Characteristics of water retention curves for unsaturated fractured rocks

Limin Zhang
Department of Civil Engineering, Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong.

D.G. Fredlund
Visiting Professor, Department of Civil Engineering, Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong.

Second Asian Conference on Unsaturated Soils
UNSAT-ASIA 2003, April 15-17, Osaka, Japan

ABSTRACT: Flow and transport in unsaturated fractured rocks have drawn a great attention over the past two decades. These issues play a key role in a number of engineering problems such as contaminants transport, nuclear waste disposal, oil extraction, and slope stability. One of the commonly used methods for simulating flows in a fractured rock involves the use of a composite dual-porosity continuum approach. In this approach, fluid storages in the matrix blocks and in the fracture network are considered separately and the fractured rock is characterized as two overlapping porous continua. If the rock mass is unsaturated, a water retention curve for the equivalent dual-porosity continuum must be known in order that the mass-conservation equation can be solved. This paper focuses on generating water retention curves based on the characteristics of the pores in the rock matrix and in the fracture network. The pore sizes of the rock matrix and the aperture of the fractures are assumed to follow lognormal distributions. The capillary law is used to link the pore size and matric suction and to derive the water retention curves for the rock matrix and the fractures. The water retention curve of the rock mass is then obtained from the composite pore-size distribution that is formed by weighing the respective distributions of the rock matrix and the fractures with their porosity values.

1 INTRODUCTION

Flow and transport in unsaturated fractured rocks have drawn a great deal of attention over the past two decades. These issues play a key role in a number of engineering problems such as contaminants transport, nuclear waste disposal, oil extraction, and slope stability. A fractured rock mass can be assumed to be a dual-porosity material consisting of the matrix phase and the fracture phase. Flows take place both through the matrix and the fractures. To simplify the flow problem, one of the commonly used methods for simulating flows in a fractured rock involves the use of a composite dual-porosity continuum approach. In this approach, fluid storages in the matrix blocks and in the fracture network are considered separately, and the fractured rock is characterized as two overlapping porous continua.

If the rock mass is unsaturated, a water retention curve and a permeability function for the equivalent continuum must be known in order that the mass-conservation equation can be solved. In reality, even in a single rock type with uniform fracture geometry, measurements of water retention curves representative of filed scale behavior are difficult to obtain (NRC 1996). Yet, the water retention curves for samples of rock matrix can be measured (Montazer and Wilson 1984; Peters and Klavetter 1988) and the water retention curves for fractures can be estimated (Wang and Narasimhan 1985). It would be preferred if the water retention curve of the equivalent continuum can be obtained by combining the curves for the respective material phases.

Fredlund and Xing (1994) established a theoretical basis for soil-water characteristic curves based on the pore-size distribution curve for the soil. The objective of this paper is to extend this method for deriving water retention curves for the rock matrix, the fractures, and the equivalent continuum of the fractured rock based on the pore-size distribution of the rock matrix and the distribution of the fracture aperture.

2 USE OF CAPILLARY LAW FOR DERIVING WATER RETENTION CURVE

Fredlund and Xing (1994) established a theoretical basis for soil-water characteristic curves based on the pore-size distribution curve for the soil. The soil was regarded as a set of interconnected pores that were randomly distributed. The pores are characterized by a pore radius r and described by a pore-size distribution function $f(r)$ with the following identity

$$\int_{R_{min}}^{R_{max}} f(r)dr = \theta_s \qquad (1)$$

where R_{max} and R_{min} are the maximum and minimum pore radii in the soil; θ_S is the volumetric water content when the soil is fully saturated; and $f(r)dr$ is the relative volume of pores of radius r to $r+dr$ in the total volume. When all the pores with radius less than or equal to R are filled with water, the volume water content $\theta(R)$ can be expressed as

$$\theta(R) = \int_{R_{min}}^{R} f(r)dr \qquad (2)$$

Equation (2) implies that water tends to fill small pores first. This assumption can be justified using the capillary law

$$(u_a - u_w) = \frac{2T}{r}\cos\alpha \quad or \quad r = \frac{C}{\psi} \qquad (3)$$

where u_a and u_w are pore-air and pore-water pressures; $\psi = (u_a - u_w)$ is suction; T is surface tention of water; α is angle of contact between water and soil particle; and $C = 2T\cos\alpha$. Based on this law, the suction is inversely proportional to the pore radius. Thus, smaller pores would be associated with a higher suction and would be filled earlier than larger pores.

According to Eq. (3), two extreme suction conditions can be defined

$$\psi_{max} = \frac{C}{R_{min}} \qquad \psi_{min} = \frac{C}{R_{max}} \qquad (4)$$

where ψ_{max} and ψ_{min} are the maximum and minimum suctions, respectively. In fact, ψ_{min} is the air-entry value of the soil (Fredlund and Xing 1994). Substituting the capillary law (i.e., Eq. (3)) into Eq. (2), the volumetric water content can be expressed as

$$\theta(\psi) = \int_{\psi_{max}}^{\psi} f\left(\frac{C}{s}\right) d\left(\frac{C}{s}\right) = \int_{\psi}^{\psi_{max}} f\left(\frac{C}{s}\right)\frac{C}{s^2}ds \qquad (5)$$

This equation can be written further into a general relationship between volumetric water content and suction (Fredlund and Xing 1994)

$$\theta(\psi) = \theta_s \int_{\psi}^{\psi_{max}} f(s)ds \qquad (6)$$

where $f(s)$ is the pore-size distribution as a function of suction.

Although Eq. (6) is initially intended for soil, it should be valid for any porous medium containing pores that can be described by the capillary law. Therefore, Eq. (6) may be extended to the water retention characteristics of a number of materials such as fractured rocks and structured soils. In the following sections, the water retention curves for the matrix blocks, the fractures and the fractured rock are derived based on the equation.

3 WATER RETENTION CURVE FOR ROCK MATRIX WITH KNOWN PORE-SIZE DISTRIBUTION

The pore-size distribution of rock matrix can be assumed to follow a modified lognormal distribution,

$$f_m(r) = \frac{\theta_{sm}}{\sqrt{2\pi}\,\zeta_m r}\exp\left[-\frac{1}{2}\left(\frac{\ln r - \lambda_m}{\zeta_m}\right)^2\right] \qquad (7)$$

where λ_m and ζ_m are the mean and standard deviation of $ln(r)$ of the rock matrix; θ_{sm} is the volumetric water content when the rock matrix is fully saturated and is used to retain the identity shown in Eq. (1). Accordingly, the water retention curve of the rock matrix can be written following Eqs. (2) and (4),

$$\theta(\psi) = \int_{R_{min}}^{R} \frac{\theta_{sm}}{\sqrt{2\pi}\,\zeta_m r}\exp\left[-\frac{1}{2}\left(\frac{\ln r - \lambda_m}{\zeta_m}\right)^2\right]dr$$

$$= \int_{\left(\frac{\ln R_{min} - \lambda_f}{\zeta_m}\right)}^{\left(\frac{\ln R - \lambda_m}{\zeta_m}\right)} \frac{\theta_{sm}}{\sqrt{2\pi}}\exp\left[-\frac{1}{2}y^2\right]dy$$

$$= \theta_{sm}\left[\Phi\left(\frac{\ln(\frac{C}{\psi}) - \lambda_m}{\zeta_m}\right) - \Phi\left(\frac{\ln(\frac{C}{\psi_{max}}) - \lambda_m}{\zeta_m}\right)\right] \qquad (8)$$

where Φ is the cumulative function of the standard normal distribution. When ψ_{max} approaches infinity, Eq. (8) reduces to

$$\theta(\psi) = \theta_{sm}\Phi\left(\frac{\ln(\frac{C}{\psi}) - \lambda_m}{\zeta_m}\right) \qquad (9)$$

4 WATER RETENTION CURVE FOR ROCK FRACTURES

If a rock fracture is not fully saturated, the water in the fracture, which is in contact with both walls of the fracture, will be held under tension. Fig. 1 shows a case with a planar fracture. If the pore-air is in connection with the air, the height of water column h_c the surface tension can hold is,

$$h_c = \frac{2T}{\gamma_w x}\cos\alpha \qquad (10)$$

where γ_w is the unit weight of water and x is half of the fracture aperture. Where the gauge air pressure u_a is not zero, an isolated water phase can exist in the fracture and the capillarity effect can be expressed as

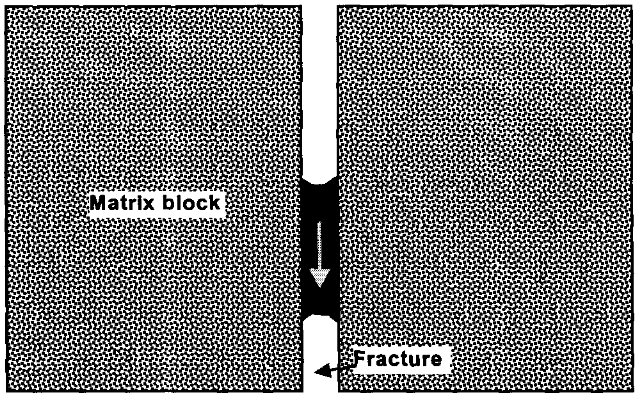

Figure 1. Capillary flow in an unsaturated fracture.

$$(u_a - u_w) = \frac{2T}{x}\cos\alpha \qquad (11)$$

This equation is similar to Eq. (3). Therefore, the general equation (Eq. (6)), with *f(s)* standing for the distribution of aperture size of rock fractures, is also valid for rock fractures.

The Poisson process has been adopted to describe the stochastic process of occurrence of a fracture (Priest and Hudson 1976). In this paper, however, this process is not considered. Instead, a volumetric content θ_{sf}, which is ratio of the volume of fractures to the total volume of the rock mass containing these fractures, is assumed. In addition, the distribution of the fracture aperture is also assumed to be lognormal

$$f_f(r) = \frac{\theta_{sf}}{\sqrt{2\pi}\,\zeta_f r}\exp\left[-\frac{1}{2}\left(\frac{\ln r - \lambda_f}{\zeta_f}\right)^2\right] \qquad (12)$$

where λ_f and ζ_f are the mean and standard deviation of *ln (r)* of the fracture aperture.

Given the distribution of the fracture aperture, the water retention curve of the fractures can be expressed as

$$\theta(\psi) = \theta_{sf}\left[\Phi\left(\frac{\ln(\frac{C}{\psi}) - \lambda_f}{\zeta_f}\right) - \Phi\left(\frac{\ln(\frac{C}{\psi_{max}}) - \lambda_f}{\zeta_f}\right)\right] \qquad (13)$$

When ψ_{max} approaches infinity, it reduces to

$$\theta(\psi) = \theta_{sf}\Phi\left(\frac{\ln(\frac{C}{\psi}) - \lambda_f}{\zeta_f}\right) \qquad (14)$$

5 WATER RETENTION CURVE FOR FRACTURED ROCK MASS

The pores in the fractured rock, which is a bi-modal material consisting of the matrix phase and the fracture phase, should be the sum of the pores in the two material phases. Therefore, the pore-size distribution of the rock mass can be expressed as

$$f(r) = f_m(r) + f_f(r) \qquad (15)$$

The distributions of the pore size of the rock matrix $f_m(r)$ and the aperture of the rock fracture $f_f(r)$ have been defined in Eqs. (7) and (12), respectively. Note that the meaning of θ_{sm} attached with $f_m(r)$ in Eq. (15) is slightly different from that in Eq. (7). In Eq. (7), θ_{sm} is the porosity of rock matrix; whereas in Eq. (15), it is the ratio of the pore volume inside the rock matrix to the total volume of the rock mass. It can be shown that $f(r)$, $f_f(r)$, and $f_m(r)$ all have the identity shown in Eq. (1).

The suction inside a fracture may not be equal to the suction inside the matrix blocks near the fracture at a non-equilibrium condition. However, the matrix and the fracture are both assumed to be continua in this paper thus the suction over the two overlapping porous continua can be assumed to be the same. Using Eqs. (2) and (15) and considering that water storages in the matrix blocks and in the fracture network constitute the total water storage, the volumetric water content of the fractured rock is

$$\theta(\psi) = \int_{R_{min}}^{R}\left[f_m(r) + f_f(r)\right]dr$$

$$= \int_{R_{min}}^{R} f_m(r)dr + \int_{R_{min}}^{R} f_f(r)dr$$

$$= \theta_{sm}\left[\Phi\left(\frac{\ln(\frac{C}{\psi}) - \lambda_m}{\zeta_m}\right) - \Phi\left(\frac{\ln(\frac{C}{\psi_{max}}) - \lambda_m}{\zeta_m}\right)\right] +$$

$$\theta_{sf}\left[\Phi\left(\frac{\ln(\frac{C}{\psi}) - \lambda_f}{\zeta_f}\right) - \Phi\left(\frac{\ln(\frac{C}{\psi_{max}}) - \lambda_f}{\zeta_f}\right)\right] \qquad (16)$$

When the suction approaches infinity, this equation reduces to

$$\theta(\psi) = \theta_{sm}\Phi\left(\frac{\ln(\frac{C}{\psi}) - \lambda_m}{\zeta_m}\right) + \theta_{sf}\Phi\left(\frac{\ln(\frac{C}{\psi}) - \lambda_f}{\zeta_f}\right) \qquad (17)$$

Equation (17) shows that the water retention curve of the rock mass is the sum of the water retention curves of the two material phases weighted by their porosities.

It can be shown that the volumetric water content of a fully saturated rock mass is,

$$\theta_s = \theta_{sm} + \theta_{sf} \qquad (18)$$

In fact, θ_s is total porosity of the rock mass, i.e., the ratio of the total pore volume to the total volume of the rock mass.

6 EXAMPLE

Let us consider a rock sample with a matrix porosity 0.3 and a fracture porosity 0.05 (i.e., the rock sample is highly fractured). The pore-size distributions of the rock matrix and the fractures are both assumed lognormal, as shown in Fig. 2. In particular, the mean diameter and the standard deviation of the matrix pores are 1.0 μm and 0.5 μm, respectively; and those of the fracture aperture are 25 μm and 3.75 μm, respectively. Since the predominant sizes of the matrix pores and the fractures are distinctly different, the two distributions have little or no overlap.

Based on Eqs. (9), (14) and (17), the water retention curves for the rock matrix, the fractures and the fractured rock consisting of the rock matrix and the fractures can be obtained and are plotted in Fig. 3. In the calculations, the surface tension of water at 20°C, T = 0.07275 N/m, and a contact angle of 0° are used. The curves can also be converted to saturation curves as shown in Fig. 4. It can be observed that the water retention and saturation curves follow more closely the curves for the rock matrix after the residual suction of the fracture is exceeded, because the fractures with large apertures tend to desaturate before the pores in the matrix blocks. Near saturation, the connected pathways across the fracture develop and the role of the fracture increases.

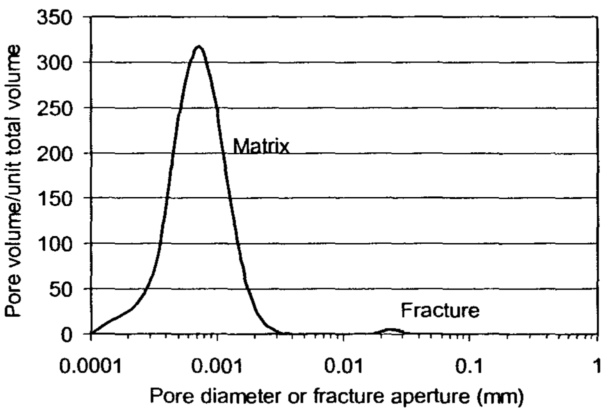

Figure 2. Pore-size distributions of the rock matrix and the fracture.

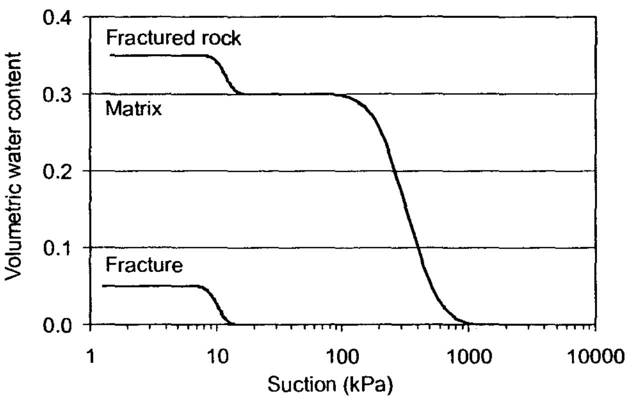

Figure 3. Water retention curves of the rock matrix, the fracture, and the fractured rock.

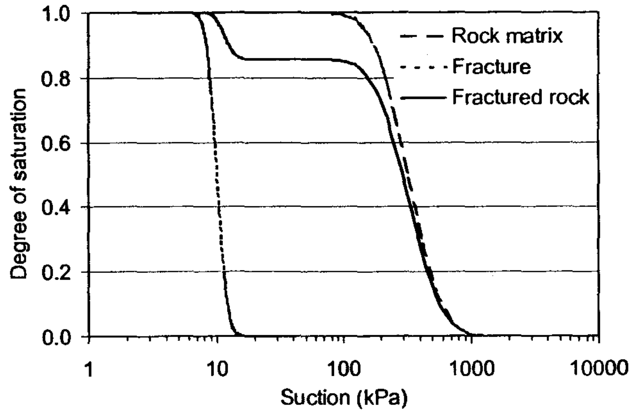

Figure 4. Saturation curves for the rock matrix, the fracture, and the fractured rock.

In reality, the fracture porosity might be far less than 0.05. Therefore, the water retention curve of the rock mass is more influenced by that of the rock matrix. However, this does not suggest the fractures play a minor role in flows through the rock mass. Instead, the fracture system can provide major flow channels because their hydraulic conductivity at full saturation is a few orders of magnitude larger than that of the rock matrix. Verification tests are needed to confirm the proposed methodology.

7 CONCLUSIONS

A method is proposed for deriving the water retention curves of fractures rocks based on the capillary law and the characteristics of the pores in the rock matrix and the fracture network. The method shows promise in that it captures the water retention ability and contributions from the matrix phase and from the fractures at different suction levels.

8 REFERENCES

Fredlund, D.G., and Xing, A.Q. 1994. Equations for the soil-water characteristic curve. *Canadian Geotechnical Journal* 31: 521-532.

Montazer, P., and Wilson, W.E. 1984. *Conceptual hydrological model of flow in the unsaturated zone, Yucca Mountain, Nevada.* USGS Water-Resources Investigation Report 84-4345, 55pp.

National Research Council. 1996. *Rock Fractures and Fluid Flow-Contemporary Understanding and Applications.* National Academy Press, Washington D.C.

Peters, R.P, and Klavetter, E.A. 1988. A continuum model for water movement in an unsaturated fractured rock mass. *Water Resources Research* 24(3): 416-430.

Priest, S.D., and Hudson, J. 1976. Discontinuity spacings in rock. *International Journal of Rock Mechanics and Mining Science* 13: 135-148.

Wang, J.S.Y., and Narasimhan, T.N. 1985. Hydrologic mechanisms governing fluid flow in a partially saturated, fractured porous medium. *Water Resources Research* 20(12): 1861-1874.