - u_0)} \]

\[ m_2^w = \frac{\partial \theta_w}{\partial (u_s - u_0)} \]

where \( \theta_w = V_w / V_0 \) is the volumetric water content.

**Flow law for water phase**

Darcy’s law relates the water flow rate to the hydraulic head (i.e., pressure head plus elevation head) as follows:

\[ v_{wi} = -k_{wi} \frac{\partial}{\partial i} \left( \frac{u_w}{\rho_w g} + Y \right) \]

where \( v_{wi} \) is Darcy’s flux in the \( i \) direction, \( k_{wi} \) is the hydraulic conductivity in the \( i \) direction, \( \rho_w \) is the density of water, \( g \) is the gravitational acceleration, and \( Y \) is the elevation.

**Governing partial differential equations for seepage in an unsaturated, swelling soil**

The water continuity equation for an unsaturated soil, assuming that water is incompressible and deformations are incrementally infinitesimal, can be written as follows (Freeze and Cherry 1979):

\[ \frac{\partial \theta_w}{\partial t} + \nabla (v_w) = 0 \]

where \( t \) is time, \( \nabla = \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k \)

is the divergence operator, and \( v_w = v_w^i i + v_w^j j + v_w^k k \) is Darcy’s flux.

The governing equation for the water phase can be obtained by substituting the time derivative of the water phase constitutive equation (eq. [9]) and Darcy’s law (eq. [12]) into the water phase continuity equation (eq. [13]):

\[ \beta_{w1} \frac{\partial e_w}{\partial t} + \beta_{w2} \frac{\partial (u_s - u_0)}{\partial t} - \nabla \left[ k_w \nabla \left( \frac{u_w}{\rho_w g} + Y \right) \right] = 0 \]

Equation [14] can be written for two-dimensional seepage in unsaturated, expansive soils:

\[ \beta_{w1} \frac{\partial e_w}{\partial t} + \beta_{w2} \frac{\partial (u_s - u_0)}{\partial t} = \frac{\partial}{\partial x} \left[ k_w \frac{\partial}{\partial x} \left( \frac{u_w}{\rho_w g} + Y \right) \right] + \frac{\partial}{\partial y} \left[ k_w \frac{\partial}{\partial y} \left( \frac{u_w}{\rho_w g} + Y \right) \right] \]

The analysis of seepage in an unsaturated, expansive soil (eq. [15]) requires the definition of the soil structure and water phase constitutive surfaces (i.e., Fig. 4), the coefficient of permeability function, and the rate of change of soil volume. Equation [14] shows the influence of the compressibility of the soil structure and the rate of the volume change of the soil structure, which are dependent on the net normal stresses, on the transient water flow process in a swelling soil. The effect of changes in stress and volume on eq. [15] has been studied and presented in Vu (2003). The solution for transient seepage through a swelling soil is more sensitive to changes in stress and volume when the soil is close to saturation, when the soil has higher compressibility or more volume change. The effect of changes in stress and volume appeared to be minimal when the soil is at high matric suction (Vu 2003).

The formulation can be simplified by not considering changes in volume and induced net normal stresses during the seepage process. The soil structure constitutive surface becomes unnecessary and the constitutive surface for the water phase can be represented using the soil-water characteristic curve. Equation [15] then takes on the following form:

\[ m_2^w \frac{\partial (u_s - u_0)}{\partial t} = \frac{\partial}{\partial x} \left[ k_w \frac{\partial}{\partial x} \left( \frac{u_w}{\rho_w g} + Y \right) \right] + \frac{\partial}{\partial y} \left[ k_w \frac{\partial}{\partial y} \left( \frac{u_w}{\rho_w g} + Y \right) \right] \]

Equation [16] applies for both saturated and unsaturated soils and for transient and steady state seepage conditions. Two unsaturated soil properties are required when solving transient seepage problem (i.e., eq. [16]), namely, the coefficient of water volume change (or coefficient of water storage) and the coefficient of permeability. Both the coefficient of water storage and the coefficient of permeability are predominantly functions of matric suction.

The coefficient of water storage is the slope of the soil-water characteristic curve and can be obtained by differentiating the soil-water characteristic curve with respect to matric suction. Numerous equations have been proposed to simulate the soil-water characteristic curve (Gardner 1958; van Genuchten 1980; Fredlund and Xing 1994). The assessment of equations to represent the soil-water characteristic curve has been presented by Sillers and Fredlund (2001) and Sillers et al. (2001). The soil-water characteristic curve used in the present study is the Fredlund and Xing (1994) equation:

\[ \theta_w = \theta_s \left[ \frac{1}{\ln(1 + (\psi/\alpha)^n)} \right]^m \]

where \( \psi \) is the soil suction (kPa), \( e \) is the natural logarithm (base 2.71828 ...), \( \theta_s \) is the volumetric water content at saturation, \( \alpha \) is a soil parameter that is related to the air-entry value of the soil (kPa), \( n \) is a soil parameter that controls the slope at the inflection point in the soil-water characteristic curve, and \( m \) is a soil parameter that is related to the residual water content of the soil.

The coefficient of permeability function can be indirectly computed or estimated from the soil-water characteristic curve and the saturated coefficient of permeability. There are several equations for the coefficient of permeability that have been proposed to represent the permeability function of an unsaturated soil (e.g., Gardner 1958; Fredlund et al. 1994; Leong and Rahardjo 1997). These equations involve finding best-fit parameters which produce a curve that fits the measured data or data estimated from the soil-water...
characteristic curve. The equation proposed by Leong and Rahardjo (1997) appeared to be satisfactory for the problem at hand. Leong and Rahardjo illustrated that the coefficient of permeability can be approximated as a power function of dimensionless volumetric water content. Using the Fredlund and Xing (1994) equation for the soil-water characteristic curve, the permeability function was shown to take the following form:

$$k_w = k_s \left[ \frac{1}{\ln[e + (\psi/a)^p]} \right]$$

[18]

The parameter p can be evaluated using the curve fitting of the coefficient of permeability data.

The transient water flow equation (eq. [16]) along with the equation of a soil-water characteristic curve (eq. [17]) and a permeability function (eq. [18]) can be used to predict pore-water pressure profiles (i.e., matric suction profiles) at different times during a seepage process. The matric suction profiles can then be used in an uncoupled manner to compute the matric suction change for the stress–deformation analysis. Deformations due to changes in matric suction during any time period can then be predicted based on initial and final matric suction profiles.

The solution for transient saturated–unsaturated seepage also requires the designation of initial soil suction conditions and the moisture flux boundary conditions. The results of the seepage analysis provide the distributions of matric suction in the soil profile with respect to time for the specified boundary conditions.

Quantification of boundary conditions for seepage analysis

Either a flux or a head boundary condition can be specified for a seepage analysis. Both the flux and head types of boundary conditions can be functions of space and time. Accurate prediction of suction conditions in soils requires boundary conditions based on a reasonable analysis of climatic conditions at particular sites. Research studies on flux boundary conditions have been presented in Wilson et al. (1994, 1997) and Blight (2003). The boundary flux conditions are characterized by considering the soil system water balance at a particular site.

Governing partial differential equations for soil structure equilibrium

The equations of overall static equilibrium for an unsaturated soil can be written as follows:

$$\frac{\partial \sigma_{ij}}{\partial x_j} + b_i = 0$$

[19]

where $\sigma_{ij}$ are components of the net total stress tensor, and $b_i$ are components of the body force vector.

Substituting the strain–displacement relationship (eq. [1]) and the stress–strain relationship (eq. [4]) into the equilibrium equation, eq. [19] gives the following governing equations for general three-dimensional problems (i.e., equations for x, y, and z directions):

$$G \nabla^2 u_i + \frac{G}{1 - 2\mu} \frac{\partial \varepsilon_x}{\partial x_i} - \beta \frac{\partial(u_a - u_w)}{\partial x_i} + \frac{\partial u_a}{\partial x_i} + b_i = 0$$

[20]

where

$$\beta = \frac{m_2^2}{m_1^2} = \frac{E/H}{1 - 2\mu}$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

is the Laplace operator

$$\varepsilon_x = \frac{\partial u_a}{\partial x_i} = \frac{\partial u_a}{\partial y} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

$$G = \frac{E}{2(1 + \mu)}$$

Equations [14] and [20] form a system of coupled equations for the theory of swelling in three dimensions for an unsaturated, swelling soil with a continuous air phase. The equations have essentially the same form as those presented by Biot (1941) for a soil with occluded air bubbles.

Equation [20] can be written for two-dimensional plane strain problems as follows:

$$\frac{\partial}{\partial x} \left( c_{11} \frac{\partial u}{\partial x} + c_{12} \frac{\partial v}{\partial y} \right) + c_{33} \frac{\partial}{\partial y} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} \right) - d_s \frac{\partial(u_a - u_w)}{\partial x} + b_x = 0$$

[21]

$$\frac{\partial}{\partial y} \left( c_{12} \frac{\partial u}{\partial x} + c_{22} \frac{\partial v}{\partial y} \right) + \frac{\partial}{\partial y} \left( c_{12} \frac{\partial u}{\partial y} + c_{22} \frac{\partial v}{\partial y} \right) - d_s \frac{\partial(u_a - u_w)}{\partial y} + b_y = 0$$

[22]

where $b_x$ and $b_y$ are body forces in x- and y-directions respectively,

$$c_{11} = c_{22} = \frac{(1 - \mu)E}{(1 + \mu)(1 - 2\mu)}$$

$$c_{12} = \frac{\mu E}{(1 + \mu)(1 - 2\mu)}$$

$$c_{33} = \frac{E}{2(1 + \mu)}$$

$$d_s = \frac{E}{(1 - 2\mu)H}$$

Equations [21] and [22] can be used to compute displacements in the horizontal and vertical directions under an applied load and (or) due to changes in matric suction.

Solutions to the seepage equation (eq. [15]) and soil structure equilibrium equations (eqs. [21] and [22]) can be obtained through either an uncoupled or a coupled analysis. Uncoupled and coupled volume change problems in expansive soils were studied, analyzed, and compared through various example problems by Vu (2003) and Vu and Fredlund (2003). Both uncoupled and coupled solutions were obtained for a general case with respect to soil properties when the
elasticity parameters \(E\) and \(H\), the coefficients of water volume change \(m_1^w\) and \(m_2^w\), and the coefficient of permeability \(k_e\) were functions of both stress state variables. Only the uncoupled approach is reported in this paper.

The solution of the stress–deformation equations (i.e., eqs. [21] and [22]) under specified boundary conditions requires the designation of initial matric suctions, initial stress conditions, the elasticity parameter functions associated with the volume change of the soil, and the results from a seepage analysis (i.e., for changes in soil suction).

### Evaluation of the elasticity parameter functions from volume change indices

The stress–deformation analysis in two and three dimensions requires the characterization of the elasticity parameters \(E\) and \(H\), which are functions of stress state. The elasticity parameters can be calculated from the coefficients of volume change, \(m_1^w\) and \(m_2^w\), by differentiating the constitutive surface for the soil structure using eqs. [6] and [7], respectively. Several testing conditions can be used to obtain the void ratio constitutive surface (i.e., \(K_0\) condition, plane strain condition, or isotropic condition). It is important to note that three fundamental elasticity parameters are required in the constitutive equations (i.e., \(E\), \(H\), and \(\mu\)). However, there are only two coefficients of volume change obtained from the constitutive surface for soil structure (i.e., \(m_1^w\) and \(m_2^w\)). Therefore, it is suggested that Poisson’s ratio be assumed (or measured) to convert the coefficients of volume change for different loading conditions to the fundamental elasticity parameters.

The elasticity parameter functions can be calculated from various testing conditions and then used for different types of analyses (Fig. 5). Table 1 presents the calculation of these elasticity parameter functions for various loading conditions (after Fredlund and Rahardjo 1993).

The coefficients obtained from one loading condition can be converted to other loading conditions using the assumed value of Poisson’s ratio \((\mu)\) (Fredlund and Rahardjo 1993):

\[
\begin{align*}
    m_1^w &= \frac{(1 - \mu)}{(1 + \mu)} m_{1,1D}^w = \frac{3}{2(1 + \mu)} m_{1,2D}^w \\
    m_2^w &= \frac{(1 - \mu)}{(1 + \mu)} m_{2,1D}^w = \frac{3}{2(1 + \mu)} m_{2,2D}^w
\end{align*}
\]

The coefficients of volume change can be obtained through the conversion of the semilogarithmic plot of void ratio to an arithmetic plot (Vu 2000). The swelling indices, \(C_s\) and \(C_m\), are the slope of the void ratio versus logarithm of net normal stress or matric suction as shown in Fig. 6. The semilogarithmic plot of the void ratio constitutive surface for an unsaturated soil is approximately linear on the extreme planes over a relatively large stress range (Ho et al. 1992). The volume change indices obtained from \(K_0\) loading have been shown to be essentially the same as those obtained from isotropic loading conditions (Graham and Li 1985; Al-Shamrani and Al-Mhaidib 2000; Vu 2003).

The elasticity parameter functions \(E\) and \(H\) can also be calculated directly from volume change indices \(C_s\) (from net normal stress plane) and \(C_m\) (from matric suction plane), respectively. The elasticity parameters are calculated from the coefficient of volume change as shown in Table 1. The elasticity parameter \(E\) can be expressed as a function of the volume change index with respect to net normal stress, \(C_s\), initial void ratio, and Poisson’s ratio. The elasticity parameter \(H\) can be expressed as a function of the volume change index with respect to matric suction, \(C_m\), initial void ratio, and Poisson’s ratio. The equations for these elasticity parameters can be written for general three-dimensional loading conditions as follows:

\[
\begin{align*}
    E &= \frac{6.908(1 - 2\mu)(1 + e_0)}{C_s} (\sigma_{mean} - u_a) \\
    H &= \frac{6.908(1 + e_0)}{C_m} (u_a - u_w)
\end{align*}
\]

The constant 6.908 arises from the conversion between the logarithmic and arithmetic scales (i.e., \(6.908 = 3/\log_{10}2.718\)).

Equations [25] and [26] can be written for two-dimensional plane strain conditions as follows:

\[
\begin{align*}
    E &= \frac{4.605(1 + \mu)(1 - 2\mu)(1 + e_0)}{C_s} (\sigma_{ave} - u_a) \\
    H &= \frac{4.605(1 + \mu)(1 + e_0)}{C_m} (u_a - u_w)
\end{align*}
\]

The constant 4.605 arises from the conversion between the logarithmic and arithmetic scales (i.e., \(4.605 = 2/\log_{10}2.718\)).

Assuming a constant value of Poisson’s ratio, the elasticity parameter \(E\) (or \(H\)) increases with an increase in net normal stress (or matric suction) and a decrease in the volume change index \(C_s\) (or \(C_m\)). Figures 7 and 8 graphically illustrate the relationship between the elasticity parameter \(E\) and net normal stress for various values of the swelling index \(C_s\) and various values of Poisson’s ratio, respectively. Figure 9 presents the variation of elasticity parameter \(H\) with matric suction for various values of swelling index \(C_m\).

### Estimation of the swelling indices

The swelling indices can be measured experimentally or estimated through correlation with the Atterberg limits (Fredlund and Rahardjo 1993; Lytton 1994). Test procedures for the estimation of swelling indices are presented in Fredlund and Rahardjo (1993) and the American Society for...
Testing and Materials (ASTM) standards. The ASTM standards related to the measuring of swelling indices include D4546 (ASTM 1996a), D2435 (ASTM 1996b), and D427 (ASTM 1998). It should be noted that these swelling indices are commonly used for a conventional heave analysis in one dimension. This paper does not present any new testing procedures to measure these soil properties for two- and three-dimensional analyses but suggests the use of conventional oedometer test results.

Poisson’s ratio

Poisson’s ratio may not be a constant for an unsaturated soil but may be a function of stress state (i.e., net normal stress and matric suction). It is generally assumed that Poisson’s ratio increases with increasing net mean stress and with decreasing matric suction (Pereira and Fredlund 2000). Several researchers (Miranda 1988; Alonso et al. 1988) have used a constant Poisson’s ratio of 0.3 in numerical simulations of the behaviour of unsaturated, collapsing soils. It was suggested that this value of Poisson’s ratio might reflect the as-compacted condition of a loosely compacted embankment. The choice of a value for Poisson’s ratio can also be related to an experimentally observed relationship between the coefficient of earth pressure at rest (\(K_0\)) and the overconsolidation ratio (OCR). In this study, the Poisson’s ratio is assumed to be a constant and is related to \(K_0\) as follows:

\[
\mu = \frac{K_0}{1 + K_0}
\]

Initial matric suction and stress conditions

Initial matric suction conditions can be measured using field methods and laboratory methods (Fredlund and Rahardjo 1993) or estimated from theoretical considerations of unsaturated soil conditions.

The net normal stress state within the soil mass can be either computed by switching on gravity due to the unit weight of the soil or estimated from the total unit weight of the soil through total stress theory. Horizontal net normal stresses can be estimated from the vertical stresses and \(K_0\):

\[
\sigma_y = \int_0^H \rho g \, dy
\]

\[
\sigma_z = K_0 \sigma_y
\]

where \(\sigma_z\) and \(\sigma_y\) are the horizontal and vertical net normal stress, respectively; \(y\) is the vertical distance from the ground surface, and \(H\) is the depth of soil under consideration.
Fig. 7. Relationship between elasticity parameter, $E$, and net normal stress for various values of swelling index, $C_s$.

Fig. 8. Relationship between elasticity parameter, $E$, and net normal stress for various values of Poisson’s ratio, $\mu$.

Fig. 9. Relationship between elasticity parameter, $H$, and net normal stress for various values of swelling index, $C_m$. 

Coefficient of earth pressure at rest

The coefficient of earth pressure can go from as low as zero to as high as the coefficient of passive earth pressure. Several procedures suggested in the literature for the estimation of the coefficient of earth pressure at rest are presented in the following equations. Fredlund and Rahardjo (1993) considered elastic equilibrium within a homogenous, isotropic soil mass and presented the following equation for the coefficient of earth pressure at rest:

\[ K_0 = \frac{\mu}{1-\mu} - \frac{E}{(1-\mu)H} \left(\sigma_v - u_s\right) \]

Equation [32] can be rewritten as follows:

\[ K_0 = \frac{\mu}{1-\mu} - \frac{(1-2\mu) C_m}{(1-\mu)} \left(\sigma_v - u_s\right) \]

Considering the effect of previous stress paths (i.e., previous wetting and drying, loading and unloading), the coefficient of earth pressure should have the following tangent value:

\[ K_0 = \frac{\mu}{1-\mu} - \frac{(1-2\mu) C_m \Delta(u_s - u_w)}{(1-\mu) C_s \Delta(\sigma_v - u_s)} \]

Jaky (1944) estimated the coefficient of earth pressure for normally consolidated soils \((K_{nc})\) from the effective stress parameter \(\phi\):

\[ K_{nc} = 1 - \sin \phi \]

Wroth (1979) proposed two empirical relationships between \(K_0, K_{nc}\), and OCR:

\[ K_0 = K_{nc} OCR - \frac{\mu}{1-\mu} (OCR - 1) \]

\[ m \left[ \frac{3(K_{nc} - K_0)}{2K_{nc}} - \frac{3(1 - K_{nc})}{2K_0} \right] = \ln \left[ \frac{(1 + 2K_{nc}) OCR}{1 + 2K_0} \right] \]

where \(m = 0.0022875\)PI + 1.22, in which PI is the plasticity index.

Equation [36] provides a reasonable fit to existing data for soils up to an OCR of about 5. The value of Poisson’s ratio necessary to fit the observed data was in the range of 0.254–0.371 for eight different soils. Equation [37] was proposed as being valid for even higher OCR values (Wroth 1979).

Mayne and Kulhawy (1982) suggested that a modified Jaky’s equation be used to estimate the coefficient of earth pressure at rest:

\[ K_0 = K_{nc} OCR \sin \phi \]

where \(\phi\) is the angle of internal friction.

Lytton (1994) presented the following typical values for coefficients of lateral earth pressure back-calculated from field observations of heave and shrinkage:

\[ \begin{array}{c|c}
0 & \text{when the soil is dry and cracked} \\
0.333 & \text{when the soil is dry and cracks are opening} \\
0.500 & \text{when cracks are closed and suction is at a steady state condition} \\
0.667 & \text{when cracks are closed and the soil is wetting} \\
1 & \text{when the soil is wetting and is in hydrostatic stress condition} \\
2–3 & \text{when the soil is approaching passive earth pressure} \\
\end{array} \]

The coefficient of earth pressure at rest presented by eq. [39] is used for the estimation of horizontal net normal stress in this study.

Case history of a slab-on-grade floor on Regina clay

The proposed method for the prediction of heave was studied using data collected by the Prairie Regional Station of the Division of Building Research (DBR), National Research Council of Canada, located in Saskatoon, Saskatchewan. In the study, a two-dimensional heave analysis was used to investigate volume change problems associated with the heave of a floor slab of a light industrial building in north-central Regina, Saskatchewan. History of the site and details on testing and monitoring programs were presented by Yoshida et al. (1983).

Construction of the building and instrumentation took place during the month of August 1961. Instrumentation installed at the site included a deep benchmark, vertical movement gauges, and a neutron moisture meter access tube. Vertical ground movement was monitored at depths of 0.58, 0.85, and 2.39 m below original ground level. The building owner noticed heave and cracking of the floor slab in early August 1962, about a year after construction. The owner also noticed an unexpected increase in water consumption of approximately 35 000 L. The loss of water was traced to a leak in a hot-water line beneath the floor slab, which was subsequently repaired. The location of the cracking and contours of heave for the floor slab are shown in Fig. 10.

Laboratory analyses were performed on samples from a borehole advanced on August 1961 for the installation of the deep benchmark. Laboratory tests evaluated the Atterberg limits, in situ water content, grain-size distribution, swelling indices, and the corrected swelling pressures of the soil. On average, the liquid limit was found to be 77%, with a plastic limit of 33% and a natural water content of 29%. The specific gravity and unit weight for the soil profile were 2.82 and 18.88 kN/m³, respectively.

Constant-volume oedometer tests on three samples were used to evaluate the initial void ratios, swelling indices, and corrected swelling pressures. Table 2 presents the oedometer test results and water content of samples collected at different depths. Although there was some variation in the initial void ratio and the swelling index, an average value of 0.962 for the initial void ratio and 0.090 for the swelling index are
used in the analyses. Figure 11 shows the distribution of the corrected swelling pressure with depth. A straight line can be used to represent the apparent distribution of the corrected swelling pressure with depth. Figure 12 shows the distribution of gravimetric water content with depth measured on 21 August 1961 using a neutron moisture meter access tube.

This case history would be best modelled as a three-dimensional problem; however, a simpler two-dimensional analysis is used to illustrate the method for the prediction of heave proposed in this paper.

**Characterization of the soil properties with the seepage analysis**

A seepage analysis was performed to predict changes in matric suction conditions in the soil. The initial matric suction conditions are estimated from the corrected swelling pressures. An approximate soil-water characteristic curve was defined using measured water contents at various values of soil suction.

The corrected swelling pressure is the sum of the overburden pressure and the matric suction equivalent and is written as follows (Fredlund and Rahardjo 1993):

\[
P'_s = \sigma_y - u_{\text{field}} + (u_a - u_w)
\]
where \( P'_s \) is the corrected swelling pressure, \( \sigma_y \) is the original overburden pressure, and \((u_h - u_w)\) is the matric suction equivalent.

Initial matric suction conditions can be estimated from the corrected swelling pressure assuming a slope for the net normal stress versus matric suction curve at a constant void ratio (Fig. 13). Let us assume that the slope can be written as a function \( f \) and, for the sake of this case history, let the function be taken as equal to the degree of saturation. Equation [40] then becomes

\[
P'_s = (\sigma_y - u_w)_{\text{field}} + f(u_h - u_w)_{\text{field}}
\]

Therefore, the initial matric suction conditions required for the seepage analysis can be estimated as follows:

\[
(u_h - u_w)_{\text{field}} = \frac{P'_s - (\sigma_y - u_w)_{\text{field}}}{f}
\]

where \( f \) is a function set equal to the degree of saturation.

The estimation of initial matric suction condition is presented in Table 3 and shown graphically in Fig. 11. A straight line can be used to represent the distribution of the initial matric suction with depth.

The initial degree of saturation and volumetric water content can also be calculated, since the initial water content profile has been established (i.e., \( S = wG_s/e \) and \( \theta_w = wG_s/(1 + e) \), where \( G_s \) is specific gravity of the soil). The measured data of water content and degree of saturation at various values of matric suction can be used to produce an estimated soil-water characteristic curve. The Fredlund and Xing (1994) equation for a soil-water characteristic curve (eq. [17]) was used with \( \theta_s = 49.3\% \), \( a = 300 \text{ kPa} \), \( n = 0.6 \), and \( m = 0.7 \) to fit the volumetric water content data, and \( a = 300 \text{ kPa} \), \( n = 0.5 \), and \( m = 0.7 \) to fit the degree of saturation data. Figure 14 presents the volumetric water content versus soil suction curve and the degree of saturation versus soil suction curve.

A coefficient of permeability function for compacted Regina clay (Shuai 1996) was used for the analysis as part of this study. The coefficient of permeability function was described using the Leong and Rahardjo (1997) equation (eq. [18]) and is presented graphically in Fig. 15.

The analysis examines the cross section A–A shown in Fig. 10. The lower boundary is selected at the 2.3 m depth, where there was no apparent tendency for swelling (see Fig. 11). This lower boundary can also be selected on the basis of the depth to which changes in matric suction appeared to be a minimum (Fredlund and Rahardjo 1993).

The geometry and boundary conditions for the seepage analysis are illustrated in Fig. 16. It was assumed that water leaked from the water line along a 2 m length of the line and that the initial suction conditions did not change outside of the concrete slab and at the lower boundary. A moisture flux equal to zero was specified elsewhere along the boundaries.

Matric suction conditions are predicted for various elapsed times (i.e., 5, 20, 50, and 100 days and at steady state conditions).

Characterization of the soil properties for the stress–deformation analysis

A coefficient of earth pressure at rest, \( K_0 \), equal to 0.667, as suggested by eq. [39], was used to determine the initial stress state conditions. A Poisson’s ratio equal to 0.40 was suggested for \( K_0 \) using eq. [29].

The elasticity parameter function with respect to changes in net normal stress, \( E \), can be calculated for two-
dimensional conditions using eq. [27] for $e_0 = 0.962$, $C_s = 0.090$, and $\mu = 0.40$ and can be written as follows:

$$E = 28.11(\sigma_{ave} - u_a)$$

The swelling index with respect to changes in matric suction, $C_m$, was not measured for this case history. It is suggested that a value of $C_s$ be used for $C_m$, and the initial stress state (IST) and final stress state (FST) from the net normal stress plane be used as the initial and final matric suctions for the stress–deformation analysis. The analysis procedure is illustrated in Fig. 17.

The elasticity parameter function with respect to changes in matric suction, $H$, can be calculated for two-dimensional

Table 3. Estimation of initial matric suction from the corrected swelling pressure.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Depth (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overburden pressure, $(\sigma_y - u_a)$ (kPa)</td>
<td>0.69</td>
</tr>
<tr>
<td>Initial void ratio, $e_0$</td>
<td>1.34</td>
</tr>
<tr>
<td>Corrected swelling pressure, $P'_s$ (kPa)</td>
<td>2.20</td>
</tr>
<tr>
<td>Gravimetric water content, $w$ (%)</td>
<td></td>
</tr>
<tr>
<td>Degree of saturation, $S$ (%)</td>
<td></td>
</tr>
<tr>
<td>Volumetric water content, $\theta_w$ (%)</td>
<td></td>
</tr>
<tr>
<td>Estimated field suction, $(u_a - u_w)$ (kPa)</td>
<td></td>
</tr>
</tbody>
</table>

Note: $S = wG/\varepsilon; \theta_w = wG/(1 + \varepsilon)$.

Fig. 14. Estimated soil-water characteristic curve for Regina clay.

Fig. 15. Computed coefficient of permeability function for Regina clay. $k_{sat}$, saturated hydraulic conductivity.
conditions using eq. [28] for \( \epsilon_0 = 0.962, C_m = 0.090, \) and \( \mu = 0.40 \) and can be written as follows:

\[ H = 140.5(u_a - u_w) \]

The in situ stress state (IST) is the sum of net normal stress and matric suction equivalent and can be written as follows for a two-dimensional analysis:

\[ \text{IST} = (\sigma_{ave} - u_w)_{f1} + f_i(u_a - u_w)_{f1} \]

or

\[ \text{IST} = \frac{1 + K_0}{2} (\sigma_y - u_a)_{f1} + f_i(u_a - u_w)_{f1} \]

The final stress state (FST) can be written as follows:

\[ \text{FST} = (\sigma_{ave} - u_w)_{f1} + f_i(u_a - u_w)_{f1} \]

or

\[ \text{FST} = \frac{1 + K_0}{2} (\sigma_y - u_a)_{f1} + \Delta(\sigma_{ave} - u_a) + f_i(u_a - u_w)_{f1} \]

Deformation of the slab associated with this case was due to applied load and wetting. Deformation of the slab and in the soil mass due to loading can be assumed to respond immediately, whereas the deformations due to wetting are a time-dependent process. Therefore, the stress–deformations due to loading and wetting need to be analyzed independently. Figure 18 shows the stress path followed in the analysis. The stress–deformation analysis was first performed to predict the displacements and induced stress due to the loading of the slab. The deformations due to changes in matric suction were then predicted for various elapsed times using matric suction profiles obtained from the seepage analysis. The stress–deformation analysis is also performed for the cases when pore-water pressure goes to zero and when the groundwater level rises to the ground surface, resulting in a hydrostatic pore-water pressure distribution. Hydrostatic conditions present the upper limits for total heave.

Figure 19 shows the geometry and boundary conditions for the stress–deformation analysis. A load equal to 5.76 kPa is applied on the surface of a 100 mm thick concrete slab. This surcharge is made up of 180 mm of fill with a unit weight of 18.88 kN/m\(^3\) and 100 mm of concrete with a unit weight of 23.6 kN/m\(^3\). An accurate perimeter load is unknown, however, a typical value of 15 kN/m was assumed. The perimeter load includes the weight of footing and the load of the upper structure. The soil is free to move in a vertical direction and fixed in the horizontal direction at the left and right sides of the domain. The lower boundary is fixed in both directions. A Young’s modulus of 10 GPa and Poisson’s ratio of 0.15 were used for the concrete slab.

A parametric study was also performed to show the effect of varying Poisson’s ratio, swelling index, initial void ratio, coefficient of earth pressure at rest, and stiffness of the concrete slab on the predicted results.
A general-purpose partial differential equation solver, called FlexPDE, was used in this study to solve the saturated–unsaturated water flow (i.e., eq. [16]) and stress–deformation (i.e., eqs. [21] and [22]) associated with the unsaturated soil.

FlexPDE is a scripted finite element model builder and numerical solver for both two- and three-dimensional problems. The user must specify the governing partial differential equations to be solved (such as eq. [16] for seepage or eqs. [21] and [22] for stress–deformation). FlexPDE performs the operations necessary to turn a description of the partial differential equations system into a finite element model, solves the system, and presents graphical output of the results (PDE Solutions Inc. 2001). FlexPDE is an automatic mesh generation and refinement, adaptive time step design and refinement program. The equations can be linear or nonlinear. Nonlinear equations are solved by applying a modified Newton–Raphson iteration process. The material properties can be described in a tabular form or as a mathematical equation. Boundary conditions can be specified as a dependent variable type or a derivative of a dependent variable type.

**Computer results and discussions**

Figure 20 presents changes of matric suction with time at various points in the soil. Matric suctions decreased significantly during the first 30 days of wetting and approached a steady state condition in about 150 days. The total amount of water that leaked from the water line with time is presented in Fig. 21. A loss of 35 m$^3$ of water is equivalent to about 3.5 m$^3$ of water over each metre of width of the floor. This amount of water is more than the amount of water required for the steady state condition attained under the specified boundary conditions. Figure 22 presents pore-water pressure profiles under the centre of the slab for various times and also for the case where the final pore-water pressure goes to zero and then a hydrostatic condition. Figure 23 shows the matric suction distribution in the soil at steady state conditions. It can be noted that, under the specified boundary conditions, the matric suction at steady state conditions is about 20 kPa under the centre of the slab.

---

FlexPDE is a proprietary product of PDE Solutions Inc., 2120 Spruce Way, Antioch, CA 94509, USA.
Figure 24 shows contours of vertical displacement due to loading. Less than 1 mm of settlement is predicted due to the loading at the centre of the slab. The induced net normal stress was used to calculate the final net normal stress state in the soil. Also, the soil was loaded at initial net normal stress and matric suction conditions in the field. Therefore, the sum of initial net normal stress and initial matric suction equivalent must be used along with the swelling index obtained on the net normal stress plane for the prediction of displacements and induced stresses due to loading.

Figure 25 compares the predicted vertical displacements at various final suction conditions with the measured total heave at the centre of the slab. The agreement between the predicted and the measured heave at different depths differs to some degree. The amounts of heave measured at depths of 0.58 and 0.85 m correspond to the predicted heave at 100 days, and the total heave of 106 mm at ground surface corresponds to the case when the pore-water pressure goes to zero under the slab. It must be noted that a heave of 106 mm represents the maximum heave observed on the slab. The maximum heave observed at the cross section under consideration is only 80 mm (see Fig. 10). The distribution of horizontal displacements at different final suction conditions right under the water line is presented in Fig. 26.

Figure 27 compares the predicted vertical displacement at various final matric suction conditions with the measured total heave at the top surface of the slab. The total heave predicted under steady state conditions agrees well with the measured heave. It should be noted that there were some unknown loads placed near the perimeter of the floor slab, and this was not considered in the study. Assuming that the final pore-water pressure increases from a negative value to zero results in a maximum predicted heave of 107 mm. A maximum heave of 130 mm is predicted for the case when the final pore-water pressures are assumed to be hydrostatic.

Figure 28 shows the measured and predicted water contents under the centre of the slab. The final water contents predicted at steady state conditions are about 3% less than...
the measured final water contents. When the pore-water pressure increased to zero, the predicted final water content is about 2% more than the measured values. The distribution of water contents in the soil predicted at steady state conditions is shown in Fig. 29.

Figures 30 and 31 present contours of horizontal and vertical displacements predicted at steady state conditions.

Results of the parametric studies

The results of the presented stress–deformation analysis depend on the calculated elasticity parameter functions (i.e., $E$, $H$, and $\mu$), initial stress state conditions, and stiffness of the concrete slab. The elasticity parameters $E$ and $H$ are calculated from the initial void ratio $e_0$, swelling index $C_s$, and assumed value of Poisson’s ratio $\mu$. The initial horizontal net normal stress is estimated using the coefficient of earth pressure at rest, $K_0$.

Several stress–deformation analyses were performed to study the effect of each of the aforementioned parameters on the solutions and to gain confidence in their significance to the analysis. For each case, only the parameter under consideration was allowed to vary; all other parameters were kept constant.

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Fig. 26. Distribution of horizontal displacements with depth at different matric suction conditions under the water line.

Fig. 27. Measured and predicted vertical displacements at the surface of the slab.

Fig. 28. Measured and predicted water content with depth near the centre of the slab.
unchanged at the values used for the “base case.” Table 4 shows the values of the parameters that were used in the parametric study. It should be noted that the relationship between Poisson’s ratio and the coefficient of earth pressure at rest (i.e., eq. [29]) was not considered in the parametric studies. The displacements are only calculated for steady state conditions of matric suction.

Figures 32 and 33 present the predicted vertical displacements at the surface of the slab and under the centre of the slab, respectively, for various values of Poisson’s ratio.

Table 4. Values of parameters used in the parametric study.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Lower values</th>
<th>Base case</th>
<th>Upper values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poisson’s ratio, μ</td>
<td>0.30, 0.35</td>
<td>0.40</td>
<td>0.45, 0.49</td>
</tr>
<tr>
<td>Swelling index, C_s</td>
<td>0.085</td>
<td>0.090</td>
<td>0.095, 0.100</td>
</tr>
<tr>
<td>Initial void ratio, e_0</td>
<td>0.920</td>
<td>0.962</td>
<td>1.000</td>
</tr>
<tr>
<td>Coefficient of earth pressure at rest, K_0</td>
<td>0.400</td>
<td>0.667</td>
<td>1.000</td>
</tr>
<tr>
<td>Young’s modulus of concrete, E_c (GPa)</td>
<td>5</td>
<td>10</td>
<td>20</td>
</tr>
</tbody>
</table>

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total heave predicted increases with an increasing value for the assumed Poisson’s ratio. For a change in Poisson’s ratio from 0.35 to 0.45, the variation in maximum total heave predicted is about ±7%.

Figures 34 and 35 present the predicted vertical displacements at the surface of the slab and under the centre of the slab, respectively, for various values of the swelling index. The total heave predicted increases with an increase in the value of the swelling index. For a change in the swelling index varying from 0.085 to 0.095, the variation in the maximum total heave predicted is about ±5%.

Figures 36 and 37 present the predicted vertical displacements at the surface of the slab and under the centre of the slab, respectively, for various values of initial void ratio. The total heave predicted increases with a decrease in the value of the initial void ratio. For a change in initial void ratio varying from 0.920 to 1.000, the variation in maximum total heave predicted is about ±2%.

Figures 38 and 39 present the predicted vertical displacements at the surface of the slab and under the centre of the slab, respectively, for various values of the coefficient of earth pressure at rest. The total heave predicted increases with a decrease in the value of the coefficient of earth pressure at rest. For a change in the coefficient of earth pressure at rest from 0.4 to 1.0, the variation in maximum total heave predicted is about ±2%.

Figures 40 and 41 present the predicted vertical displacements at the surface of the slab and under the centre of the slab, respectively, for various values of Young’s modulus for concrete slab. The maximum total heave predicted increases with a decrease in the value of Young’s modulus for concrete. For a change in Young’s modulus for the concrete from 5 to 20 GPa, the variation in maximum total heave predicted is about ±3%. The distribution of heave at the surface of the slab changes considerably with changes in the stiffness of the slab, however. The concrete slab with the lowest
Fig. 34. Measured and predicted vertical displacements at the surface of the slab for various values of swelling index.

Fig. 35. Measured and predicted vertical displacements with depth near the centre of the slab for various values of swelling index.

Fig. 36. Measured and predicted vertical displacements at the surface of the slab for various values of initial void ratio.
Fig. 37. Measured and predicted vertical displacements with depth near the centre of the slab for various values of initial void ratio.

![Graph showing measured and predicted vertical displacements with depth for various initial void ratios.]

Fig. 38. Measured and predicted vertical displacements at the surface of the slab for various values of the coefficient of earth pressure at rest.

![Graph showing measured and predicted vertical displacements at the surface for various coefficients of earth pressure at rest.]

Fig. 39. Measured and predicted vertical displacements with depth near the centre of the slab for various values of the coefficient of earth pressure at rest.
Young’s modulus would suffer more differential heave. The cracking of the slab may result in a larger amount of maximum heave.

The results of the parametric study are summarized in Table 5. It can be seen that the swelling index and Poisson’s ratio are the factors that have the greatest effect on the solution. The value of Young’s modulus for concrete appears to control the shape of the deformed slab.

### Table 5. Results of the parametric study.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
<th>Variation of predicted heave (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poisson’s ratio, $\mu$</td>
<td>0.35–0.45</td>
<td>±7</td>
</tr>
<tr>
<td>Swelling index, $C_s$</td>
<td>0.085–0.950</td>
<td>±5</td>
</tr>
<tr>
<td>Initial void ratio, $e_0$</td>
<td>0.92–1.00</td>
<td>±2</td>
</tr>
<tr>
<td>Coefficient of earth pressure at rest, $K_0$</td>
<td>0.4–1.0</td>
<td>±2</td>
</tr>
<tr>
<td>Young’s modulus of concrete, $E_c$ (GPa)</td>
<td>5–20</td>
<td>±3</td>
</tr>
</tbody>
</table>

**Conclusions**

The proposed method for the prediction of heave based on the general theory of unsaturated soil provides a practical means of predicting multidimensional heave in unsaturated, expansive soils. Changes in matric suction (or pore-water pressure) in the soil mass are estimated through a saturated–unsaturated seepage analysis. Displacements due to loading...
and changes in matric suction are predicted using a stress-deformation analysis. The elasticity parameter functions required for the stress-deformation analysis can be calculated from conventional oedometer test results.

Verification of the proposed method is accomplished using data collected for the case history of a floor slab in a light industrial building on Regina clay. The predicted results appear to be in reasonable agreement with measured values. The results of the analysis show that the predicted heave in two dimensions is somewhat sensitive to the assumed value of Poisson’s ratio and the measured swelling index. The predicted heave for the case history presented in this study is insignificantly affected by the initial void ratio measured in the laboratory and the coefficient of earth pressure at rest.

References