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Challenges of Using Unsaturated Soil Mechanics in Engineering Practice

Professor Delwyn G. Fredlund
University of Saskatchewan
Saskatoon, SK, Canada

Tokyo, Japan

From the Challenges to the Usage of Unsaturated Soil Mechanics in Engineering Practice

- *Introduction*
- *Challenges to Implementation*
- *Description of the Stress State*
- *Fundamental Constitutive Relations*
- *Role of the Soil-Water Characteristic Curve*
- *Use of SWCC in the Constitutive Relations*
- *Solution of a Series of PDEs*
- *Modeling Unsaturated Soils Problems*

Objectives

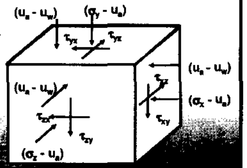
- *To illustrate the progression from theories and formulations to practical engineering protocols for unsaturated soil mechanics problems (e.g., seepage, shear strength and volume change), through the use of "indirect" characterization of unsaturated soil property functions*
- *To describe Challenges faced and the Solutions Generated in moving towards the Implementation of Unsaturated Soil Mechanics*

Gradual Emergence of Unsaturated Soil Mechanics

- *1950s: Independent measurement of pore-air and pore-water pressure through use of high air entry ceramic disks*
- *1960s: Laboratory testing of unsaturated soils*
- *1970s: Constitutive relations proposed and tested for unsaturated soils*
- *1980s: Solving formulations for classic Boundary Value Problems*
- *1990s: Establishing protocols for determination of unsaturated soil property functions*
- *2000+: Implementation into routine engineering practice*

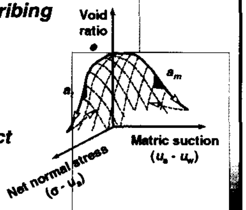
Challenges to the Implementation of Unsaturated Soil Mechanics

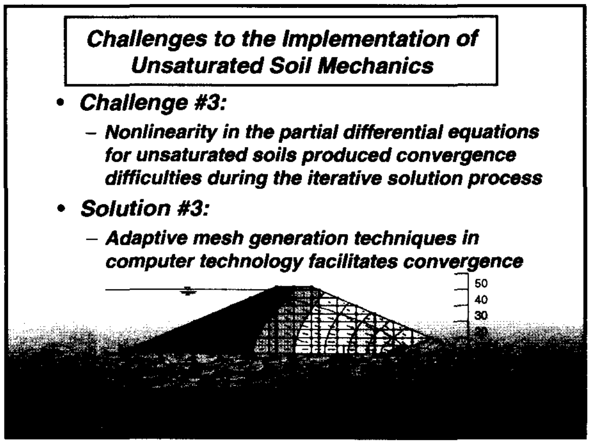
- *Challenge #1:*
  - *Produce a theoretical basis for describing the physical behavior of unsaturated soils*
- *Solution #1:*
  - *Use of independent stress state variables based on multiphase continuum mechanics principles*



Challenges to the Implementation of Unsaturated Soil Mechanics

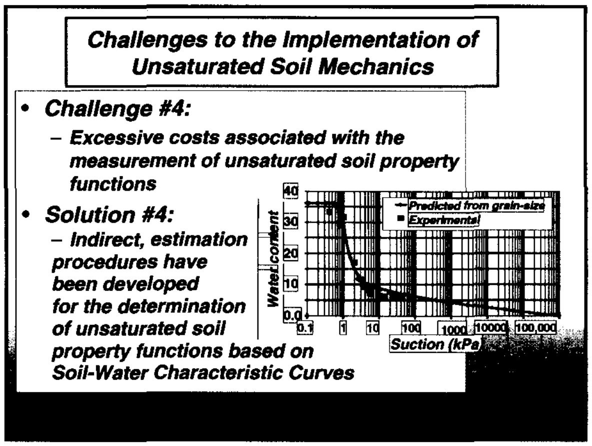
- *Challenge #2:*
  - *Constitutive relations for saturated soils were unacceptable for describing unsaturated soil behavior*
- *Solution #2:*
  - *Constitutive relations for saturated soil needed to be extended to embrace the effect of changing degrees of saturation*





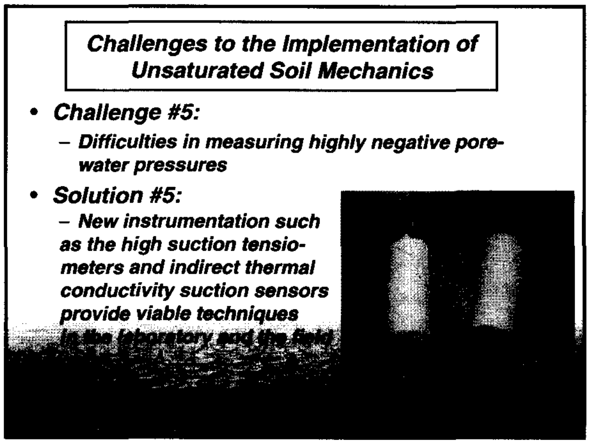
Challenges to the Implementation of Unsaturated Soil Mechanics

- ***Challenge #3:***
  - ***Nonlinearity in the partial differential equations for unsaturated soils produced convergence difficulties during the iterative solution process***
- ***Solution #3:***
  - ***Adaptive mesh generation techniques in computer technology facilitates convergence***



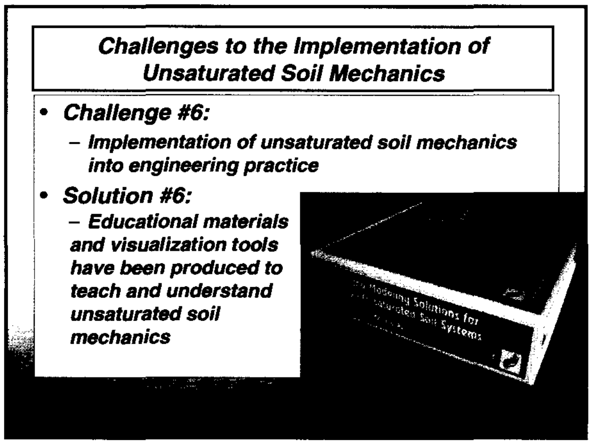
Challenges to the Implementation of Unsaturated Soil Mechanics

- ***Challenge #4:***
  - ***Excessive costs associated with the measurement of unsaturated soil property functions***
- ***Solution #4:***
  - ***Indirect, estimation procedures have been developed for the determination of unsaturated soil property functions based on Soil-Water Characteristic Curves***



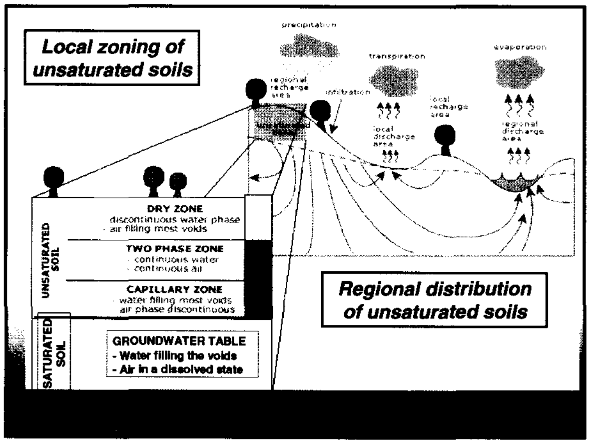
Challenges to the Implementation of Unsaturated Soil Mechanics

- ***Challenge #5:***
  - ***Difficulties in measuring highly negative pore-water pressures***
- ***Solution #5:***
  - ***New instrumentation such as the high suction tensiometers and indirect thermal conductivity suction sensors provide viable techniques in the laboratory and the field***



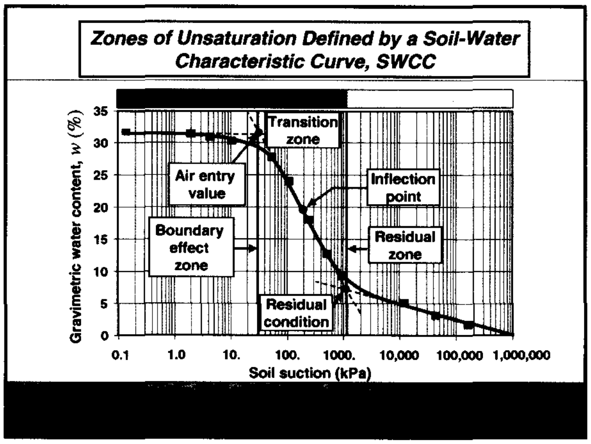
Challenges to the Implementation of Unsaturated Soil Mechanics

- ***Challenge #6:***
  - ***Implementation of unsaturated soil mechanics into engineering practice***
- ***Solution #6:***
  - ***Educational materials and visualization tools have been produced to teach and understand unsaturated soil mechanics***

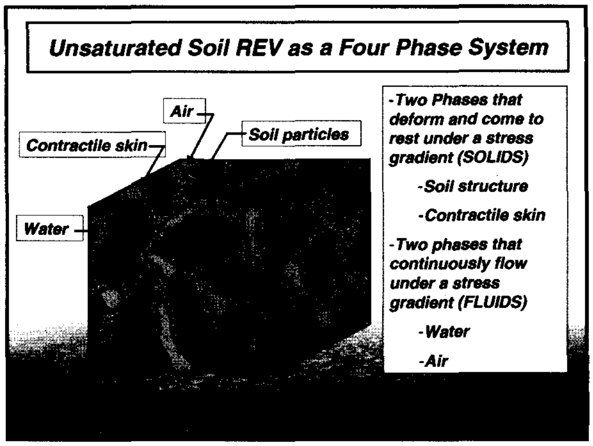


Local zoning of unsaturated soils

Regional distribution of unsaturated soils

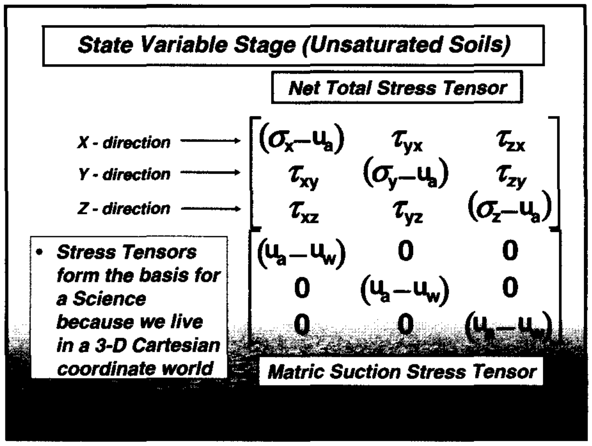


Zones of Unsaturation Defined by a Soil-Water Characteristic Curve, SWCC



Unsaturated Soil REV as a Four Phase System

- Two Phases that deform and come to rest under a stress gradient (SOLIDS)
  - Soil structure
  - Contractile skin
- Two phases that continuously flow under a stress gradient (FLUIDS)
  - Water
  - Air



State Variable Stage (Unsaturated Soils)

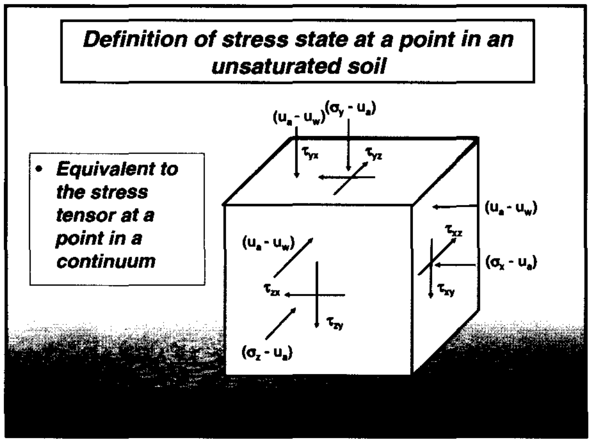
Net Total Stress Tensor

$$\begin{array}{l} \text{X - direction} \rightarrow \\ \text{Y - direction} \rightarrow \\ \text{Z - direction} \rightarrow \end{array} \begin{bmatrix} (\sigma_x - u_a) & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & (\sigma_y - u_a) & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & (\sigma_z - u_a) \end{bmatrix}$$

$$\begin{bmatrix} (u_a - u_w) & 0 & 0 \\ 0 & (u_a - u_w) & 0 \\ 0 & 0 & (u_a - u_w) \end{bmatrix}$$

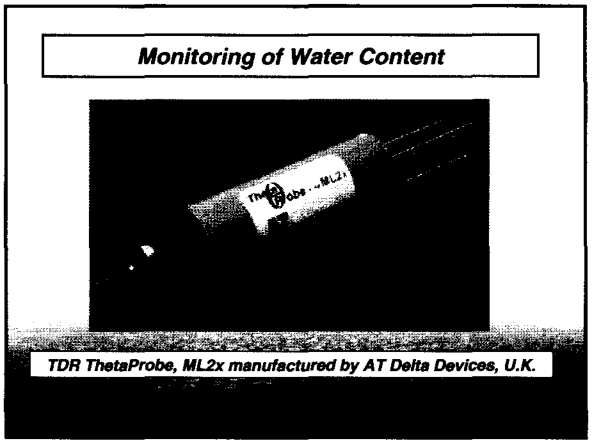
- Stress Tensors form the basis for a Science because we live in a 3-D Cartesian coordinate world

Matric Suction Stress Tensor



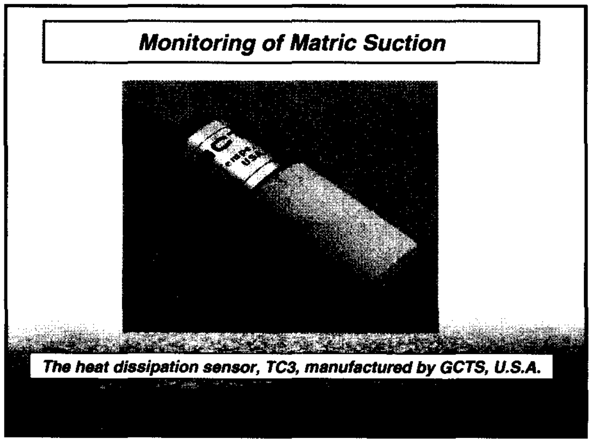
Definition of stress state at a point in an unsaturated soil

- Equivalent to the stress tensor at a point in a continuum



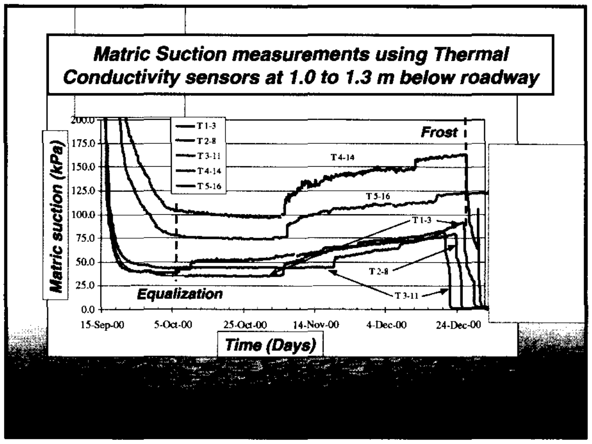
Monitoring of Water Content

TDR ThetaProbe, ML2x manufactured by AT Delta Devices, U.K.



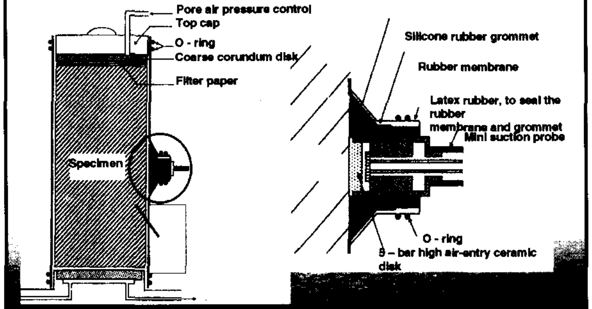
Monitoring of Matric Suction

The heat dissipation sensor, TC3, manufactured by GCTS, U.S.A.



Matric Suction measurements using Thermal Conductivity sensors at 1.0 to 1.3 m below roadway

Direct, high suction sensor used to measure suctions greater than one atmosphere on the side of a triaxial specimen (Meilani, 2004)



Fundamental Constitutive Relations for Unsaturated Soils

- ***Constitutive behaviors in Classical Soil Mechanics:***
  - ***Seepage (Liquid water and Vapor)***
  - ***Shear strength***
  - ***Volume-mass changes: Void ratio, water content changes***
- ***Other topics in soil mechanics:***
  - ***Heat flow***
  - ***Air flow***
  - ***Contaminant transport***
- ***Processes to be solved:***
  - ***"Independent" processes***
  - ***"Uncoupled" processes***
  - ***"Coupled" processes***

Each constitutive relationship requires a linear or nonlinear soil property

Water Seepage Constitutive Relations

$$h = \frac{u_w}{\rho_w g} + Y$$

Driving potential for water flow is hydraulic head, h

$$v_x = \boxed{-k_{wx}} \frac{dh}{dx}$$

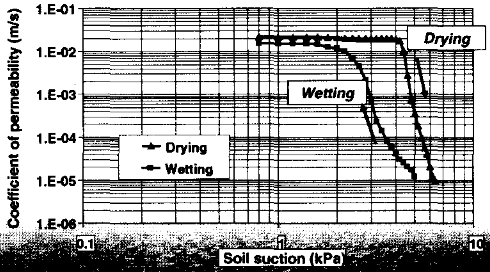
$$v_y = \boxed{-k_{wy}} \frac{dh}{dy}$$

$$v_z = \boxed{-k_{wz}} \frac{dh}{dz}$$

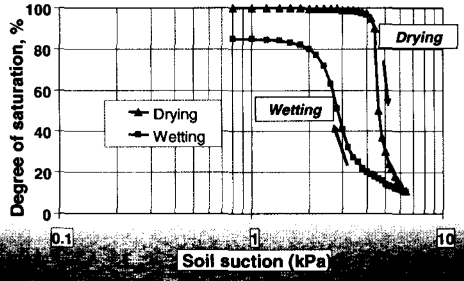
Darcy's law for flow in the x-, y-, and z-direction

Coefficient of permeability, k_w is a function of matric suction; therefore, the flow law is nonlinear

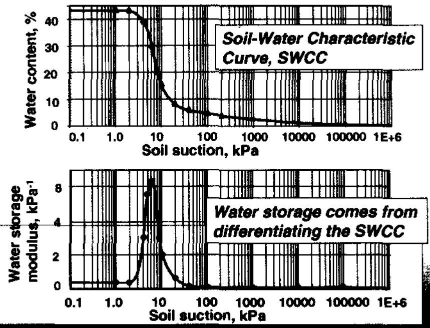
Shape of the water permeability function for glass beads tested by Mualem (1976)



The SWCC for the glass beads showing hysteresis during drying and wetting (Maulem 1976)



Storage (Water, Air or Heat)



Soil-Water Characteristic Curve, SWCC

Water storage comes from differentiating the SWCC

Air Flow Constitutive Relations

$$v_{ax} = \boxed{-k_{ax}}\frac{du_a}{dx}$$

Driving potential for air flow is Pore-air pressure, u_a

$$v_{ay} = \boxed{-k_{ay}}\frac{du_a}{dy}$$

Fick's law for flow in the x-, y-, and z-direction

$$v_{az} = \boxed{-k_{az}}\frac{du_a}{dz}$$

Coefficient of permeability, k_a is a function of matric suction; therefore, the flow law is nonlinear

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Shear Strength Constitutive Relations

$$\tau = c' + (\sigma_n - u_a)\ \tan\phi' + (u_a - u_w) f_1$$

f_1 = function showing the rate of increase in shear strength with matric suction

$$\tau = c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)\tan\phi^b$$

Linear form of the extended Mohr-Coulomb shear strength equation

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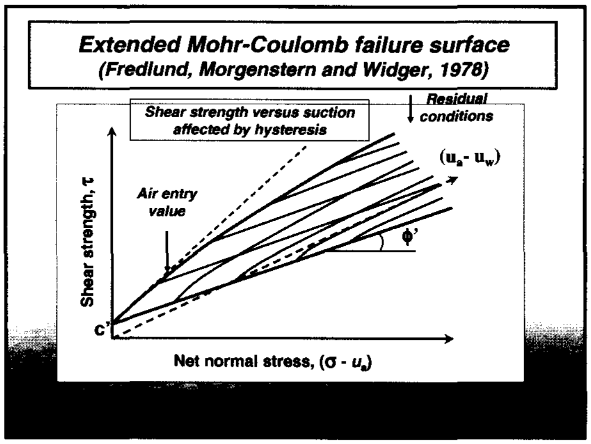
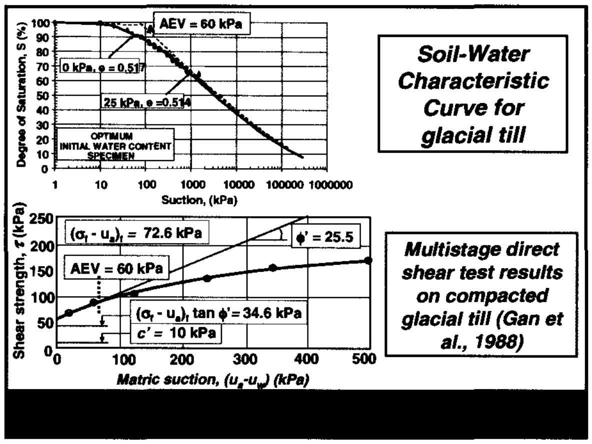
Extended Mohr-Coulomb failure surface

(Fredlund, Morgenstern and Widger, 1978)

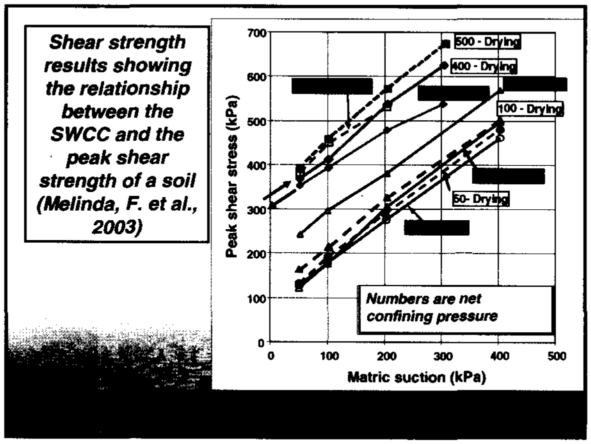


Soil-Water Characteristic Curve for glacial till

Multistage direct shear test results on compacted glacial till (Gan et al., 1988)



Shear strength results showing the relationship between the SWCC and the peak shear strength of a soil (Melinda, F. et al., 2003)

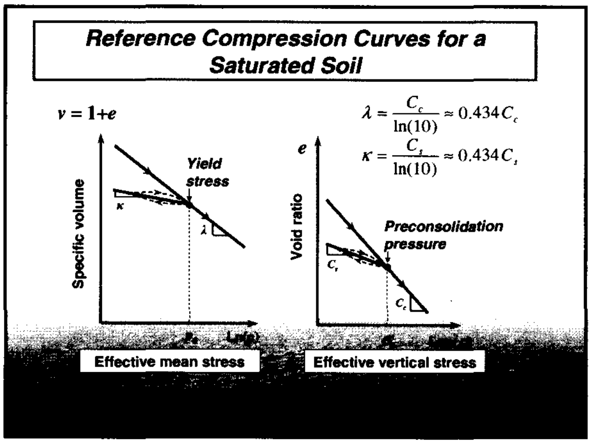


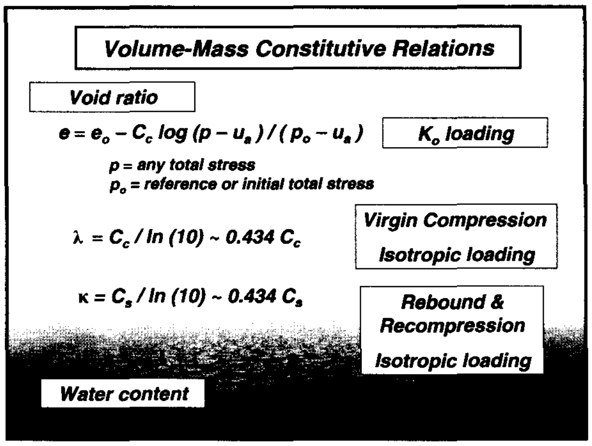
Reference Compression Curves for a Saturated Soil

$$v = 1 + e$$

$$\lambda = \frac{C_c}{\ln(10)} \approx 0.434 C_c$$

$$\kappa = \frac{C_s}{\ln(10)} \approx 0.434 C_s$$





Volume-Mass Constitutive Relations

Void ratio

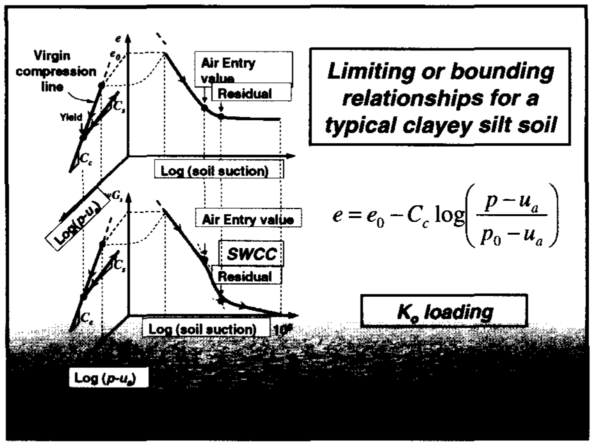
$e = e_o - C_c \log(p - u_a)/(p_o - u_a)$ **K_o loading**

p = any total stress
p_o = reference or initial total stress

$\lambda = C_c / \ln(10) \sim 0.434\, C_c$ **Virgin Compression Isotropic loading**

$\kappa = C_s / \ln(10) \sim 0.434\, C_s$ **Rebound & Recompression Isotropic loading**

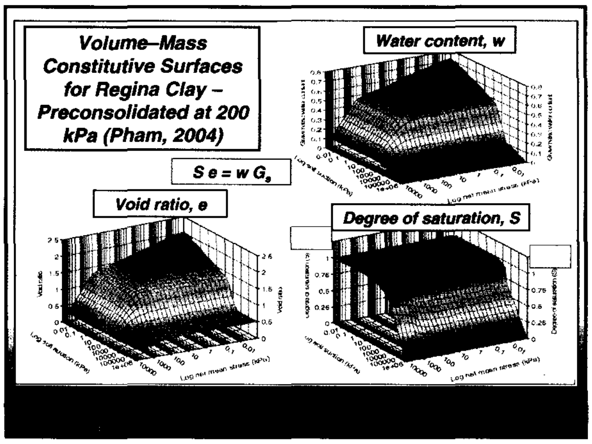
Water content



Limiting or bounding relationships for a typical clayey silt soil

$$e = e_0 - C_c \log\left(\frac{p - u_a}{p_0 - u_a}\right)$$

K_o loading



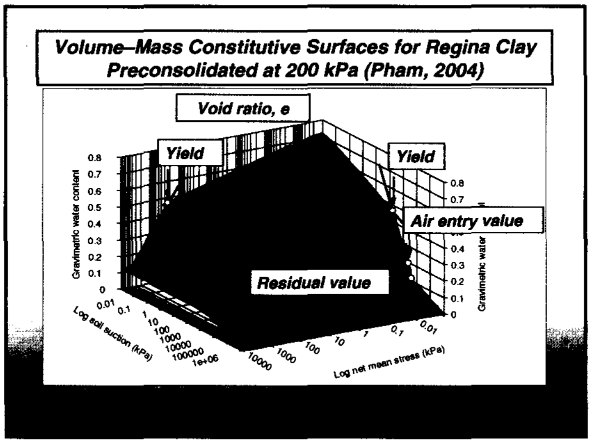
Volume–Mass Constitutive Surfaces for Regina Clay – Preconsolidated at 200 kPa (Pham, 2004)

Water content, w

$S\,e = w\,G_s$

Void ratio, e

Degree of saturation, S



Volume–Mass Constitutive Surfaces for Regina Clay Preconsolidated at 200 kPa (Pham, 2004)

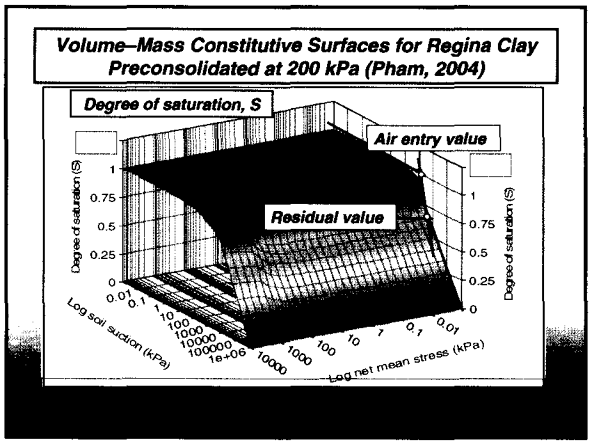
Void ratio, e

Yield

Yield

Air entry value

Residual value

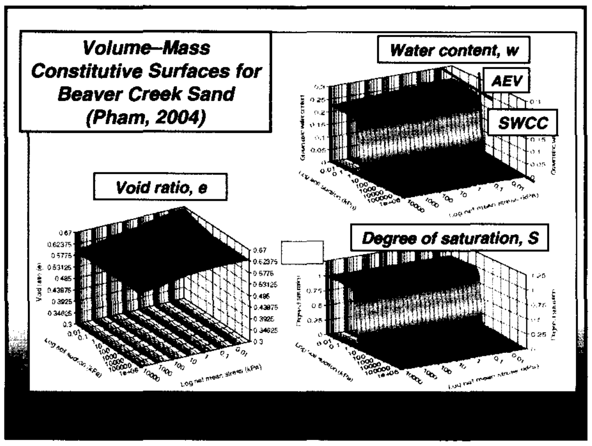


Volume–Mass Constitutive Surfaces for Regina Clay Preconsolidated at 200 kPa (Pham, 2004)

Degree of saturation, S

Air entry value

Residual value



Volume–Mass Constitutive Surfaces for Beaver Creek Sand (Pham, 2004)

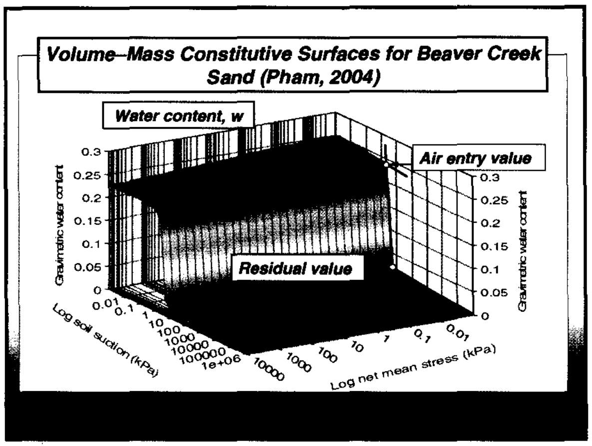
Water content, w

AEV

SWCC

Void ratio, e

Degree of saturation, S



Volume–Mass Constitutive Surfaces for Beaver Creek Sand (Pham, 2004)

Direct Measurement of Unsaturated Soil Property Functions

- ***Excessive cost and demanding laboratory testing techniques***
- ***Testing must allow control of total stresses and matric suction***
- ***High air entry ceramic disks must be used as separators between air and water pressures***
- ***Several measurements must be made to determine the shape of the function***

Costly Laboratory Testing Creates a Need for Alternative Procedures to Obtain Unsaturated Soil Property Functions

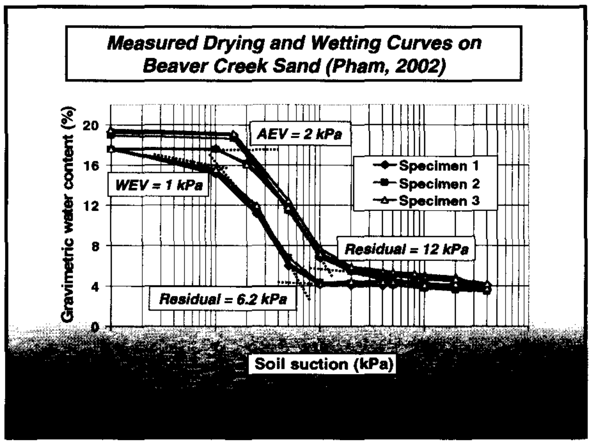
Nature and Role of the Soil-Water Characteristic Curve

-Initially viewed as a means of estimating in situ soil suction

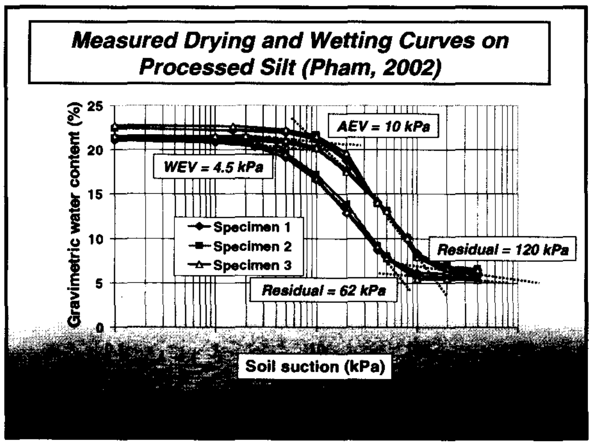
- ***Unsuccessful: hysteresis between the drying and wetting curves***

-SWCC later used for the estimation of Unsaturated Soil Property Functions

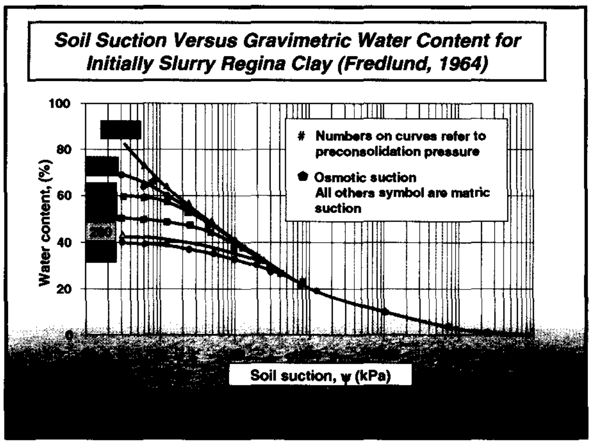
- ***Successful: for all unsaturated soil properties***
  - ***Water permeability function***
  - ***Air permeability function***
  - ***Shear strength function***
  - ***Thermal flow functions***
  - ***Incremental elasticity functions***



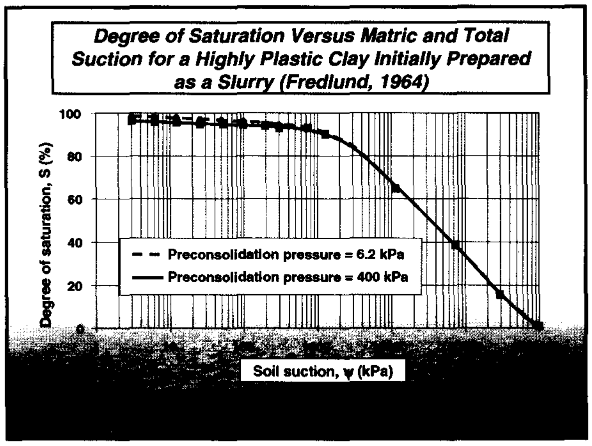
Measured Drying and Wetting Curves on Beaver Creek Sand (Pham, 2002)



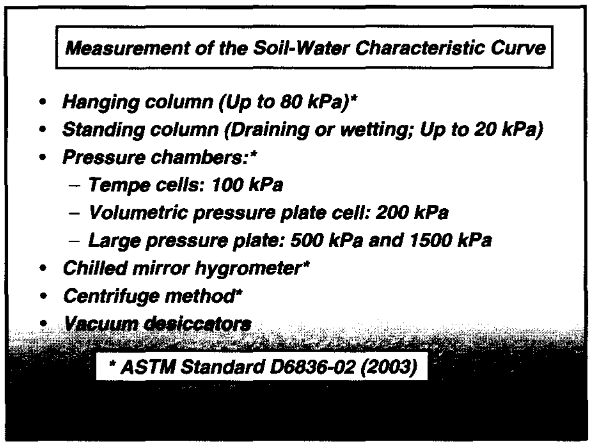
Measured Drying and Wetting Curves on Processed Silt (Pham, 2002)



Soil Suction Versus Gravimetric Water Content for Initially Slurry Regina Clay (Fredlund, 1964)



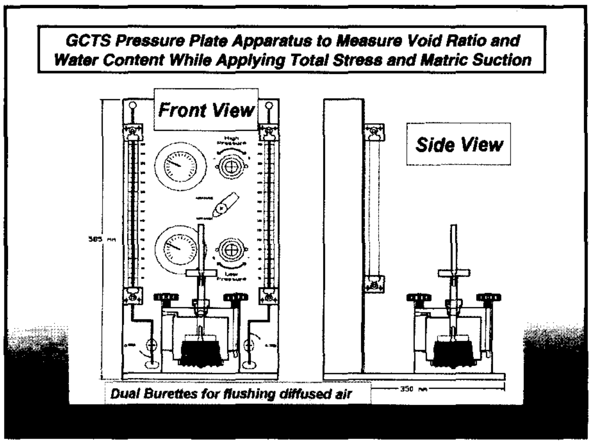
Degree of Saturation Versus Matric and Total Suction for a Highly Plastic Clay Initially Prepared as a Slurry (Fredlund, 1964)



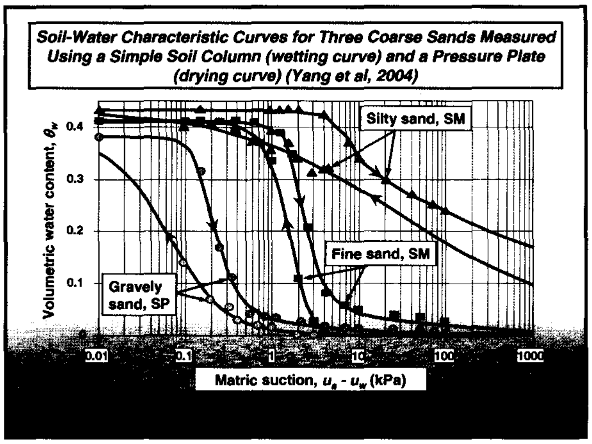
Measurement of the Soil-Water Characteristic Curve

- *Hanging column (Up to 80 kPa)**
- *Standing column (Draining or wetting; Up to 20 kPa)*
- *Pressure chambers:**
  - *Tempe cells: 100 kPa*
  - *Volumetric pressure plate cell: 200 kPa*
  - *Large pressure plate: 500 kPa and 1500 kPa*
- *Chilled mirror hygrometer**
- *Centrifuge method**
- *Vacuum desiccators*

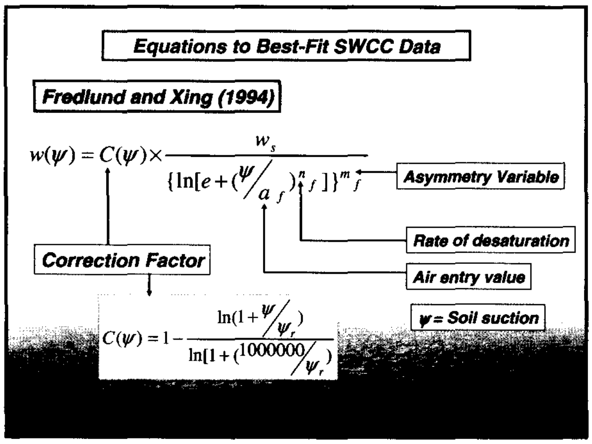
** ASTM Standard D6836-02 (2003)*



GCTS Pressure Plate Apparatus to Measure Void Ratio and Water Content While Applying Total Stress and Matric Suction



Soil-Water Characteristic Curves for Three Coarse Sands Measured Using a Simple Soil Column (wetting curve) and a Pressure Plate (drying curve) (Yang et al, 2004)

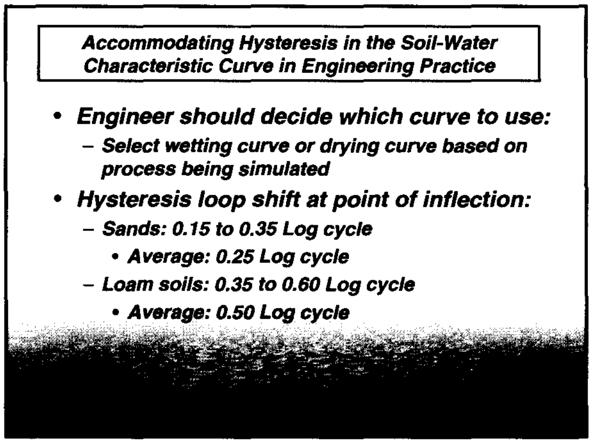


Equations to Best-Fit SWCC Data

Fredlund and Xing (1994)

$$w(\psi) = C(\psi) \times \frac{w_s}{\{\ln[e + (\psi/a_f)^{n_f}]\}^{m_f}}$$

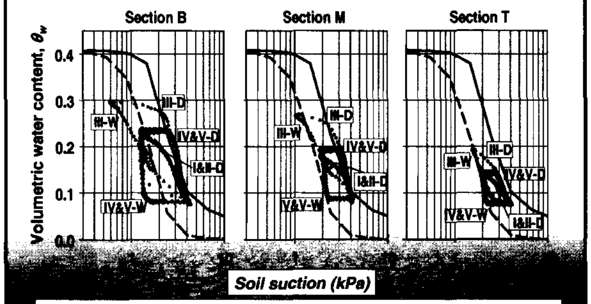
$$C(\psi) = 1 - \frac{\ln(1 + \psi/\psi_r)}{\ln[1 + (1000000/\psi_r)]}$$



Accommodating Hysteresis in the Soil-Water Characteristic Curve in Engineering Practice

- *Engineer should decide which curve to use:*
  - *Select wetting curve or drying curve based on process being simulated*
- *Hysteresis loop shift at point of inflection:*
  - *Sands: 0.15 to 0.35 Log cycle*
    - *Average: 0.25 Log cycle*
  - *Loam soils: 0.35 to 0.60 Log cycle*
    - *Average: 0.50 Log cycle*

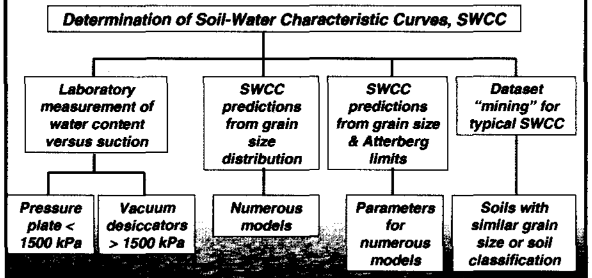
Model measurements of water content and matric suction showing the SWCC relationship from water contents and matric suctions during wetting and drying simulations (Tami et al, 2004)



Suctions with Tensiometers

Water content with TDR

Approaches that can be used to obtain the Soil-Water Characteristic curves

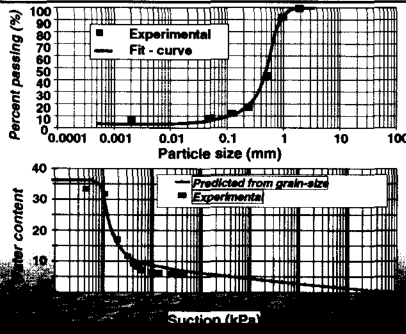


Determination of Soil-Water Characteristic Curves, SWCC

- **Laboratory measurement of water content versus suction**
  - **Pressure plate < 1500 kPa**
  - **Vacuum desiccators > 1500 kPa**
- **SWCC predictions from grain size distribution**
  - **Numerous models**
- **SWCC predictions from grain size & Atterberg limits**
  - **Parameters for numerous models**
- **Dataset "mining" for typical SWCC**
  - **Soils with similar grain size or soil classification**

Decreasing Accuracy ⟶

Soil-Water Characteristic Curve computed from a Grain Size Distribution Curve.

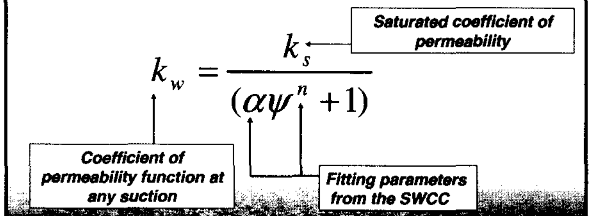


Incorporation of SWCC into the Constitutive Relations for Unsaturated Soils

- *Give rise to INDIRECT procedures for the estimation of unsaturated soil property functions*
- *Procedures view unsaturated soil characterization as an extension of saturated soil properties*
- *Unsaturated soil property functions rely on the saturated soil properties and the soil-water characteristic curve, SWCC*
- *Unsaturated soil property functions result in a nonlinear partial differential equation to solve using numerical modeling techniques*

Seepage Constitutive Relations

Gardner (1958)



$$k_w = \frac{k_s}{(\alpha\psi^n + 1)}$$

Saturated coefficient of permeability

Coefficient of permeability function at any suction

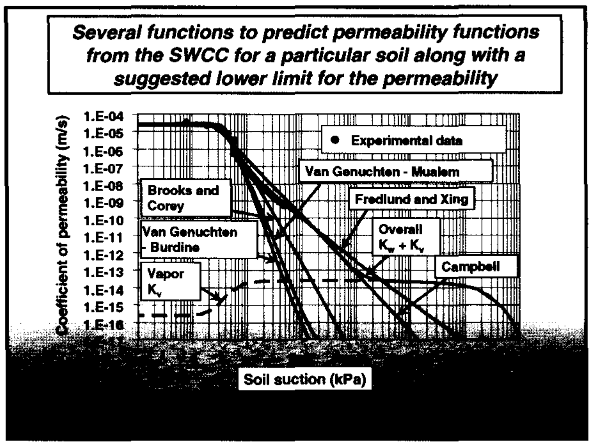
Fitting parameters from the SWCC

Seepage Constitutive Relations in Terms of SWCC

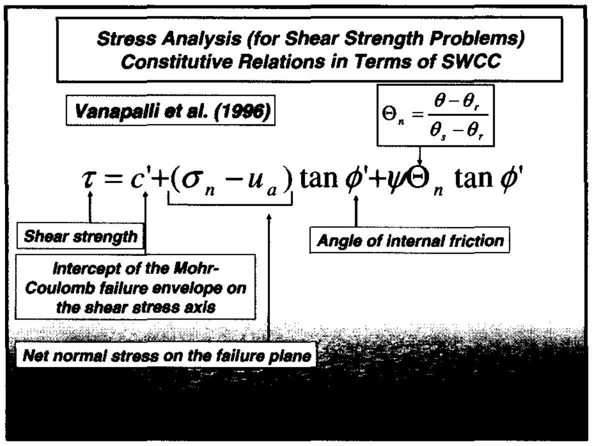
Permeability Models	References for the Soil-Water Characteristic Curve	
	Fredlund and Xing (1994)	Campbell (1974)
Child and Collis – Georg (1950)	$k_r = \dfrac{\int_{\ln(\psi)}^{b} \frac{\theta(e^y) - \theta(\psi)}{e^y}\theta'(e^y)dy}{\int_{\ln(\psi_{aev})}^{b} \frac{\theta(e^y) - \theta_s}{e^y}\theta'(e^y)dy}$	$k_r = (\frac{\psi}{\psi_{aev}})^{-2-\frac{2}{b}}$

b = Ln (1000000) **θ (ψ) = Soil water content**

y = Dummy variable of integration representing the logarithm of integration



Several functions to predict permeability functions from the SWCC for a particular soil along with a suggested lower limit for the permeability



Stress Analysis (for Shear Strength Problems) Constitutive Relations in Terms of SWCC

Vanapalli et al. (1996)

$$\Theta_n = \frac{\theta - \theta_r}{\theta_s - \theta_r}$$

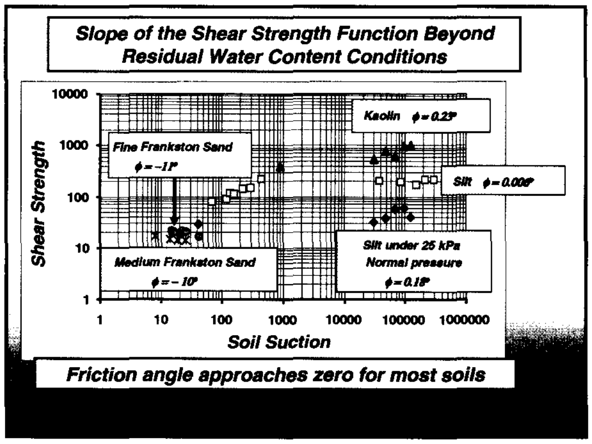
$$\tau = c' + \underbrace{(\sigma_n - u_a)} \tan\phi' + \psi\Theta_n \tan\phi'$$

Shear strength

Intercept of the Mohr-Coulomb failure envelope on the shear stress axis

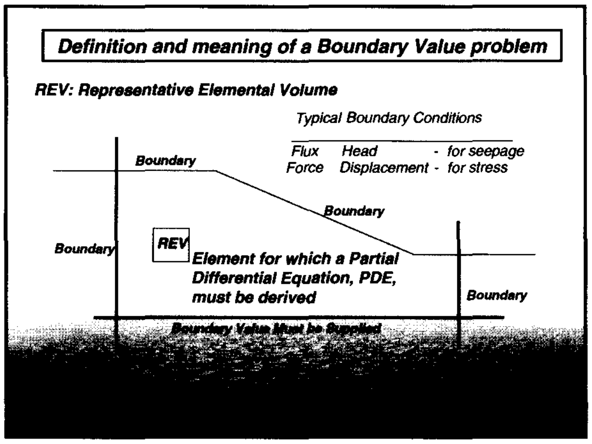
Angle of internal friction

Net normal stress on the failure plane



Slope of the Shear Strength Function Beyond Residual Water Content Conditions

Friction angle approaches zero for most soils



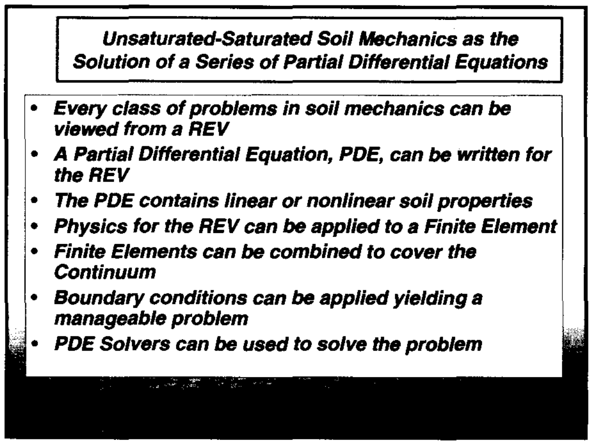
Definition and meaning of a Boundary Value problem

REV: Representative Elemental Volume

Typical Boundary Conditions

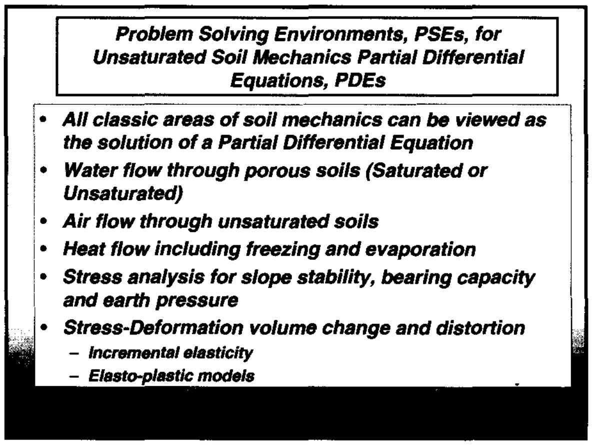
Flux	*Head*	*- for seepage*
Force	*Displacement*	*- for stress*

Element for which a Partial Differential Equation, PDE, must be derived



Unsaturated-Saturated Soil Mechanics as the Solution of a Series of Partial Differential Equations

- Every class of problems in soil mechanics can be viewed from a REV
- A Partial Differential Equation, PDE, can be written for the REV
- The PDE contains linear or nonlinear soil properties
- Physics for the REV can be applied to a Finite Element
- Finite Elements can be combined to cover the Continuum
- Boundary conditions can be applied yielding a manageable problem
- PDE Solvers can be used to solve the problem



Problem Solving Environments, PSEs, for Unsaturated Soil Mechanics Partial Differential Equations, PDEs

- All classic areas of soil mechanics can be viewed as the solution of a Partial Differential Equation
- Water flow through porous soils (Saturated or Unsaturated)
- Air flow through unsaturated soils
- Heat flow including freezing and evaporation
- Stress analysis for slope stability, bearing capacity and earth pressure
- Stress-Deformation volume change and distortion
  - Incremental elasticity
  - Elasto-plastic models

Convergence of Nonlinear Partial Differential Equations

- ***Single most pressing problem facing modelers***
- ***Most successful solution has been Adaptive Grid Refinement methods, ADR (Oden, 1989; Yeh, 2000)***
- ***Automatic, dynamic mesh assignment is based on error estimates***
- ***ADR becomes extremely important when solving the nonlinear PDEs associated with Unsaturated Soil Mechanics***

Partial Differential Equation for Saturated-Unsaturated Water Flow Analysis

Head variable to be solved

$$k_x^w \frac{\partial^2 h}{\partial x^2} + \frac{\partial k_x^w}{\partial x}\frac{\partial h}{\partial x} + k_y^w \frac{\partial^2 h}{\partial y^2} + \frac{\partial k_y^w}{\partial y}\frac{\partial h}{\partial y} = -m_2^w \gamma_w \frac{\partial h}{\partial t}$$

Water coefficient of permeability (function of soil suction)

Water storage (function of soil suction)

Time

Partial Differential Equation for Saturated-Unsaturated Stress-Deformation Analysis

$$\frac{\partial}{\partial x}\left[D_{11}\frac{\partial u}{\partial x} + D_{12}\frac{\partial v}{\partial y}\right] + \frac{\partial}{\partial y}\left[D_{44}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right] = 0 \quad \boxed{X\text{-}}$$

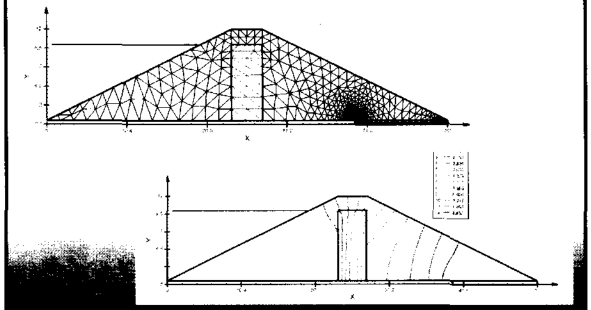
$$\frac{\partial}{\partial x}\left[D_{44}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right] + \frac{\partial}{\partial y}\left[D_{12}\frac{\partial u}{\partial x} + D_{11}\frac{\partial v}{\partial y}\right] + \gamma_t = 0 \quad \boxed{Y\text{-}}$$

D_{11}, D_{12}, D_{44} = Combination of E and μ which are function of soil suction and net total stresses

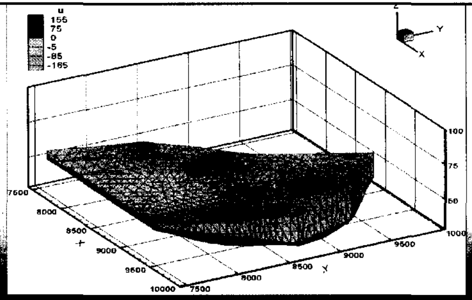
Problem Solving

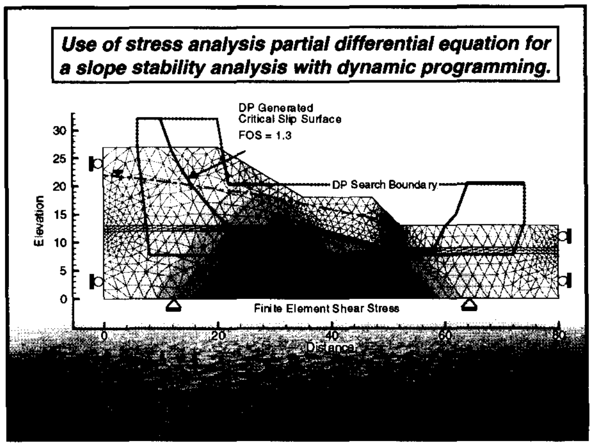
- ***Decide on category of "Boundary Value Problem" to be solved based on the constitutive behavior involved***
- ***Select the "Partial Differential Equation(s)" to solve***
- ***Determine (or estimate) the required unsaturated soil property functions***
- ***Assess the "Initial Conditions" and the "Boundary Values" for the problem***
- ***Undertake numerous simulations (Parametric Study or Probabilistic Study)***

Two-dimensional seepage analysis through an earthfill dam with a clay core.

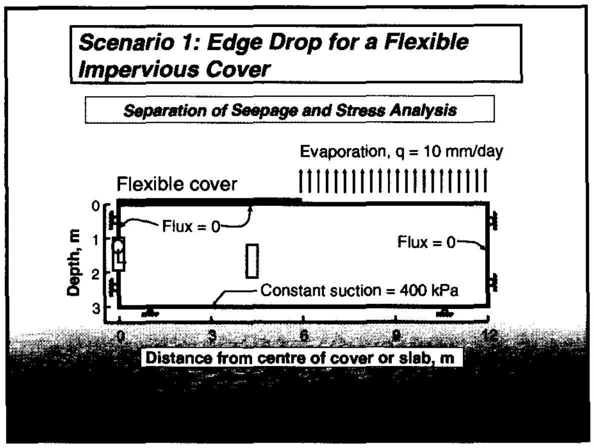


Example problem utilizing the solution of the saturated-unsaturated seepage partial differential equation for a three dimensional solution.



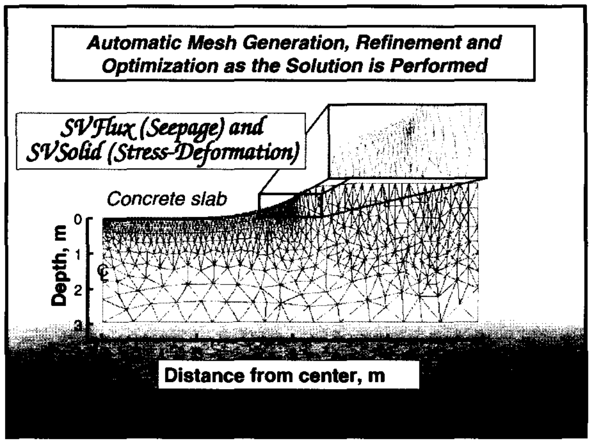


Use of stress analysis partial differential equation for a slope stability analysis with dynamic programming.



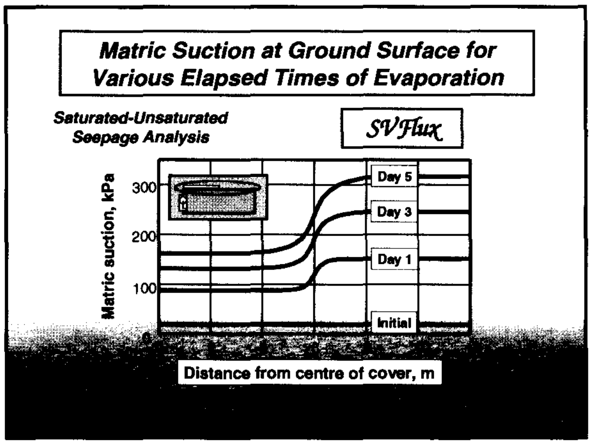
Scenario 1: Edge Drop for a Flexible Impervious Cover

Separation of Seepage and Stress Analysis



Automatic Mesh Generation, Refinement and Optimization as the Solution is Performed

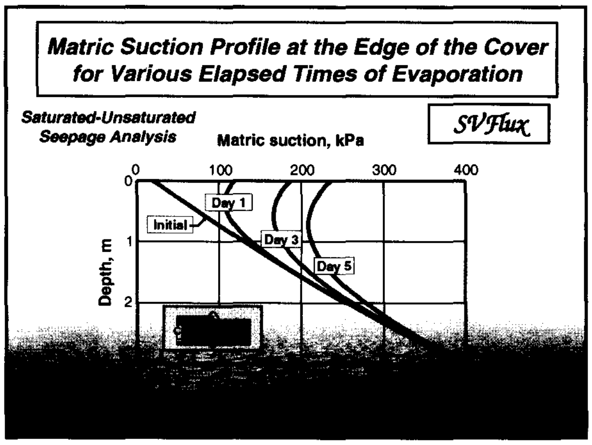
SVFlux (Seepage) and SVSolid (Stress-Deformation)



Matric Suction at Ground Surface for Various Elapsed Times of Evaporation

Saturated-Unsaturated Seepage Analysis

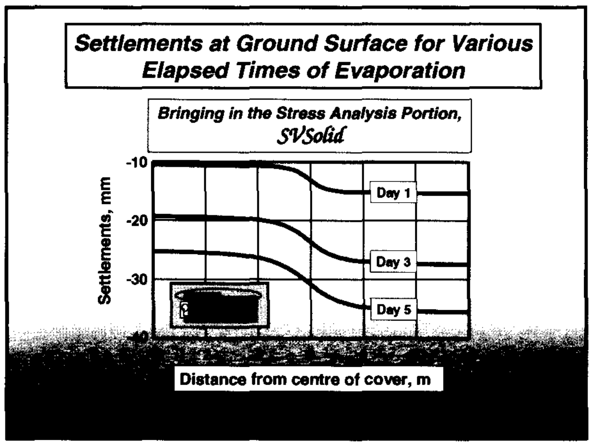
SVFlux



Matric Suction Profile at the Edge of the Cover for Various Elapsed Times of Evaporation

Saturated-Unsaturated Seepage Analysis

SVFlux



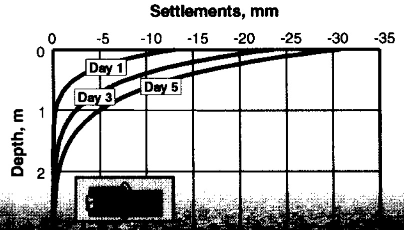
Settlements at Ground Surface for Various Elapsed Times of Evaporation

Bringing in the Stress Analysis Portion, *SVSolid*

Settlements versus Depth at the Edge of the Cover

Stress-Deformation Analysis

SVSolid



Concluding Remarks

- Unsaturated Soil Mechanics needs to be first understood from the standpoint of the Constitutive Equations describing soil behavior
- Constitutive Equations can be written in terms of the SWCC for the soil which are then known as Unsaturated Soil Property Functions, USPF
- Direct and Indirect procedures are available for the assessment of the SWCC
- It is always possible to obtain an estimate of the required Unsaturated Soil Property Functions for geotechnical engineering applications