Uncoupled and coupled solutions of two-dimensional swelling in expansive soils

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ABSTRACT: This paper presents coupled and uncoupled solutions of swelling in an expansive soil. The governing partial differential equations are formulated based on the general theory of consolidation/swelling for an unsaturated soil. The research results presented in this paper show that the answers from uncoupled solutions compared well with the answers from coupled solutions. This paper suggests that uncoupled solutions may be adequate for the analysis of many volume change problem involving unsaturated, expansive soils. A typical two-dimensional problem associated with deformations in an expansive soil resulting from water infiltration into the soil is presented to illustrate and verify the above findings.

1 INRODUCTION

The volume change in an unsaturated, expansive soil involves interactions amongst several physical proc- ;es. The primary interactive processes are stress-ueformation and water flow. The soil properties associated with unsaturated soils are highly nonlinear. Most volume change problems are under isothermal condition in engineering practice and the air phase can be assumed to be continuous and at atmospheric conditions.

The stress-deformation process is governed by the mechanical equilibrium equation while the water flow process is governed by the water continuity equation. A rigorous solution for volume change in an expansive soil requires that the equilibrium equation and the continuity equation be solved. When the two equations are solved simultaneously, the approach is referred to as a coupled solution. Coupled solutions are difficult to obtain for expansive soils, in part, because of the nonlinear soil property functions associated with both water flow and stress-deformation.

Solutions can also be obtained by independently considering the two processes, in which case it is known as an uncoupled solution. Uncoupled solutions can be more easily obtained than coupled solu- ᵗⁱons because the soil property functions involved in :h process (i.e., water flow or stress-deformation) are considered independently. The coupling between stress-deformation and water flow processes takes place under transient conditions. At steady state conditions, the coupling disappears and the pore-water pressure condition can be determined through the solutions of the continuity equation.

This paper presents formulations for swelling and compares coupled and uncoupled solutions for the prediction of heave. The governing partial differential equations are formulated based on the general theory of consolidation/swelling for an unsaturated soil. A finite element computer program, called COUPSO (Pereira, 1996), was used to obtain coupled solutions. Uncoupled solutions were obtained by using a general partial differential equation solver, called FlexPDE[1].

A typical two-dimensional problem associated with deformations in an expansive soil resulting from water infiltration is analyzed.

2 BACKGROUND

Two independent stress state variables are needed to describe the mechanical behavior of unsaturated soils (Fredlund and Morgenstern, 1977). The two stress state variables are net normal stress, $(\sigma - u_a)$ and matric suction, $(u_a - u_w)$. The use of these stress state variables allows the combination of volume changes of both the soil structure and water phase to be predicted using constitutive soil properties.

The continuity requirement shows that the volume changes associated with any two of thee phases (i.e., soil structure, water phase and air phase) must be known, while the third phase can be calculated. In practice, the overall and water volume changes are usually measured.

Using a Cartesian coordinate system and referencing deformation to an elemental volume of soil solids, the total incremental volumetric strain of an unsaturated soil element, $d\varepsilon_v$, can be written as the sum of the incremental normal strains:

$$d\varepsilon_v = \frac{dV_v}{V_0} = d\varepsilon_x + d\varepsilon_y + d\varepsilon_z \qquad (1)$$

where $d\varepsilon_x$, $d\varepsilon_y$, $d\varepsilon_z$ = normal strain components in x-, y-, and z-direction, respectively.

3 GOVERNING PARTIAL DIFFERENTIAL EQUATIONS FOR SWELLING

Equations of overall static equilibrium for an unsaturated soil are:

$$\sigma_{ij,j} + b_i = 0 \qquad (2)$$

where σ_{ji} = components of the net total stress tensor, and b_i = components of the body force vector.

The water continuity equation in an unsaturated soil can be written as follows:

$$\frac{\partial(\rho_w V_w / V_o)}{\partial t} + \nabla(\rho_w q) = 0 \qquad (3)$$

where V_0 = initial overall volume of the referential soil element, V_w = volume of water, ρ_w = water density, $\nabla = \frac{\partial}{\partial x}i + \frac{\partial}{\partial y}j + \frac{\partial}{\partial z}k$, the divergence operator, $q = q_x i + q_y j + q_z k$, the Darcy's flux.

Assuming that the soil behaves in an incrementally isotropic and linear elastic material, the soil structure stress-strain constitutive relationships can be written in the index notation (Fredlund and Rahardjo, 1993):

$$d\varepsilon_{ij} = \frac{1+\mu}{E} d(\sigma_{ij} - \delta_{ij} u_a) - \frac{\mu}{E} d(\sigma_{kk} - 3u_a)\delta_{ij} + \frac{d(u_a - u_w)}{H}\delta_{ij} \qquad (4)$$

where ε_{ij} = components of the strain tensor for the soil structure, σ_{ij} = components of the total stress tensor for the soil structure, δ_{ij} = the Kronecker delta, μ = Poisson's ratio, E = elastic modulus for the soil structure with respect to a change in net normal stress, and H = elastic modulus for the soil structure with respect to a change in matric suction.

The water phase constitutive relationship can also be formulated in a semi-empirical approach as follows (Fredlund and Rahardjo, 1993):

$$\frac{dV_w}{V_0} = \frac{1}{E_w} d(\sigma_{ii} - 3u_a) + \frac{1}{H_w} d(u_a - u_w) \qquad (5)$$

where E_w = volumetric water modulus associated to a change in net normal stress, and H_w = volumetric water modulus associated to a change in matric suction.

Darcy's law can be used to describe water flow through soils in both saturated and unsaturated soil condition. For the case where the Cartesian coordinates are the same as the direction of the major and minor hydraulic conductivity, Darcy's law is written as follows:

$$q_i = -k_{wi} \frac{\partial}{\partial x_i}\left(\frac{u_w}{\gamma_w} + Y\right) \qquad (6)$$

where q_i = Darcy's flux in i-direction; k_{wi} = hydraulic conductivity in i-directions γ_w = unit weight of water; and Y = elevation.

The horizontal displacement, u; the vertical displacement, v; and the pore-water pressure, u_w are chosen as primary unknown for the problem. The equilibrium equations (Eq. 2) and continuity equation (Eq. 3) can be expressed using three primary unknowns by the constitutive relationships (Eqs. 3, 5, and 6) (Pereira, 1996; Hung et al., 2002).

$$\frac{\partial}{\partial x}\left(c_{11}\frac{\partial u}{\partial x} + c_{12}\frac{\partial v}{\partial y}\right) + c_{33}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) - d_s\frac{\partial(u_a - u_w)}{\partial x} + b_x = 0 \qquad (7a)$$

$$c_{33}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial y}\left(c_{12}\frac{\partial u}{\partial x} + c_{22}\frac{\partial v}{\partial y}\right) - d_s\frac{\partial(u_a - u_w)}{\partial y} + b_y = 0 \qquad (7b)$$

$$\beta_{w1}\frac{\partial \varepsilon_v}{\partial t} + \beta_{w2}\frac{\partial(u_a - u_w)}{\partial t} = \frac{\partial}{\partial x}\left(k_x\frac{\partial u_w}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y\frac{\partial u_w}{\partial y}\right) \qquad (8)$$

where b_x, b_y = body forces in x- and y-direction,

$$c_{11} = c_{22} = \frac{(1-\mu)E}{(1+\mu)(1-2\mu)}, \quad c_{33} = \frac{E}{2(1+\mu)},$$

$$c_{12} = \frac{\mu E}{(1+\mu)(1-2\mu)}, \quad d_s = \frac{E}{(1-2\mu)H},$$

$$\beta_{w1} = \frac{E}{E_w(1-2\mu)}, \quad \beta_{w2} = \frac{1}{H_w} - \frac{3E/H}{(1-2\mu)E_w}.$$

Equations (7a) and (7b) are stress equilibrium (i.e., deformation) equations in the x- and y-direction, respectively, and Eq. (8) is water phase continuity (i.e., seepage) equation. Assuming a constant value for Poisson's ratio of the soil, the solution of the system of equations (7a), (7b) and (8) requires the definition of the elastic parameters E, H,

E_w and H_w for the soil structure and the water phase as well as the hydraulic conductivity function, k_w.

The elastic parameters, E, H, E_w and H_w are calculated from the compressibility parameters. The compressibility parameters are obtained by differentiating constitutive surfaces for void ratio and degree of saturation (Fredlund and Rahardjo, 1993).

Equations (7a), (7b) and (8) can be solved using either an uncoupled or a coupled approach.

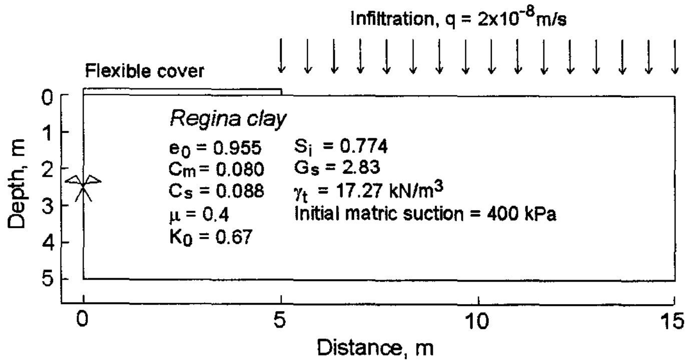

Figure 1. Illustration of the geometry and key variable for the example problem

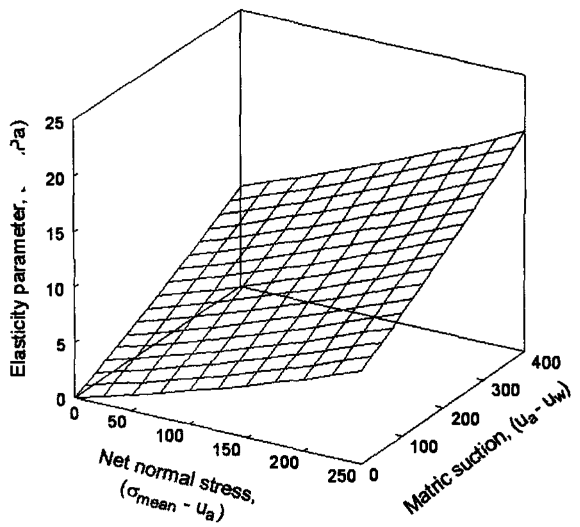

Figure 2. Elasticity parameter for soil structure, E

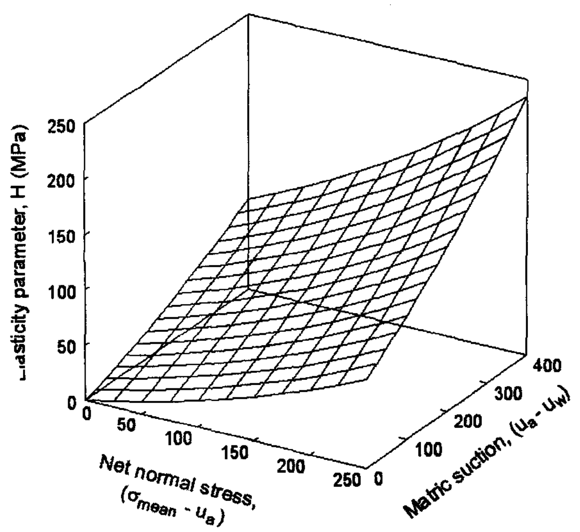

Figure 3. Elasticity parameter for soil structure, H

4 EXAMPLE PROBLEM

This example considers the hypothetical case of a 5-m thick deposit of swelling clay. The surface is partially covered with a flexible cover. Figure 1 presents the geometry and key variable for this problem. Experimental data obtained from tests on compacted specimens of Regina clay are used for the analysis. Poisson's ratio is assumed to be 0.4. The elasticity parameters are presented graphically in Figures 2-5.

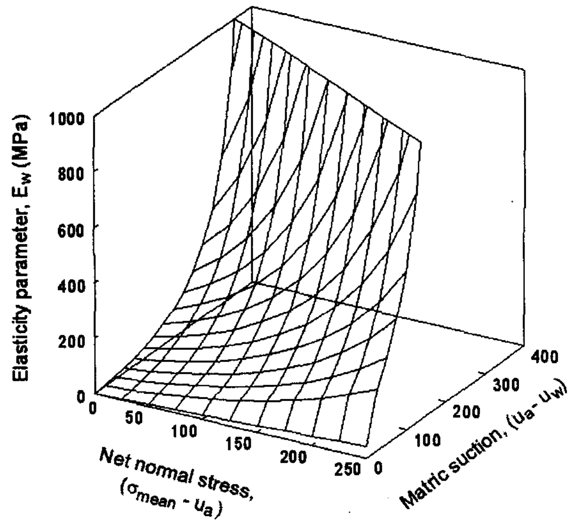

Figure 4. Elasticity parameter for water phase, E_w

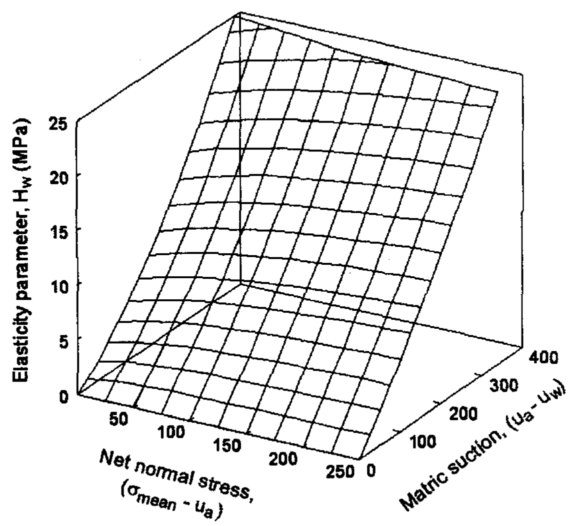

Figure 5. Elasticity parameter for water phase, H_w

The initial matric suction in the soil mass is assumed to be constant and equal to 400 kPa. The coefficient of earth pressure at-rest, K_0, is assumed to be 0.67 for the calculation of initial stress condition. The transient wetting process is introduced by imposing a water infiltration rate equal to 2×10^{-8} m/s at the uncovered portion of the ground surface. Such a wetting condition simulates the water infiltration into the soil mass due to the watering of a lawn or a light rain. The analysis is performed to track both the swelling soil behaviour and matric suction changes as the transient wetting front advances into the soil mass.

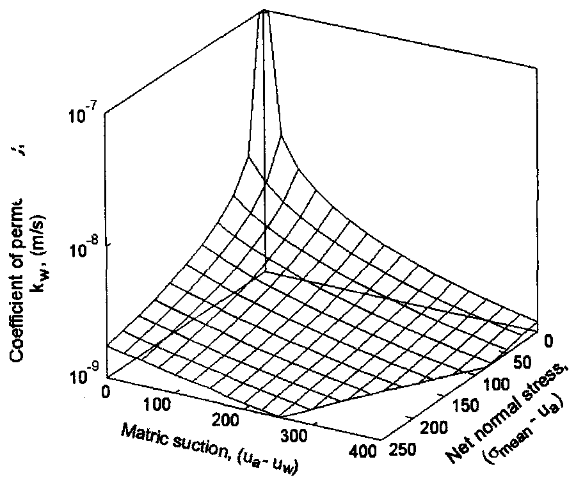

Figure 6. Coefficient of permeability constitutive surface

4.1 *Coupled approach*

In the coupled approach, deformation equations and seepage equation are solved simultaneously, the dynamic interdependence between the seepage and deformation problems is fully considered. The physical effect arising from the interaction between the stress-deformation and water flow processes can only found through coupled analysis, however, coupled analysis may requires orders-of-magnitude increases in computational requirements.

The boundary conditions associated with Equaons (7) and (8) are of the following four types; prescribed displacements, prescribed boundary stresses, prescribed pore-water pressure, and prescribed boundary flow. The boundary conditions for coupled analysis of this example is as follows. The transient wetting process is induced by imposing a water infiltration rate equal to 2.0x10^{-8} m/s at the uncovered portion of the ground surface. A value of matric suction equal to 400 kPa is specified at lower boundary, zero flux is specified on the other parts of the boundary. The soil was free to move in the vertical direction and fixed in horizontal direction at the left and right sides of the domain. The lower boundary was fixed in both directions.

The coupled solutions are obtained through the use of the COUPSO computer program. A finite element mesh of 75 nine-node quadrilateral elements and 431 node was used.

4.2 *Uncoupled approach*

In the uncoupled approach, the water phase continuity (i.e., seepage) equation is solved separately from he equilibrium (i.e., stress-deformation) equations. The interdependence of the equations is made in an iterative manner where the flow portion of the formulation is solved for a given time period and the resultant pore-water pressure changes are used as input in a deformation analysis. In turn, volume changes and induced stresses from the deformation analysis are used in the computation of the soil properties for the next time period in the seepage analysis.

The involvement of dependent variables and a number of non-linear soil properties are separated into two analyses; namely, a seepage analysis and a stress-deformation analysis. For seepage analyses, the dependent variable is pore-water pressure (or hydraulic head). At each given time period, the elasticity parameters, E and H, for soil structure are calculated at the initial conditions of current period, and assumed to remain unchanged over the current time increment. Net normal stress is assumed to be unchanged in the seepage analysis, therefore the elasticity parameters, E_w and H_w, for water phase and coefficient of permeability, k_w, are function of only matric suction, rather than both matric suction and net normal stress. Boundary conditions for seepage can be either pore-water pressure (or hydraulic head) type or water flux type. The results of seepage analysis provide the development of pore-water pressure and water flux with time in the time period considered, therefore changes in pore-water pressure can be obtained. These changes in pore-water pressure are then used in the stress-deformation. For this example, the transient wetting process is induced by imposing a water infiltration rate equal to 2.0x10^{-8} m/s at the uncovered portion of the ground surface. A value of matric suction equal to 400 kPa is specified at lower boundary, zero flux is specified on the other parts of the boundary.

For a stress-deformation analysis, dependent variables are horizontal displacement, u, and vertical displacement, v. In addition to Poisson's ratio, only two elasticity parameters, E and H, for soil structure need to be described as functions of matric suction at unchanged initial net normal stress. The elasticity parameters, E_w and H_w, for water phase and the coefficient of permeability, k_w, are no longer required for stress deformation analysis. Boundary conditions for the stress-deformation analyses can be of the displacement type or load type. For this example, the soil was free to move in the vertical direction and fixed in horizontal direction at the left and right sides of the domain. The lower boundary was fixed in both directions. Results of the stress-deformation analysis provide the displacements and induced stresses due to applied boundary condition and changes in pore-water pressure.

The uncoupled solutions are obtained through the use of the general partial equation solver, FlexPDE.

Solutions using the uncoupled approach depends on the magnitude of the selected times from seepage analysis. Short time period allows the stress state in the soils and the soil properties to be described more accurately with time and the results are more accurate for pore-water pressures and displacements. For the current problem, the rate of change in volumetric strain and net normal stress is updated at the following elapsed time: 13 days, 33 days, 53 days, 93 days, 133 days, 175 days, and after that at each 100 day period. Further refinement in the time periods does not produce significant change in the results.

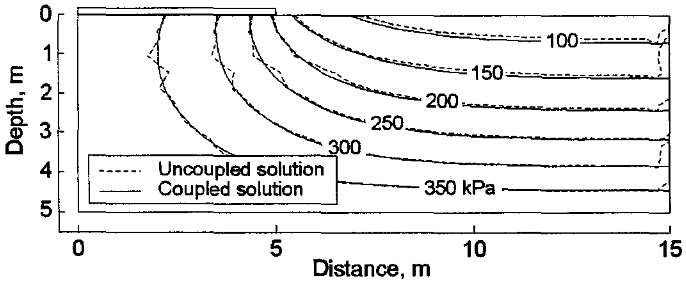

Figure 7. Comparison of matric suction distribution at day 53

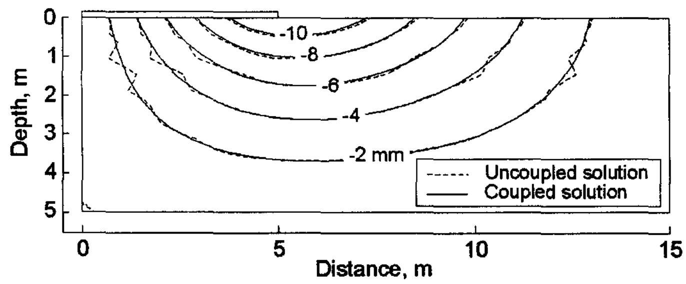

Figure 8. Comparison of horizontal displacement distribution day 53

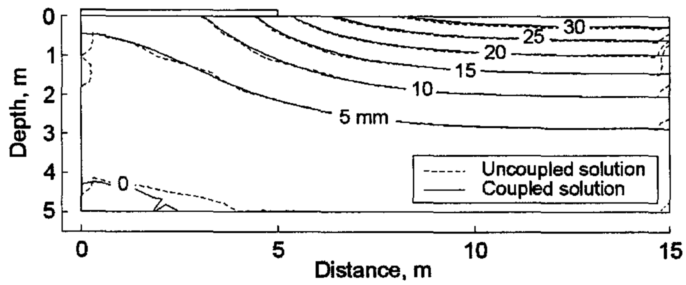

Figure 9. Comparison of vertical displacement distribution at day 53

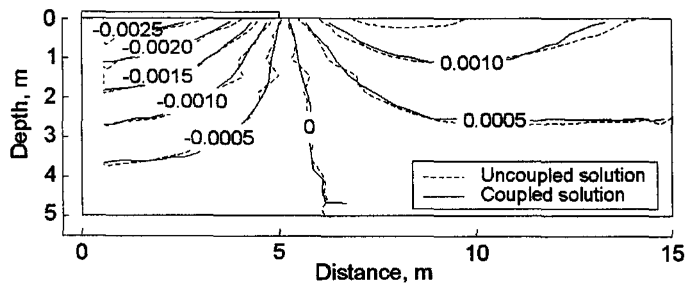

Figure 10. Comparison of horizontal strain distribution at day 53

4.3 *Presentation and discussion of the results*

Figures 7-11 compare the results of the uncoupled and coupled solutions at day 53 after infiltration begins. The comparisons are presented for the distribution of matric suction, horizontal displacement, vertical displacement, horizontal strain and vertical strain. Immediately after wetting was introduced into the soil from the uncovered surface, water flowed downward and to the left of the soil domain. Matric suction reduced to less than 100 kPa near ground surface. Most of the soil suction changes occurred below the uncovered portion where infiltration took place. The soil was displaced horizontally and vertically because of the decrease in matric suction. Horizontal displacements decreased with depth, with a maximum value of 13 mm at ground surface near to the cover. About 34 mm of heave took place at the uncovered location. The horizontal strain pattern shown in Figure 10 indicated that at this moment, soil displaced to the left in the left half of the soil mass, and displaced to the right in the other half of the soil mass. Most of the vertical strain took place in the uncovered portion of the soil mass, the magnitude of vertical strain decreased with depth.

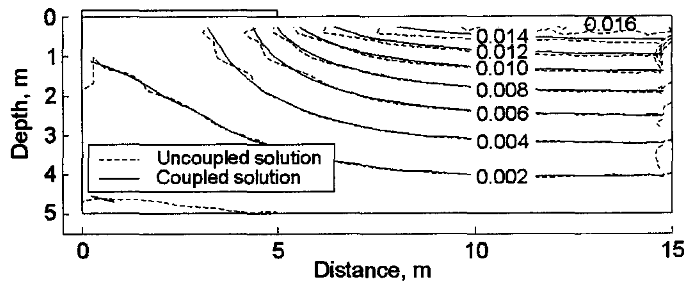

Figure 11. Comparison of vertical strain distribution at day 53

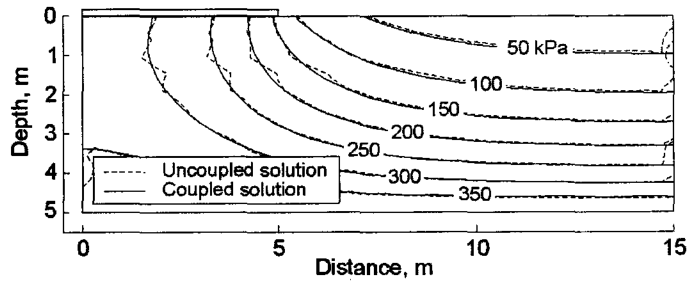

Figure 12. Comparison of matric suction distribution at day 175

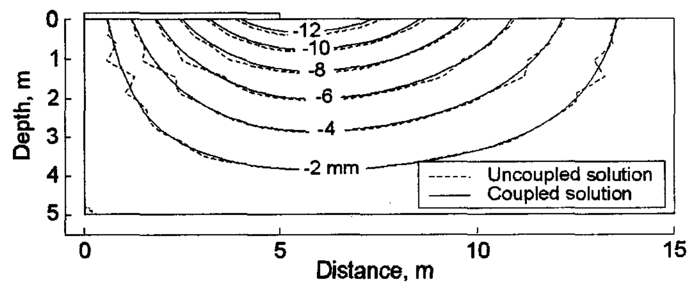

Figure 13. Comparison of horizontal displacement distribution at day 175

Figures 12-14 compare the results of the uncoupled and coupled solutions at day 175. At this time, matric suction below the uncovered surface reduced to less than 50 kPa. Cumulative horizontal and vertical displacements had the same patterns as those at day 53. A maximum cumulative heave of 52 mm took place at the upper-right corner of the soil domain. About 12 mm of heave could be observed at [illegible]

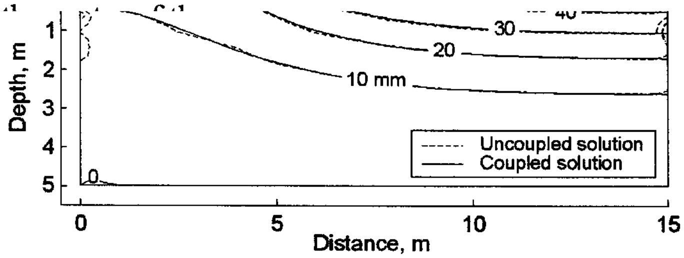

Figure 14. Comparison of vertical displacement distribution at day 175

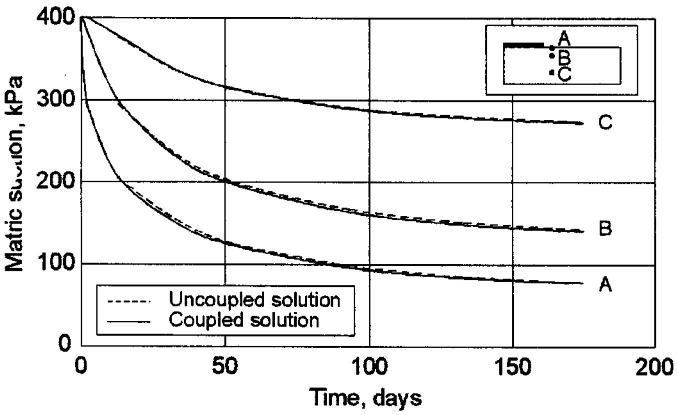

Figure 15. Comparison of suction development with time for point A, B and C

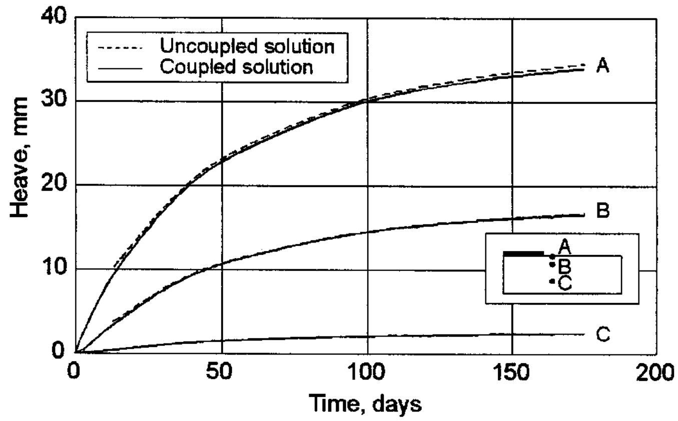

Figure 16. Comparison of vertical displacement development with time for point A, B and C

Figure 15 compares the developments of matric suction with time at the monitoring points for uncoupled and coupled solutions. Matric suction below the cover reduced rapidly in the first 50 days. It can be noted that the matric suction in the soil mass has not stabilized in the first 175 days; however, the rate of change in matric suction in the soil mass reduced rapidly with time.

Figure 16 presents the development of vertical displacement with time for the monitoring points. Most of the heave at the monitoring points occurred [illegible] pled solution. Uncoupled analysis in this study used elastic parameters at net normal stresses that are lower than the actual net normal stresses. The stiffness of the expansive soil decreases with the decrease in net normal stress, resulting in a slightly larger amount of heave in uncoupled analysis.

5 CONCLUDING REMARKS

Solutions to the volume change problems associated with unsaturated, swelling soils can be obtained through either an uncoupled or a coupled analysis. The research results presented in this paper show that the answers from uncoupled solutions compared well with the answers from coupled solutions. It is suggested that uncoupled solutions may be adequate for the analysis of volume change predictions (i.e., heave analyses) for unsaturated, swelling soils.

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[1] FlexPDE is a propriety product of PDE Solutions Inc., 2120 Spruce Way, Antioch, CA 94509, USA