

Assessment of unsaturated soil properties for seepage modeling through tailings and mine wastes

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ABSTRACT: The most common modeling performed on tailings and mine wastes involves seepage and the movements of contaminants. Most emphasis in geotechnical engineering is related to determining the saturated soil properties of soils; however, the unsaturated soil properties are generally required when considering tailings and mine wastes. This paper focuses on means whereby unsaturated soil property functions can be determined for seepage modeling purposes. It is proposed the assessment of unsaturated soil properties be divided into a hierarchical system involving four *Levels*. The *Levels* of assessment will focus primarily on seepage related problems involving tailings and mine wastes. The intent is to illustrate that there should be no situation where it is not possible to obtain an assessment of the unsaturated soil property functions required for modeling purposes.

1 INTRODUCTION

Modeling studies are a part of the design and management of tailings or the waste rock associated with mining operations. The accuracy of the results of modeling depends upon the ability of the modeler to input reasonable soil parameters into the model. Both the tailings and the mine wastes generally have a portion of the profile that is unsaturated and therefore it is necessary to be able to determine suitable unsaturated soil property functions for seepage and contaminant transport. It might initially seem to be a formidable task to determine unsaturated soil property functions such as the permeability function and the water storage function for the project at-hand. However, geotechnical engineering practice has increasingly endorsed the use of indirect methods and estimation techniques for the determination of unsaturated soil properties (Fredlund, 2000, 2002). It is important to provide a contextual framework or a protocol for the assessment of unsaturated soil property functions because of the wide variety of estimation techniques that have become prevalent in geotechnical engineering practice.

The objective of this paper is to investigate the use of a set of hierarchical protocol *Levels* for the assessment of unsaturated soil properties for geotechnical engineering practice. The engineering protocols are a response to research developments in unsaturated soil mechanics over the past few decades. The intent is to provide geotechnical engineers with more theoretically-based, analytical tools in order that unsaturated soil mechanics problems can be better addressed in response to specific site conditions. The scope of the paper is limited to the assessment of soil properties for seepage problems involving tailings and mine wastes.

2 UNSATURATED SOIL PROPERTY FUNCTIONS

The quantification of unsaturated soil property functions, more than any other single factor, provides the key to the implementation of unsaturated soil mechanics in geotechnical engineering practice (Fredlund, 2000). The main challenge before engineers involves the determination of economically viable

procedures for the assessment of unsaturated soil property functions.

The general types of unsaturated soil property functions that are required for modeling seepage through tailings and mine waste materials are the hydraulic conductivity or the permeability function, and the water storage function for each of the materials involved. It may also be necessary to model the seepage behavior of the surrounding or underlying soil strata in order to fully understand the water balance as it relates to the mining operation. It is necessary to have an appreciation of the *saturated* soil properties and the *unsaturated* soil properties. The *unsaturated* soil properties take on the form of a nonlinear equation and are therefore generally referred to as “functions”.

The soil–water characteristic curve has played a dominant role in the study of unsaturated soils in disciplines such as soil science, soil physics, agronomy and agriculture. The soil–water characteristic curve is a relationship between the amount of water in the soil and soil suction.

2.1 Measurement of the soil–water characteristic curve

A variety of pressure plate apparatuses can be used to measure the soil–water characteristic curve in the range up to 1500 kPa. Figure 1 shows a pressure plate apparatus recently developed by GCTS¹. This apparatus is capable of applying matric suctions up to 1500 kPa with the measurement of water content and volume change.

2.2 Equations for the soil–water characteristic curve and the unsaturated soil property functions

A series of empirical equations have been proposed to best-fit to data for the soil–water characteristic curve (Sillers, 1997, Leong and Rahardjo, 1997a; van Guenuchten, 1980). Most of the equations are asymptotic to horizontal lines in the low soil suction range as well as soil suctions beyond residual conditions. As such, these equations are not forced through zero water content at 1,000,000 kPa of suction. A correction factor has been applied to the typically sigmoidal mathematical equation by Fredlund and Xing (1994) to give an equation that can best-fit the entire soil suction range and pass through a soil suction of 1,000,000 kPa at a water content of zero.

All of the proposed equations appear to provide a reasonable fit of soil–water characteristic data in the low and intermediate soil suction ranges (Leong and

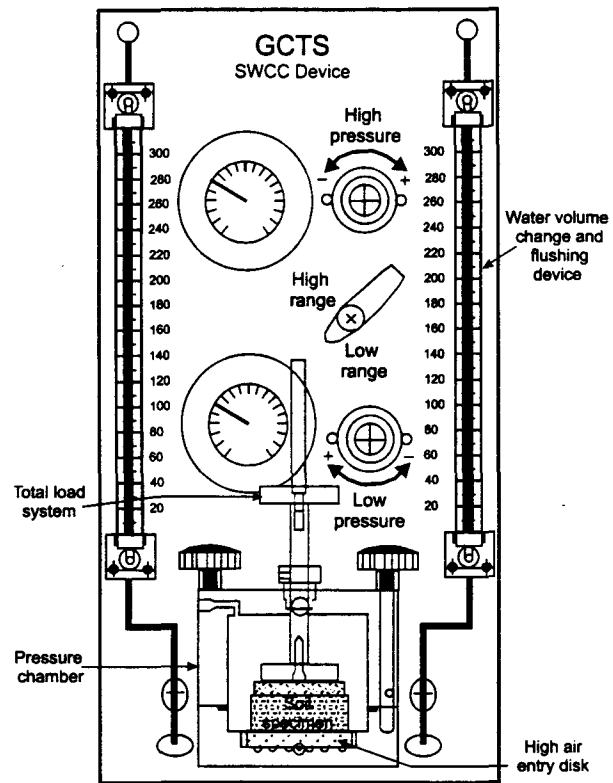


Figure 1. Pressure plate developed by GCTS for the measurement of the soil–water characteristic curve.

Rahardjo, 1997a). In all cases, the first soil fitting parameter bears a relationship to the air entry value of the soil and usually refers to the inflection point along the curve. The second soil fitting parameter corresponds to the slope of the straight line portion (i.e., at the inflection point) of the main desorption (or adsorption) portion of the soil–water characteristic curve.

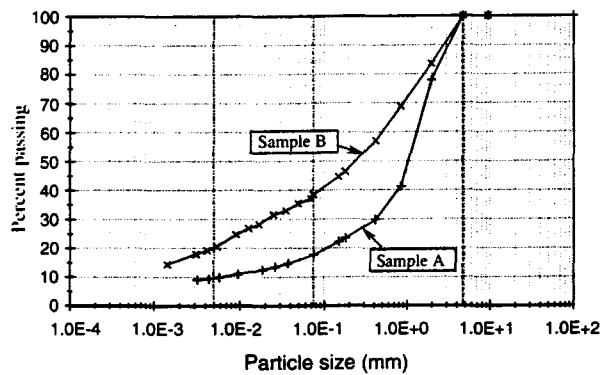
Several unsaturated hydraulic conductivity functions have been proposed and numerous studies have been done to assess the function that best-fits measured laboratory data on unsaturated soils (Fredlund and Xing, 1994; Leong and Rahardjo, 1997b). Common to all methods is the existence of a mathematical relationship between hydraulic conductivity and the soil–water characteristic curve.

The water storage function is usually taken as the change in water content with respect to a change in soil suction and is consequently the result of differentiating the SWCC. The assumption is generally made that the change in soil suction has the dominant influence when solving unsaturated soils problems.

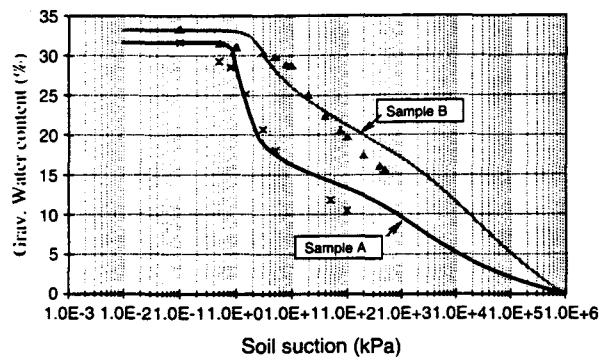
Figure 2 shows the grain-size distribution curves and the soil–water characteristic curves for two tailings

¹The SWCC Pressure Plate apparatus was developed and manufactured by GCTS, Geotechnical Consulting and Testing Systems, Tempe, Arizona, USA.

²SoilVision is a proprietary software program developed and maintained by SoilVision Systems Ltd., Saskatoon, SK., Canada.



2a) Grain size for waste rocks samples A and B



2b) SWCC for waste rocks samples A and B

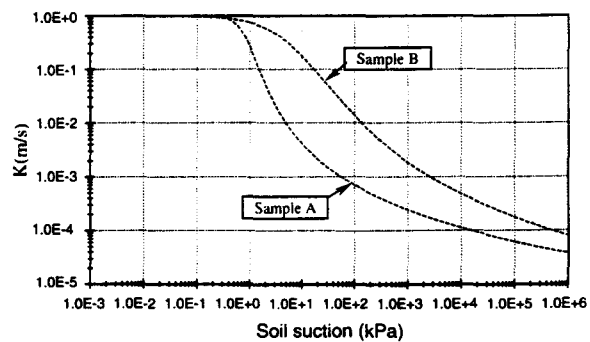
Figure 2. Classification and soil-water characteristic curves for two waste rock samples.

materials retrieved from the SoilVision² knowledge-based software (Fredlund, 1996). The hydraulic conductivity (or permeability) function for each of the tailings materials is shown on Figure 3a while the water storage function is shown on Figure 3b. It is necessary to have a measured (or estimated) value for the saturated coefficient of permeability in order to compute the permeability function (Fredlund et al., 1994).

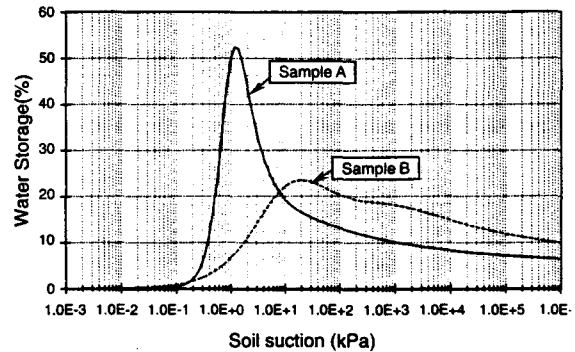
3 HIERARCHICAL APPROACH TO UNSATURATED SOIL PROPERTY ASSESSMENT

The determination of unsaturated soil property functions requires a contextual framework that categorizes various methods available according to accuracy and cost of implementation. The challenge is to find measurement, estimation and approximation procedures that can result in an adequate engineering simulation of the unsaturated portion of tailings, mine wastes and the unsaturated portion of the soil profile.

One of several general approaches can be taken in the determination of unsaturated soil property functions as shown in Figure 4. Laboratory tests can be used as a direct measure of the unsaturated soil property



3a) Permeability function for samples A and B



3b) Water storage function for samples A and B

Figure 3. Unsaturated soil functions for waste rock samples A and B.

functions required when solving unsaturated soil mechanics problems. However, for many mining related problems it may be sufficient to perform an indirect laboratory test (i.e., pressure plate test) in order to obtain data that can be used to estimate the unsaturated soil property functions. It is also possible to obtain an indication of the SWCC from classification soil properties, correlations or from "mining" the results of past soil test (Fredlund et al., 1996).

Measurement of the soil-water characteristic curve for a soil can be used as an indirect laboratory test to compute *unsaturated* soil property functions. The soil-water characteristic curve can then be used in conjunction with the *saturated* soil properties to estimate the *unsaturated* soil property functions with an acceptable level of accuracy for many engineering projects.

Figure 5 illustrates the use of a classification test (i.e., grain-size distribution curve) for the prediction of the desired unsaturated soil property function. A grain-size distribution curve can be used to estimate the soil-water characteristic curve that is then used to determine the *unsaturated* soil property function (Fredlund et al., 1997). There may be a reduction in the accuracy of the estimated unsaturated soil property function when using this procedure. The engineer must assess whether or not the approximated unsaturated soil property function is satisfactory for the analyses to be performed.

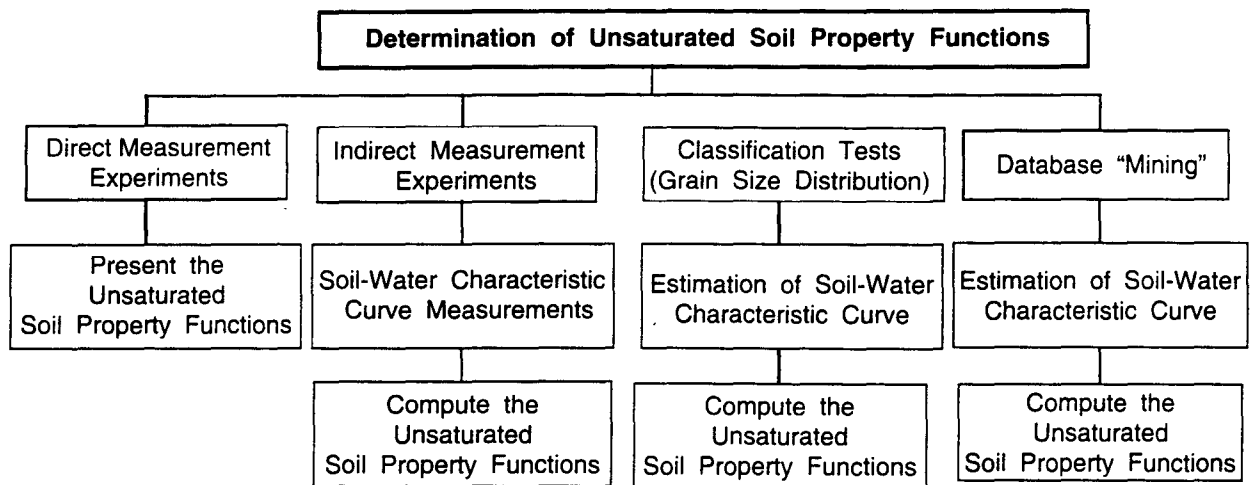
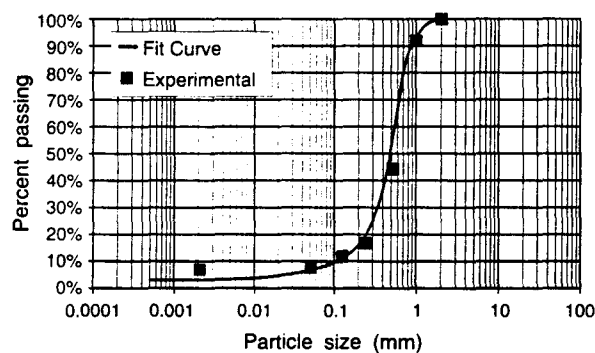
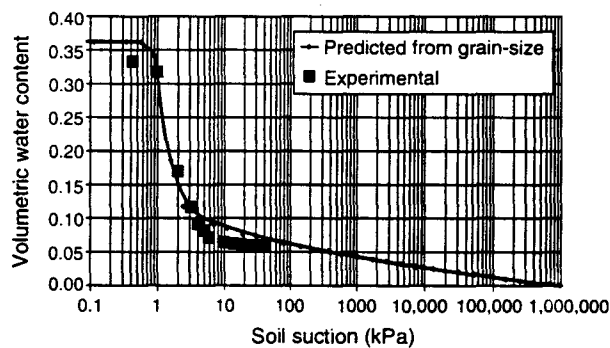


Figure 4. Some approaches that can be used to determine the unsaturated soil property functions.



a. Grain-size distribution for sand



b. Comparison between experimental and predicted soil-water characteristic curves for a sand

Figure 5. Illustration of the use of a grain-size distribution curve for the estimation of the soil-water characteristic curve

4 DETAILS OF HIERARCHICAL LEVELS OF UNSATURATED SOIL PROPERTY ASSESSMENT

The implementation of unsaturated soil mechanics within the classical, historical paradigm involves direct measurement of unsaturated soil properties while using sophisticated soil testing equipment with soil

suction measurements (or controls). The tests are complex and time-consuming. Consequently, other approaches are necessary for determining unsaturated soil property functions the unsaturated soils portions are going to be modeled. There has gradually been an emergence and an acceptance of more empirical, approximation procedure for the determination of unsaturated soil property functions. Common to all of these procedures is the use of the soil-water characteristic curve, SWCC, along with saturated soil parameters for the estimation of the unsaturated soil properties (Fredlund, 1995). These indirect measurements and estimation procedures open the way for a hierarchical approach to the application of unsaturated soil mechanics into geotechnical engineering practice.

The keywords used to describe the categories or *Levels* of the hierarchical approach applied to the assessment of unsaturated soil properties can be listed as follows.

Level 1: DIRECT measurement of unsaturated soil properties. *Level 1 (L1)* is intended for high profile designs involving significant cost or projects with profound implications in case of failure. This level would also be appropriate for research applications and large, costly projects of high risk, or where there could be potential cost savings associated with a *Level 1* investigation. At this *Level*, advanced unsaturated soil testing equipment and procedures are used, such as triaxial and hollow cylinder devices capable of following a variety of soil suction and total stress paths along with the measurement of total and water volume changes.

Level 2: INDIRECT measurements of a soil property (e.g., soil-water characteristic curve) can be used to compute the unsaturated soil properties. *Level 2 (L2)* is intended to represent a more commonly used, but economically viable approach, in which some

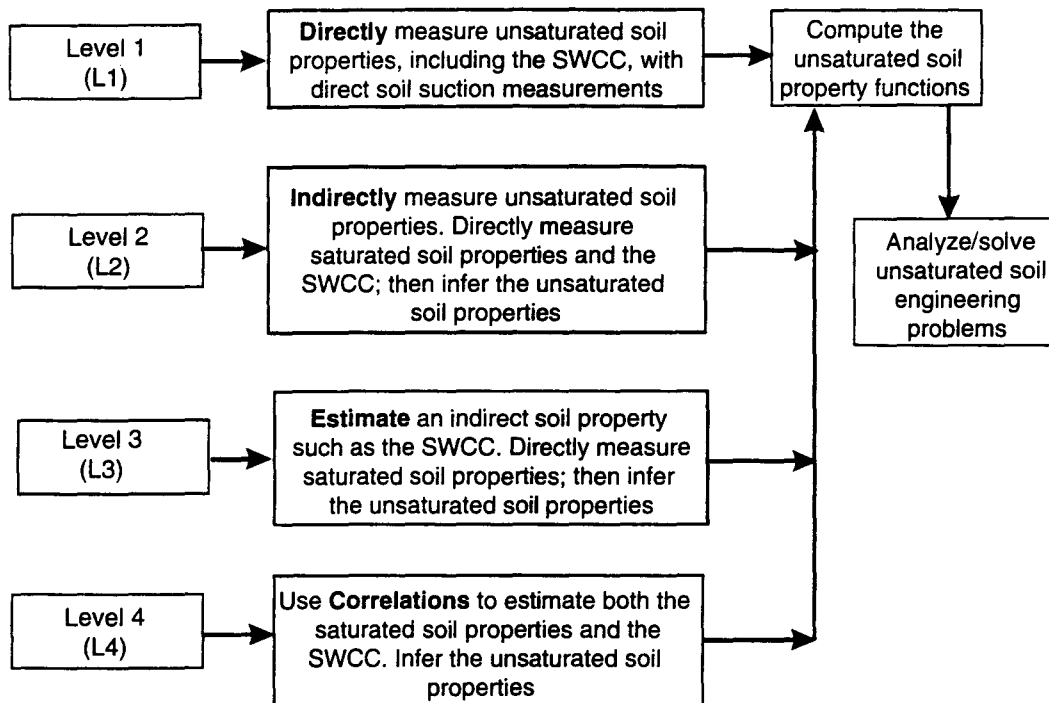


Figure 6. Flow-chart for hierarchical approach.

laboratory (or *in situ*) testing is used in conjunction with some degree of empiricism. At this *Level*, conventional laboratory tests are performed on saturated test specimens while soil suction measurements may be associated with the determination of the soil–water characteristic curve, SWCC.

Level 3: ESTIMATION of the indirect soil property (e.g., Soil–Water Characteristic Curve), that can be used to compute the unsaturated soil properties. *Level 3 (L3)* is intended to involve index properties and correlations with index properties to estimate the soil–water characteristic curve, SWCC. The required information may also be derived from a database of available curves or from a database of previous test results. It is assumed that the saturated soil properties are known or measured. This *Level* may represent the most common scenario used for preliminary studies on tailings and mine wastes engineering projects.

Level 4: CORRELATIONS can be used to allow the estimation of the unsaturated soil property functions or an indirect soil property (e.g., SWCC). *Level 4 (L4)* is intended to represent the lowest level of sophistication, and is expected to be appropriate only for routine structures of low risk or for preliminary design purposes. At this *Level*, index soil properties are measured and all other needed functions and values are derived from correlations with index properties such as plasticity or grain-size distribution.

The development of a hierarchical approach to solving unsaturated soil, geotechnical or geo-environmental problems is not intended to encourage substandard or

low-level practice, but rather to provide a framework that will allow the rational implementation of unsaturated soil theories at an appropriate level of sophistication (or *Levels* of difficulty). Figure 6 illustrates the hierarchical approach in flow-chart form for the determination of unsaturated soil properties required for solving seepage and contaminant transport engineering problems.

The hierarchical approach should *not* be viewed as being fixed and final. Rather, it is anticipated that changes and refinements will need to be made to the four *Levels* in response to new developments in geotechnical engineering. There may also be other measurement and estimation techniques that need to be given consideration. However, it would now appear that presently proposed procedures for unsaturated soil property assessment fit well within the four hierarchical *Levels*. The proposed hierarchical *Levels* should form the contextual framework for moving geotechnical engineering towards routine usage in geotechnical engineering practice.

5 IMPLEMENTATION UNDER LEVEL, L1

Implementation under *Level 1* involves testing the unsaturated soil in the laboratory (or *in situ*) in a manner that has become known as the classic paradigm for saturated soil mechanics. The emphasis is on the direct measurement of the entire unsaturated soil property function in terms of the complete stress

state. Laboratory testing equipment and suitable testing procedures have been developed for the measurement of unsaturated soil property functions. In each case, the laboratory equipment and testing procedures have taken on the character of being an extension of the procedures that have become generally accepted for testing saturated soils.

Usually 5 to 10 points need to be measured in order to define an unsaturated soil property function. Consequently, the laboratory testing period is substantially increased; commonly by about one order of magnitude. This means that a test on a *saturated soil* that might cost \$1000 will likely now cost \$10,000 on an *unsaturated soil*. These exorbitant costs will prove to be unacceptable for many routine engineering projects and can only be acceptable at the research level or in situations where understanding the behavior of the unsaturated soil is of great importance.

In reality, each of the unsaturated soil property functions are a function of one primary stress state variable and one secondary stress state variable (Fredlund and Rahardjo, 1993). It is suggested that the measurement of an unsaturated soil property function in terms of either a primary stress state variable or the primary and secondary stress state variables qualifies as a *Level 1* implementation. As an example, the hydraulic conductivity function for an unsaturated soil is a function of both the net normal stress and matric suction (Fredlund et al., 1994). However, the hydraulic conductivity function will generally be a primary function of matric suction and a secondary function of net normal stress. Measuring the hydraulic conductivity function in terms of either one or two of the stress state variables qualifies for a *Level 1* implementation.

6 IMPLEMENTATION UNDER LEVEL, L2

Implementation under *Level 2* suggests that the unsaturated soil property function can be computed with sufficient accuracy after a laboratory or *in situ* measurement has been made of a soil property other than that of the unsaturated soil property function of direct concern. The most common situation involves the measurement of the soil–water characteristic curve that can subsequently be used to compute the required unsaturated soil property function(s). In other words, a laboratory measurement is still undertaken but additional theory must be added to obtain the required unsaturated soil property functions.

The SWCC relates water content to matric and total suction over the range from zero to 1,000,000 kPa. A graph of laboratory data generally shows a plot of matric suction from zero to 1500 kPa and total suction for the remainder of the soil suction range. Pressure plate apparatuses of a variety of designs are used to apply matric suctions up to 1500 kPa (e.g., GCTS

Pressure Plate). The equilibrium water content of small soil specimens placed over salt solutions of varying concentrations, in a vacuum desiccators, is used for the total suction range.

The SWCC is one of the volume–mass constitutive relationships and is a function of two independent stress state variables. The SWCC also exhibits hysteresis (i.e., a wetting and a drying curve). Although the complete SWCC is complex, it is generally deemed adequate to measure the drying curve and this is sufficient to qualify as a *Level 2* implementation.

There may also be indirect laboratory measurements other than the SWCC that can be used as part of a *Level 2* implementation. It is also assumed that the *saturated* soil properties are known.

7 IMPLEMENTATION UNDER LEVEL, L3

Implementation under *Level 3* requires some means of estimating a representative soil–water characteristic curve. The use of operations research techniques holds promise in the search for a suitable soil–water characteristic curve. A large volume of soil–water characteristic curve data has been collected in several disciplines (e.g., soil science, agronomy, agriculture and engineering) and in many countries. A compiled database can be used to select an approximate soil–water characteristic curve. The grain-size distribution curves for a soil can be matched to other grain-size curves in order to select an approximate soil–water characteristic curve. It is also possible to use the classification of a soil when searching for an appropriate SWCC (Fredlund et al., 1996) (e.g., SoilVision).

There are two primary approaches that can be used to obtain a soil–water characteristic curve from a database; namely, there can be a match of the soil classification or the grain-size distribution curve. The resulting estimated SWCC can subsequently be used for the determination of unsaturated soil property functions. The classification soil properties and previously measured soil–water characteristic curves can be used in conjunction with a knowledge-based database to assist the user in arriving at a reasonable soil–water characteristic curve (Fredlund et al., 1996).

It is also possible to compare a measured soil–water characteristic curve with soil–water characteristic curves already in the database for a similar soil. The measured soil–water characteristic curve can be used either to compute unsaturated soil property functions or to select unsaturated soil property functions already in the database.

As mentioned above, it is possible to match measured classification properties (i.e., grain-size distribution curves) with classification properties already in the database. Once one or more similar soils have been found, corresponding soil–water characteristic

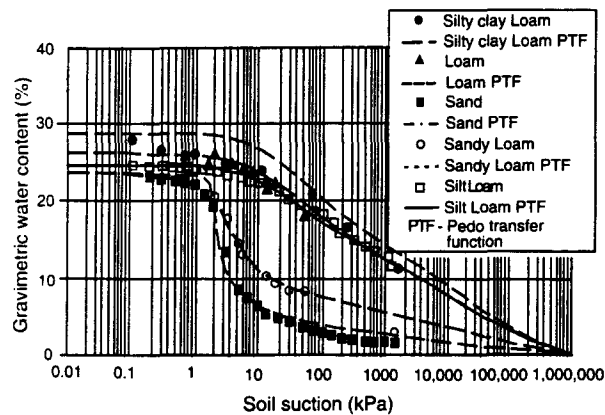


Figure 7. A series of soil–water characteristic curves estimated from a variety of grain-size distribution curves (Fredlund et al., 2002)

curves can be retrieved from the database. The soil–water characteristic curve data can be used to compute suitable unsaturated soil property functions or existing unsaturated soil property functions can be retrieved from the database.

It is also possible to make direct use of the measured grain-size distribution curve for the estimation of the SWCC. The grain-size distribution curve for a given soil can be compared to grain-size curves already in the database. Soil–water characteristic curves can then be computed from the grain size curves and compared to soil–water characteristic curves in the database. An engineering decision must be made regarding a reasonable soil–water characteristic curve and then the unsaturated soil property functions can be computed. In general, each of the above procedures becomes increasingly less accurate for the estimation of unsaturated soil property functions.

The last suggested procedure involves the use of physico-empirical SWCC models based on the grain-size distribution curve. There are a number of approaches that have been proposed (Fredlund et al. 1997, 2002). A mathematical equation similar to that used for describing the soil–water characteristic curve has been modified to fit the grain-size distribution curve (Fredlund et al., 1997). It is possible to estimate the soil–water characteristic curve from a grain-size distribution curve provided the procedure is “trained” with the assistance of a knowledge base. Figure 7 shows a series of SWCCs that have been computed from the grain-size distribution curves for several soil types. The results are encouraging for sands and silts, but more research is required when using this procedure for structured and clayey soils.

8 IMPLEMENTATION UNDER LEVEL, L4

Implementation under *Level 4* involves the use of the classification properties of a soil to estimate

unsaturated soil property functions. Several attempts have been made to correlate the fitting parameters for a soil–water characteristic curve with the plasticity and/or grain size distribution classification properties of a soil (Ahuja et al., 1985). These correlations are based on limited data, but further studies may prove these relationships to be of value in engineering practice.

The general procedure is to use of the soil classification properties to obtain an indication of the soil–water characteristic curve. The SWCC is then used to empirically compute the unsaturated soil property functions with the assistance of the saturated soil properties. The soil classification property generally used for the estimation of the SWCC is the grain-size distribution curve but Atterberg limits could also be used as well as density or other initial volume–mass soil properties. These functions are called Pedo-Transfer functions.

There are two procedures whereby soil classification properties have been used to provide an indication of the SWCC for a soil. First, statistical estimates of water contents corresponding to various soil suctions can be obtained based on the analysis of soils in a database. Attempts have been made to correlate the fitting parameters for a soil–water characteristic curve with the plasticity and/or grain-size distribution classification properties of a soil. These correlations are based on limited data.

Second, the soil parameters corresponding to various SWCC models can be estimated based on correlation from previously tested soils.

Zapata (1999) undertook a study related to the reliability and reproducibility of commercially available soil–water characteristic curve data. It was noted that less than 20% of commercial geotechnical laboratories performed suction measurements or applied a known suction to the soil. It was also noted that there was a high degree of variability in the experimental suction measurements and it was observed that even experienced researchers had difficulties in measuring a unique SWCC for a soil (Zapata et al., 2000). In part, the high variability was related to a lack of uniformity in the laboratory testing procedures. Given the high variability that is still present in the experimental determination of the SWCC, it was proposed that a model based on statistical correlations of simple and easy to measure soil index properties with the fitting parameters of the SWCC function defined by Fredlund and Xing (1994). A database characterizing approximately 190 soils was assembled from research papers and a knowledge-database developed by SoilVision Systems². The soils were divided into two categories; namely, soils having a Plasticity Index (*PI*) greater than zero and soils having a *PI* equal to zero. Data for approximately 70 soils with *PI* values greater than zero and 120 soils with *PI* values equal to zero were collected. The resulting average SWCCs are summarized in

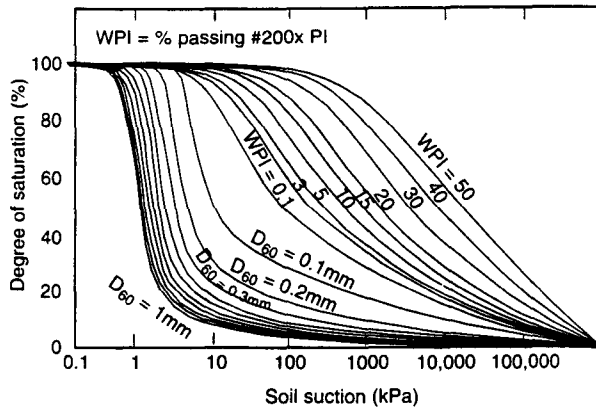


Figure 8. Correlation of the classification properties of a soil with previously measured soil–water characteristic curves (Zapata, 1999)

Figure 8. The results of the above study produce a correlation type model that fits a *Level 4* type assessment for unsaturated soil property functions.

It is also possible to have a direct correlation between classification soil properties (and possibly some other volume–mass property) and the desired unsaturated soil property.

9 USE OF THE UNSATURATED SOIL PROPERTY FUNCTIONS IN MODELING

Most geotechnical engineering problems such as those related to understanding the behavior of tailings and mine wastes can be reduced to the solution of a series of partial differential equations. This is true for saturated–unsaturated seepage and contaminant transport problems as well as thermal and stress analyses. The concept of a computer problem solving environment, PSE, for geotechnical engineering has led to the development of complimentary software packages such as SVFlux for seepage analysis (Fredlund, 2000), ChemFlux for contaminant transport analysis (Fredlund, 2001), as well as other computer software for other applications in geotechnical engineering. In all cases, the solution of a partial differential equation, PDE, forms the basic algorithm that needs to be solved. Each partial differential equation contains material properties that can be input as constants or mathematical functions (e.g., unsaturated soil property functions). In other words, in order to use partial differential equation based software, it is necessary to be able to either estimate or measure the material properties for all materials involved.

10 SUMMARY ON THE HIERARCHICAL LEVEL APPROACH

There are many factors that may influence the hierarchical *Level* selected for the assessment of the soil

properties on a particular engineering project. The main point to realize is that it is acceptable from an engineering standpoint to use different *Levels* of assessment of the unsaturated soil properties for different engineering projects. Attempt should be made to use the highest possible *Level* of assessment that each case can warrant. There is always a means whereby the unsaturated soil properties can be determined; whether crudely estimated from correlations or estimated by some other means.

This paper has set forth a hierarchical level system related to the assessment of unsaturated soil property functions. Some engineering problems can justify extensive expenditures to directly measure the unsaturated soil properties. However, in most cases it is sufficient to use a *Level 2, 3, or 4* assessment of the unsaturated soil properties. It is anticipated that each increase in the *Level* number will decrease the reliability of the soil property being determined. The geotechnical engineer needs to assess the *Level* of assessment that is adequate for the problem at-hand. The point is also made that it is always possible to obtain some assessment of the unsaturated soil properties and consequently, there is the possibility of undertaking an analysis of the entire saturated–unsaturated soil system.

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