ESTIMATION OF THE HYSTERETIC SOIL-WATER CHARACTERISTIC CURVES FROM THE *BOUNDARY DRYING* CURVE

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ABSTRACT: A statistical analysis of the key features of hysteresis Soil-Water Characteristic Curves (SWCC) is presented in this paper. The air entry value, the slope of the SWCC and the distance between *boundary wetting* and the *boundary drying* curves are compared for a number of soil types and suggested values for these relationships are presented for different types of soils. A simple scaling method is also presented for estimating the *initial drying* curve, the *boundary drying* curve and the *boundary wetting* curve when one of the other three curves is known.

RESUME: Cet article présente une analyse statistique des caractéristiques principales des hystérésis des Courbes Caractéristiques de Sol-Eau (SWCC). La valeur d'entrée d'air, la pente du SWCC et la distance entre les courbes de limite mouillée et sèche sont comparées pour un certain nombre de types de sol ; et les valeurs suggérées par relations sont présentées pour les différents types des sols. Une méthode simple de scaling est aussi présentée afin d'estimer la courbe initiale sèche, la courbe de limite séchée et la courbe de limite mouillée lorsqu'une des trois autres courbes est connue.

1. INTRODUCTION

The SWCC is used in a number of geotechnical engineering analyses (Fredlund and Rahardjo, 1993; Barbour, 1999; Fredlund, 2000); however, hysteretic nature of the SWCC is rarely taken into account. Methods of characterizing hysteretic SWCC are quite complex and the measurement of hysteresis curves are costly and time consuming. In many engineering applications only one branch of the SWCC is used.

The SWCC is generally measured in the laboratory. For tests on slurry specimens, the initial drying curves are most commonly measured and for compacted samples which depend on the saturation process the boundary drying curves may be measured. In order to make use of the measured curve for the prediction of the other curves (i.e., boundary wetting curve), a comprehensive model for hysteresis SWCC is required.

Numerous hysteresis models have been proposed for predicting the scanning curves between the boundary wetting and boundary drying curves. These models require at least two boundary curves for calibration (Pham, 2001). There are a few models that are capable of predicting boundary wetting curve from the boundary drying curve and vice versa; however, these methods require several measured points on both curves (Parlange, 1976, 1980; Mualem, 1977, 1984; Nimmo, 1992; Pham et al., 2003). These models are not applicable if the proper experimental data is not available. In this study, a statistical analysis was implemented to establish relationships between key features of the following three hysteretic SWCCs (i.e., the Initial drying curve, the boundary drying curve, and the boundary wetting curve). Using typical values from these relationships, it is possible to estimate one of the boundary curves by using the other boundary curve.

2. RELATIONSHIPS BETWEEN THE HYSTERESIS SWCC CURVES

There are four key parameters on any soil-water characteristic curve. These are the water content at zero soil suction, the air entry value, the slope of the curve and the residual water content. The residual water content obtained from the *initial drying* curve, the *boundary drying* curve, and the *boundary wetting* curve are essentially the same. The water content at zero soil suction on the *initial drying curve* is the water content at saturation, while the water content at zero soil suction on the *boundary hysteresis* curve is approximately equal to 90% of that at saturation (Rogowski, 1971). The following features of SWCCs were studied; namely, the distance between the *boundary drying* and the *boundary wetting* curves, the slopes of the *boundary drying* and the *boundary wetting* curves, and the air entry value of *initial drying* and *boundary drying* curves. The distance between the *boundary drying* curve and the *boundary wetting* curve is defined as the horizontal distance between the inflection point of the *boundary drying* curve and the point having the same water content on the *boundary wetting* curve (Figure 1).

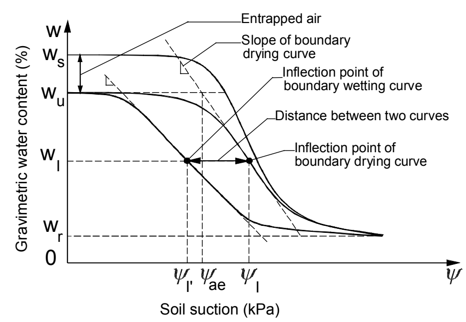

Figure 1. Schematic illustration of the slope and distance between the two boundary hysteretic soil-water characteristic curves.

The Feng and Fredlund (1999) curve-fitting equation, was used to investigate these relationships because the equation provides an approximate slope of the SWCC and has an inflection point that lies midway between saturation and residual water content conditions (i.e., at a soil suction of $b^{1/d}$ and 50% of effective water content). The inflection points of the two SWCC curves have the same water content and the line joining the two points is horizontal. The Feng and Fredlund (1999) curve-fitting equation can be expressed in equation [1].

$$w(\psi) = \frac{w_u b + c\psi^d}{b + \psi^d} \qquad [1]$$

where: w_u is water content on the *boundary drying* curve at zero soil suction, and *b, c, d* are curve-fitting parameters.

The differentiation of the Feng and Fredlund (1999) equation at a particular soil suction, ψ, can be expressed as follows:

$$S_L = \frac{dw(\psi, w_u, b, c, d)}{d\psi} = c\psi^d \frac{d}{\psi(\psi^d + b)} - \frac{w_u b + c\psi^d}{(\psi^d + b)^2}\psi^d \frac{d}{\psi} \qquad [2]$$

where: S_L is slope of the curve in the arithmetic soil suction coordinate system, and *b, c, d* are fitting parameters for the soil-water characteristic curve. The water content at zero soil suction is w_u .

The slope of the soil-water characteristic curve in the semi-logarithmic suction coordinate system can be calculated as follows:

$$S_{L\log} = \frac{dw(\psi)}{d(\log(\psi))} = \frac{dw(\psi)}{d\psi}\psi \ln(10) = S_L \psi \ln(10) \qquad [3]$$

where: S_{Llog} is slope of the curve in the semi-logarithmic soil suction coordinate system.

The soil suction value of a point on the *boundary wetting* curve that has the same water content as the inflection point on the *boundary drying* curve can be calculated as follows:

$$\psi_{I'} = \sqrt[d_w]{\frac{b_w (w_s^w - w_I)}{w_I - c_w}} \qquad [4]$$

Table 1. List of soils used in the study of hysteresis.

No	Soil name	Data[2]	Research authors
1	Adelaide Dune Sand	D, W	Talsma (1970)
2	Avondale Clay Loam	D, W	Watson et al. (1975)
3	Caribou Silt Loam	D, W	Topp (1971b)
4	Ceramic (No. 1)	I, D, W	Feng (1999)
5	Coarse Sand	D, W	Viaene et al. (1995)
6	Dune Sand	D, W	Gillham, Klute and Heermann (1976)
7	Glass Bead (4D)[1]	D, W	Nimmo and Miller (1986)
8	Glass Bead (20D)[1]	D, W	Nimmo and Miller (1986)
9	Glass Bead (35D)[1]	D, W	Nimmo and Miller (1986)
10	Glass Bead (50D)[1]	D, W	Nimmo and Miller (1986)
11	Glass Beads	D, W	Bomba and Miller (1967)
12	Glass Beads Ballotini (mixed)	D, W	Poulovassilis (1962)
13	Mixed Sand Fraction	D, W	Poulovassilis (1970b)
14	Mixed Sand Fraction	D, W	Poulovassilis and Ghamry (1978)
15	Molongo Sand	D, W	Talsma (1970)
16	Norfolk Sandy Loam (15D)[1]	D, W	Hopmans and Dane (1986)
17	Norfolk Sandy Loam (32D)[1]	D, W	Hopmans and Dane (1986)
18	Packed Sand	D, W	Vachaud and Thony (1971)
19	Plainfield Sand Loam (20D)[1]	D, W	Nimmo and Miller (1986)
20	Plainfield Sand Loam (35D)[1]	D, W	Nimmo and Miller (1986)
21	Plainfield Sand Loam (50D)[1]	D, W	Nimmo and Miller (1986)
22	Plano Silt Loam (4D)[1]	D, W	Nimmo and Miller (1986)
23	Plano Silt Loam (35D)[1]	D, W	Nimmo and Miller (1986)
24	Plano Silt Loam (50D)[1]	D, W	Nimmo and Miller (1986)
25	Porous body I (sand)	I, D, W	Poulovassilis (1970a)
26	Porous body II (sand)	I, D, W	Poulovassilis (1970a)
27	Rideau Clay Loam	D, W	Topp (1971b)
28	Rubicon Sandy Loam	D, W	Topp (1969)
29	Sand	I, D, W	Poulovassilis and Childs (1971)
30	Sand (No.17)	D, W	Perrens and Watson (1977)
31	Sand (R8)	D, W	Ayers and Watson (1977)
32	Wray Dune Sand	I, D, W	Mualem and Klute (1984)
33	Beaver Creek sand (Packed)	I, D, W	Pham, Fredlund and Barbour (2003)
34	Processed Silt (Packed)	I, D, W	Pham, Fredlund and Barbour (2003)

[1] Some soils are tested at different temperatures, the notations: 4D, 15D, 20D, 32D, 35D and 50D mean the temperatures of the samples are 4, 15, 20, 32, 35 and 50 degrees Celsius, respectively.

[2] I = Initial Drying Curve; D = Boundary Drying Curve; W = Boundary Wetting curve;

where: w_s^w is water content at zero suction on the *boundary wetting* curve, b_w, c_w, d_w are fitting parameters for the boundary wetting curve, and w_I is water content at the inflection point on the *boundary drying* curve.

The distance between the two boundary hysteresis curves in the normal soil suction coordinate system, D_S, can be expressed as follows:

$$D_S = \psi_I - \psi_{I'} \quad [5]$$

where: ψ_I is the soil suction at the inflection point on the *boundary drying* curve, and $\psi_{I'}$ is the soil suction at the same water content on the *boundary wetting* curve.

The distance between the two boundary hysteresis curves in the logarithmic soil suction coordinate system, $D_{S\log}$, can be calculated as follows:

$$D_{S\log} = \log(\psi_I) - \log(\psi_{I'}) \quad [6]$$

3. SOIL DATABASE

The soil data for this study was obtained from laboratory tests on two soils from Saskatchewan as well as a database of 32 soils studied by other researchers. The database includes the results from 6 different samples of glass beads, a ceramic stone, 13 sands, 6 sandy loams, 4 silt loams and 2 clay loams. Detail procedures for measuring these curves using Tempe cells can be found in Pham et al., (2003). The soil datasets were first collected in the form of volumetric water content and then converted to the gravimetric water content form. The literature references for the SWCCs are presented in Table 1. It should be noted that the soils listed in Table 1 likely show negligible volume change as a result of changes in soil suction.

4. STATISTICAL ANALYSIS OF THE HYSTERESIS CURVES

A statistical analysis was undertaken to investigate the relationships between features on the *initial drying* curve, the *boundary drying* curve and the *boundary wetting* curve for the 34 soils (Table 1).The slopes of the two boundary curves and the distance between the two boundary curves were calculated using both algebraic and semi-logarithmic scales coordinate systems. The calculated results are shown in Table 2. From the statistical analysis, some suggested values for the slope ratio and the distance between the boundary curves and the *boundary wetting* curve are presented in Table 3. Equation [7] was used to calculate the percentage deviation (*PD*) between the suggested values in Table 3 and the calculated values from the statistical analysis.

$$PD(\%) = \sum_{i=1}^{n_{soil}} Abs\left(\frac{v_{sg}(i) - v_c(i)}{v_c(i)}\right) \quad [7]$$

where: PD is percentage deviation of relationship value, n_{soil} is number of soil datasets in the soil category, $v_{sg}(i)$ is the suggested calculated relationship value (slope or distance) for soil *i*, and $v_c(i)$ is the calculated relationship value (slope or distance) for soil *i*.

There are many soil datasets that have information of the *initial drying* curve. However, calculations based on the collected datasets show that the volume of *air entrapped* in the soil, (i.e., the difference in volumetric water content between the *initial drying* curve and *boundary drying* curve at zero soil suction), is about 5% to 15% of the volume of water at saturation (Figure 1). This information agrees with the findings of Rogowski (1971). The results also show that the air entry value on the *boundary drying* curve is approximately equal to that of the *initial drying* curve.

5. SCALING METHOD

A simple scaling method is proposed for approximating the hysteretic SWCC for a soil. It is suggested that the Feng and Fredlund (1999) SWCC-fitting equation be used along with assumptions on the fixed relationships between the two boundary curves. The scaling method uses one of the three following curves; namely, the two boundary curves and *initial drying* curve, to estimate the other two curves. The *boundary drying* curve and the *initial drying* can be interchanged by changing the parameter that controls the water content at zero soil suction (i.e., w_u and w_s). Similarly, the *boundary drying* curve and the *boundary wetting* curve can be interchanged by changing the slope and stretching the entire curve to the left or to the right.

Let us assume that the *initial drying* curve was best-fit using the Feng and Fredlund (1999) equation (Equation 8). The water contents along the *boundary drying* curve can be calculated by changing the curve fitting parameter (i.e., w_u = 0.9w_s) that controls water content at zero soil suction and keeping the other curve fitting parameters the same (Equation 9).

$$w_I(\psi) = \frac{w_s b_i + c_i \psi^{d_i}}{b_i + \psi^{d_i}} \quad [8]$$

$$w_d(\psi) = \frac{w_u b_i + c_i \psi^{d_i}}{b_i + \psi^{d_i}} \qquad [9]$$

where: w_s is gravimetric water content at zero soil suction on the *initial drying* curve, w_u is gravimetric water content at zero soil suction on the *boundary drying* curve, and b_i, c_i, d_i are curve-fitting parameters of the *initial drying* curve. The water content along the *boundary wetting* curve can be described by using the distance between two boundary curves on a semi-logarithmic suction scale (D_{Slog}) and the slope ratio between the slope of the *boundary drying* curve and the slope of the *boundary wetting* curve on the semi-logarithmic suction coordinate (S_{LR}).

Let us assume that the water content along the *boundary wetting* curve can be described using the Feng and Fredlund (1999) equation (Equation 10). The curve-fitting parameters of the *boundary wetting* curve (b_w, c_w, d_w) can be calculated using the curve-fitting parameters of the *boundary drying* curve and the distance, and slope ratio between two boundary curves in equations (11), (12) and (13).

$$w_w(\psi) = \frac{w_u b_w + c_w \psi^{d_w}}{b_w + \psi^{d_w}} \qquad [10]$$

where: w_u is gravimetric water content at zero soil suction on the *boundary drying* curve, and b_w, c_w, d_w are curve-fitting parameters of the *boundary wetting* curve.

The curve-fitting parameters for the *boundary wetting* curve can be calculated as follow:

$$c_w = c_d \qquad [11]$$

$$b_w = \left(\frac{b_d}{\left(10^{D_{S\log}}\right)^{d_d}} \right)^{\frac{1}{S_{LR}}} \qquad [12]$$

$$d_w = d_d / S_{LR} \qquad [13]$$

Table 2. Distance between the two boundary hysteresis curves and the slopes of the boundary hysteresis curves on the arithmetic soil suction coordinate system

No.	Soil name	Arithmetic coordinate system			Semi-logarithmic soil suction coordinate system		
		Slope drying	Slope wetting	Distance (kPa)	Slope drying	Slope wetting	Distance (log-cycle)
1	Adelaide Dune Sand	2.93	2.97	2.24	48.62	33.75	0.16
2	Avondale Clay Loam	0.47	0.62	5.33	12.47	9.08	0.27
3	Caribou Silt Loam	0.57	0.87	9.46	17.99	8.71	0.51
4	Ceramic (No. 1)	0.21	0.27	13.70	18.68	16.28	0.19
5	Coarse Sand	4.06	2.64	1.70	39.83	16.67	0.22
6	Dune Sand	7.20	6.89	1.33	55.42	31.92	0.22
7	Glass Bead (4D)[1]	7.87	7.52	1.74	74.01	40.08	0.24
8	Glass Bead (20D)[1]	8.99	9.47	1.61	77.17	45.77	0.25
9	Glass Bead (35D)[1]	9.45	9.98	1.45	72.84	43.04	0.25
10	Glass Bead (50D)[1]	9.80	10.27	1.31	69.46	41.09	0.24
11	Glass Beads	16.87	9.58	1.21	142.24	54.05	0.17
12	Glass Beads Ballotini (mixed)	10.29	9.75	0.94	41.51	18.96	0.33
13	Mixed Sand Fraction	8.70	6.95	0.94	56.54	30.00	0.18
14	Mixed Sand Fraction	11.05	8.37	1.42	107.85	53.74	0.18
15	Molongo Sand	5.11	6.09	0.64	16.82	10.80	0.26
16	Norfolk Sandy Loam (15D)[1]	1.62	0.90	3.99	30.61	9.89	0.29
17	Norfolk Sandy Loam (32D)[1]	1.55	0.94	3.90	25.88	8.65	0.33
18	Packed Sand	6.32	12.50	2.74	78.85	82.49	0.31
19	Plainfield Sand Loam (20D)[1]	3.19	2.63	0.86	23.14	14.34	0.14
20	Plainfield Sand Loam (35D)[1]	4.19	3.25	0.75	27.18	15.79	0.13
21	Plainfield Sand Loam (50D)[1]	5.31	3.97	0.70	30.53	17.05	0.14
22	Plano Silt Loam (4D)[1]	0.23	0.48	6.15	4.36	2.86	0.59
23	Plano Silt Loam (35D)[1]	0.23	0.68	6.19	4.74	3.57	0.53
24	Plano Silt Loam (50D)[1]	0.22	0.57	7.59	4.99	3.38	0.61
25	Porous body I (sand)	7.40	7.14	0.75	41.88	28.64	0.16
26	Porous body II (sand)	8.40	6.50	0.94	55.17	28.83	0.17
27	Rideau Clay Loam	1.03	1.32	4.17	16.23	8.79	0.40
28	Rubicon Sandy Loam	2.40	1.88	6.15	51.92	15.06	0.46

29	Sand	10.49	7.41	1.61	101.12	42.84	0.21
30	Sand (No.17)	12.38	11.54	1.44	112.33	65.59	0.20
31	Sand (R8)	8.66	14.95	2.43	96.93	83.44	0.30
32	Wray Dune Sand	6.87	8.91	1.37	56.26	45.14	0.21
33	Beaver Creek sand (Packed)	1.34	2.78	2.98	16.90	16.59	0.34
34	Processed Silt (Packed)	0.17	0.34	23.99	15.95	14.83	0.37

where: b_d, c_d, d_d are curve-fitting parameters of the *boundary wetting* curve, S_{LR} is the ratio between the slope of the *boundary drying* curve and the *boundary wetting* curve on the semi-logarithmic suction scale, and D_{Slog} is the distance between the two boundary curves on the semi-logarithmic suction scale.

The application of the scaling method to engineering practice can be studied by applying the suggested procedure to three independent soils. These soils are: Aiken Clay Loam soil (Richards and Fireman, 1943), Chalks (Croney and Coleman, 1954), and silty sand (Croney and Coleman, 1954). These soils represent various types of soils presented in Table 1. The estimation for the *boundary drying* curve from the *initial drying* curve for the Aiken clay loam can be seen in Figure 2. The measured *wetting* curve for Aiken clay loam is not truly a *boundary wetting* curve because the soil was wetted from a water content higher than residual water content (i.e., the initial drying process did not dry the soil to the residual water content); therefore, no prediction of the *boundary wetting* curve is available. The estimation of the *boundary wetting* curves were presented for the Chalks and the silty sand. The grain-size distribution of soft chalks is similar to that of the silt or silt loam soil; therefore, the SWCC for the soft chalks is similar to that of silt or silt loam soil and the SWCC of the silty sand is in between that of sand and a silt loam soil. Values of S_{LR} and D_{Slog} for the soft chalks and the silty sand (Croney and Coleman, 1954) were selected and are presented in Table 4. Equations (11), (12), and (13) were used to calculate the curve-fitting parameters for the *boundary wetting* curve from the curve-fitting parameters of the *boundary drying* curve. The estimated *boundary wetting* curve for chalks and the silty sand (Croney and Coleman, 1954) are shown in Figures 3 and 4. The results are also presented in Table 4. The residual values between the estimated and the actual boundary curves, *RE*, were calculated using equation (14).

$$RE = \frac{\sum_{i=1}^{n} Abs\left(w_{es}(i) - w_{ms}(i)\right)}{n} \qquad [14]$$

where: w_{es} is water content on the estimated curve, and w_{ms} is water content on the measured (actual) curve, while n is number of measured point.

Figure 2 shows that the estimated *boundary drying* curve for the Aiken Clay Loam (Richards and FireMan, 1943) is quite close to the measured data. The water content at zero soil suction on the *boundary drying* curve should be approximately 85% of that at saturated conditions; however, if a value of 90% of water content at saturation is used, reasonable results are obtained. Figures 3 and 4 show estimated *boundary wetting* curves for chalks and silty sands that appear to be reasonable. The minimum R squared value is 0.94 and the maximum residual value of 1.44. It seems that, the estimation of the scaling method for the *boundary wetting* curve at high soil suctions is better than that at low soil suctions. It is suggested that the scaling method is reasonable for geotechnical engineering practice.

Table 3. Suggested value and the deviation of the slope ratio and the distance between the two boundary curves for different soil types.

Soil type	Ratio of slopes in arithmetic coordinate system		Ratio of slopes in semi-logarithmic coordinate system		Distance in semi-logarithmic coordinate system	
	Suggested value	Percentage deviation (%)	Suggested value	Percentage deviation (%)	Suggested value (*log-cycle*)	Percentage deviation (%)
Sand	1	18.7	2	22.6	0.2	16.9
Sandy loam	1.5	15.8	2.5	33.8	0.25	54.3
Silt loam and clay loam	0.5	32.0	1.5	11.9	0.5	24.8
Compacted silt or compacted sand	0.5	2.63	1	4.50	0.35	7.54

Table 4. Chosen values for the slope ratio and the distance between the two boundary curves for Chalks and silty sand and the calculated results.

Soil type		Soft chalks	Hard chalks	Silty sand (low initial density)	Silty sand (high initial density)
Ratio of slopes in semi-logarithmic coordinate system		1.5	1	2	1
Distance in semi-logarithmic coordinate system (*log-cycle*)		0.5	0.35	0.35	0.35
Curve fitting parameters for the boundary drying curve	b_d	6.19E+07	1.08E+08	9.20E+02	54.455
	c_d	1.262	0.897	2.216	5.632
	d_d	3.163	3.003	5.66	2.85
Calculated curve fitting parameters for the boundary wetting curve	b_w	1.38E+04	9.62E+06	3.10E+00	5.475
	c_w	1.262	0.897	2.216	5.632
	d_w	2.109	3.003	2.83	2.85
R squared between the estimated and the actual curve		0.975	0.941	0.941	0.983
Residual between the estimated and the actual curve (*RE*)		1.442	0.831	1.365	0.567

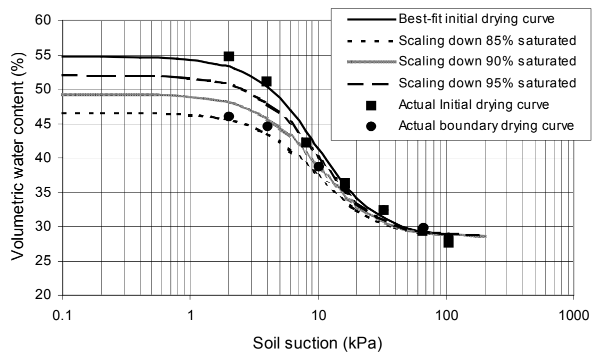

Figure 2. Estimated *boundary drying* curves for the Aiken Clay Loam (Richards and FireMan, 1943) obtained by scaling from the initial drying curve at 85%, 90% and 95% saturated water content.

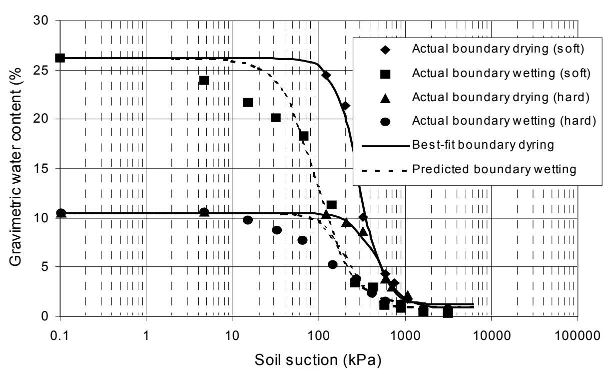

Figure 3. Estimated *boundary wetting* curve for the soft and hard chalks (Croney and Coleman, 1954).

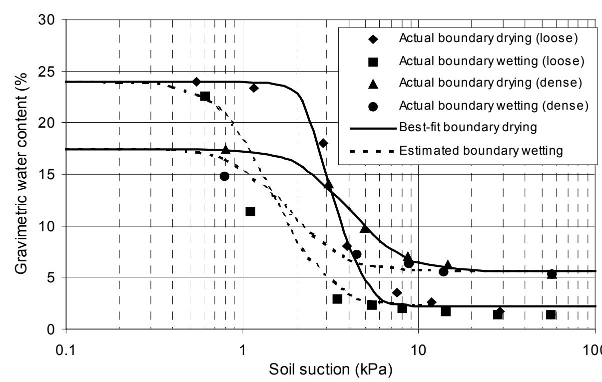

Figure 4. Estimated *boundary wetting* curve for the loose and dense silty sands (Croney and Coleman, 1954).

6. CONCLUSION

The following conclusions can be drawn from the study:

- For each soil type there seem to be a particular range of values for the distance and slope ratios between the two boundary curves. Values for the distance and slope ratio between the two boundary curves are presented, based on the limited soil datasets presented in this paper.
- Air entrapped in the soil during wetting appears to be about 5% to 15% of the volume of the soil. It is suggested that for engineering practice, a value of 10% can be assumed.
- It is suggested that the recommended scaling method be used to estimate other SWCC curves when only one of the following curve are known; namely, the *initial drying* curve; the *boundary drying* curve; the *boundary wetting* curve.
- The analysis is limited to the current with a database of 34 soils. Hopefully, more soils will be tested with time in order that the suggested values for the relationships between variables can be updated and applied to a wider range of soils.

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