NUMERICAL MODELLING OF TWO-DIMENSIONAL HEAVE FOR SLABS-ON-GROUND AND SHALLOW FOUNDATIONS

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ABSTRACT: This paper presents a numerical method for performing a two-dimensional heave for slabs-on-ground and shallow foundations. The proposed method is an extension of a commonly used method for the prediction of one-dimensional heave to two-dimensions. The method is based on oedometer test results. Measurements of soil suctions in the field and the swelling index with respect to changes in soil suction are not required. An example involving a concrete slab constructed on expansive soils with external loads and infiltration of water into the soil is analyzed to illustrate the proposed method.

RÉSUMÉ: Cet article présente une méthodenumérique pour simuler le gonflement dans deux dimensions pour les dalles sur le sol et les fondations peu profondes. La méthode proposée est une extension d'une méthode ordinairement utilisée pour la prédiction de gonflement dans un-dimension aux deux-dimensions. La méthode est basée sur les tests de oedometer. Les mesures de succions de sol dans le champs et l'index de changement de volume par rapport aux changements dans la succion de sol ne sont pas exigées. Un exemple classique associé avec la déformation d'une dalle concrète construite sur les sols expansifs grâce aux chargements externes et l'infiltration d'eau dans les sols est analysé pour illustrer la méthode proposée.

1. BACKGROUND

The prediction of heave in two-dimensions requires a definition of the initial total stresses and matric suction conditions, the elasticity parameters related to changes in net normal stress and matric suction conditions and an assumed Poisson's ratio (Hung and Fredlund, 2002). Soil suction conditions in the field can be measured, estimated or assumed. A saturated-unsaturated seepage analysis can be performed to predict changes in soil suction. The soil properties required for a transient seepage analysis are the soil-water characteristic curve and the coefficient of permeability function. The soil-water characteristic curve and the coefficient of permeability function for the volume change analysis involving an unsaturated soil are described using the Fredlund and Xing (1994) equation and the Leong and Rahardjo (1997) equation, respectively. The net normal stress conditions in the field can be estimated from a total stress theory and the coefficient of earth pressure at-rest. The elasticity parameter with respect to net normal stress, E, and elasticity parameter with respect to soil suction, H, can be obtained by differentiating the equation for constitutive surface for the soil structure (Fredlund and Rahardjo, 1993) or calculated directly from the volume change index with respect to net normal stress, C_s, and the volume change index with respect to soil suction, C_m, (Hung and Fredlund, 2002).

This paper suggests that a commonly used method for the prediction of one-dimensional heave (i.e., the Fredlund, Hasan and Filson method, 1980), can be extended for the prediction of two-dimensional heave. The Fredlund et al. method (1980) is briefly reviewed in the following section to serve as a background and the terminology is set forth for the two-dimensional procedure suggested in this paper.

A two-dimensional example problem involving a slab-on-ground is used to illustrate the suggested procedure for a heave analysis. The numerical solutions are obtained using a general-purpose partial differential equation solver, FlexPDE[1]. The results of the seepage analysis include the distributions of soil suction in the soil profile with respect to time for specified boundary conditions. The results of stress/deformation analysis include the distribution of horizontal and vertical displacements that occur due to changes in applied load and matric suction. The numerical procedure can be used for the analysis of a wide variety of two-dimensional heave problems associated with expansive soils. The results show that it is possible to compute the moments and shears in the concrete slab due to external loading to the slab and swelling in the soil.

2. FREDLUND, HASAN AND FILSON METHOD (1980) FOR ONE-DIMENSIONAL HEAVE ANALYSIS

The Fredlund et al. (1980) method is based on the constant volume oedometer test results performed on undisturbed samples. The basic data required from the laboratory test are the rebound or swelling index, C_s, and the corrected swelling pressure, P'_s. The data must be corrected for the effects of compressibility of the apparatus prior to its interpretation.

The amount of total heave is computed from changes in void ratios corresponding to the initial and final stress states and the swelling index, C_s. The initial and final stress states are projected onto the net normal stress plane, as shown in Fig. 1. The stress path follows a constant void ratio path from the *in situ* stress state to the initial stress state (i.e., the corrected swelling pressure, P'_s) on net normal stress plane, and then follows the rebound curve from the initial stress state to the final stress state. The equation for the rebound portion of the oedometer test data is written as follows:

$$\Delta e = C_s \log \frac{FST}{IST} \qquad [1]$$

where: C_s = swelling index with respect to net normal stress measured at saturation, FST = final stress state, and IST = initial stress state.

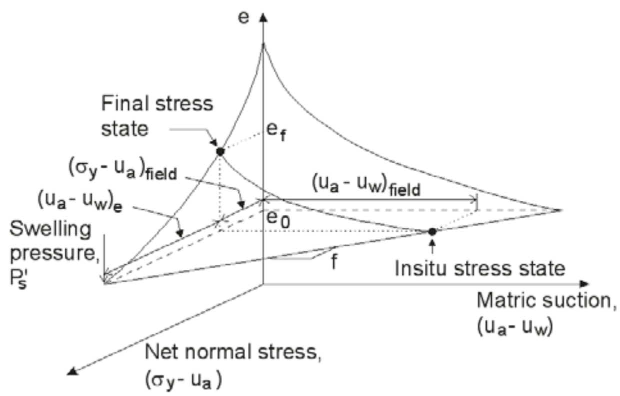

Figure 1. Illustration of total stress path for heave analysis and assumed relationship between matric suction and matric suction equivalent

The initial stress state, *IST*, or the corrected swelling pressure, P'_s, can be formulated as the sum of the overburden pressure and the matric suction equivalent (Fig. 1) as follows:

$$IST = P_s^{'} = (\sigma_y - u_a)_i + (u_a - u_w)_e \qquad [2]$$

The final stress state must account for total stress changes and the final matric suction conditions. The final matric suction condition can either be predicted or estimated. At low suction conditions, the matric suction equivalent can be assumed to be equal to the actual matric suction, and the final stress state can be estimated as follows:

$$FST = (\sigma_y - u_a)_i + \Delta\sigma_y + (u_a - u_w)_f \qquad [3]$$

where: f = subscript indicating final condition, i = subscript indicating initial condition, and $\Delta\sigma$ = change in total stress due to applied loads.

3. PROPOSED METHOD FOR TWO-DIMENSIONAL HEAVE ANALYSIS

The solution of a heave problem associated with unsaturated, expansive soils involves the solution of a saturated-unsaturated seepage model and a stress-deformation model. The models can be formulated based on the general theory of unsaturated soil behaviour (Hung and Fredlund, 2002). The use of data from oedometer tests and concepts of matric suction equivalent suggested in the Fredlund et al. (1980) method will be presented for seepage analysis and stress-deformation analysis.

3.1 Seepage analysis

A seepage analysis is required to predict changes in the matric suction conditions in the soil. Initial matric suction conditions can be estimated from the corrected swelling pressure assuming a slope for the net normal stress versus suction curve at a constant void ratio. Let us assuming that the slope can be written as the function, f, and for the sake of this example it can be taken as being equal to the degree of saturation (Fig. 1), Eq. 2 becomes:

$$P_s^{'} = (\sigma_y - u_a)_i + f(u_a - u_w)_i \qquad [4]$$

Therefore, the initial matric suction conditions required for seepage analysis can be estimated as follows:

$$(u_a - u_w)_i = \frac{P_s^{'} - (\sigma_y - u_a)_i}{f} \qquad [5]$$

where: f = a function set equal to the degree of saturation.

3.2 Stress-deformation analysis

The stress-deformation analysis can be performed to predict displacements and induced stresses due to external loads and wetting. Deformation in the soil mass due to external loads can be assumed to respond immediately, while deformation due to wetting is a time dependent process. Therefore, the deformations due to loading and wetting need to be analyzed independently. The suggested stress path for the stress-deformation analysis is illustrated in Fig. 2. The analysis is first performed to predict the displacements and induced stresses due to the loading. The displacements due to changes in matric suctions are then predicted for various elapsed times using matric suction profiles obtained from the seepage analysis.

A set of initial stress state (*IST*), final stress state (*FST*) and the swelling index (C_s) from the net normal stress plane can be used for the actual suction stress path as illustrated in Fig. 3. The *in situ* stress state (*IST*) for a two-dimensional analysis can be written as follows:

$$IST = (\sigma_{ave} - u_a)_i + f_i(u_a - u_w)_i \quad [6]$$

where: $\sigma_{ave} = (\sigma_x + \sigma_y)/2$

or

$$IST = \frac{1+K_0}{2}(\sigma_y - u_a)_i + f_i(u_a - u_w)_i \quad [7]$$

where: K_0 = coefficient of earth pressure at-rest, f_i = a slope set equal to the final degree of saturation.

The final stress state (*FST*) can be written as follows:

$$FST = (\sigma_{ave} - u_a)_f + f_f(u_a - u_w)_f \quad [8]$$

or

$$FST = \frac{1+K_0}{2}(\sigma_y - u_a)_i + \Delta(\sigma_{ave} - u_a) + f_f(u_a - u_w)_f \quad [9]$$

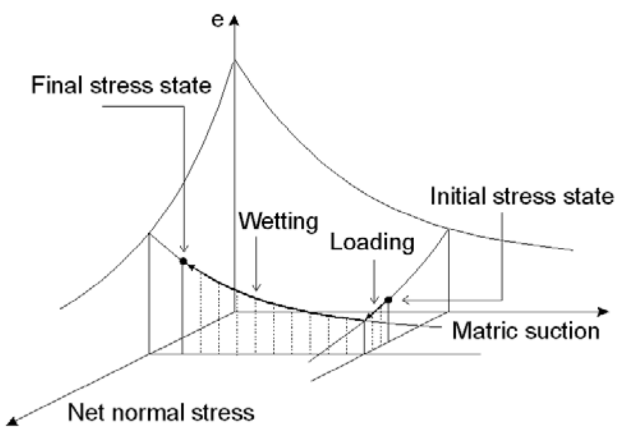

Figure 2. Stress path followed in the stress-deformation analysis

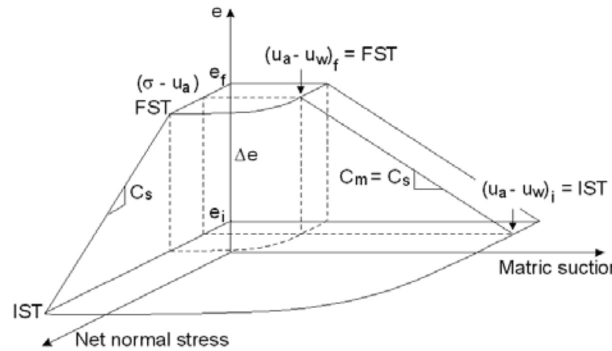

Figure 3. Illustration of the use of initial stress state (*IST*), final stress state (*FST*), and swelling index obtained at net normal stress plane (C_s) for suction stress path

The coefficient of lateral earth pressure at-rest, K_0, can be selected from typical values that have been back-calculated from field observations of heave and shrinkage (Lytton, 1994):

$$K_0 = \begin{cases} 0 & \text{when the soil is dry and cracked} \\ 0.333 & \text{when the soil is dry and cracks are opening} \\ 0.500 & \text{when cracks are closed and suction is at a steady state condition} \\ 0.667 & \text{when cracks are closed and the soil is wetting} \\ 1 & \text{when the soil is wetting and is in hydrostatic stress condition} \\ 2-3 & \text{when the soil is approaching passive earth pressure} \end{cases} \quad [10]$$

The elasticity parameter functions can be written for two-dimensional plane strain conditions as follows (Hung and Fredlund, 2002):

$$E = \frac{4.605(1+\mu)(1-2\mu)(1+e_0)}{C_s}(\sigma_{ave} - u_a) \quad [11]$$

$$H = \frac{4.605(1+\mu)(1+e_0)}{C_m}(u_a - u_w) \quad [12]$$

It is suggested that Poisson's ratio be assumed to be a constant that is estimated from the coefficient of earth pressure at-rest, K_0, as follows:

$$\mu = \frac{K_0}{1+K_0} \quad [13]$$

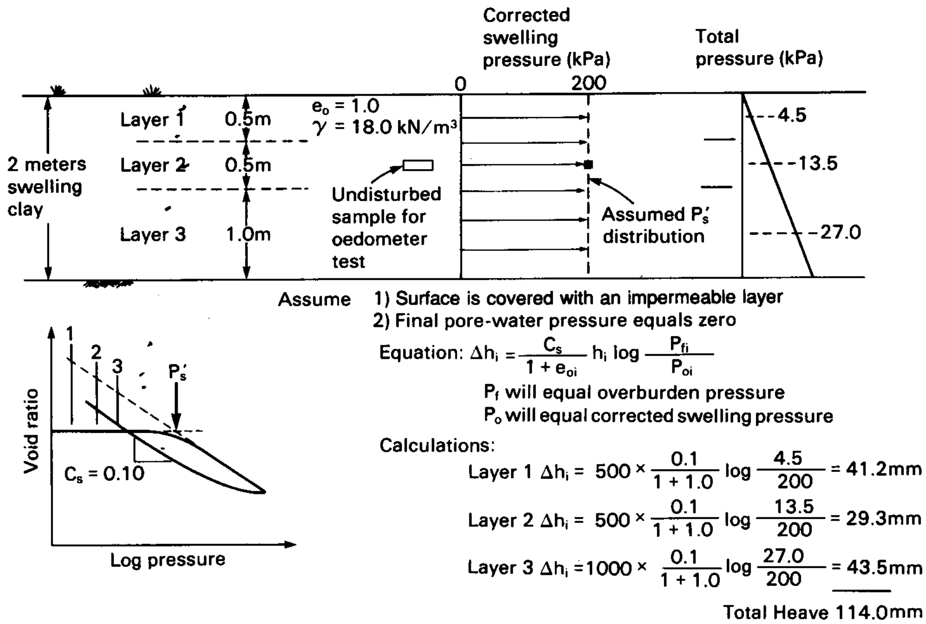

Figure 4. Illustration of the example problem and the one-dimensional solution (Fredlund and Rahardjo, 1993)

3.3 Calculation of moments and shear forces in the slab

Assuming that displacements at the edge of the slab are small in comparison to its thickness, the loads applied on the slab can be assumed to be normal to the slab surface. The bending moments can be calculated from the displacements or bending stresses, and the shear force can be calculated from the moments. Timoshenko and Woinowsky-Krieger (1959) presented the following equations for computing bending moments and shear.

$$M = -\frac{E_c h_s^3}{12(1-\mu_c^2)}\frac{\partial^2 v}{\partial x^2} \qquad [14]$$

or

$$M = \frac{\sigma_{max} h_s^2}{6} \qquad [15]$$

$$Q = \frac{\partial M}{dx} \qquad [16]$$

where: v = vertical displacement of the slab; M = bending moment per unit length; Q = shear force per unit length; h_s = thickness of the slab; σ_{max} = maximum bending stress; E_c = elastic modulus of concrete; and μ_c = Poisson's ratio of concrete.

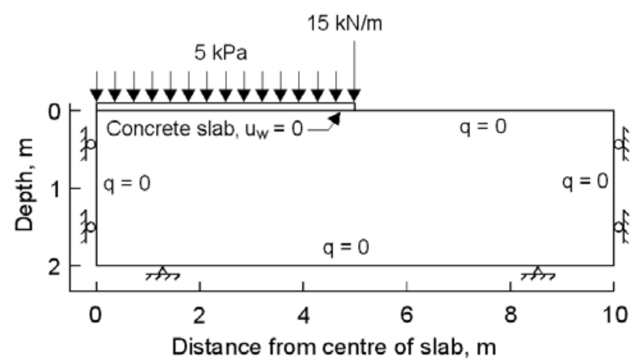

Figure 5. Illustration of the example problem and the boundary conditions for the seepage and stress-deformation analysis

4. EXAMPLE

An example problem presented in Fredlund and Rahardjo (1993) was analyzed to illustrate the suggested procedure for predicting two-dimensional heave. The problem and its one-dimensional solution using the Fredlund et al. method is presented in Fig. 4. The clay layer is 2 m in thickness. The initial void ratio of the soil is 1.0, the total unit weight is 18.0 kN/m^3, and swelling index, C_s, is 0.1. Only one oedometer test was performed on a sample taken from a depth of 0.75 m. Test data showed a corrected swelling pressure of 200 kPa and an initial degree of saturation of 70%. It is assumed that the corrected swelling pressure is constant throughout the entire soil layer. A total heave of 114 mm was predicted from one-dimensional analysis when no external load was considered.

The problem was modified to show two-dimensional behavior by placing a concrete slab of 100 mm thickness at the surface (Fig. 5). Displacements due to the external loads, and leakage of water under the cover will be predicted for various suction conditions (i.e., elapsed times) in the soil mass. A Young modulus of 10 MPa and a Poisson’s ratio of 0.15 was used for the concrete slab.

Figure 5 shows the geometry and boundary conditions for both the seepage and stress-deformation analyses. Zero pore-water pressure was specified under the slab and a moisture flux equal to zero was specified elsewhere along the boundaries. A load equal to 5 kPa and a perimeter load of 15 kN/m were applied on the surface and at the perimeter of the concrete slab. The soil is free to move in a vertical direction and fixed in the horizontal direction at the left and right sides of the domain. The lower boundary is fixed in both directions.

The soil-water characteristic curve and the permeability function presented in Hung and Fredlund (2002) were assumed for the seepage analysis (Fig. 6). A relationship between degree of saturation and matric suction was assumed to estimate the matric suction equivalent from matric suction and is presented in Fig. 7. Initial matric suction conditions in the field were estimated from the corrected swelling pressure using Eq. 5. Figure 8 shows distribution of overburden pressure, corrected swelling pressure, matric suction equivalent and the estimated initial matric suction conditions.

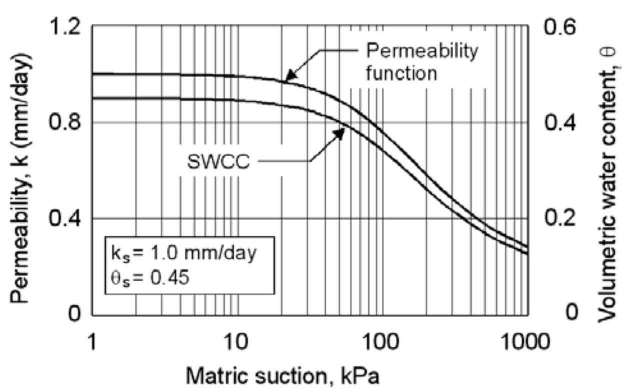

Figure 6. Assumed permeability function and soil-water characteristic curve

The coefficient of earth pressure at-rest, K_0, equal to 0.667 was used to determine the initial stress state conditions (Eq. 10). A Poisson’s ratio of 0.40 was calculated from K_0 using Eq. 13.

The elasticity parameter function with respect to changes in net normal stress, E, can be calculated for two-dimensional conditions using Eq. 11 for e_0 = 1.0; C_s = 0.10; and μ = 0.4 and can be written as follows:

$$E = 25.8(\sigma_{ave} - u_a) \quad [17]$$

The elasticity parameter function with respect to changes in matric suction, H, can be calculated for two-dimensional condition using Eq. 12 for e_0 = 1.0; C_m = 0.10; and μ = 0.4 and can be written as follows:

$$H = 128.9(u_a - u_w) \quad [18]$$

5. COMPUTER RESULTS AND DISCUSSIONS

Figure 9 presents matric suction profiles at the centre of the slab for various elapsed times of wetting. Figure 10 shows the matric suction distribution in the soil at day 45. It can be seen that under the specified boundary conditions, the matric suctions at day 45 approaches zero below centre of the slab and about 50 kPa below the edge of the slab.

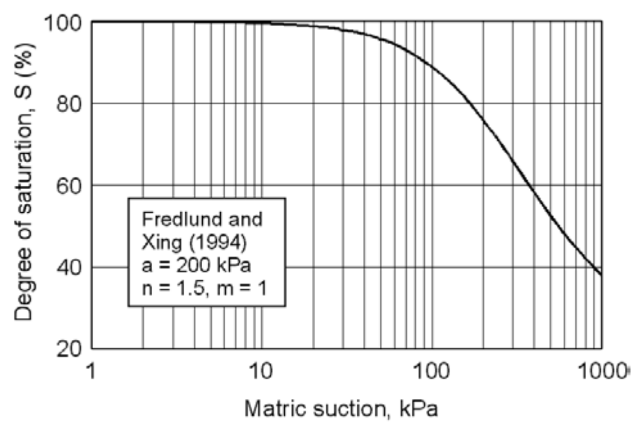

Figure 7. Assumed relationship between degree of saturation and matric suction

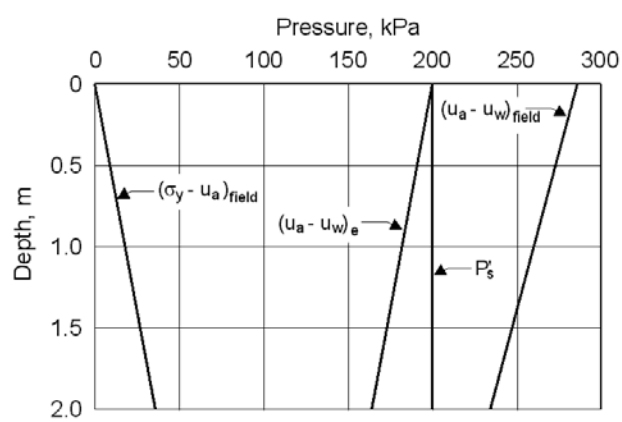

Figure 8. Initial stress state conditions

Figure 11 shows contours of vertical displacements due to loading. About 4 mm of settlement due to loading is predicted at the edge of the slab. The induced net normal stress is used to calculate the final net normal stress state in the soil. The soil was loaded at the initial net normal stress and matric suction conditions in the field. Therefore, the sum of initial net normal stress and initial matric suction equivalent must be used along with the swelling index obtained on the net normal stress plane for the prediction of displacements and induced stresses due to loading.

Figure 12 presents the horizontal displacements versus depth at the edge of the slab after loading and various final pore-water pressure conditions. A maximum horizontal displacement of 17 mm was computed for the 0.25 m depth at day 45. Insignificant values of horizontal displacements were observed when the pore-water pressure was raised uniformly to zero throughout the entire soil profile.

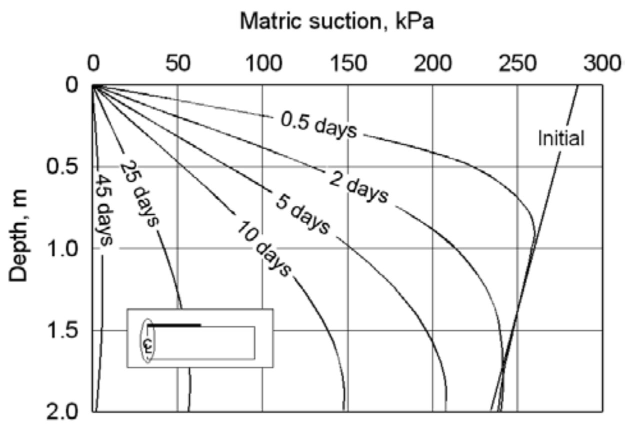

Figure 9. Matric suction profiles at the centre of the slab for various elapsed times

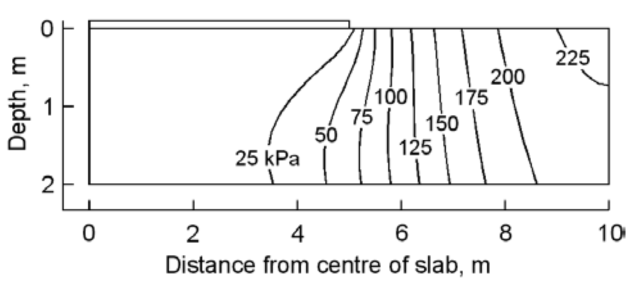

Figure 10. Matric suction conditions after 45 days of wetting

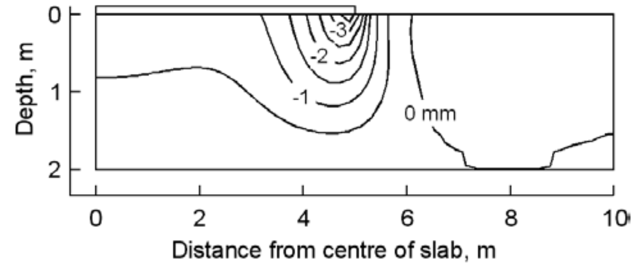

Figure 11. Contours of vertical displacements due to loading

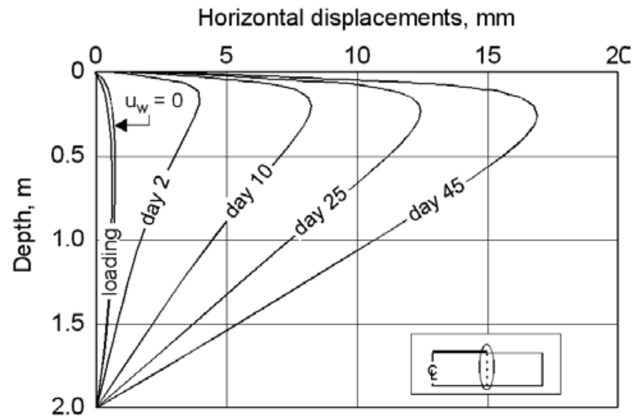

Figure 12. Horizontal displacements versus depth at the edge of the slab after loading and various final pore-water pressure conditions

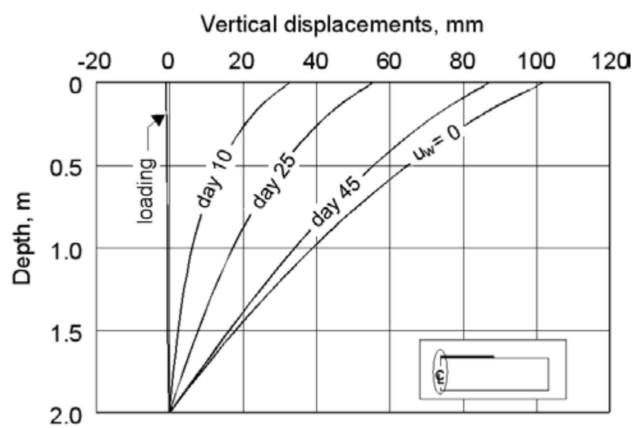

Figure 13. Vertical displacements versus depth at centre of the slab after loading and various final pore-water pressure conditions

Figure 13 presents the predicted vertical displacements versus depth at the centre of the slab after loading and various final pore-water pressure conditions. A total heave of 102 mm predicted for the case when the final pore-water pressure is equal to zero and this compares well with the total heave of 114 mm predicted from the one-dimensional analysis. It must be noted that the predicted one-dimensional heave for this example did not consider the external load applied on the slab.

Figure 14 shows vertical displacements of the slab after loading and various final pore-water pressure conditions. Figures 15 and 16 presents contours of horizontal and vertical displacements, respectively. Total heaves of 88 mm and 51 mm were predicted for day 45 at the centre and the edge of the slab, respectively. A differential heave of 37 mm was observed at day 45. When the pore-water pressure was raised uniformly to zero throughout the entire soil profile, the differential heave of the slab is minimal.

The results of the stress-deformation analysis also include the distribution of the resulting stresses in the slab. Figure 17 shows the predicted flexural stresses at the top and bottom of the slab at day 10 after wetting. Figure 18 presents the flexural stresses at the top of the slab after loading and various elapsed times of wetting.

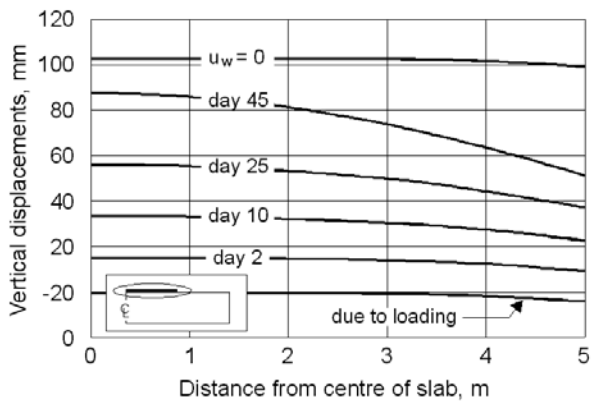

Figure 14. Vertical displacements of the slab after loading and wetting with times

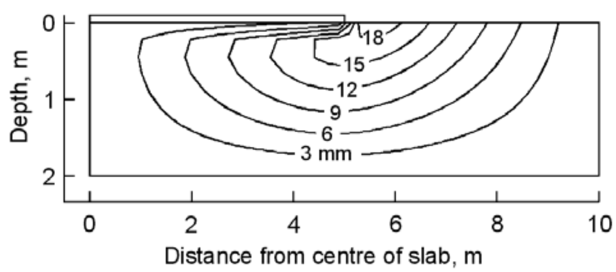

Figure 15. Contours of horizontal displacements at day 45

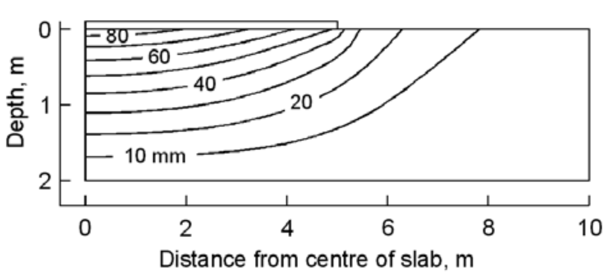

Figure 16. Contours of vertical displacements at day 45

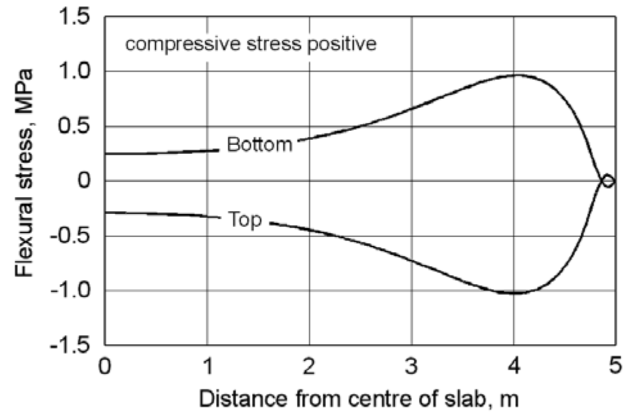

Figure 17. Flexural stresses at top and bottom of the slab at day 10

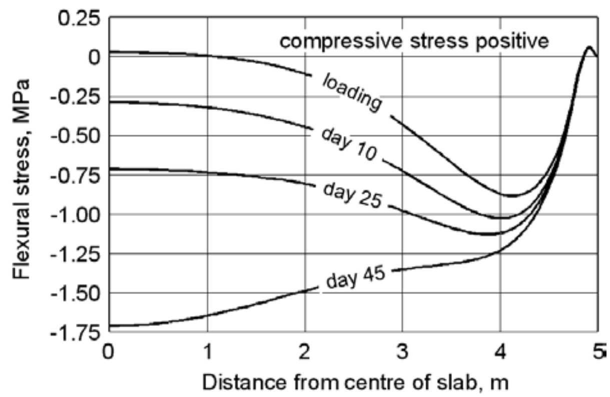

Figure 18. Flexural stresses at top of the slab after loading and various elapsed times of wetting

Cumulative bending moments in the slab after loading and various elapsed times of wetting are presented in Fig. 19. It was assumed that the perimeter load was applied after the concrete was hardened. The bending moments predicted for this example at early periods of wetting (i.e., less than 25 days) are due mainly to the loading of the slab. The bending moments resulted from wetting took place in a latter period.

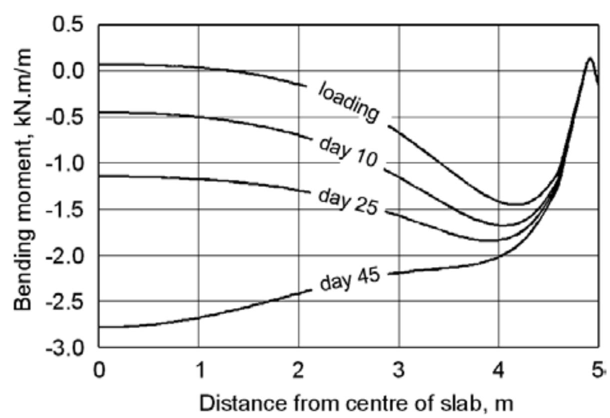

Figure 19. Cumulative bending moments of the slab after loading and various elapsed times of wetting

6. SUMMARY

The method for the prediction of two-dimensional heave is proposed based on general theory of unsaturated soils using conventional oedometer test results. Initial soil suction conditions in soils can be estimated from the corrected swelling pressure. Changes in soil suctions can be estimated through saturated-unsaturated seepage analysis. Soil properties obtained in net normal stress plane along with the concepts of matric suction equivalent can be used for the stress-deformation analysis for both external loads and changes in soil suction. Calculated displacements, flexural stresses and bending moments of the concrete slab appears to be reasonable and consistent with those generally observed in the field. Total heave predicted from two-dimensional analysis appears to agree well with the analytical one-dimensional heave predicted using the Fredlund et al. (1980) method for heave prediction.

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[1] FlexPDE is a proprietary product of PDE solutions Inc., 2120 Spruce Way, Antioch, CA 94509, USA