A study of critical state on an unsaturated silty soil

Q. Wang, D.E. Pufahl, and D.G. Fredlund

Abstract: Critical state models for unsaturated soils have been proposed in recent years; however, the proposed models have been based on limited experimental data. Compacted specimens have generally been used for research, and the complications of soil fabric resulting from the compaction procedures have brought difficulties into the interpretation of fundamental soil behavior. There is a need to undertake further laboratory research studies on unsaturated soils, particularly to obtain a fundamental understanding of the shear strength and critical state of unsaturated soils by testing soil specimens with simple soil structures. Suction-controlled triaxial drained shear tests on an unsaturated silt were carried out as part of this research program. Specimens were prepared by gradually consolidating the initially slurried soil. The resulting specimens had a relatively simple soil fabric and stress history. The results showed that applying suction to an initially saturated specimen has a similar influence on the stress–strain behavior and critical state characteristics as does increasing its density by applying a higher confining pressure. The critical state lines for the unsaturated soil corresponding to different soil suctions are parallel to those for the saturated soil on the $(q : p'')$, $(v : p'')$, and $(v_w : p'')$ planes.

Key words: critical state, shear strength, unsaturated soils, volume change, triaxial testing.

Résumé : Des modèles d'état critique ont été proposés au cours des récentes années; cependant, les modèles proposés ont été basés sur des données expérimentales limitées. Des spécimens compactés ont généralement été utilisés pour la recherche, et la complexité de la fabrique du sol résultant des procédures de compactage ont présenté des difficultés dans l'interprétation du comportement fondamental du sol. Il est nécessaire d'entreprendre d'autres recherches en laboratoire sur des sols non saturés, particulièrement pour acquérir une compréhension fondamentale de la résistance au cisaillement et de l'état critique des sols non saturés en faisant des essais sur des spécimens de sols ayant des structures simples. Des essais de cisaillement triaxial drainé avec succion contrôlée sur un silt non saturé ont été réalisés dans le cadre de ce programme de recherche. Les spécimens ont été préparés en consolidant graduellement le sol initialement sous forme de boue. Les spécimens de sol qui en résultent ont une fabrique et une histoire des contraintes relativement simples. Les résultats montrent que l'application d'une succion à un spécimen de sol initialement saturé a une influence sur le comportement contrainte–déformation et sur les caractéristiques de l'état critique semblable à l'accroissement de sa densité par l'application d'une pression de confinement plus élevée. Les lignes d'état critique pour le sol non saturé correspondant à différentes succions sont parallèles à celles du sol saturé sur les plans $(q : p'')$, $(v : p'')$, et $(v_w : p'')$.

Mots clés : état critique, résistance au cisaillement, sols non saturés, changement de volume, essais triaxiaux.

[Traduit par la Rédaction]

Introduction

The critical state frameworks for unsaturated soil mechanics have been proposed and compared with those of saturated soil mechanics (Sun et al. 2000; Wheeler and Sivakumar 1995; Maatouk et al. 1995; Alonso et al. 1990; Toll 1990). The smooth transition from the stress state variables for an unsaturated soil, $(\sigma - u_a)$ and $(u_a - u_w)$, (where σ is mean normal stress, u_a is pore air pressure, and u_w is pore-water pressure) to the single stress variable $(\sigma - u_w)$ for a saturated soil (Fredlund and Morgenstern 1977), forms the basis for this extension. The concepts of yielding, hardening, and critical state are the key elements comprising the critical state framework for saturated soils. These concepts can be extended using two independent stress state variables, $(\sigma - u_a)$ and $(u_a - u_w)$, to build a critical state framework for unsaturated soils. However, laboratory tests to determine critical state behaviors for unsaturated soils are still limited in number due to the complex and time-consuming nature associated with testing unsaturated soils. Moreover, previous research shows diversity in the results, for example, the reported critical state lines were different in patterns (Wheeler and Sivakumar 1995; Maatouk et al. 1995; Toll 1990).

Differences in the soil fabric could be at least partly responsible for the diversity in the test results. Compacted specimens were used in most previous research studies. The soil fabric resulting from compaction adds a complexity to isolating the fundamental shear strength component related to matric suction (Mancuso et al. 2000). The complex soil fabric also makes it difficult to determine whether a true critical state has been reached (Wheeler and Sivakumar 1995; Toll 1990). As a result, there remains a need to carry on additional

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Table 1. List of test series and specimen numbers.

				Initial		After consolidation and desaturation	
Test series	Specimen number	Matric suction, $(u_a - u_w)$ (kPa)	Net confining pressure, $(\sigma_3 - u_a)$ (kPa)	Water content, w (%)	Density, γ (g/cm^3)	Water content, w (%)	Density, γ (g/cm^3)
Sat-0	Sat-0–100	0	100	26.15	1.978	19.64	2.093
	Sat-0–200	0	200	27.37	1.994	17.08	2.191
	Sat-0–400	0	400	25.80	1.943	15.14	2.187
	Unsat-30–100-b	30	100	26.57	1.976	21.88	—
Unsat-100	Unsat-100–100-b	100	100	27.79	1.971	15.55	—
	Unsat-100–200-b	100	200	27.48	1.945	14.47	—
	Unsat-100–300-a	100	300	27.04	1.938	13.50	2.105
Unsat-200	Unsat-200–100-b	200	100	26.86	1.963	13.89	—
	Unsat-200–200-a	200	200	28.03	1.929	13.95	1.995
	Unsat-200–300-a	200	300	27.23	1.933	12.28	2.044

Fig. 1. Stress–strain curves for the saturated soil specimens (test series: Sat-0; matric suction, s = 0).

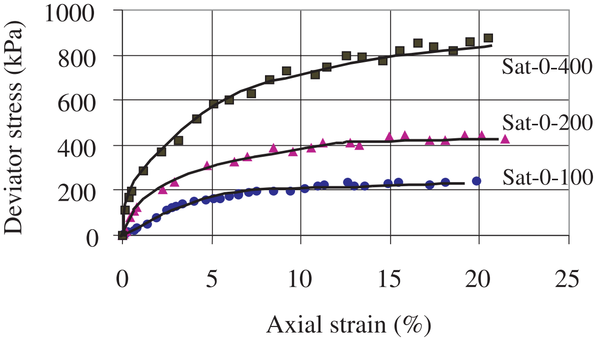
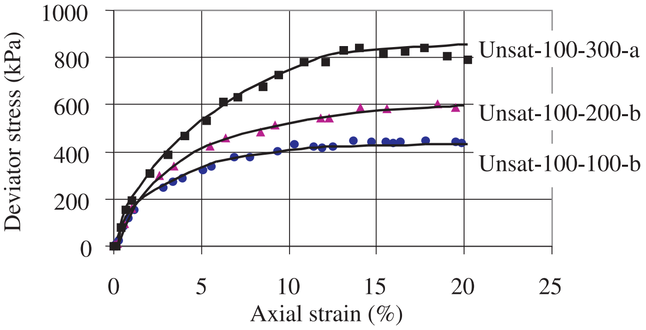


Fig. 2. Stress–strain curves for the unsaturated soil specimens (test series: Unsat-100; matric suction, s = 100 kPa).



laboratory research studies on unsaturated soils, particularly to obtain a fundamental understanding of the shear strength and critical state of unsaturated soils by testing soil specimens with a simple structure and stress history.

In this research study, suction-controlled triaxial drained shear tests on unsaturated "Botkin silt" were carried out to study the behavior of an unsaturated soil (Wang 2000). Specimens were prepared by gradually consolidating an initially slurried soil. The resulting specimens have a relatively simple soil fabric and stress history. This paper provides a description of the laboratory procedure used in the test program, presents the results, and provides an analysis of the data.

Soil sample and specimen preparation

The soil selected for the laboratory testing program consists of 41.8% sand and 58.2% silt and clay. The index properties (liquid limit, w_L; plastic limit, w_p; and plastic index, I_p) are 29.4, 13.8, and 15.6%, respectively. The soil is classified as CL according to the Unified Classification System.

Specimens were prepared by consolidating the soil slurry using a cylindrical mould (i.e., 70.5 mm diameter by 220 mm high). Approximately 1150 g of air-dried soil passing through a 0.75 mm sieve was mixed with distilled, deaired water to make a soil slurry, and sufficient water was added to allow entrapped air to escape easily. The slurry was then poured into the cylindrical mould and one-dimensional pressure was applied in increments from 2 kPa up to 100 kPa. The consolidation process took at least 10 days and the slurry was eventually consolidated to form a specimen for triaxial testing with an average length of 175 mm. During consolidation, water was added to the top of the specimen to ensure that saturation was maintained.

Specimens prepared in this way should have had a relatively simple structure and stress history. The specimens were saturated and assumed to be nearly uniform in terms of initial water content and density. Table 1 shows that for all ten specimens, the initial water content ranges from 25.8 to 28.0%; the initial density ranges from 1.929 to 1.994 g/cm^3.

The modified triaxial apparatus

A conventional strain-controlled triaxial apparatus was modified for the axially loaded compression tests on the soil specimens. A linear variable differential transformer (LVDT), a loading cell, three pressure transducers, a data collection system, and a computer were originally in place before the modification of the equipment. The modifications included: (1) a five-bar-high air entry ceramic disk glued around the perimeter to the top of the base pedestal, to allow the axis-translation technique to be used in applying matric suction; (2) a digital pressure–volume controller (GDS) to control pore-water pressure and to measure the

Fig. 3. Stress–strain curves for the unsaturated soil specimens (test series: Unsat-200; matric suction, s = 200 kPa).

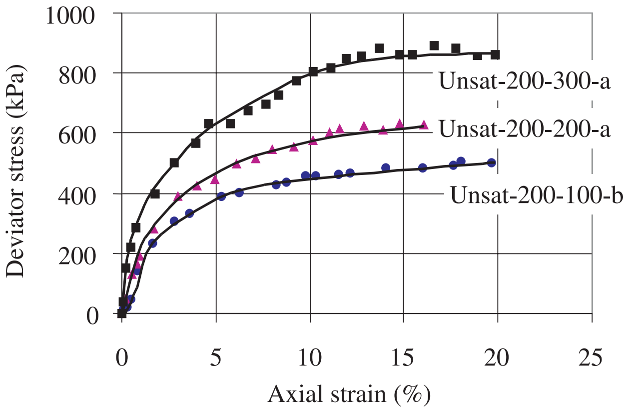
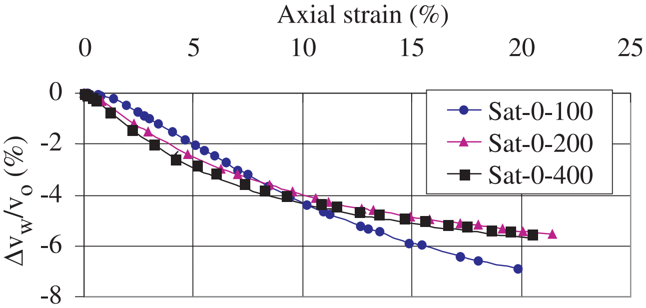


Fig. 4. Pore-water volume change versus axial strain for the saturated soil specimens (test series: Sat-0; matric suction, s = 0).



volume change of the pore water in the specimen during the test; (3) another GDS device to apply the cell pressure and measure the total volume change of the specimen by measuring the volume of water entering or exiting from the triaxial cell; and (4) a diffused air volume indicator to measure the amount of air diffused through the high air entry ceramic disk.

The GDS digital controllers used in this research had a pressure range of 0–2000 kPa and a volume change range of 0–900 mL. The reading is accurate to 1 kPa for pressure and 1 mm^3 for volume.

Testing program and procedures

The testing program can be roughly grouped into three series according to the magnitude of applied matric suction (Table 1).

For the saturated series of tests, the specimen number consists of the series number (Sat-0) followed by a number representing the net confining pressure, $(\sigma_3 - u_a)$ where σ_3 is the confining pressure. For the unsaturated series, in addition to the series numbers (Unsat-100 and Unsat-200) and the number representing the net confining pressure, a letter "a" or "b" is also included in the specimen number to represent the procedure used to carry out the consolidation and drying of the triaxial specimen. The letter "a" represents the procedure for drying (i.e., applied soil suction) after consolidation, while the letter "b" represents the procedure for drying simultaneously with consolidation. In the "a" procedure, the initially saturated soil specimen was first consolidated under the applied confining pressure. The consolidation process took about 12 days. Then, the soil suction was applied to desaturate the specimen. The drying process took about 10 more days. The "b" procedure took about 12 days to simultaneously consolidate and desaturate the specimen. In comparison, procedure "b" took less time, whereas procedure "a" yielded more information. The final water content and density after consolidation and drying were not influenced by the consolidation procedures. The final water content and density appeared to depend only on the final magnitude of the combined applied net confining pressure and matric suction (Table 1).

Fig. 5. Pore-water volume change versus axial strain for the unsaturated soil specimens (test series: Unsat-100; matric suction, s = 100 kPa).

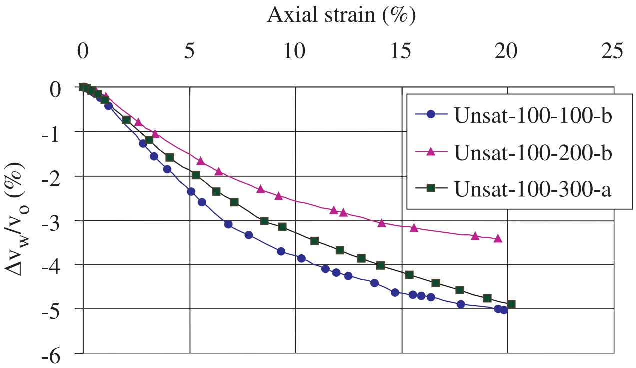
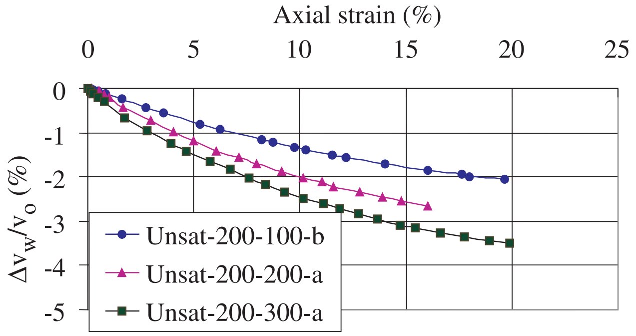


Fig. 6. Pore-water volume change versus axial strain for the unsaturated soil specimens (test series: Unsat-200; matric suction, s = 200 kPa).



After the consolidation and desaturation stages, the specimen was loaded axially to a maximum vertical strain of about 20%. The strain rate for the axial loading was 2.5×10^{-5} % per second. This drained-loading stage took 9–10 days.

Stress–strain behavior and critical state characteristics

The definition of critical state used herein is a condition of constant volume, constant water content, and constant

Fig. 7. Total and pore-water volume change versus axial strain for unsaturated soil specimens (test series: Unsat-100 and Unsat-200).

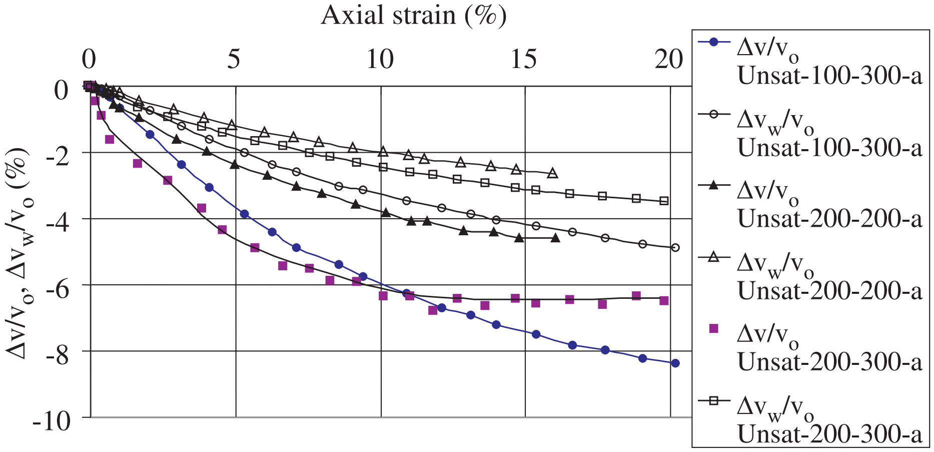
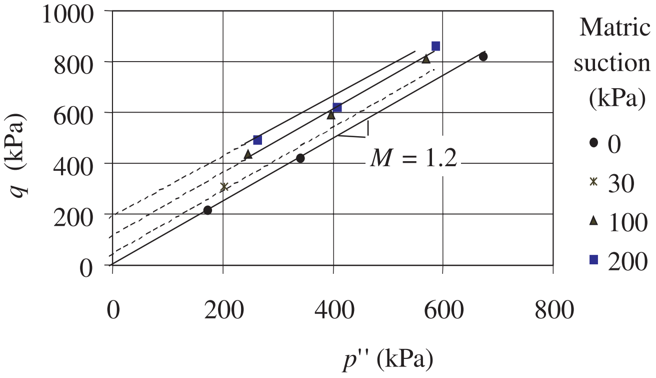


Table 2. Suggested critical state variables for unsaturated soils.

Name	Notation	Definition
Net mean stress	p''	$p'' = (1/3(\sigma_1 + \sigma_2 + \sigma_3) - u_a)$
Deviator stress	q	$q = (\sigma_1 - \sigma_3)$
Matric suction	s	$s = (u_a - u_w)$
Specific volume	v	$v = 1 + e$
Specific water volume	v_w	$v_w = 1 + wG_s$

Note: e is void ratio and G_s is specific gravity of soil solids.

Fig. 8. Critical state lines on the (q : p'') plane for the "Botkin silt" (test Series: Sat-0, Unsat-100, and Unsat-200).



shear strength that a soil reaches when subjected to triaxial loading under different test paths. The results of this research program will be described and discussed in this context. Figure 1 illustrates the stress–strain curves for saturated specimens (test series Sat-0). The stress–strain curves demonstrate typical behavior for a saturated and normally consolidated soil in triaxial loading. The deviator stress increases with strain and reaches a maximum or stable value at an axial strain range between 10 and 15%, indicating a critical state. The loading was terminated when the axial strain was about 20%. Specimens bulged at failure and no distinct shearing planes were observed. In some cases, particularly at higher net confining pressures, the stress–strain curves were not smooth. Since the loading cell was located outside of the triaxial cell, this phenomenon is believed to be due to friction along the loading ram.

Figures 2 and 3 are the stress–strain curves for the unsaturated specimens (test series Unsat-100 and Unsat-200). For the same net confining pressure, the strength of the unsaturated specimens is significantly greater than the strength of a saturated specimen. For example, the shear strength for specimen number (Unsat-100–100-b) is roughly twice that for specimen number (Sat-0–100). An increase in soil suction increased the shear strength. However, increases in soil suction did not affect the general shape of the stress–strain relationship. The shapes of the stress–strain curves for the unsaturated specimens are similar to those for saturated specimens. Similar to the saturated soil specimens, the shear stress of the unsaturated specimens stabilizes at an axial strain between 10 and 15%, again without distinct failure planes being observed in the specimen.

Figure 4 shows the pore-water volume change versus axial strain for the saturated soil specimens. In most cases, the pore-water volume changes tend to stabilize close to the end of loading (i.e., 20% strain). The volume of specimen number (Sat-0–100) was still changing at the termination of loading. It is not clear why this specimen behaved this way.

The pore-water volume change versus strain curves for the unsaturated soil specimens (Figs. 5 and 6) are also similar to those for the saturated soil tests (Fig. 4), with little change occurring close to the termination of loading. Pore-water volume change decreases with increasing suction. The ranges of the final pore-water volume change are 5.6–7.0%; 3.4–5.0%, and 2.1–3.5% for matric suction of 0, 100, and 200 kPa, respectively. However, once again matric suction does not have an influence on the general shape of the stress–strain curve, but it does influence the maximum value of the shear stress.

Figure 7 presents both the total and pore-water volume changes during loading for specimens numbered (Unsat-100–300-a), (Unsat-200–200-a), and (Unsat-200–300-a). The trends of the total volume changes are similar to those for pore-water volume changes. Both relationships become stable close to the termination of loading. As expected, the total volume change is higher than the pore-water volume change. For example, for specimen number (Unsat-200–200-a), the total

Fig. 9. Critical state lines on the (v : p'') plane for the "Botkin silt" (test series: Sat-0 and Unsat-200).

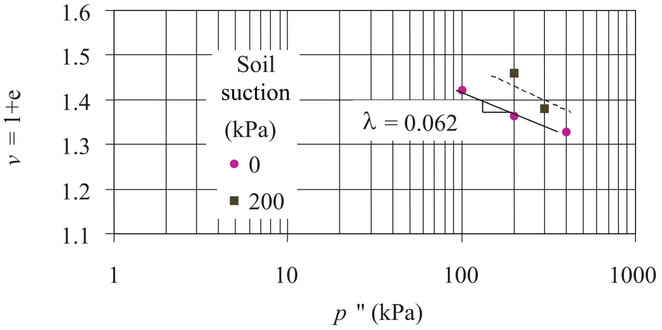
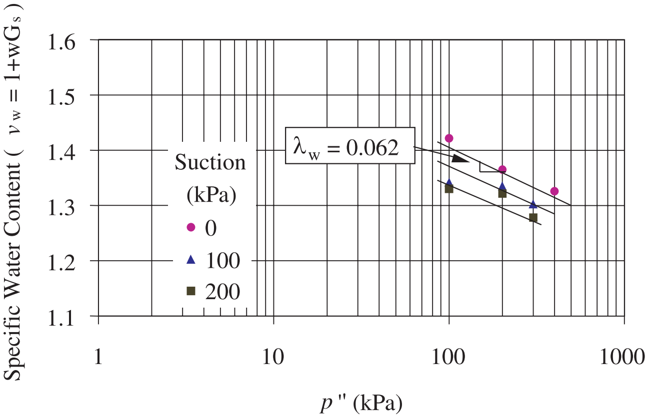


Fig. 10. Critical state lines on the (v_w : p'') plane for the "Botkin silt" (test series: Sat-0, Unsat-100, and Unsat-200).



volume change is 4.6% and the pore-water volume change is only 2.7%.

Critical state lines for unsaturated soils

The critical states for the unsaturated soils are shown on stress–strain curves (Figs. 2 and 3), pore-water volume change versus strain curves (Figs. 5 and 6), and total volume change versus strain curves (Fig. 7). These test results seem to support that the net mean stress, p'', deviator stress, q, specific volume, v, and specific water volume, $v_{w,}$ can be used as critical state variables for unsaturated soils (Table 2). The variables p'', q, and v have been suggested as the critical state variables for unsaturated soils by several researchers (Wheeler and Sivakumar 1995; Maatouk et al. 1995; Rampino et al. 1998; and Adams and Wulfsohn 1997). In comparison, data on v_w at critical state is scarce. The v_w measured by Wheeler and Sivakumar (1995) was not stable at critical state, but Rampino et al. (1998) measured stabilized v_w at critical state. Figure 7 indicates that v_w tends to be stable when v tends to be stable, supporting the view that v_w should be treated as a critical state variable for unsaturated soils, at least in the low suction ranges. However, since the amount of water coming out of a specimen during triaxial loading at high suctions can be small, it may not be reliable to use v_w as an indicator of critical state. More data are required before a conclusion can be reached on this matter.

Figures 8, 9, and 10 show the CSLs on the (q : p''), (v : p''), and (v_w : p'') planes. The unsaturated critical state lines are parallel to the saturated lines. Only one critical state line for the unsaturated soil on the (v : p'') plane was obtained in this research study (Fig. 9). The total volume changes during loading were measured with acceptable accuracy for three specimens, i.e., (Unsat-100–300-a), (Unsat-200–200-a), and (Unsat-200–300). The critical state equations are:

[1] $q = Mp'' + q_o(s)$

[2] $v = v_o(s) - \lambda \ln p''$

[3] $v_w = v_{wo}(s) - \lambda_w \ln p''$

where

M, λ, and λ_w (where $\lambda = \lambda_w$) are the slopes of the CSL for saturated soils on the (q : p''), (v : p''), and (v_w : p'') planes, respectively;

$q_o(s)$ is the final intercept of the CSL with the q axis;

$v_o(s)$ is the specific volume of the soil at critical state with p'' = 1 kPa (a reference pressure); and

$v_{wo}(s)$ is the specific pore-water volume of the soil at critical state with p'' = 1 kPa.

The slope of the CSL on the (q : p'') and (v : p'') planes, M, and λ are independent of matric suction and can be determined from the saturated critical state line. The intercepts $q_o(s)$, $v_o(s)$, and $v_{wo}(s)$ are functions of matric suction. The results from this research are essentially in agreement with Alonso et al. (1990), Wheeler and Sivakumar (1995), Rampino et al. (1998), and Adams and Wulfsohn (1997). Equations [1], [2], and [3] have simpler forms because these equations are proposed based on the testing of specimens with a relatively simple soil fabric and stress history. M and λ may be dependent on matric suction in other cases of more complex soil failure.

Conclusions and recommendations

Based on the triaxial drained shear tests on the soil used in this research program, conclusions on critical states for unsaturated soils can be reached.

(1) The shearing behavior and critical state characteristics for unsaturated soil specimens are similar to those for saturated soil specimens. Applying a suction to a saturated soil specimen has a similar influence on the general shearing behavior and critical state characteristics as increasing its density by applying a higher confining pressure.

(2) The critical state lines for the unsaturated soils corresponding to different soil suctions are lines parallel to those for the saturated soil on the (q : p''), (v : p''), and (v_w : p'') planes. More experimental results on simple soil fabric specimens would be of value.

The soil specimens used in this research study were consolidated from a slurry and had relatively simple soil fabrics and stress history. The extension of the conclusions from this soil to other soil types should take into account the influence of soil fabric and stress history on soil behavior. It would also be beneficial to expand the testing program on the "Botkin silt" in order to obtain a larger database of results.

References

Adams, B.A., and Wulfsohn, D. 1997. Variation of the critical-state boundaries of an agriculture soil. European Journal of Soil Science, **48**: 739–758.

Alonso, E.E., Gens, A., and Josa, A. 1990. A constitutive model for partly saturated soils. Géotechnique, **40**(3): 405–530.

Fredlund, D.G., and Morgenstern, N.R. 1977. Stress state variables for unsaturated soils. ASCE Journal Geotechnical Engineering Division, **103**(GT5): 447–466.

Maatouk, A., Leroueil, S., and Rochelle, P.L.A. 1995. Yielding and critical state of a collapsible unsaturated silty soil. Géotechnique, **45**(3): 465–477.

Mancuso, C., Vassallo, R., and Vinale, F. 2000. Effects of moulding water content on the behaviour of an unsaturated silty sand. *In* Proceedings of the 1st Asian Conference on Unsaturated Soils (UNSAT-ASIA 2000), Singapore, 18–19 May 2000. *Edited by* H. Rahardjo, D.G. Toll, and E.C. Leong. Balkema, Rotterdam, ISBN 90 5809 1392, pp. 545–550.

Rampino, C., Mancuso, C., and Vinale, F. 1998. Behavior of a compacted silty sand during suction controlled tests. *In* Proceedings of the 2nd International Conference on Unsaturated Soils UNSAT2, Beijing, China. Vol. 1, pp. 108–113.

Sun, D.A., Matsuoka, H., Yao, Y.P., and Ichihara, W. 2000. Three-dimensional elasto-plastic model for unsaturated soils. *In* Proceedings of the 1st Asian Conference on Unsaturated Soils (UNSAT-ASIA 2000), Singapore, 18–19 May 2000. *Edited by* H. Rahardjo, D.G. Toll, and E.C. Leong. Balkema, Rotterdam, ISBN 90 5809 1392, pp. 153–158.

Toll, D.G. 1990. A framework for unsaturated soil behavior. Géotechnique, **40**(1): 31–44.

Wang, Q. 2000. A laboratory study on the critical state for an unsaturated silt. M.Sc. thesis, Department of Civil Engineering, University of Saskatchewan, Saskatoon, Canada.

Wheeler S.J., and Sivakumar, V. 1995. An elasto-plastic critical state framework for unsaturated soil. Géotechnique, **45**(1): 35–53.