An Illustration of the Use of Unsaturated Soil Mechanics in Hazard Management

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Hong Kong
November 8, 2003
Introduction

Decision Analysis framework for Weather-Related Geo-Hazards Assessments, W-GHA

Numerical Model for the problem

Unsaturated Soil properties assessment

Frequency and sensitivity analyses

Final remarks
Introduction

- **W-GHA model**
- **Weather-Related Geo-Hazard Assessment model**
  - *Problem background*: decision situation, benefits from the W-GHA model implementation
  - *Decision Analysis framework*: deterministic and probabilistic components within one framework
  - *Unsaturated soil theories*: to solve the problem
  - *Theory implementation*: from experimental evidence towards practice
Canadian Pacific Railway network and main landslide areas according to the type of terrain (terrain data from Office of Critical Infrastructure Protection and Emergency Preparedness 2001)

Tens of Thousands of Kilometers
Terrain Through the Rocky Mountains
A True Hazard; Difficult to Predict!
Main-track derailment data provided by the Transportation Safety Board of Canada (1994 and 2001)

- In 1982: 3.8 derailments per 1,000,000 miles
- In 1991: 1.3 derailments per 1,000,000 miles
Main- and non-main track derailments by assigned factors and injuries/fatalities due to derailments (data from Transportation Safety Board of Canada 2001)

Most Fatalities Occur at Railway Crossings; Not Shown
Derailment Resulting from an Embankment Failure
Bearing capacity type failures are due to a loss of soil suction and subsequent shear strength.
Problem Background

- **Geometry and Roadbed** factors have a significant effect on the number of derailments.

- First step towards managing geo-hazard risks associated with the railway: **Categorization of the Geo-Hazards**

  *Unsaturated Soil Mechanics Related Issues*
Categorization of Geo-Hazards.

Geo-hazard categories

- Slope and subgrade: failure or serviceability problems
  - Debris flow
  - Embankment failure
  - Rock fall
  - Subsidence and frost heave

- Erosion washout
  - Crossing
  - Parallel
Use of Unsaturated Soil Mechanics in Hazard Management

- Use the **Soil-Water Characteristic Curve (SWCC)** as a **Hazard Gauge**
  - Means of assessing the impact of weather-related geo-hazards.
  - **Soil-water characteristic curve**, SWCC, can be thought of as a **Water Volume Gauge** for the soil and therefore as a **Hazard Gauge**
Soil-Water Characteristic Curve Conceptualization

- Saturated zone
- Desaturation zone
- Air-entry value, \( \psi_b = 5 \text{ kPa} \)
- Residual suction, \( \psi_{\text{res}} = 250 \text{ kPa} \)
- Residual degree of saturation, \( S_{\text{res}} = 0.2 \)
- Bending point shapness, \( a = 0.05 \)
SWCC as a Hazard Gauge

When the **Gauge** approaches **EMPTY**, there is the **ability to accept More Water**

**EMPTY IS GOOD!**

When the **Gauge** approaches **FULL**, there is **NO ability to accept More Water**
SWCC as a Water Level Gauge for Embankment Hazard Level

Soil suction, kPa

Degree of saturation, $S$, %

Factor of Safety

High hazard level

Intermediate hazard level

Low hazard level

Factor of safety

SWCC
Analysis Levels

- Deterministic assessment
  - Absolute prediction of rupture
- Hazard assessment is reliability-based
  - Probability of rupture
- Risk assessment
  - Probability and consequences of rupture
- Decision analysis
  - Decision based on risk assessment
**Fundamental Objectives Hierarchy of the W-GHA Model**

Maximize railway system performance

- Minimize financial loss
  - Minimize injuries and loss of life
    - Minimize release of dangerous goods
  - Minimize disruption of rail service (delays)
    - Minimize all types of geo-hazards
  - Minimize equipment and facility damage
    - Minimize all types of geo-hazards

Fundamental Objective: Maximize Railway Performance Through Minimization of Geo-Hazards
Means-Objectives Network of the W-GHA Model

Focus of This Study

Maximize railway system performance

Prevent and/or protect against failures in advance

(Proactive)

Improve weather forecast

Improve ground and hazard characterisation

Improve analytical models

Improve computational tools and methods

Develop database systems

Improve response and/or remediation measures

(Reactive)

Develop and/or obtain better equipment

Training of technical and field personnel

Improve field procedures

Improve field procedures
Deterministic Model for the Railway Embankment

- Geotechnical Engineering Approach:
  - Stability = $f(\text{total stresses; pore-water pressures; shear strength})$
- Utilize a Series of Partial Differential Equations to Describe Soil Behaviour
- Changes are Driven by Atmospheric Boundary Conditions
- Factor of Safety
  - “the factor by which the shear strength must be reduced to bring the soil to a state of limiting equilibrium”
Factors Affecting the Stability of an Embankment

- precipitation
- soil A
- soil B
- bedrock
- actual evaporation
- run-off
- infiltration
- slip surface
- water table

Water Balance Considerations Gives Rise to the Water Storage in the Soil
Water Storage in Soil Linked to the SWCC
W-GHA Model Flowchart

Initial Pore-Water Pressure Distribution $u_w0$

Transient Moisture and Heat Flow Analysis

Transient Soil-Atmosphere Boundary Flux

Stress Analysis at $t = t_i$

$u_w$ at $t = t_i$

$\sigma$ at $t = t_i$

Slope Stability Analysis (Dynamic Programming)

Factor of Safety, $F_s$, at $t = t_i$

Computations for Changes in Pore-Water Pressure

Computations for Changes in Factor of Safety
Partial Differential Equations Governing Soil Behaviour

- Continuum Mechanics or Phenomenological Approach to Soil Mechanics Formulations
- Independent Stress State Variables:
  - \((\sigma - u_a)\) and \((u_a - u_w)\)
- Conservation Laws: mass of water (moisture), energy (heat), momentum (equilibrium of forces)
- Constitutive Relationships for Soil Behavior
Conservation and Flow of Moisture

Soil representative elemental volume and water mass fluxes

Liquid Water (Advection)

Water Vapor (Diffusion)

Water Vapor carried By Bulk Air Diffusion
**Formulation for Flow of Moisture**

**Conservation of mass of water**

- Liquid Water
  
  \[ q^w_x = -\rho^w k^w \frac{\partial h}{\partial x} = -\rho^w k^w \partial \left( \frac{u^w}{\gamma^w} \right) \]

- Water Vapor
  
  \[ q^w_y = -\rho^w k^w \frac{\partial h}{\partial y} = -\rho^w k^w \partial \left( \frac{u^w + y}{\gamma^w} \right) \]

**SWCC**

\[ \frac{dV^w}{V_0} = m^w_d (u_a - u_w) \]

**Darcy’s law**

\[ \frac{\partial u^w}{\partial x} = \frac{\partial (u^w / \gamma^w)}{\partial x} \]

**Fick’s law**

- Liquid Water
  
  \[ q^v_x = q^{dv}_x + q^{av}_x = -D_v \frac{\partial p^v}{\partial x} - \frac{\rho^v}{\rho^d} D^d \frac{\partial \bar{u}_a}{\partial x} \]

- Water Vapor
  
  \[ q^v_y = q^{dv}_x + q^{av}_x = -D_v \frac{\partial p^v}{\partial y} - \frac{\rho^v}{\rho^d} D^d \frac{\partial \bar{u}_a}{\partial y} \]

\[ \bar{u}_a = u_{air} + p_v + u_{atm} \]
Formulation for Flow of Moisture

Partial Differential Equation

$$\frac{\partial}{\partial x} \left[ k_w \frac{\partial (u_w / \gamma_w)}{\partial x} + \frac{\bar{u}_a + p_v}{\rho_w} \frac{D^v}{\partial x} \frac{\partial p_v}{\partial x} \right]$$

$$+ \frac{\partial}{\partial y} \left[ k_w \frac{\partial (u_w / \gamma_w + y)}{\partial y} + \frac{\bar{u}_a + p_v}{\rho_w} \frac{D^v}{\partial y} \frac{\partial p_v}{\partial y} \right] = -m_w^w \frac{\partial u_w}{\partial t}$$

where:

$k_w = \text{hydraulic conductivity}, \ k_w = f(u_a - u_w)$

$D^v = \text{vapour diffusion coefficient}, \ D^v = f(u_a - u_w)$

$m_w^w = \text{coefficient of water volume change with respect to changes in soil suction}, \ m^w = f(u_a - u_w)$

Three Unknowns: $u_w, p_v$ and Temperature
**Lord Kelvin’s equation:**

\[ p_v = p_{\text{vsat}} \exp \left[ \frac{u_w g W_v}{\gamma_w RT} \right] \]

**Relationship between** \( p_{\text{vsat}} \) **and Temperature is fixed**

\[ \nabla p_v = \frac{g W_v p_v}{\gamma_w RT} \left( \nabla u_w - \frac{u_w}{T} \nabla T \right) \]

where:
- \( p_{\text{vsat}} = \) saturation vapour pressure in the soil at \( T \)
- \( T = \) temperature
- \( W_v = \) molecular weight of water
- \( R = \) universal gas constant

**Link Between** \( p_v \) **and Temperature**
Combining Vapor Equilibrium with Flow of Moisture

**PDE based on** $u_w$ **and** $T$, using Lord Kelvin’s equation

\[
\frac{\partial}{\partial x} \left[ \left( \frac{k^w}{\gamma_w} + D^{v*} \right) \frac{\partial u_w}{\partial x} - D^{v*} \frac{u_w}{T} \frac{\partial T}{\partial x} \right] \\
+ \frac{\partial}{\partial y} \left[ \left( \frac{k^w}{\gamma_w} + D^{v*} \right) \frac{\partial u_w}{\partial y} + k^w - D^{v*} \frac{u_w}{T} \frac{\partial T}{\partial y} \right] = -m^w \frac{\partial u_w}{\partial t}
\]

where:

\[
D^{v*} = \left( \frac{(\bar{u}_a + p_v)p_v}{\rho_w \bar{u}_a} \right) \left( \frac{W_v}{\rho_w RT} \right) D^v
\]

**Two Unknowns:** $u_w$ and Temperature
Heat Flow in Terms of Temperature and Pore-Water Pressure

PDE based on $u_w$ and $T$, using Lord Kelvin’s equation

$$\frac{\partial}{\partial x} \left[ L_v D_v^* \rho_w \frac{\partial u_w}{\partial x} + \left( \lambda - L_v D_v^* \rho_w \frac{u_w}{T} \right) \frac{\partial T}{\partial x} \right]$$

$$+ \frac{\partial}{\partial y} \left[ L_v D_v^* \rho_w \frac{\partial u_w}{\partial y} + \left( \lambda - L_v D_v^* \rho_w \frac{u_w}{T} \right) \frac{\partial T}{\partial y} \right] = \zeta \frac{\partial T}{\partial t}$$

where:

$$D_v^* = \left( \frac{\overline{u}_a + p_v}{\rho_w \overline{u}_a} \right) p_v \left( \frac{W_v}{\rho_w RT} \right) D_v$$

Two Unknowns: $u_w$ and Temperature
Coupled Moisture and Heat Flow

Reduced to Two Variables: $u_w$ and $T$

$$\frac{\partial}{\partial x}\left[\left(\frac{k^w}{\gamma_w} + D^v*\right)\frac{\partial u_w}{\partial x} - D^v* \frac{u_w}{T} \frac{\partial T}{\partial x}\right]$$

$$+ \frac{\partial}{\partial y}\left[\left(\frac{k^w}{\gamma_w} + D^v*\right)\frac{\partial u_w}{\partial y} + k^w - D^v* \frac{u_w}{T} \frac{\partial T}{\partial y}\right] = -m^w_2 \frac{\partial u_w}{\partial t}$$

Conservation of moisture

$$\frac{\partial}{\partial x}\left[L_v D^v* \rho_w \frac{\partial u_w}{\partial x} + \left(\lambda - L_v D^v* \rho_w \frac{u_w}{T}\right) \frac{\partial T}{\partial x}\right]$$

$$+ \frac{\partial}{\partial y}\left[L_v D^v* \rho_w \frac{\partial u_w}{\partial y} + \left(\lambda - L_v D^v* \rho_w \frac{u_w}{T}\right) \frac{\partial T}{\partial y}\right] = \zeta \frac{\partial T}{\partial t}$$

Conservation of heat
Equilibrium of Forces for Stress Analysis

- Equilibrium on x- and y- directions
- Hooke’s generalised stress-strain law
- Small strains

\[
\frac{\partial}{\partial x} \left[ D_{11} \frac{\partial u}{\partial x} + D_{12} \frac{\partial v}{\partial y} \right] + \frac{\partial}{\partial y} \left[ D_{44} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right] = 0
\]

\[
\frac{\partial}{\partial x} \left[ D_{44} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right] + \frac{\partial}{\partial y} \left[ D_{12} \frac{\partial u}{\partial x} + D_{11} \frac{\partial v}{\partial y} \right] + \gamma = 0
\]

where:

\[
D_{11} = \frac{E(1-\mu)}{(1+\mu)(1-2\mu)} \quad D_{12} = \frac{E\mu}{(1+\mu)(1-2\mu)}
\]

\[
D_{44} = \frac{E}{2(1+\mu)}
\]

\[
E \text{ is the Young modulus, kPa}
\]

\[
\mu \text{ is the Poisson ratio}
\]

\[
\gamma = \left( G_s + Se \right) \frac{\gamma_w}{1 + e}
\]
Solution of PDE’s governing the Thermo-Hydro-Mechanical Behaviour of Saturated-Unsaturated Soil Comprising a Railway Embankment

- General Purpose PDE Solver called FlexPDE (PDE Solutions, 2003)
- Analyst can concentrate on the physical modelling rather than on the details of numerical modelling
- Use a *(Finite Element + Finite Difference)* formulation
- Adaptative mesh and time step, and Newton-type methods for solution of non-linear equations
- Coupled systems
Weather-Related Boundary Conditions

Truly a Flux Driven Boundary Value Problem

- **Moisture flux (hydrological cycle)**
  - Rainfall, evaporation, and runoff are of primary importance
  - Depression storage, interception, and transpiration can also be considered

- **Heat flux**
  - Net radiation
  - Latent Heat
  - Sensible heat
Moisture Flux Boundary Condition

- Mass Balance for Water at Ground Surface

\[
NF = P - AE - R
\]

where:
- \( NF \) = net moisture flux (Positive or Negative)
- \( P \) = precipitation
- \( AE \) = actual evaporation
- \( R \) = runoff
Moisture Flux Boundary Condition

Precipitation, \( P \), based on Weather Data

Actual Evaporation, \( AE \): weather data + soil properties

\[
AE = PE \left[ \frac{RH - \left( \frac{p_{air}^{\text{sat}}}{p_{vsat}} \right) RH_{air}}{1 - \left( \frac{p_{air}^{\text{sat}}}{p_{vsat}} \right) RH_{air}} \right]
\]

where:

\( PE \) = potential evaporations (water surface)
\( RH = \frac{p_s}{p_{vsat}} \) = relative humidity at the soil surface
\( RH_{air} \) = relative humidity of the air near the soil surface
\( p_{air}^{\text{sat}} \) = saturation vapour pressure of the air
\( p_{vsat} \) = saturation vapour pressure of the air
Stability Analysis Using Dynamic Programming

- **Dynamic Programming Method**
  - Minimization of linear (additive) functionals (Bellman 1957)
  - Baker (1980)
  - Yamagami and Ueta (1988)
  - Zou et al. (1995)
  - Pham et al. (2001)

- Slip surface shape restrictions are relaxed
- Combination with FE stress fields provides further benefits
Analytical Scheme for Stability Analysis Using Dynamic Programming Method

- Initial stage, $1$
- Final stage, $n+1$
- Search grid crossing ground surface
- Slip surface (segment 2 of $n$)
- Grid cell
- Train load
- State points on stage $n$
- Final stage, $n+1$
- Soil A
- Soil B
- Bedrock
- Stages: $2..., i, i+1...$
Stability Analysis Using Dynamic Programming

Factor of safety

\[ F_s = \frac{\sum_{i=1}^{n} R_i}{\sum_{i=1}^{n} S_i} = \frac{\sum_{i=1}^{n} \tau_{i} \Delta L_i}{\sum_{i=1}^{n} \tau_{i} \Delta L_i} \]

Auxiliary (additive) functional

\[ G = \sum_{i=1}^{n} (\tau_{i} \Delta L_i - F_s \tau_{i} \Delta L) \]

Optimum function

\[ H_{i+1}(j) = \min[H_i(k) + DG_{i+1}(j,k)] \quad DG_{i+1}(j,k) = \tau_{i} \Delta L_i - F_s \tau_{i} \Delta L \]
Unsaturated Soil Properties Assessment

- W-GHA model requires a large quantity of information
  - Soil properties
  - Weather parameters
- Unsaturated soil properties
  - Important because unsaturated flow and unsaturated shear strength play a major role in the stability of an embankment
  - Particularly difficult to measure
  - Bear a close relationship to the SWCC
Approaches to Determine the Unsaturated Soil Property Functions

Determination of unsaturated soil property functions

Direct measurement

Laboratory experiment

Field experiment

Interpretation & presentation

Interpretation & presentation

Indirect Prediction

Soil-water characteristic curve measurement

Unsaturated soil property = Saturated property $\times$ SWCC$^{\text{power}}$

Classification test (grain-size distribution)

SWCC = $f$(pore size distribution) = $f$(grain-size distribution)

Focus in this Study
Water Movement Related Soil Properties Required by the W-GHA Model

All Unsaturated Soil Properties are a Function of SWCC
Heat Movement Related Soil Properties Required by the W-GHA Model

Unsaturated Soil Properties are a Function of SWCC
Thermal Conductivity Related to SWCC

Volume

\[ S = 100\% \]

\[ S = 0\% \]

\[ V_v \]

\[ V_s \]

\[ V_a \]

\[ V_w \]

Soil solids

Thermal conductivity

\[ \lambda_a F_a S (1 - n) \sum F_i (V_i / V) \]

\[ \lambda_w F_w S n \sum F_i (V_i / V) \]

\[ \lambda_s F_s (1 - n) \sum F_i (V_i / V) \]
Soil solids $S = 100\%$

S = 0$

\begin{align*}
\zeta_a &= 0 \\
\zeta_w \left( \frac{V_w}{V} \right) &= \zeta_w nS \\
\zeta_s \left( \frac{V_s}{V} \right) &= \zeta_s (1 - n)
\end{align*}
Density and Shear Strength Related Soil Properties Required by the W-GHA Model

Unsaturated Soil Properties are a Function of SWCC
Shear Strength

Calculated based on the reduction in the effective wetted area of contact past the air-entrapment value:

\[ \tau_{f_i} = c' + (\sigma_{n_i} - u_a) \tan \phi' + (u_a - u_w) \Theta^\kappa \tan \phi' \]

where:
- \( c' \) = effective cohesion
- \( \phi' \) = effective friction angle
- \( \kappa \) = fitting parameter to account for non-linearities
- \( \Theta \) = \( S \) (Fredlund et al. 1996) or \( \Theta = S_e = (S - S_{res})/(1 - S_{res}) \) (Vanapalli et al. 1996)
An Example of the Shear Strength Function for an Unsaturated Soil

Shear strength parameters for saturated soil:
\[ c' = 2.0 \text{ kPa} \]
\[ \phi' = 28.0^\circ \]
An Influence Diagram for W-GHA Model

Circles: Uncertainties  
Squares: Known Values

- Relative hydraulic conductivity
- Potential Evaporation
- Saturated hydraulic conductivity
- Initial pore-water pressure distribution
- Geometry
- Shear strength parameters
- SWCC: Air-entry value
- External load
- Stressed distribution
- Factor of Safety, Fs
- Water storage: m2w
- Vapour diffusion: Dvap
- Thermal properties
- Precipitation

OUT-COME

SWCC

Residual saturation
Residual suction
Residual saturation
Residual suction

SWCC

Residual saturation
Residual suction

Residual saturation
Residual suction
Frequency Analysis

- **Factor of Safety**: dependent variable (outcome)
- How to determine frequency distributions?
  - Decision Trees
  - Discrete stochastic analysis (smart sampling)
  - DPL *(Applied Decision Analysis LLC 1998)*
A Decision Tree for the W-GHA Model

- **SWCC:** Air-entry value
  - Low
  - Nominal
  - High

- **SWCC:** Residual suction
  - Low
  - Nominal
  - High

- **SWCC:** Residual saturation
  - Low
  - Nominal
  - High

- Saturated hydraulic conductivity
  - Low
  - Nominal
  - High

- Precipitation
  - Low
  - Nominal
  - High

- Shear strength parameters
  - Low
  - Mean
  - Factor_of_Safety
  - High

- **Factors of Safety**
  - Mean
  - Factor_of_Safety

- **Branches suppressed for clarity**

- **Decision tree above:** 729 scenarios ($3^6$)

- **Difficulties with extreme low frequency problems**
Sensitivity Analysis

- Large number of parameters involved
- What variables need to be considered as uncertain variables? **Answer: perform sensitivity analyses**
  - Rainbow diagrams
  - Expected tornado diagrams
  - Event tornado diagrams
- “Value of Information” analyses
- **Type of Result:** Precipitation proven to be the most sensitive parameter for certain regions. Evaporation is potentially important in some regions
Conclusions

- **Unsaturated Soil Mechanics** has an important role to play in Hazard Management
- **Imminent Failures** are based on being able to predict when pore-water pressures approach zero and a significant portion of the embankment approaches saturation
- **Soil-Water Characteristic Curves** play a primary role in estimating the unsaturated soil property functions
Acknowledgments

- Canadian Pacific Railway
- Natural Sciences and Engineering Research Council of Canada – NSERC
- “Conselho Nacional de Desenvolvimento Científico e Tecnológico – CNPq, Brasil”
Danke!