

## FINITE ELEMENT SIMULATIONS OF TEMPERATURE CHANGES IN A HIGHWAY SUBGRADE

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### ABSTRACT

This paper studies the simulation of temperature readings beneath a thin membrane pavement system using a two-dimensional finite element solution method. The accuracy of the results of the numerical simulations are verified using temperature readings obtained from thermal conductivity sensors installed at Torquay and Bethune, Saskatchewan. The purpose of this portion of the study was to verify the ability and accuracy of simulating heat transfer using a finite element simulation. A parametric study was also conducted to investigate the effect of changes in the geotechnical properties on the thermal conductivity and the volumetric specific heat capacity of the soil. Boundary conditions used in the simulation included surface temperatures taken from thermistors and matric suctions recorded by the thermal conductivity suction sensors. This paper also demonstrates the ability of the SVHeat software in accommodating a large amount of data describing the surface temperature conditions, while conducting the thermal simulations.

### RESUME

Cet article étudie la simulation des lectures de température sous une mince chaussée par une méthode bi-dimensionnelle d'élément fini. La précision des résultats des simulations numériques sont vérifiés en utilisant les lectures de température obtenues à partir de senseurs de conductivité thermique installés à Torquay et Bethune en Saskatchewan. L'objectif de cette section de l'étude est de vérifier la capacité et l'exactitude de simulation du transfert de chaleur en utilisant une méthode d'élément fini. De plus, une étude paramétrique est réalisée afin d'évaluer l'effet des changements de propriétés géotechniques sur la conductivité thermique et sur la capacité calorifique volumétrique spécifique du sol. Les conditions limites utilisées dans la simulation incluent les températures de surface obtenues à partir de thermistores ainsi que les suctions interstitielles enregistrées par des senseurs de succion par conductivité thermique. Cet article démontre aussi l'utilité du logiciel SVHeat dans la manipulation d'une grande quantité de données définissant les conditions de température de surface tout en réalisant les simulations thermiques.

## 1. INTRODUCTION

Thin membrane surface pavement (TMS) roads have been used for secondary roads in the province of Saskatchewan, Canada. With the closure of grain elevators in the province, farmers have to haul grain across longer distances increasing the usage of the secondary roads. For most of the year, the thin membrane roads are able to fulfill the purpose of providing a dust-free driving environment for low volume loads. However, with the arrival of spring, thawing occurs and subsequently decreases the matric suction and the bearing capacity of the road.

Sixteen thermal conductivity matric suction sensors were installed beneath two TMS sites at Torquay and Bethune, Saskatchewan. These sensors record both soil temperature and matric suction every four hours with the exception of winter when readings are taken every 12 hours. Thermistors were also installed at the sites to record the surface temperature of the road. Readings for surface temperatures were taken every hour.

The following section will describe how the surface temperatures obtained from the thermistors were used to predict the soil temperature at 0.3 meters below the surface of the pavement using a two-dimensional non-coupled thermal analysis. The results of this simulation are compared to the soil temperature readings obtained from the thermal conductivity matric suction sensors. A favorable comparison will enable a reasonable description of the thermal regime beneath the road

subgrade to be made and also provide a more accurate prediction of the freezing period of the ground.

## 2. BOUNDARY CONDITIONS AND MATERIAL PROPERTIES

The geometry of the site in Torquay and Bethune are based on information provided by Marjerison (2001). The elevation of the sites, cross-section and location of the sensors at each of the site were obtained from Marjerison (2001) and other unavailable data (i.e., the gradient of the shoulder and the side-slope) were provided by the Saskatchewan Department of Highways and Transportation. The cross-section for both sites have roadways with 3% gradients, shoulders with gradients of 4:1 and 3% gradients for the side-slopes.

The program used for this simulation is SVHeat<sup>1</sup> which has an automated mesh and time step refinement feature. The surface temperature applied along the entire surface of the problem is the only changing boundary condition while the remainder of the boundary conditions is no flux boundaries. The surface temperature is entered in a tabular form and changes every hour. Where there are missing surface temperature data, an interpolation method is used to fill in the missing information.

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<sup>1</sup> SVHeat is a proprietary product of SoilVision Systems Ltd., Saskatoon, SK., Canada.

The limited information available for the properties of the soil at the sites resulted in the need to reference the literature for typical values. The material properties used in the simulation, such as the soil-water characteristic curve, the soil freezing curve, the porosity and the thermal properties were taken from examples of similar soils found in the literature. The volumetric water content was obtained from the matric suction readings taken by the thermal conductivity sensors. Matric suction readings were taken at 4 hours intervals with the exception when the ground was frozen.

Vanapalli et al. (1996) used a best-fit equation [Eq. 1] to represent a Soil-Water Characteristic Curve obtained from laboratory testing of an unsaturated glacial till. For the purpose of this simulation program, a slight modification is made to the 'm' parameter of the Soil-Water Characteristic Curve equation to create a more gradual descent of the curve to zero saturation at  $10^6$  kPa of suction. This enables a more reasonable soil freezing curve to be obtained.

$$\theta = C(\psi) \left[ \frac{\theta_s}{\ln \left[ \left( e + (\psi / a)^n \right)^m \right]} \right]$$

[Eq. 1]

where:

- $\theta$  = volumetric water content,
- $\theta_s$  = saturated volumetric water content,
- $a$  = suction related to the air-entry value of the soil,
- $n$  = soil parameter related to the slope at the inflection point on the soil-water characteristic curve,
- $\psi$  = soil suction,
- $m$  = soil parameter related to the residual water content,
- $\theta_r$  = volumetric water content at residual conditions,
- $e$  = natural number and
- $C(\psi)$  = correction function.

The correction function is defined as:

$$C(\psi) = \left[ 1 - \frac{\ln \left( 1 + \frac{\psi}{\psi_r} \right)}{\ln \left( 1 + \frac{1000000}{\psi_r} \right)} \right]$$

[Eq. 2]

where,  $\psi_r$  is the suction value corresponding to residual water content,  $\theta_r$ .

The soil freezing curve used in this simulation was derived from the soil-water characteristic curve fit function by Fredlund and Xing (1994). The relationship between the soil-water characteristic curve and soil freezing curve proposed by Koopmans and Miller (1966) for pure adsorption forces in soil was also taken into consideration to obtain the soil freezing curve shown in Eq. 3;

$$\theta_u = C(T') \left[ \frac{1}{\ln \left[ \exp(1) + \left[ \frac{C_f 1110.T'}{a} \right]^n \right]^m} \right] \theta_{sat}$$

[Eq. 3]

where,  $C_f$  is the volumetric water content at residual suction,  $n$ ,  $m$  and  $a$  are curve fit parameters,  $T'$  is the temperature in  $^{\circ}\text{C}$  and  $C(T')$  is a curve fit correction factor given by:

$$C(T') = \left[ 1 - \frac{\ln \left[ 1 + \frac{C_f 1110.T'}{C_r} \right]}{\ln \left[ 1 + \frac{C_f 1110.T'}{C_r} \right]} \right]$$

[Eq. 4]

and  $C_r$  is an additional curve fit parameter.

The additional curve fit parameter,  $C_r$ , was obtained by trial and error to create a reasonable soil freezing curve. The slope of the soil freezing curve,  $m_1^2$ , was obtained by differentiating the soil freezing curve (Pentland, 2000).

The thermal conductivity and specific heat capacity of the soil is based on the typical values published by Andersland and Ladanyi (1994) and de Vries (1963).

The unsaturated glacial till samples tested by Vannapalli et al. (1996) yielded dry densities of approximately  $1.8 \text{ Mg/m}^3$  with void ratios of about 0.47. A list of general material properties from various soils compiled by Andersland and Ladanyi (1994) gave a range of porosity for glacial till from 0.46 to 0.12. A method of trial and error was used in determining the porosity for this simulation. Two layers or porosities were used. The top layer nearer to the surface of the road is considered to have a lower porosity compared to the bottom layer of the subgrade.

For the parametric study, variations were made to the values for porosity and volumetric water content to observe the effects of these properties on the output.

### 3. RESULTS AND DISCUSSION

Fig. 1 shows the comparison of temperature results for the site in Torquay obtained from field data taken by the sensors and results from the numerical simulation conducted. The readings were taken at a depth of 0.3 meters below the surface of the road. The results are shown for a period of 500 days from February 12, 2001 to June 27, 2002 with an interval of 100 days on the x-axis.

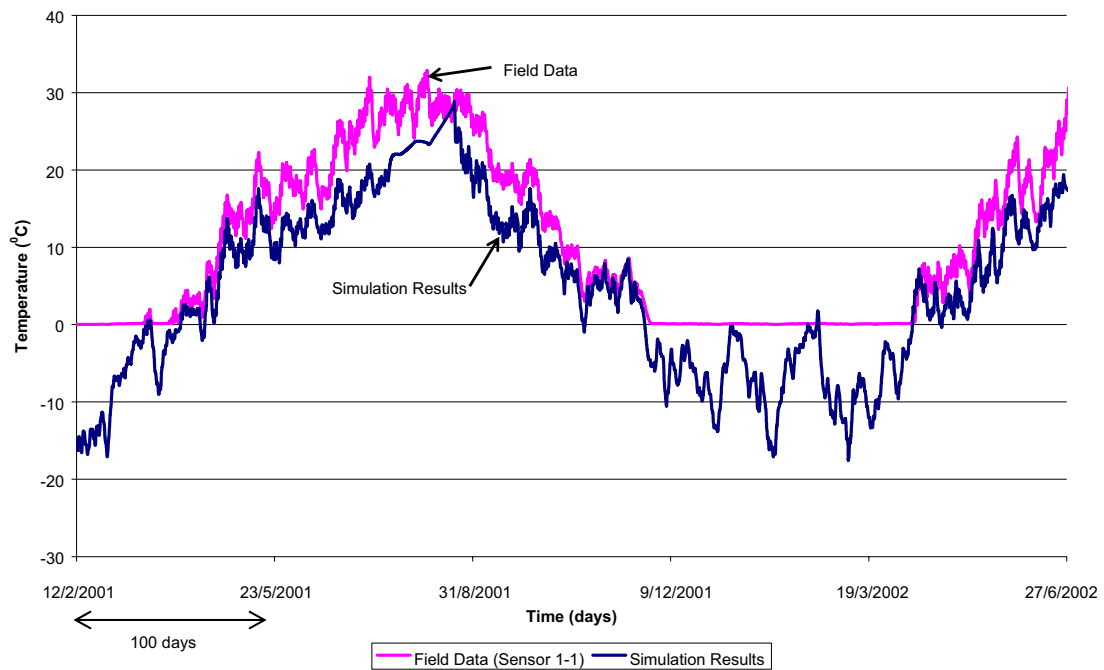


Figure 1. Comparison between simulation results and field data 0.3 meters below the surface of the road for Torquay, SK.

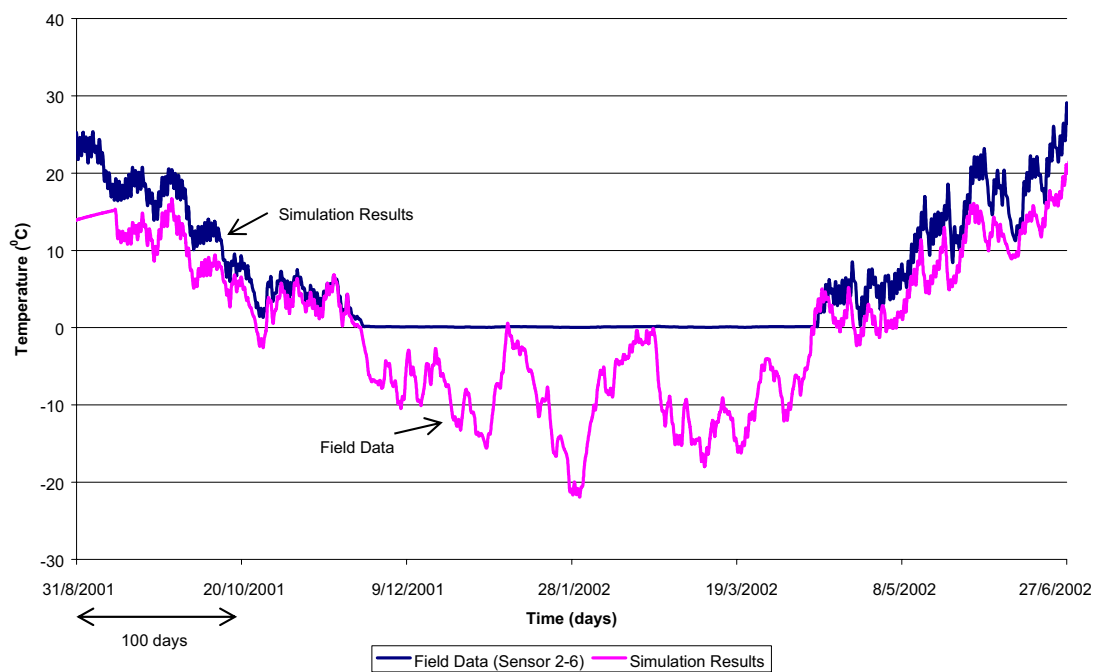


Figure 2. Comparison between simulation results and field data 0.3 meters below the surface of the road for Bethune, SK.

The readings taken by the sensors and the results from the simulation are comparable. The magnitude of the fluctuations appear to be similar but the overall amplitude is lower for the simulated results compared to the field data. The difference in overall amplitude could be influenced by the stress applied to the road by vehicles and this was not considered in the simulation. The missing surface temperature data between July 19, 2001 to August 25, 2001 could also influence the overall amplitude of the graph obtained.

The period when frost enters and leaves the ground seem to correspond with the field data. Readings below freezing temperatures (i.e., 0°C) are not recorded by the sensors and a value of zero is assigned. The predictions obtained from the simulations show the ground freezing and thawing at about the same time as provided by the readings from the thermal conductivity suction sensors.

Results for the site at Bethune shown in Fig. 2 exhibit the same behavior. The freezing period, measures fluctuations and the magnitude of the fluctuations from the simulation results are comparable but the overall amplitude is lower than for the field data.

Fig. 3 shows the results obtained by varying the average volumetric water content at constant porosity for Torquay. The change in volumetric water content with constant porosity

resulted in an increase in amplitude of the overall soil temperature curve with increasing volumetric water content. Volumetric water content is a significant factor in determining the thermal conductivity and the volumetric specific heat capacity of the soil. With an increase of volumetric water content, both the thermal conductivity and the volumetric specific heat capacity of the soil increase. The increase in thermal conductivity results in an increasing amount of heat passing a unit cross-sectional area of the soil over a unit time under a unit temperature gradient. This increases the efficiency of heat transfer. The increase in volumetric specific heat, however, increases the amount of heat required to raise the temperature of one unit volume of soil by one degree. This decreases the efficiency of heat transfer.

Fig. 4 shows the increase in thermal conductivity and volumetric specific heat capacity with an increase in volumetric water content is not in direct proportion with each other. The volumetric specific heat capacity appears to increase in a linear pattern while the thermal conductivity increases rapidly with increasing water content. This leads to the non-linear decrease in soil temperature shown in Fig. 4 for Torquay at February 25, 2001 for temperatures below 0°C. The same trend can also be observed for temperatures above 0°C. The soil temperature increases in a non-linear manner with increasing volumetric water content and appear to take on the same shape as the thermal conductivity curve.

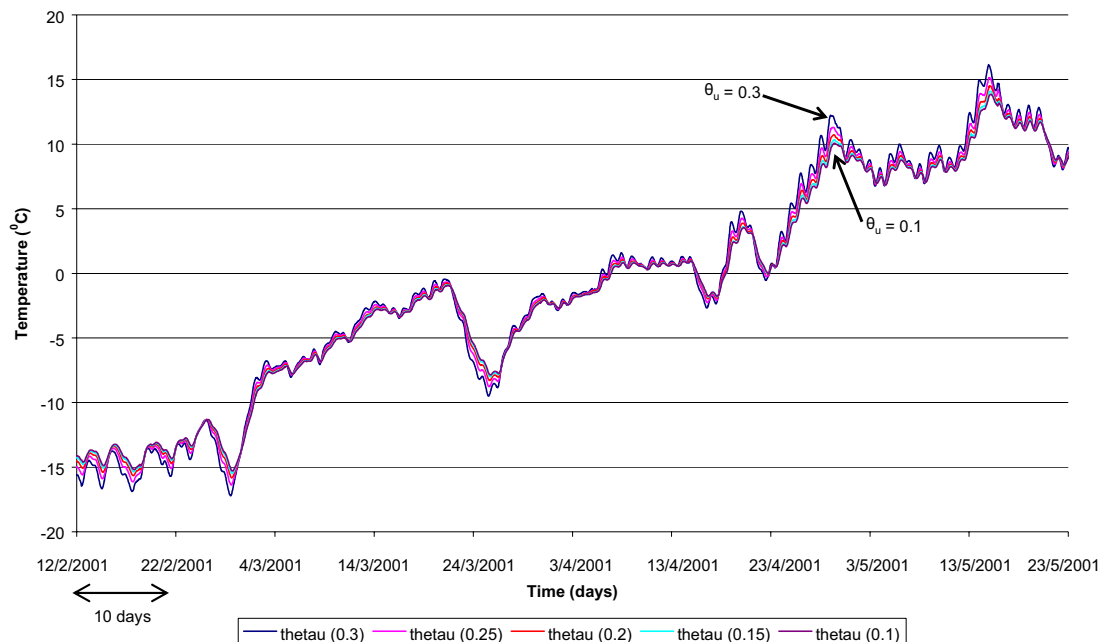


Figure 3. Comparison of soil temperature results obtained by varying the volumetric water content at constant porosity for Torquay

Temperature and Thermal Properties versus Volumetric Water Content for Torquay  
(Temperature < 0°C)

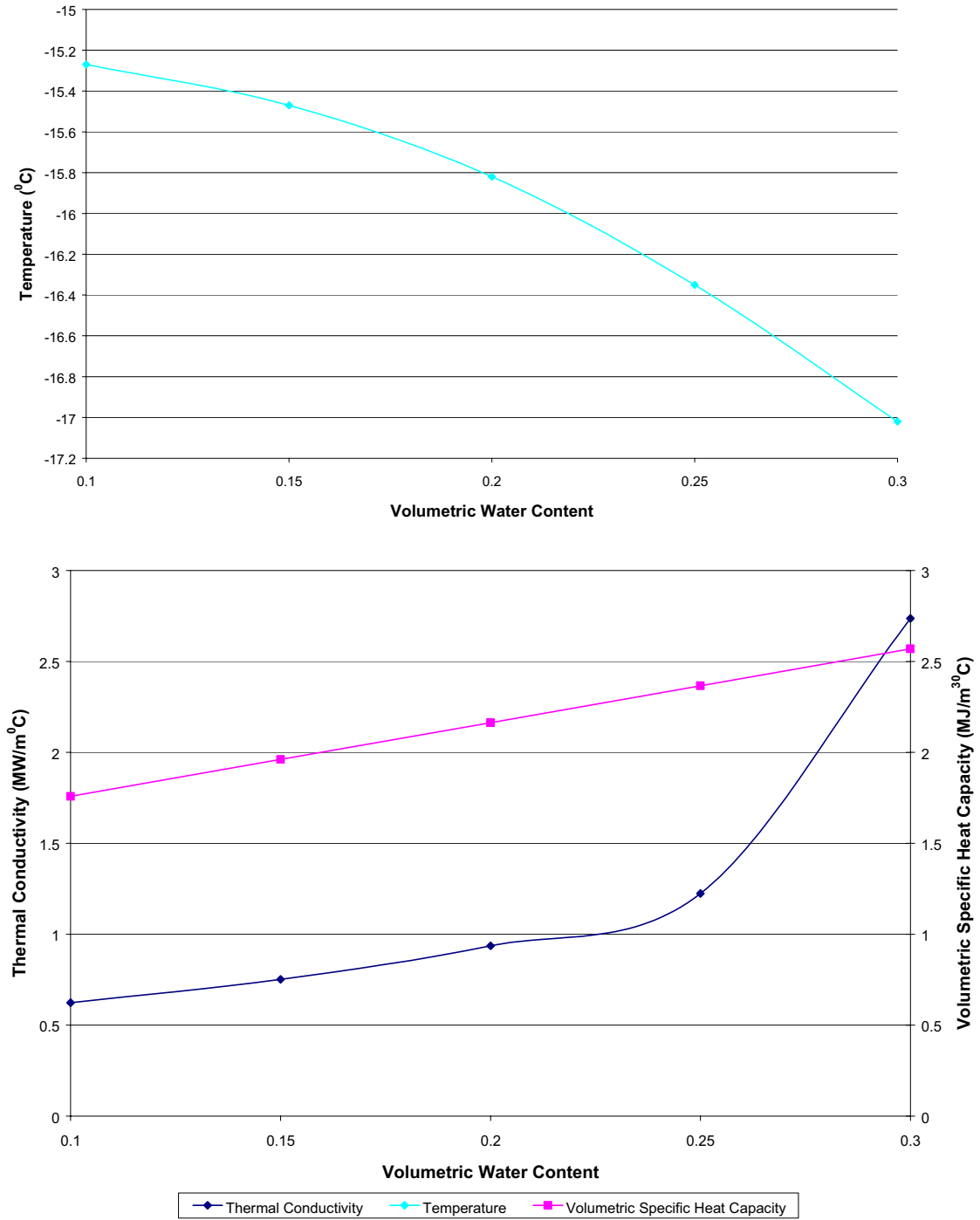


Figure 4. Temperature and thermal properties versus volumetric water content for Torquay at February 25, 2001 (Temperature < 0°C)

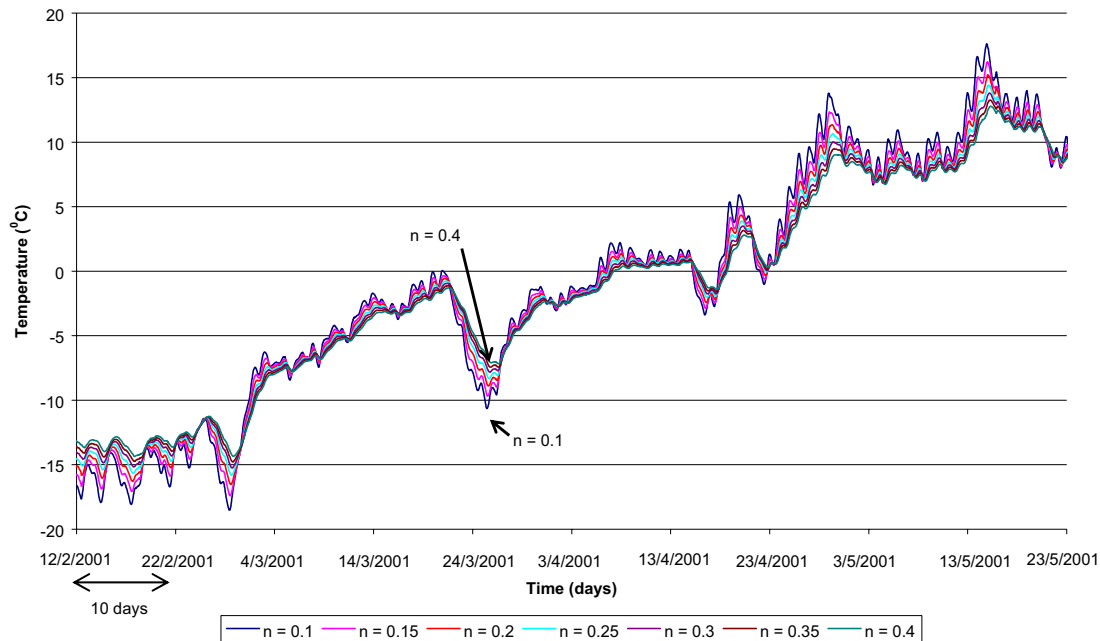


Figure 5. Comparison of temperature results obtained by varying the porosity at constant volumetric water content for Torquay, SK

Fig. 5 shows the effect of varying porosity for Torquay while keeping volumetric water content constant. At decreasing porosity with a constant water content, the overall amplitude of the temperature output also increases. The increase in porosity, at constant volumetric water content, also increases the thermal conductivity and the volumetric specific heat capacity of the soil. It is observed that the volumetric specific heat capacity of the soil increases at a rate that is lower than the increase in thermal conductivity. This results in temperature outputs that vary in a non-linear pattern with increasing porosity. At temperatures below  $0^{\circ}\text{C}$  taken from Torquay, the decrease in computed temperatures obtained with increasing water contents occur nonlinearly. The same can be observed from results for temperatures above  $0^{\circ}\text{C}$  where the increase in temperature occurs in a nonlinear manner.

#### 4. CONCLUSION

Reasonable predictions for soil temperatures that correspond to field data from the thermal conductivity sensors were obtained even in the absence of detailed soils information at the site. The results generated by SVHeat indicates that the computer software is capable of handling large amounts of data input (i.e., surface temperature conditions and volumetric water content) while conducting thermal simulations.

#### 5. ACKNOWLEDGEMENT

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