Analysis of the Post-Filling Performance of Small Collapsing Dams

J.H.F. Pereira & R. P. Cunha
University of Brasília, Brasília, DF, Brazil

Delwyn G. Fredlund & G. F. N. Gitirana Junior
University of Saskatchewan, Saskatoon, Saskatchewan, Canada

Second Asian Conference on Unsaturated Soils
UNSAT-ASIA 2003, April 15-17, Osaka, Japan

ABSTRACT: An incremental procedure was developed for performing finite element analysis on the behavior of a small collapsing dam during transient unsaturated-saturated seepage which follows the first filling of the reservoir. A computer program, named COUPSO, has been coded based on a procedure that couples stress equilibrium and water flow using a generalized form of the theory of consolidation for unsaturated soils. In this form, a metastable-structured soil has a non-linear and stress-induced anisotropic behavior in response to a decrease in matric suction. The extended Mohr-Coulomb failure criterion for unsaturated soil defines the failure conditions in points within the small dam. The model takes into account the varying permeability of the collapsing soil when following a wetting stress path. Analyses were performed to study the post-filling performance and failure mechanisms of small dams constructed in Northeast Brazil. The results indicate that the procedure developed is a potentially useful tool.

1 INTRODUCTION

The principles governing mechanical behavior of a saturated soil are well established with respect to both theoretical and practical aspects. The theories for unsaturated soils have been experimentally investigated to accommodate the key aspects involved with their mechanical behaviors. These theories are consistent with a multiphase, continuum mechanics approach and describe the mechanical behavior of an unsaturated soil as a function of two independent sets of stress variables. These are: the net normal stress (σ - u_a) and the matric suction (u_a - u_w) where σ is the total normal stress, u_a is the pore-air pressure and u_w is the pore-water pressure. The collapsing behavior of soils during saturation is one of the complex aspects to be developed through application of the unsaturated soil theory. The prediction of the post-filling performance of small collapsing earth dams is one of the engineering problems depending upon these developments.

The primary objective of this study is to develop a better understanding of the mechanical behavior of small dams (height smaller than 10 meters) constructed using collapsing soils (i.e., collapsing dams). The condition of primary importance is the first reservoir filling.

After the first reservoir impounding, water flows through the dam in a transient manner. As the water flows, the soil in the dam undergoes volume changes in response to changes in total stress and matric suction. Volume changes imply changes in both the mechanical and hydraulic properties of the materials within the dam. In addition, volume changes can generate pore-water pressures and alter the transient flow regime within the dam embankment. This dynamic process puts in action a complex process in terms of both mechanical behavior and water flow. The dynamic process is transient and occurs until the establishment of steady-state conditions. Therefore, the solution to the problem requires a mathematical model coupling mechanical equilibrium and water flow in collapsing dams.

2 BACKGROUND

Studies have experienced difficulties in reproducing available experimental results in terms of the stress-strain behavior of collapsing soils during saturation. This is particularly true for the condition of collapse of the soil structure under Ko-conditions. Under these conditions, a collapsing soil undergoes an increase in mean net confining stress when following a wetting stress path and the applied vertical stress is kept constant (Maswoswe, 1985).

Miranda (1988) reported that failures resulting from cracks initiated by wetting-induced collapse in earth dams located in the semiarid region of Northeast Brazil are so common that these dams are publicly referred to as "Alka-Seltzer" dams. Such dams are normally constructed with residual soil derived from gneiss. Due to local conditions, these dams often are constructed without the necessary amount of water being added and with inadequate compaction (i.e., poorly compacted). There is a need for a better understanding of the failure mechanism of "Alka-Seltzer" dams to allow the adequate design of such collapsing earth dams.

3 THEORY

The theory for the analysis of the mechanical behavior of "Alka-Seltzer" dams during first reservoir filling is based on a general coupled solution for consolidation of unsaturated soils as presented by Fredlund and Rahardjo (1993). The problem of an "Alka-Seltzer" dam is here analyzed considering the two-dimensional plane strain condition. It is assumed that the pore air pressure is always atmospheric during the transient water flow process. The soil is considered to be an incrementally isotropic, linear and elastic material in terms of mechanical properties related to changes in net normal stresses. The soil is assumed to be an incrementally stress-induced anisotropic, linear and elastic material in terms of mechanical properties related to changes in matric suction (Pereira 1996). The soil is assumed to be anisotropic in terms of its hidraulic properties.

The solution for the plane strain case is solved using the finite element method. Galerkin's residual weighted method is used for the spatial discretization of the continuum. A finite difference scheme is used for the temporal discretization. The theory for the behavior of a small earth dam during its first reservoir filling, requires:

a.) a definition of the basic equations governing the phenomena involved;

b.) the constitutive relationships for the component phases of the soil in terms of both mechanical behavior and pore-fluid flow properties;

c.) the initial stress state conditions in the dam immediately after the first filling of the reservoir ;

d.) the essential and natural boundary conditions required for the basic equations governing the phenomena involved;

3.1 *Basic Equations of Physics*

A rigorous analysis of the mechanical behavior of an unsaturated soil requires the coupling of the following system of equations: a.) water phase continuity equation; b.) air phase continuity equation; c.) static equilibrium of the overall soil medium. In this paper, only the static equilibrium of the soil medium and the water phase continuity equation are considered.

3.1.1 *Water continuity equation*

The water flow continuity equation can be expressed, according to Freeze and Cherry (1979), as follows:

$$\frac{\partial(\rho_w nS)}{\partial t} + \nabla \cdot (\rho_w \mathbf{v}_w) = 0 \qquad (1)$$

where: n = porosity, S = degree of saturation, ρ_w = mass density of water, $\mathbf{v}_W = v_w^x \mathbf{i} + v_w^y \mathbf{j} + v_w^z \mathbf{k}$, macroscopic velocity of the water.

3.1.2 *Equilibrium of the soil element*

The static equilibrium of a soil element can be expressed in a condensed form, as follows:

$$\frac{\partial \sigma_{ij}}{\partial x_j} + b_i = 0 \qquad (2)$$

where: σ_{ij} = total stresses, x_i = the directional system coordinates, b_i = the body forces.

3.2 *Constitutive Relationships and Laws of Motion*

The displacement vector, $\boldsymbol{u}$ and the water pore-pressure (i.e., u_w) are chosen as the basic variables of the flow/deformation problem.

For the soil element equilibrium equation it is necessary to define constitutive relationships linking volume/displacement changes to the stress state variables. Additionally, the shear strength constitutive relationship is necessary in problems where there is a potential risk of failure of the soil. A post-failure behavior have also to be defined.

3.2.1 *Soil structure constitutive relationship*

An stress induced anisotropic formulation for the elasticity form of the constitutive equations for a collapsing soil has been proposed (Pereira 1996, Pereira et al. 1997). By using a xyz-Cartesian system the proposed equations, for the normal strains, are as follows:

$$\varepsilon_x = \frac{(\sigma_x - u_a)}{E} - \frac{\mu}{E}(\sigma_y + \sigma_z - 2u_a) + \frac{(u_a - u_w)}{H_x} \qquad (3)$$

$$\varepsilon_y = \frac{(\sigma_y - u_a)}{E} - \frac{\mu}{E}(\sigma_x + \sigma_z - 2u_a) + \frac{(u_a - u_w)}{H_y} \qquad (4)$$

$$\varepsilon_z = \frac{(\sigma_z - u_a)}{E} - \frac{\mu}{E}(\sigma_x + \sigma_y - 2u_a) + \frac{(u_a - u_w)}{H_z} \quad (5)$$

where: $H_i = H/(1+H\chi_i)$, is the elasticity modulus for the soil structure in the i-direction relative to a change in matric suction, H = isotropic elasticity modulus (function of the mean net total stress) for the soil structure relative to a change in $(u_a - u_w)$, χ_i = stress induced anisotropic collapse factor in the i-direction (function of the stress ratios σ_i/σ_j and σ_i σ_k) for the soil structure relative to a change in $(u_a - u_w)$. i, j, k = directions of a 3-orthogonal coordinate system (e.g., x, y, z).

3.2.2 *Water Phase Constitutive Relationship.*

This constitutive equation can be written as a linear combination of the stress state variable changes as follows (Fredlund et al. 1993):

$$\frac{dV_w}{V_o} = \frac{d(\sigma_x - u_a)}{E_w} + \frac{d(\sigma_y - u_a)}{E_w} + \frac{d(\sigma_z - u_a)}{E_w} + \frac{d(u_a - u_w)}{H_w} \quad (6)$$

where: Vw/Vo = volumetric water content; E_w = water volumetric modulus associated with a change in $(\sigma - u_a)$; H_w = water volumetric modulus associated with a change in $(u_a - u_w)$.

3.2.3 *Flow laws and hydraulic properties*

Water flow through a saturated/unsaturated soil can be described by a generalized Darcy's law as follows (with z-coordinate as the vertical direction):

$$\mathbf{v}_w = -\mathbf{k}_w \nabla(u_w/\gamma_w + z) \quad (7)$$

For an unsaturated soil the coefficient of permeability, $\mathbf{k_w}$, can be expressed as a function either of the matric suction or the volume-mass soil properties (Fredlund et al. 1993).

3.2.4 *Shear Strength Behavior*

The shear strength equation can be written as follows (Fredlund et al. 1978):

$$\tau_{ff} = c' + (\sigma_f - u_a)_f \tan\phi' + (u_a - u_w)_f \tan\phi^b \quad (8)$$

where: c' = effective cohesion, $(\sigma_f - u_a)_f$ = net normal stress state on the failure plane, u_{af} = pore-air pressure at failure, ϕ' = angle of internal friction related to the net normal stress, $(u_a - u_w)_f$ = matric suction on the failure plane at failure and ϕ^b = angle indicating the rate of increase of shear strength relative to the matric suction.

4 CONSTITUTIVE MODELLING OF THE COLLAPSING SOIL

The collapsing soil specimens were statically compacted at a gravimetric water content of 6.5 % (i.e., minus 4 % dry of optimum conditions) and at a dry density of 14.75 kN/m^3.(i.e., 90 % of the maximum dry density). Table 1 shows the soil properties.

Double-oedometer tests, on the range of vertical stresses from 0 to 800 kPa, were performed on the compacted soil specimens. The soil presented low compressibility when loaded under unsaturated conditions. The soil specimens did not present any collapsing behavior when saturated under vertical stresses lower than 50 kPa. A vertical stress of 100 kPa produced soil collapse amounting to 3.0 %. The volumetric collapse reached about 7.2% and 11 % when loaded under vertical stresses of 200 kPa and 400 kPa, respectively. Triaxial permeameter and modified direct shear tests allowed the definition of the constitutive soil models (Pereira 1996).

Table 1 Index properties of the soil.

Soil	Residual silty sand
Grain size distribution	Sand - 52 %, Silt - 35 % Clay - 13%
Atterberg limits	Liquid limit, w_l = 29 Plastic limit, w_p = 17 Plasticity index , PI = 12
Specific Gravity	G_s = 2.64
USCS	SW-SM:

Equation 9 is the mathematical model for the void ratio state surface of the collapsing soil resulting from the best-fit analysis of the available data.

$$e = e_u + \frac{e_f - e_u}{\left[1 + \left(\frac{(u_a - u_w)}{c}\right)^b\right]} \quad (9)$$

where: e_u=0.77 - 0.007ln(σ_m), e_f=0.752 - 0.142 / [1+(σ_m /75)$^{-3.5}$]; $c = c_1(\sigma_m)^2 + c_2(\sigma_m) + c_3$;. $b = b_1(\sigma_m)^{b_2}$; $c_1 = 9.4\text{x}10^{-4}$; $c_2 = 7.46\text{x}10^{-2}$; $c_3 = 11.0$; $b_1 = 39.0$; $b_2 = -6.10\text{x}10^{-1}$; σ_m= net confining/mean stress.

Figure 1 illustrates the best-fit model in terms of void ratio versus matric suction relationships under different net confining stresses. The available experimental data are also illustrated in this figure and show a comparison of the best-fit surface obtained. Each soil specimen, under an isotropic stress state applied, followed a wetting stress path by gradually reducing the matric suction in stages of 90, 60, 30, 10, 5 and 0 kPa. Null-tests indicated an initial matric suction value of about 370 kPa for the as-compacted specimens.

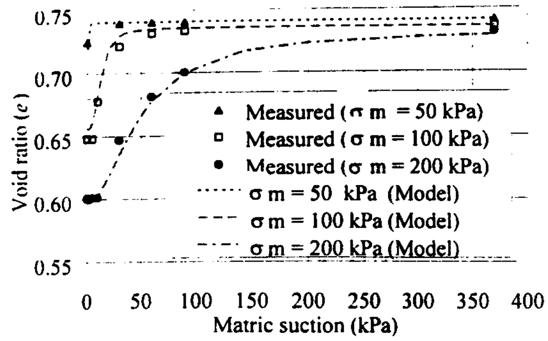

Figure 1. Void ratio best fit modeling.

At unsaturated conditions it is assumed that the Poisson ratio increases with the soil collapse. This assumption implies a variation of the Poisson ratio of the collapsing soil from an initial value of 0.3 (i.e., at the as-compacted conditions) to a value calculated, by using Eq. 10, when the soil reaches saturation (Pereira 1996).

$$\mu = 0.3 + (\mu_s - 0.3) \Bigg/ \left[1 + \left(\frac{(u_a - u_w)}{c} \right)^b \right] \quad (10)$$

where: μ_s= 0.092 ln (σ_m)-0.021, is the Poisson ratio for the saturated soil.

In Eq. 10 the parameters "b" and "c" are the same as used in Eq. 9. The anisotropic factors χ_i's (i.e., χ_x, χ_y, and χ_z) were defined in a trial and error process using the computer program COUPSO and the available double-oedometer tests. In this process, equal values were assumed for the horizontal-direction factors (i.e., χ_x and χ_z), simulating the confining oedometric conditions. The closed-form relationship χ_y equal to minus (χ_x + χ_z) defined the anisotropic factor in the vertical direction. A value of minus 1.95 was calculated for the anisotropic factors χ_x and χ_z from the above trial and error procedure (Pereira 1996).

The degree of saturation state surface was also defined using a best-fit analysis of the available data. The mathematical model obtained is expressed as follows:

$$S = S_0 + (1 - S_0) \Bigg/ \left[1 + \left(\frac{(u_a - u_w)}{c} \right)^d \right] \quad (11)$$

where: S_0 = 0.375; a = 0.354; b = 3.65x10^{-3}; c = 20; d =0.977.

Brooks and Corey's (1964) equation provided a satisfactory best-fitting relationship for the hydraulic conductivity of the collapsing soil as expressed in Equation 12.

$$k_w = k_p \left(\frac{\psi_{cr}}{(u_a - u_w)} \right)^{\lambda} \quad (12)$$

where: $k_w \leq k_s$; k_p = -1.39×10^{-7} + 6.26×10^{-8} ln(σ_m); k_s = 1.17×10^{-6} - 1.8x10^{-7} ln(σ_m), is the saturated coefficient of permeability; ψ_{cr}= 3.0; λ = 2.10.

Equation 13 expresses the best-fit mathematical model for the extended Mohr-Coulomb failure envelope for the collapsing soil (Pereira 1996).

$$\tau_{ff} = a_1 + b_1(\sigma - u_a) + c_1(u_a - u_w) + d_1(\sigma - u_a)(u_a - u_w)^p \quad (13)$$

where: a_1 =-7.89, b_1 = 0.194, c_1 = 0.324; d_1 = 0.093; p = 0.043.

5 POST-FILLING PERFORMANCE OF AN ALKA-SELTZER DAM

The computer program COUPSO (Pereira 1996), developed by using the finite element method, was utilized to analyze the post-filling performance of a small collapsing dam.

5.1 *Description of the problem*

Figure 2 shows a section of a small dam (i.e., h < 10.00 meters) typically constructed in poor areas of Northeast Brazil. Such dams are constructed as homogenous embankments, often without internal drainage.

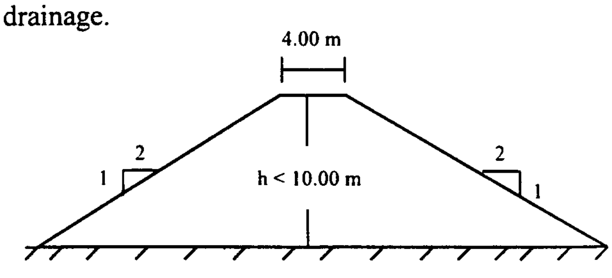

Figure 2. Typical cross section of a small dam in Northeast Brazil.

5.2 *Analysis procedure*

The initial conditions for the post-filling phase of the homogeneous embankment were defined as follows:

a.) The net normal stress distributions corresponding to the end-of-construction phase and first impounding of the reservoir were calculated.

b.) The initial matric suction of the compacted soil in the entire cross-section was assumed to be equal to 370 kPa.

At any stage of the transient analysis the shear strength mobilized at internal points of the dam is calculated as follows:

$$SMOB = \left(\frac{(\sigma_1 - \sigma_3)}{(\sigma_1 - \sigma)_f} \right) \qquad (14)$$

where: $SMOB$ = shear strength mobilization; (σ_1- σ_3)= acting deviatoric stress; (σ_1- $\sigma_3)_f$= $2\times(c\times\cos\phi + \sigma_3\times sin\phi)$ is the deviatoric stress at failure;

A high compressibility is assumed for a soil element which reaches failure within the dam. Such assumption reflects the plastic response of the soil when sheared by using the modified shear box.

5.3 *Analysis of an "Alka-Seltzer" dam*

Pereira (1996) presents the finite element mesh (90 elements and 407 nodes) used in the analysis. A quadrilateral Lagrangian element with 9 nodes was utilized in the finite element model.

Emphasis is given to the transient seepage phase which follows the first reservoir filling phase. "Alka-Seltzer" dams present satisfactory structural stability for the construction and first reservoir filling phases.

After the dam has been constructed, its reservoir is filled to an elevation of 8.0 meters. In the analysis, it is assumed that the water is raised in a short period of time.

Figure 3 shows the shear strength mobilized within the dam after the reservoir filling phase. This figure shows that the shear strength mobilized reflects satisfactory stability of the dam.

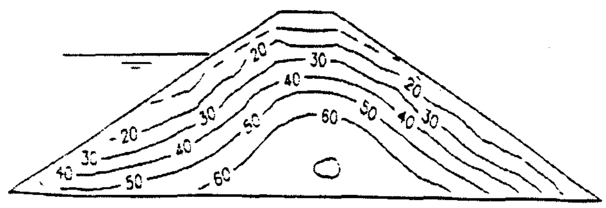

Figure 3. Shear strength mobilized (%) distribution within the dam after the reservoir-filling phase.

Figure 4 shows the mean net normal stress (i.e., σ_m - u_a) distribution within the dam immediately after the reservoir-filling phase.

The small values of mean net stress (i.e., values less than 70 kPa) within the dam reflect the combined effect of the low dry density and low Poisson ratio of the collapsing soil at unsaturated conditions. These values also suggest that upon saturation. higher collapse deformations should be expected at the central part of the dam.

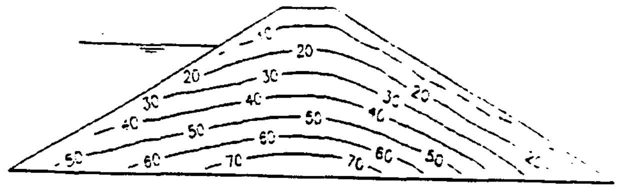

Figure 4. (σ_m - u_a) (kPa) distribution within the dam after the reservoir filling phase.

The small dam was considered to be anisotropic with a horizontal coefficient of permeability equal to 10 times the vertical coefficient of permeability.

The transient seepage analysis was performed in accordance with a time discretization consisting of time steps varying from 0.70 to 0.20 days depending upon the convergence requirements.

Pereira (1996) shows results in terms of pore pressures, displacements, stresses, and shear strength mobilization within the dam; corresponding to periods of 30 days. 55 days, 100 days and 145 days after the reservoir filling of an "Alka-Seltzer" dam. However, this paper concentrates only on results at 145 days after the first reservoir filling.

Figures 5 shows the pore-water pressure distribution within the dam 145 days after its first reservoir filling. At this stage the upstream slope of the dam has been saturated and the upper part of the phreatic line is near the downstream slope face. This pattern is a consequence of the hydraulic anisotropy of the soil combined with the high hydraulic gradient which is driving the transient water flow through the dam embankment.

Miranda (1988) reports that failure of Alka-Seltzer dams is related to piping and hydraulic fracturing within the dam embankment. Such statement was based on information from local people living in areas where an "Alka-Seltzer" dam had reached failure conditions. In summary, it had been reported that the water had emerged from the downstream slope and after a short time the entire dam was being carried out, as a mudflow, by the running water. The present study has demonstrated that the above condition is fairly well depicted in Figure 5.

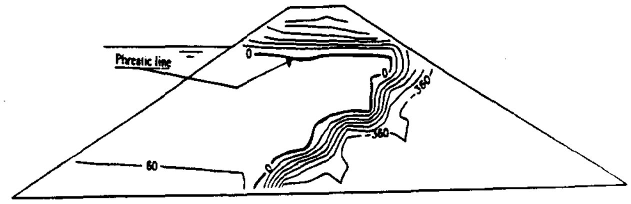

Figure 5. Pore-water pressure (kPa) distribution 145 days after first reservoir filling.

Figure 6 shows the displacement pattern within the dam 145 days after its first reservoir filling. This stage reflects the complete failure of the upstream slope of the dam embankment. At this stage. the lower central part of the dam has collapsed due to saturation and the upstream slope is sliding towards the dam reservoir. The unsaturated zone remains at high matric suctions.

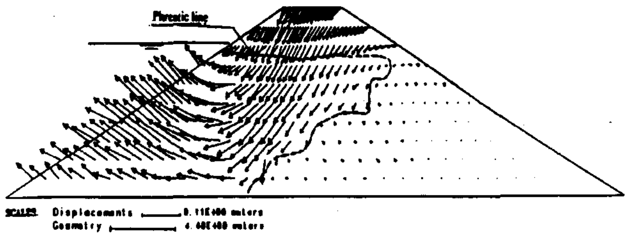

Figure 6. Displacement distribution 145 days after first reservoir filling.

Figure 7 presents the distribution of shear strength mobilized within the collapsing dam 145 days after its first reservoir filling. At this stage, the failed zone within the dam has been spread throughout the saturated zone. The downstream slope of the dam suffered minor changes of shear strength mobilized due to the high values of matric suction existing there.

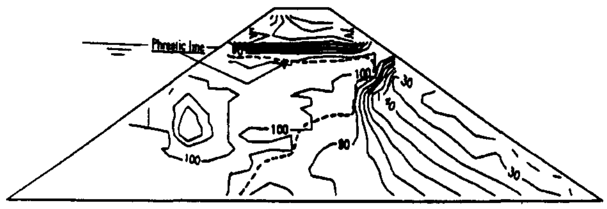

Figure 7. Percent of shear strength mobilized distribution 145 days after first reservoir filling.

Figures 6 and 7 also show that the upstream slope has reached failure conditions and presented a pronounced sliding towards the dam reservoir.

Figure 8 shows the mean stress distribution within the dam at the 145 days stage. There is an increase of mean net normal stress within the central part of the dam. Such an increase reflects the load transference from the highly compressible and saturated zone to the rigid unsaturated zone.

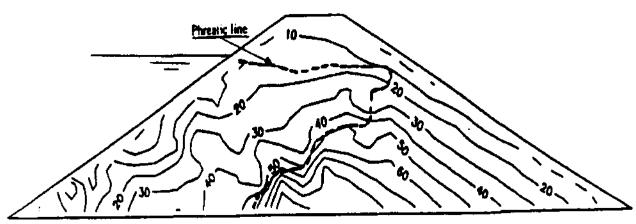

Figure 8. σ_m (kPa) distribution 145 days after first reservoir filling.

6 CONCLUSIONS

1. Fredlund's & Rahardjo's (1993) theory for consolidation of unsaturated soils in its more generalized form (i.e.. introducing an stress-induced anisotropic behavior of an unsaturated soil). can be used to simulate the stress-strain behavior of a collapsing soil during saturation.
2. COUPSO is sufficiently versatile to allow the incorporation of the most relevant theoretical aspects involved in the consolidation of both saturated and unsaturated soils.
3. Results obtained from COUPSO demonstrate the structural instability of an "Alka-Seltzer" dam during transient seepage flow following the first filling of the reservoir.
4. These analysis procedures may be used to predict pore-water pressures, stresses and movements in small collapsing dams during any stage after the beginning of construction of the dam. Perhaps the greatest value of these procedures is in connection with instrumentation studies.

ACKNOWLEDGEMENTS

The authors express their gratitude to the Brazilian Research Agency (CAPES) and to the Department of Civil Engineering of the University of Saskatchewan.

REFERENCES

Brooks R. H. and Corey, A. T. (1964). Hydraulic Properties of Porous Media. Colorado State Univ. Hydrol. Paper, No.3, 27 p.

Fredlund, D. G., Morgenstern, N. R. and. Widger, R. A. (1978). Shear Strength of Unsaturated Soils. Canadian Geotechnical Journal, vol. 15, No. 3, pp. 313-321.

Fredlund, D. G. and Rahardjo, H. (1993). Soil Mechanics for Unsaturated Soils. John-Wiley and Sons, New York , 486p.

Freeze and Cherry (1979). Groundwater. Prentice Hall.Englewood Cliffs, N.J., 604 p.

Maswoswe, J. (1985). Stress Paths for a Compacted Soil During Collapse Due to Wetting. PhD Dissertation, Imperial College of Science and Technology, London.

Miranda, A. N. (1988). Behavior of Small Earth Dams During Initial Filling. Ph.D. Thesis. Colorado State University, Fort Collins. Colorado, United States, 223p.

Pereira, J.H.F. (1996) Numerical Analysis of the Mechanical Behavior of Collapsing Earth Dams During First Reservoir Filling, Ph.D. Thesis. University of Saskatchewan, Saskatoon. Sakatchewan., Canada, 449p.

Pereira, J.H.F. and Fredlund, D.G. (1997) Constitutive Modeling of a Metastable-Strutured Compacted Soil. Proceedings of the International Symposium on Recent Developments in Soil and Pavement Mechanics, A. A. Balkema. Rio de Janeiro, Brazil, June, pp. 317-326.