

## A Lower Limit for the Water Permeability Coefficient

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### ABSTRACT

The unlimited decrease in the liquid water coefficient of permeability,  $k_w$ , with increasing soil suctions produces a shut off of water flow. It is known, however, that significant quantities of water flow occur even at relatively high soil suctions and the extremely small values for  $k_w$  can cause serious difficulties in the numerical solution of seepage problems. The objective of the present study is to suggest an appropriate lower limit for  $k_w$  in order to ameliorate numerical difficulties. Two approaches for imposing lower limit for  $k_w$  were evaluated. First, the residual conditions were used for the calculation of the minimum value of  $k_w$ . The values of  $k_w$  at the residual conditions were found to provide an inappropriate lower limit for  $k_w$ . The second approach was based on the water vapour flow theory. A vapour permeability function was developed and used to determine the change in water vapour permeability with suction. Based on the comparison of liquid water and overall permeability functions, a lower limit for  $k_w$  was established. The results presented herein suggest that a constant value of  $1 \times 10^{-14}$  m/s can be used as a minimum coefficient of liquid water permeability in liquid water flow analyses.

### RÉSUMÉ

La diminution illimitée du coefficient de la perméabilité de l'eau liquide,  $k_w$ , avec l'augmentation des suctions de sol produit l'interruption de l'écoulement de l'eau. On le sait, cependant, que les quantités significatives d'écoulement de l'eau se produisent même aux suctions relativement élevées de sol et les valeurs extrêmement petites du  $k_w$  peuvent produire de graves difficultés dans la solution numérique des problèmes d'écoulement de l'eau. L'objectif de la présente étude est de suggérer une valeur minimum appropriée pour le  $k_w$  afin d'améliorer telles difficultés numériques. Deux approches pour la définition d'une valeur minimum de  $k_w$  ont été évaluées. D'abord, les conditions résiduelles ont été employées pour le calcul de la valeur minimum du  $k_w$ . Les valeurs du  $k_w$  aux conditions résiduelles se sont avérées pour fournir une indication inadéquate de la valeur minimum du  $k_w$ . La deuxième approche a été basée sur la théorie d'écoulement de vapeur d'eau. Une fonction de perméabilité à vapeur a été développée et employée pour déterminer le changement de la perméabilité à vapeur d'eau avec la suction dans un sol. Basé sur la comparaison du coefficient de la perméabilité de l'eau liquide et de vapeur, une valeur minimum pour le  $k_w$  a été établie. Les résultats présentés suggèrent qu'une valeur constante de  $1 \times 10^{-14}$  m/s peut être employée comme coefficient minimum de la perméabilité de l'eau liquide dans des analyses de flux liquides de l'eau.

### 1. INTRODUCTION

The liquid water coefficient of permeability,  $k_w$ , for an unsaturated soil takes the form of a primary function of soil suction (Fredlund and Rahardjo, 1993). The value of  $k_w$  remains essentially equal to the saturated coefficient of permeability until the air-entry value of the soil is reached. The liquid water coefficient of permeability decreases on a logarithmic scale past the air-entry value, as the water content decreases on an arithmetic scale.

The decrease in the value of  $k_w$  with an increase in soil suction can cause numerical instability because of the high nonlinearity and the computing difficulties associated with extremely small numbers. Significant advances have been made in the solution of highly nonlinear partial differential equations (Fredlund et al, 2002). Nevertheless, numerical models can still benefit from the use of a lower limit for the liquid water coefficient of permeability. Most importantly, however, the unlimited decrease in the value of  $k_w$  fails to simulate actual water flow, where other moisture transfer mechanisms cause moisture flow at

relatively high soil suctions (Wilson et al.,1994, Gitirana Jr. and Fredlund, 2003).

The objective of the present study is to determine an appropriate lower limit for the value of  $k_w$  in an unsaturated soil. Two approaches for the determination of an appropriate lower limit for  $k_w$  were evaluated. First, the residual point was assumed to correspond to the lower limit of  $k_w$ . A variety of predictive procedures were used to obtain estimates of the liquid water coefficient of permeability of a soil at residual conditions. The second approach was based on the water vapour flow theory. At high soil suctions, a significant portion of the overall water flow occurs by vapour flow. A vapour permeability function was developed and used to determine the change in water vapour permeability with suction in a soil. A lower limit for the coefficient of liquid water permeability was established by comparing liquid water, vapour, and overall permeability coefficients.

### 2. BACKGROUND

A brief review of the conceptualization of residual water content conditions and a review of methods of predicting the liquid water coefficient of permeability is presented in this section.

### 2.1 The Relationship Between Residual Water Content and Water Coefficient of Permeability

Various definitions for water content at residual conditions can be found in the literature. Brooks and Corey (1964) defined residual water content as the water content at which suction reaches infinity. Luckner et al. (1989) stated that the residual water content specifies the maximum water content that will not contribute to liquid flow. Van Genuchten et al. (1991) defined the residual water content as the water content at which both the slope of the soil-water characteristic curve and coefficient of permeability become "zero" when soil suction becomes large.

The definition of residual water content provided by Brooks and Corey (1964) is inadequate since zero water content is reached at soil suction of approximately 1,000,000 kPa (Edlefsen and Anderson, 1943). Experimental data suggests that water can flow as films on solid surfaces even below the residual water condition (Dullien, 1986, Nitao and Bear, 1996).

The residual water content,  $\theta_r$ , may be interpreted also as an empirical fitting parameter that describes the shape of the soil-water characteristic curve. Gitirana Jr. and Fredlund (2004) defined the residual point as the centre of rotation of a hyperbole that fits the lower portion of the

soil-water characteristic curve. A graphical definition of the residual water content that is not associated with a fitting equation is given by Vanapalli et al., (1998).

### 2.2. Methods of Prediction of the Water Coefficient of Permeability

Numerous models have been proposed for the representation or estimation of the liquid water permeability function,  $k_w(\psi)$ . These models can be placed into two categories; namely, empirical equations and theoretical models including macroscopic and microscopic (statistical) models (Mualem 1986, 1992).

Many statistical models have been proposed by various individuals. The models proposed by Childs and Collis-George (1950), Burdine (1953), and Mualem (1976) are frequently referred in the literature. To solve the integral form of the statistical models and compute the water permeability functions, the saturated permeability and soil-water characteristic curves are required. Numerous equations have been proposed to represent the soil-water characteristic curve. Some of the equations used in this study are given in Table 1.

Fredlund et al. (1994) used the Fredlund and Xing (1994) SWCC equation and solved the Childs and Collis-George's (1950) model to find the water permeability coefficient. The procedure involves integration, which can be done numerically (Table 2). The closed forms of permeability functions proposed by van Genuchten (1980), Brooks and Corey (1964) and Campbell (1974) are also given in Table 2.

Table 1. Equations for the soil-water characteristic curve

References	Equation	Fitting Parameters
Brooks and Corey (1964)	$S_e = 1$ , $\alpha\psi \leq 1$ $S_e = (\alpha\psi)^{-\lambda}$ , $\alpha\psi > 1$	$\theta_r$ , $\theta_s$ , $\alpha$ , and $\lambda$ $\theta_r \geq 0$
Campbell (1974)	$S_e = \left(\frac{\psi}{\psi_e}\right)^{-b}$ , $\psi > \psi_e$	$\theta_r = 0$ , $\theta_s$ , $\psi_e$ , and $b$ $\theta_r \geq 0$
van Genuchten (1980)	$S_e = \frac{1}{[1 + (\alpha\psi)^n]^m}$ , $m = 1 - \frac{1}{n}$	$\theta_r$ , $\theta_s$ , $\alpha$ , and $n$ $\theta_r \geq 0$
van Genuchten (1980)	$S_e = \frac{1}{[1 + (\alpha\psi)^n]^m}$ , $m = 1 - \frac{2}{n}$	$\theta_r$ , $\theta_s$ , $\alpha$ , and $n$ $\theta_r \geq 0$
Fredlund and Xing (1994)	$\theta(\psi) = C(\psi) \times \frac{\theta_s}{\{\ln[e + (\frac{\psi}{\alpha})^n]\}^m}$ $C(\psi) = 1 - \frac{\ln(1 + \frac{\psi}{\psi_r})}{\ln[1 + (1000000/\frac{\psi}{\psi_r})]}$	$a$ , $n$ , and $m$

$S_e = \frac{\theta - \theta_r}{\theta_s - \theta_r}$  , where  $S_e$  is the effective degree of saturation,  $\theta_s$  and  $\theta_r$  are saturation and residual water contents respectively

Table 2. Statistical permeability predictive models

Permeability Models	References for the Soil-Water Characteristic Curve			
	van-Genuchten (1980)	Fredlund and Xing (1994)	Brooks & Corey (1964)	Campbell (1974)
Child and Collis-George (1950)	_____	$k_r = \frac{\int_{\ln(\Psi_{aev})}^b \frac{\theta(e^y) - \theta(\Psi)}{e^y} \theta'(e^y) dy}{\int_{\ln(\Psi_{aev})}^b \frac{\theta(e^y) - \theta_e}{e^y} \theta'(e^y) dy}$	_____	$k_r = \left(\frac{\Psi}{\Psi_e}\right)^{-2-\frac{2}{b}}$
Burdine (1953)	$k_r(\Psi) = \frac{1 - (\alpha\Psi)^{n-2} [1 + (\alpha\Psi)^n]^{-m}}{[1 + (\alpha\Psi)^n]^{2m}}, \quad m = 1 - \frac{2}{n}$	_____	$k_r(\Psi) = (\alpha\Psi)^{-2-3.2}$	_____
Mualem (1976)	$k_r(\Psi) = \frac{\{1 - (\alpha\Psi)^{n-1} [1 + (\alpha\Psi)^n]^{-m}\}^2}{[1 + (\alpha\Psi)^n]^{0.5}}, \quad m = 1 - \frac{1}{n}$	_____	_____	_____

$K_r(\Psi) = k(\Psi)/k_s$  is relative permeability,  $\Psi$  is soil suction,  $\Psi_{aev}$  is air entry value,  $b$  in the integration equals to  $\ln(1000000)$

### 3. THEORY

This section presents the theory for the lower limit of liquid water coefficient for permeability. Two approaches are presented. The first approach is based on the definition of residual water conditions. A graphical construction technique proposed by Vanapalli et al. (1998) is selected to find the soil water content and the soil suction at residual conditions. The second approach is based on a theory for water vapour flow.

#### 3.1. Determination of the Coefficient of Permeability at Residual Conditions

As shown in the previous section, numerous researchers have suggested that a relationship exists between the lower limit of  $k_w$  and the residual water content. The value of  $k_w$  has been computed herein as the value of  $k_w$  obtained using a method of prediction of the liquid water coefficient of permeability function and the soil suction at residual conditions.

#### 3.2. Lower Limit for $k^w$ Based on Water Vapour Flow Theory

The transfer of moisture takes place as both liquid and water vapour. The water mass flux by liquid flow is traditionally described by Darcy's law. The mass flux of water vapour and bulk air may be described by a modified form of Fick's law (Philip and de Vries, 1957 and Dakshanamurthy and Fredlund, 1981), as follows:

$$q_y^v = -D^v \frac{\partial C_v}{\partial y} - \frac{\rho_v}{\rho_a} D^a \frac{\partial C_a}{\partial y} = -D^{v*} \frac{\partial p_v}{\partial y} - \frac{\rho_v}{\rho_a} D^{a*} \frac{\partial \bar{u}_a}{\partial y} \quad [1]$$

where:  $q_i^v$  = flux rate of mass of water vapour within the air phase in the  $i$  direction per unit of total area,  $\text{kg}/(\text{m}^2\text{s})$ ;  $D^v$  = molecular diffusivity of vapour in air,  $0.229 \cdot 10^{-4} (1 + T/273.15)^{1.75}$ ,  $\text{m}^2/\text{s}$ ;  $D^a$  = coefficient of diffusion of air,  $D^a \approx D^v$  (Wilson et al, 1997);  $T$  = temperature,  $\text{K}$ ;  $C_v$ ,  $C_a$  = concentration of water vapour and air, respectively, in terms of the mass of vapour per unit volume of soil,  $C_v = \rho_v(1 - S)n$ ,  $C_a = \rho_a(1 - S)n$ ;  $S$  = degree of saturation,  $S = V_w/V_v$ ;  $n$  = porosity,  $n = V_v/V_0$ ;  $V_0$  = total volume,  $\text{m}^3$ ;  $V_w$ ,  $V_v$  = volume of water and voids in the elemental volume, respectively,  $\text{m}^3$ ;  $\rho_v$  = density of the water vapour,  $\rho_v = W_v \bar{u}_v / (RT)$ ,  $\text{kg}/\text{m}^3$ ;  $\rho_a$  = density of the bulk air phase,  $\rho_a = W_a \bar{u}_a / (RT)$ ,  $\text{kg}/\text{m}^3$ ;  $W_v$ ,  $W_a$  = molecular weight of water vapour and pore-air, respectively,  $\text{kg}/\text{kmol}$ ;  $p_v$  = partial pressure of water vapour,  $\text{kPa}$ ;  $\bar{u}_a$  = total pressure in the bulk air phase,  $u_a + u_{atm}$ ,  $\text{kPa}$ ;  $u_a$  = pore-air pressure,  $\text{kPa}$ ;  $u_{atm}$  = atmospheric pressure,  $101.325 \text{ kPa}$ ;  $R$  = universal gas constant,  $8.314 \text{ J}/(\text{mol}\cdot\text{K})$ ;  $D^{v*} = (1 - S)nD^v W_v / RT$ ,  $(\text{kg}\cdot\text{m})/(\text{kN}\cdot\text{s})$ ;  $D^{a*} = (1 - S)nD^a W_a / RT$ ,  $(\text{kg}\cdot\text{m})/(\text{kN}\cdot\text{s})$ .

Equation 1 presents a definition for the diffusion coefficient of vapour through soil,  $D^{v*}$ , based on Fick's law. However, an important factor has not been taken into account; namely, the tortuosity factor of the diffusion of vapour within the soil pores (Lai et al., 1976). Taking into account tortuosity, the diffusion coefficient of water vapour through soil,  $D^{v*}$ , can be predicted using the following equation:

$$D^{v*} = \alpha \beta D^v W_v / RT \quad [2]$$

where:  $D^{v*}$  = diffusion coefficient of vapour through soil,  $(\text{kg}\cdot\text{m})/(\text{kN}\cdot\text{s})$ ;  $\alpha$  = tortuosity factor of the soil,  $\alpha = \beta^{2/3}$  (Lai

et al., 1976);  $\beta$  = cross sectional area of soil available for vapour flow per total area;  $\beta = (1-S)n$ ;  $D^v$  = molecular diffusivity of water vapour in air;  $D^v = 0.229 \times 10^{-4} (1+T/273.15)^{1.75}$ , m<sup>2</sup>/s (Kimball et al., 1976);  $W_v$  = molecular weight of water, 18.016 kg/kmol;  $R$  = universal gas constant, 8.314 J/(mol.K);  $T$  = temperature, K.

Neglecting the gradient in atmospheric pressure and assuming the air phase is continuous and in direct contact with the atmosphere, gradients of  $u_a$  will be equal to gradients in the partial pressure of water vapour,  $p_v$ . Therefore, Eq. 1 can be re-written as follows:

$$q_y^v = -D^{v*} \frac{\partial p_v}{\partial y} - \frac{p_v}{\bar{u}_a} D^{v*} \frac{\partial p_v}{\partial y} = -\frac{\bar{u}_a + p_v}{\bar{u}_a} D^{v*} \frac{\partial p_v}{\partial y} \quad [3]$$

Equation 3 presents the vapour flow based on partial vapour pressure gradients. A comparison between liquid flow and Eq. 3 cannot be directly made, since different gradients are considered in each equation. Therefore, a comparison between the values of  $k_w$  and  $D^{v*}$  is meaningless. The next section will present how such comparison can be made, by modifying Eq. 3 using Lord Kelvin's thermodynamic equation.

### 3.2.1. Total Moisture Flow Based on Pore-Water Pressure Gradients

Based on the thermodynamic theory of soil moisture (Edlefsen and Anderson, 1943),  $p_v$  can be expressed as a function of the total potential of the liquid pore-water and temperature. Assuming local thermodynamic equilibrium, neglecting the effects of the osmotic suction, and assuming that the air pressure is equal to the atmospheric pressure, such relationship is as follows:

$$p_v = p_{vsat} e^{\frac{u_w g W_v}{\gamma_w R T}} \quad [4]$$

where:  $p_{vsat}$  = saturation vapour pressure of the soil water at temperature  $T$ , kPa;  $W_v$  = molecular weight of water, 0.018016 kg/mol;  $g$  = acceleration of gravity, 9.81 m/s<sup>2</sup>;  $R$  = universal gas constant, 8.314 J/(mol.K);  $T$  = temperature, K;

Eq. 4 is known as Lord Kelvin's equation. Values of saturation soil vapour pressure were experimentally obtained by Kaye and Laby (1973) apud Fredlund and Rahardjo (1993) for various temperatures. The other parameters are constants very well defined. A relationship between the gradients of  $p_v$  and the gradients of the other two variables,  $u_w$  and  $T$ , is determined by deriving Eq. 4 with respect to  $y$ . The relationship is as follows:

$$\frac{\partial p_v}{\partial y} = \frac{g W_v p_v}{\gamma_w R T} \left( \frac{\partial u_w}{\partial y} - \frac{u_w}{T} \frac{\partial T}{\partial y} \right) \quad [5]$$

The total moisture flow (liquid and vapour),  $q_y^m$  (kg/(m<sup>2</sup>s)), is obtained by summing Eq. 5 and Darcy's law and using Eq. 5 in order to express vapour flow in terms of pore-water pressure and temperature gradients. Therefore, the following equation is obtained:

$$q_y^m = -\frac{1}{g} (k_w + \gamma_w D^m) \frac{\partial u_w}{\partial y} - \rho_w k_w \frac{\partial Y}{\partial y} + \rho_w D^m \frac{u_w}{T} \frac{\partial T}{\partial y} \quad [6]$$

$$\text{where: } D^m = \frac{\bar{u}_a + p_v}{\bar{u}_a} \frac{g}{\gamma_w} \frac{W_v p_v}{RT} \frac{D^{v*}}{\rho_w}, \text{ (m/s)/(kN/m}^3\text{)}.$$

Equation 6 is the final equation governing the one-dimensional flow of moisture by liquid water and water vapour flow based on one single gradient. The comparison between the terms  $k_w$  and  $k_v = \gamma_w D^m$  is equivalent to the comparison between the amount of liquid and vapour flow. The terms  $k_w$  and  $k_v$  can be readily compared if isothermal conditions are considered, and gravimetric component is neglected, since they are both based on the gradient of pore-water pressure.

Temperature gradients required to render Eq. 6 solvable in non-isothermal conditions can be obtained by solving a PDE governing conservation of thermal energy. Further details about the derivation of these equations and a more comprehensive modelling of two-dimensional thermo-hydro-mechanical behaviour of saturated /unsaturated soils considering liquid water and water vapour flow can be found in Gitirana Jr. and Fredlund (2003).

Equation 2 shows that  $D^{v*}$  is a function of  $S$  and  $n$ , which in turn are function of soil suction. The soil property,  $D^{v*}$  is equal to zero when the soil is saturated and begins to increase as the air starts occupying part of the soil pores. On the other hand, the hydraulic conductivity,  $k_w$ , is at its highest value when the soil is saturated and starts declining as the air starts entering the soil pores. As the soil dries,  $k_w$  becomes lower than  $k_v$ . At this point, vapour flow begins to dominate over liquid water flow.

## 4. METHODOLOGY

### 4.1. Water Liquid Permeability Function, $k_w$ ( $\psi$ )

Forty five soil samples with different textures; namely, Sand, Loamy Sand, Sandy loam, and Silty Loam were selected from the data base SoilVision 3.34 (SoilVision Systems Ltd., 2003). Each soil sample contained the required experimental data for this study such as; soil-water characteristic curve, saturated and unsaturated permeability coefficients, and volume-mass properties.

Several permeability predictive models (see Table 2) were used to calculate the water permeability coefficient at residual conditions for each sample. The procedure has been done as follows: 1) The fitting parameters were determined for the various equations representing the

soil-water characteristic curve, for each sample. In all equations except Campbell's and Fredlund and Xing's, both saturation and residual water contents,  $\theta_s$  and  $\theta_r$ , were assumed as fitting parameters. The restriction  $\theta_r \geq 0$  was used in the fitting procedure. 2) Perform the construction procedure on the Fredlund and Xing's soil-water characteristic curve to find the value of soil suction at residual conditions. 3) Use the value of soil suction at residual conditions along with the SWCC parameters for each of the different water permeability predictive models in order to calculate the water permeability at residual conditions for each sample.

All above procedures were performed using Microsoft Excel for the Campbell, Brooks and Corey, van Genuchten- Mualem and van Genuchten-Burdine models which had a closed form for the permeability function. The software MathCad2000 was required in order to perform

the numerical integrations present in the prediction model proposed by Fredlund et al. (1994). The procedure for the computation of the coefficient of water permeability at residual condition is visualized in Figure 1.

#### 4.2. Vapour Permeability Function $k_v(\psi)$

The variation of vapour permeability coefficient with suction was determined using the equations 2, 4, and 6 for a loamy sand. To study the effect of tortuosity factor on vapour permeability function, different tortuosity models (Table 3) were used in equation 2. Since degree of saturation versus soil suction, SWCC, was required in the procedure, the van Genuchten and Mualem's SWCC equation was used. The procedure with using only the tortuosity model proposed by Lai et al., 1974 was then duplicated for two other different types of soil, sand and silty loam.

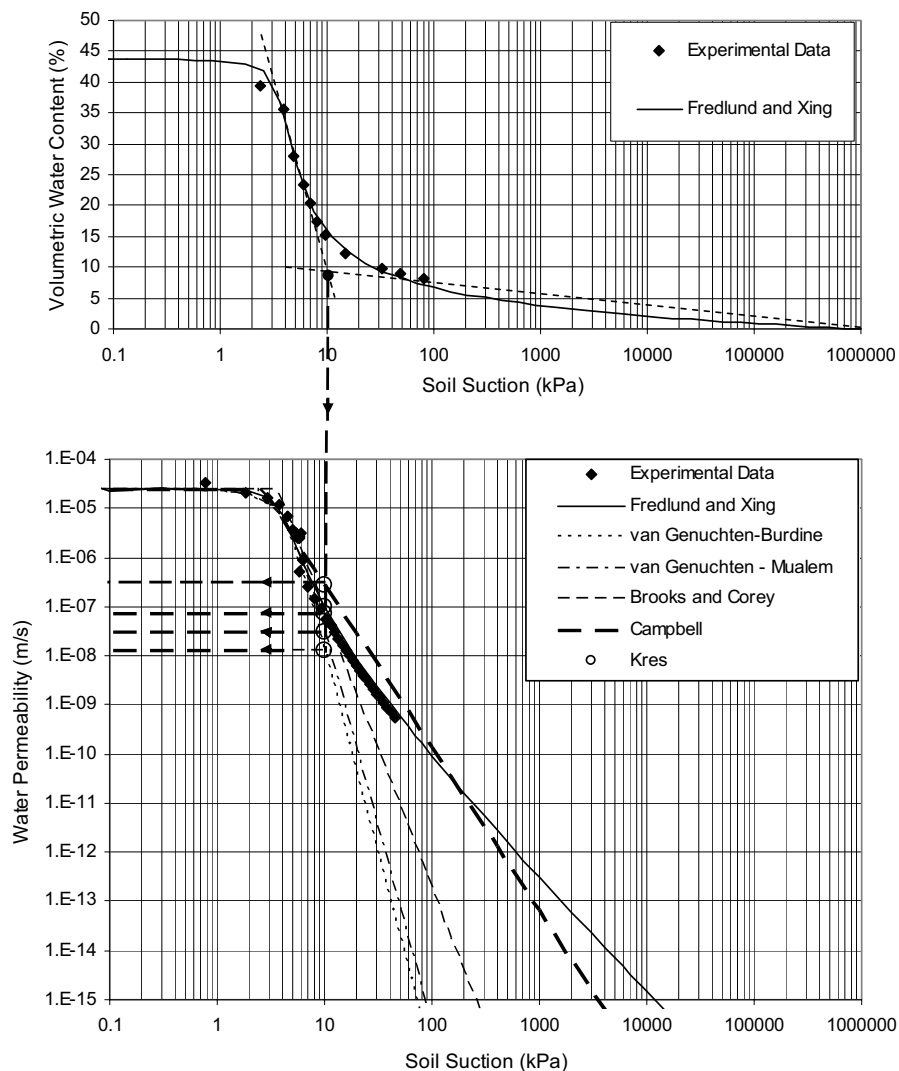


Figure 1. a) Fredlund and Xing's soil-water characteristic curve for loamy sand and b) Various water permeability models for loamy sand and water permeability at residual water content for each model (soil counter: 11178, SoilVision 3.34)

## 5- RESULTS AND ANALYSIS

### 5.1. Liquid Water Permeability at Residual Water Conditions

Computed water permeability coefficients at residual conditions using the water permeability predictive procedure proposed by Fredlund et al. (1994) are shown in Figure 2. Approximately the same distributions for coefficient of permeability at residual conditions were found with the other permeability predictive approaches but the results are not shown. In all types of soil, wide ranges of variations for water permeability coefficient at residual conditions were obtained with values as large as  $1 \times 10^{-6}$  m/s. As will be shown later, most of the values computed for the water permeability at residual conditions have been placed where the water liquid permeability coefficient still dominates over the coefficient of vapour permeability. Therefore the water permeability at residual conditions,  $k_{res}$ , can not be considered an appropriate lower limit for the liquid water permeability coefficient.

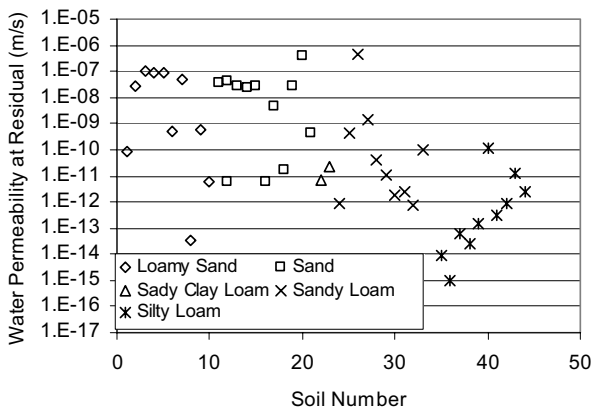


Figure 2. Computed coefficients of water permeability at residual conditions for water permeability predictive procedure proposed by Fredlund et al., 1994

Table 3. Tortuosity coefficients obtained from the literature

Reference	$\alpha\beta$
Lai et al. (1976)	$\beta^{5/3}$
Millington and Quirk(1961)	$\beta^{19/3} / n^2$
Millington (1959)	$\beta^{4/3}$
Marshal (1959)	$\beta^{3/2}$
Penman (1940)	0.66 n
Abu-El-Sha'r & Abriola (1997)	0.435 n

$\alpha\beta$  = as defined in Eq. 2, n = porosity

### 5.2. Effect of the Different Tortuosity Models on Vapour Permeability Functions

The results of using different tortuosity models in the definition of the vapour permeability function are shown in Figure 3. Six tortuosity formulations were used (see Table 3). The effect of tortuosity factor on vapour permeability coefficient was significant in low suctions where the liquid water permeability dominates and the vapour permeability coefficient can be neglected. Therefore, the tortuosity with respect to vapour flow does not have a significant effect on the overall moisture flow.

### 5.3. Vapour Permeability Function for Different Soils

The variation of vapour permeability coefficient,  $k_v$ , are shown for three different soils, Sandy, Loamy Sand, and Silty Loam in Figure 4. The effect of soil type on the vapour permeability coefficient is also significant in portion of curve where the water liquid permeability dominates and the vapour permeability coefficient can be neglected. Like tortuosity, types of soil dose not have significant effect on the vapour permeability function. The equation Lai et al. (1976) was used to find the variation of vapour permeability coefficient with soil suction for the soil samples. Maximum values of this parameter for the samples are given in Table 4.

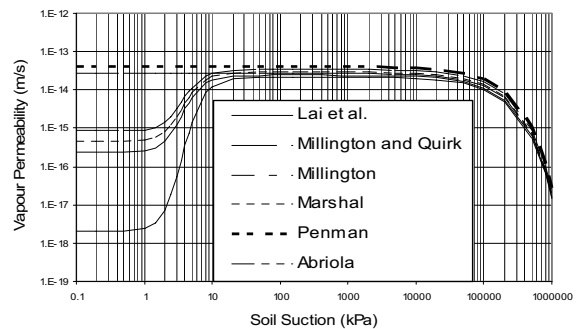


Figure 3. Variation of the coefficient of vapour permeability with soil suction using different tortuosity functions for loamy sand

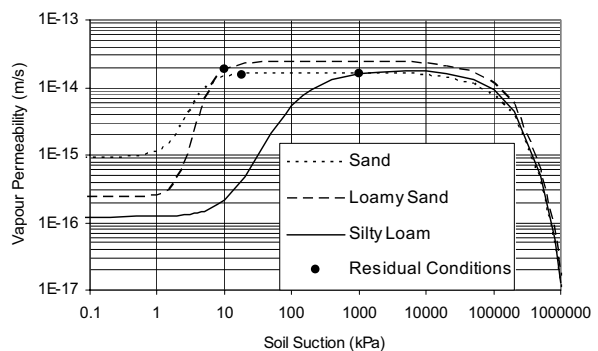


Figure 4. Vapour permeability curves for three soils.

Table 4. Maximum values of vapour permeability coefficient for different textures

Soil texture	$(k^v)_{\max}$ , m/s
Loamy Sand	$2.42 \times 10^{-14}$
Silty Loam	$1.76 \times 10^{-14}$
Sandy	$1.63 \times 10^{-14}$

#### 5.4. Comparison of the Liquid Water, Vapour, and Overall Permeability Functions

Figure 5 shows the variations of water, vapour, and overall permeability coefficients for the loamy sand on the same scale. The coefficient of vapour permeability,  $k_v$ , has its lowest value when the soil is saturated and begins to increase as the air starts occupying part of the soil pores. On the other hand, the liquid water permeability,  $k_w$ , is at its highest value when the soil is saturated and starts declining as the air starts entering the soil pores. When  $k_w$  reaches the residual water conditions,  $k_v$  becomes the maximum. However, the  $k_w$  at residual conditions still dominates and  $k_v$  is negligible. The thick curve in the Figure 5 shows the overall permeability function ( $k_w + k_v$ ) when Campbell's model is used to predict the  $k_w$ . The intersection point of liquid water and vapour permeability curves is the point after that the vapour permeability

coefficient begins to dominate over the liquid water permeability coefficient. As a result the intersection point shows the limit for the liquid water permeability coefficient.

As shown before, the maximum values for the vapour permeability coefficients for different soils (Table 4) are approximately  $2 \times 10^{-14}$ . By considering maximum values for the vapour permeability coefficients, regardless the type of soil, a value of  $1 \times 10^{-14}$  m/s is suggested as an appropriate minimum value for the liquid water permeability coefficient.

#### 6. SUMMARY AND CONCLUSION

This paper presented a study of the lower limit for the water permeability coefficient,  $k_w$ , for an unsaturated soil based on water content at residual conditions and the theory of vapour flow. First, the values of  $k_w$  at residual conditions were calculated. These values were found to suggest an inappropriate lower limit for  $k_w$ . The second approach was based on the water vapour flow theory. At high soil suctions, a significant portion of the overall water flow takes place as water vapour. A vapour permeability function was developed and used to determine the change in water vapour permeability with suction in a soil. Based on the comparison of liquid water, water vapour, and overall water coefficients of permeability functions, a lower limit for  $k_w$  was established. The results presented herein suggest that a constant value of  $1 \times 10^{-14}$  m/s can be used as a lower limit for the coefficient of liquid water permeability in liquid water flow analyses.

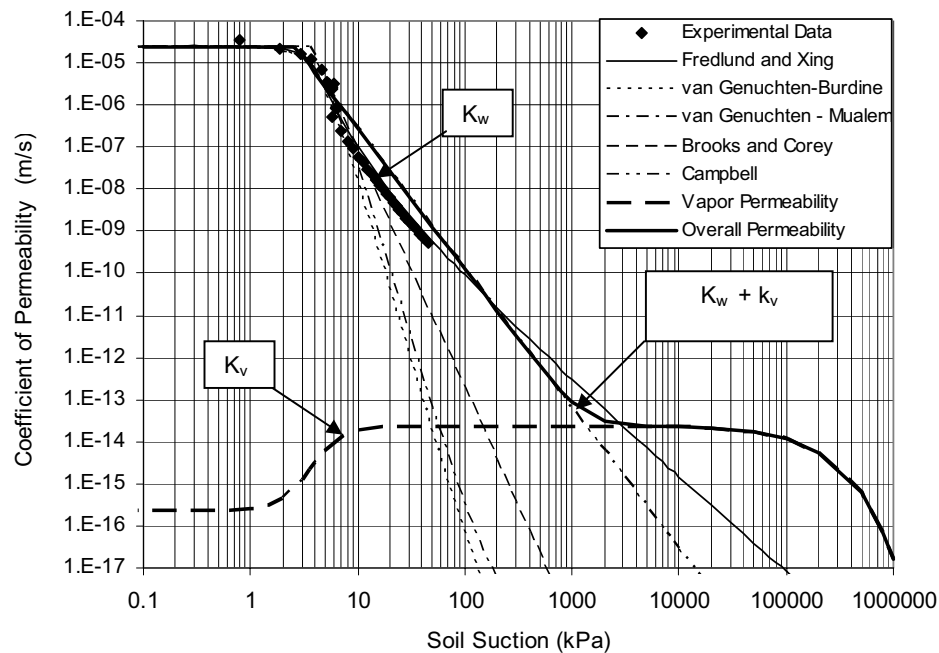


Figure 5. Variations of liquid water, vapour, and overall permeability coefficients with soil suction for a loamy sand soil

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