DISCUSSION

An equation to represent grain-size distribution: Reply[1]

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The authors wish to thank the discusser for putting together his case for the possibility of using a log-normal type representation to characterize particle size distributions. Log-normal type plots have found a degree of acceptability in the area of hydrology; however, similar attempts to use log-normal distributions have not been met with the same enthusiasm in the soil mechanics discipline. Geotechnical engineers do not view particle size distribution as statistical or probability-based information related to the number of particles associated with each particle size range.

In the original paper, the authors point out that "Gardner (1956) proposed a two-parameter, log-normal distribution to provide representation of grain-size distribution data." The idea of using log-normal distributions to characterize grain-size distribution curves has been around for a long time. It does not appear to have gained significant acceptability in geotechnical engineering nor engendered further research interest. It was against this background that the present research study was formulated. It was decided to further investigate the use of an empirical, mathematically based equation where the fitting parameters had some physical meaning. The equations were focused on the generally accepted "percent passing" versus the logarithm of the particle size plots.

There has been a recent proliferation of studies related to the estimation of the soil-water characteristic curve from a grain-size distribution curve in the soil science and related disciplines. This also encouraged further attempts to mathematically characterize the grain-size distribution curve. The use of a continuous mathematical equation that can be differentiated and integrated has appeal because it can be used for further analysis in soil mechanics.

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[1]Discussion by D. Hansen. This issue. Canadian Geotechnical Journal, **39**: 1433–1436.

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There appears to be a fundamental difficulty in viewing a grain-size distribution curve as a purely statistical phenomenon since the grain-size distribution is commonly plotted as a "percent passing" by mass (as opposed to a cumulative distribution based on the number of particles) versus the logarithm of the particle size. The "percent passing" by mass does not lend itself to a statistical evaluation in the same way as if the number of particles was being assessed (Ang and Tang 1975). The largest particles have the dominance with respect to mass but in general, have the least influence on soil behavior. For example, only 30% clay size particles render the behavior of the soil like that of a clay soil. Hazen (1930) suggested that the D_{10} size of particles best represented the particle size for the assessment of the coefficient of permeability of a soil. In other words, conventional statistical measures based strictly on frequency do not provide the geotechnical engineer with the most meaningful parameters required when attempting to foresee the mechanical and hydraulic behavior of a soil. As a result, D_{50} does not have special meaning over other "percent passing" values in soil mechanics.

Part of the natural bias towards the coarse end of the particle size spectrum (i.e., because of the mass basis for presentation of the data) is overcome in soil mechanics by plotting particle size on a logarithm scale. The differentiation of the percent passing grain size curve gives a log-normal type distribution, but this is not often used by geotechnical engineers. The particle size scale is equally spaced on a logarithm scale, and while this presents some difficulties in interpretation, it still appears to be a meaningful way to visualize the particle sizes in the distribution.

The discusser states that he feels "that this paper represents a continued divergence from available approaches." The authors maintain that it might represent a divergence from approaches quite common to hydrology; however, the approach proposed is consistent with the historical and present interpretive procedures used in soil mechanics.

The authors used a four-parameter equation in the case of a unimodal grain-size distribution curve and a seven-parameter equation in the case of a bimodal grain-size distribution curve. This may appear to be a large number of parameters, but the parameters come out of a regression analysis and allow a consistently accurate representation of the entire grain-size curve. When used in conjunction with a database, this means that essentially any question can be asked regarding the grain-size distribution curve, and an accurate answer can be obtained based upon the best-fit parameters stored in the database. In this way, the primary purpose of the proposed grain-size distribution equation is fulfilled. It appears that even with an increase in the number of parameters used for the log-normal distribution proposed by the discusser, there is no assurance of an accurate fit over the entire particle size range.

The discusser states that "if a sample is bimodal, two populations may be present and should probably be described separately." He goes on to say that the geotechnical engineer should "ask why a soil is gap-graded..." However, it must be remembered that the engineer is dealing with one soil that is gap-graded and not two independent soils. Also, the question that needs to be asked by the geotechnical engineer is not "Why is the soil gap-graded?" but rather, "How is the gap-graded soil going to behave?"

The discusser raises four issues at the end of his discussion. These are briefly responded to below.

(1) The convention in soil mechanics has been to use "percent finer than" or "percent passing" rather than "cumulative probability" when plotting grain-size distribution curves. I think that this will continue to be the case because it is a mass representation that is being measured rather than a number of particles.

(2) As stated in the original paper, the log-normal distribution is symmetrical in a log scale. But this does not mean that the D_{50} is going to be the most important particle size to the geotechnical engineer. The D_{50} can readily be computed as can the D_{10}, D_{20}, etc., when using the proposed equation for the grain-size distribution curve.

(3) The discusser suggests that the "direction being taken by the authors represents an undesirable departure from presently available, widely understood, and simpler approaches for handling data of a statistical nature." Once again, the authors would like to emphasize that handling the data associated with a grain-size distribution curve is NOT fundamentally a statistics issue. This goes back to the fact that it is a mass representation of particles that is measured and not the number of particles associated with each particle size range. This is NOT a departure from the way the grain-size distribution data is handled in geotechnical engineering. Other ways of handling the data may be common in hydrological engineering, but they cannot necessarily be directly transferred to soil mechanics. And the user should not be discouraged by the number of soil parameters used to fit the grain-size distribution curve. Regardless of the number of soil parameters, a curve-fitting routine must be used, and it is the accuracy and reliability of the fit that is more important.

(4) The authors used the words "dominant particle size" to represent the particle size with the highest frequency on the probability density curve (i.e., the peak probability density).

It is obvious that there are different approaches that have been used in different disciplines when attempting to understand measured data such as that associated with grain-size distribution curves. The authors want to thank the discusser for bringing a number of points forward for discussion.

References

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