Unsaturated Soils, Jucá, de Campos & Marinho (eds)
© 2002 Swets & Zeitlinger, Lisse, ISBN 90 5809 371 9

Representation and estimation of the shrinkage curve

M.D. Fredlund
Soil Vision Systems Ltd., Saskatoon, SK., Canada

G.W. Wilson
Department of Mining and Mineral Process Engineering University of B.C., Vancouver, BC, Canada

D.G. Fredlund
University of Saskatchewan, Saskatoon, SK, Canada

Third International Conference on Unsaturated Soils, UNSAT 2002
March 10-13, 2002, Recife, Brazil

ABSTRACT: The shrinkage characteristics of a large number of soils was taken from the research literature and used to develop a mathematical relationship to model the shrinkage behavior of (i) slurried, (ii) compacted, and (iii) undisturbed soils. The shrinkage equation was shown to fit a wide variety of soils with different stress histories. The shrinkage curve can be used in conjunction with the soil–water characteristic curve for the calculation of the void ratio versus soil suction relationship. The void ratio relationship forms the soil suction boundary for the void ratio constitutive surface. An estimation method was also proposed for the estimation of the shrinkage curve based on the soil–water characteristic curve air-entry value, volume–mass properties and the initial state of the soil.

1 INTRODUCTION

The shrinkage curve for a soil can be used in the determination of volume–mass unsaturated soil property functions (Fredlund 2000). The volume–mass unsaturated soil property functions are required when undertaking numerical modeling studies involving unsaturated soils. A fixed form for a mathematical representation of the shrinkage curve allows other volume–mass constitutive surfaces to be readily computed. This paper presents methods for representing and estimating the shrinkage curve as well as strategically linking the shrinkage curve to the constitutive relations for an unsaturated soil.

The shrinkage curve is part of the constitutive behavior of the soil and as such has a relationship to other unsaturated soil property functions. A mathematical equation describing the shrinkage curve, as well as a theoretical method for estimating the shrinkage curve, is presented in this paper.

Numerous mathematical equations have been presented to represent the soil-water characteristic curve (Sillers 1996). This paper uses the soil-water characteristic curve in conjunction with the shrinkage curve to give the volume change (i.e. void ratio or specific volume) versus soil suction relationship.

2 SHRINKAGE THEORY FOR SOILS

Shrinkage curves can be represented as either the specific volume (or the inverse of dry density of a soil) versus water content, or void ratio versus water content (Haines 1923). A plot showing a typical representation of the shrinkage curve is shown in Figure 1. Marinho (1994) identified three basic types of shrinkage behavior that an initially saturated specimen can exhibit and these

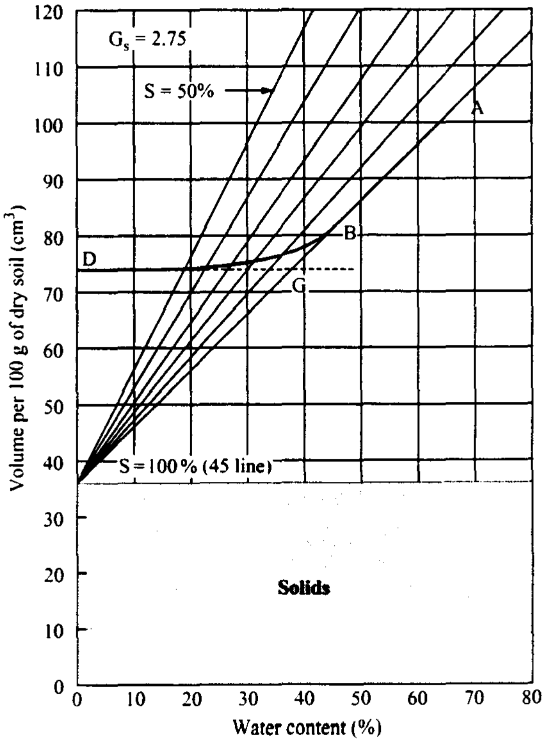

Figure 1. Drying phenomenon with theoretical lines for a constant degree of saturation (Marinho 1994).

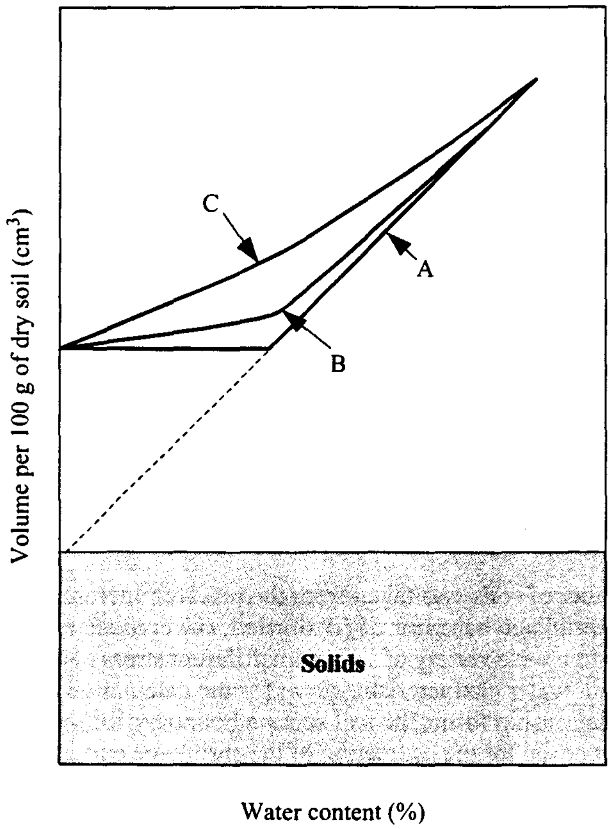

Figure 2. Three basic characteristics of shrinkage curves (Marinho 1994).

are shown as curves A, B, and C in Figure 2. Particle size distribution and stress history are the primary factors controlling shrinkage behavior.

The data collected for analysis in this paper represent three different initial states. Soils were classified as follows: (i) *undisturbed samples from the field*, (ii) *compacted specimens*, or (iii) *slurred specimens from near the liquid limit.*

The typical behavior of the drying of a slurred soil can be seen in Figure 1. The soil specimen is initially fully saturated and follows the saturation line until air begins to enter the soil voids. This point is an indication of the *Air-Entry Value* (AEV) point as shown in Figure 3. As the soil continues to dry, it reaches the minimum void ratio at which there is no further volume change as the soil suction increases to 1,000,000 kPa. The water content at the intersection between the minimum void ratio and the saturation line is referred to as the *Shrinkage Limit.* A somewhat different pattern of shrinkage behavior is observed in soils that are dried from degrees of saturation less than 100% as shown in Figure 4.

3 DEVELOPMENT OF THE SHRINKAGE EQUATION

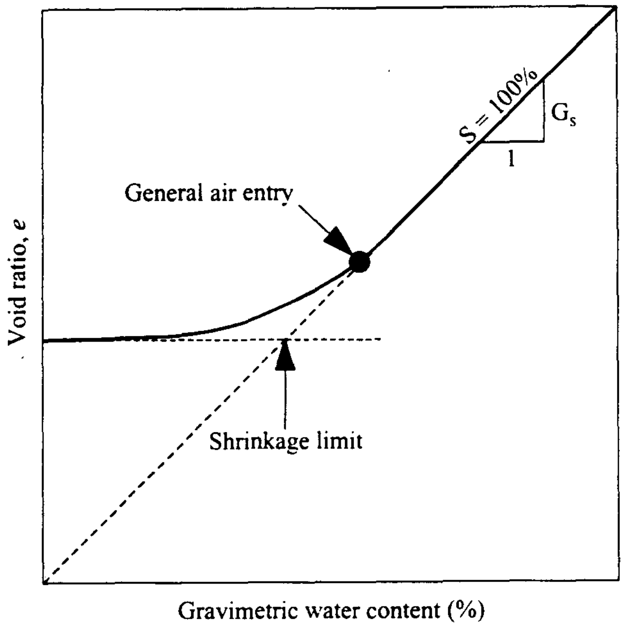

Figure 3. Volume–mass relationships for the drying curve of an initially slurried soil specimen.

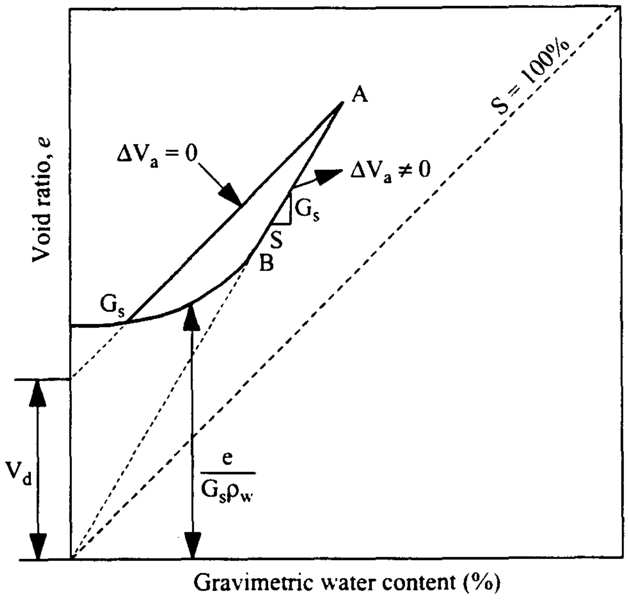

Figure 4. Volume–mass shrinkage relationship for an unsaturated soil specimen. G_s = specific gravity, S = degree of saturation, e = void ratio. V_a = volume of air.

A database of soil shrinkage curves was assembled from research literature to characterize a variety of possible shrinkage curves. Soils were organized according to the soil state at the beginning of the shrinkage test. Because of the influence of structure on the test results, the soils were grouped as follows: (i) *undisturbed*, (ii) *compacted near the plastic limit*, or (iii) *slurried near the liquid limit.* Figure 5 shows a group of typical soils from the dataset used in the analysis.

The hyperbolic nature of the shrinkage curve forms the basis for developing a suitable mathematical equation. The desire is to develop an equation having

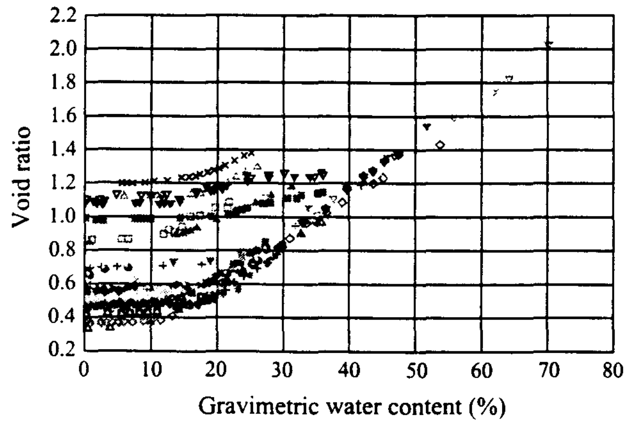

Figure 5. Subset of dataset used to characterize soil shrinkage behavior.

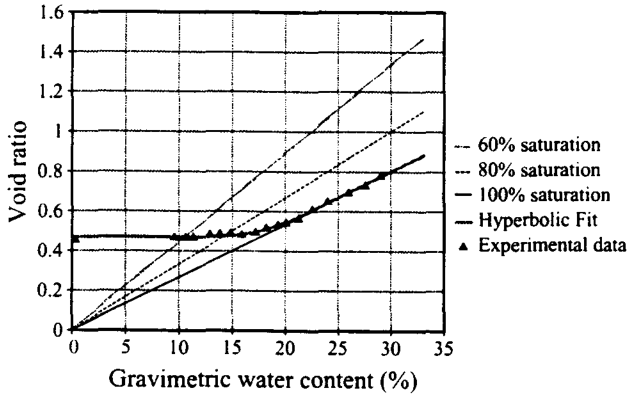

Figure 6. Fit of London clay from Croney & Coleman (1953) using the proposed shrinkage equation with parameters; $a_{sh} = 0.47$, $b_{sh} = 0.176$, $c_{sh} = 10.56$, $R^2 = 0.994$.

parameters with physical meaning. The proposed equation can be written as follows:

$$e(w) = a_{sh}\left[\frac{w^{c_{sh}}}{b_{sh}^{c_{sh}}} + 1\right]^{1/c_{sh}} \tag{1}$$

where a_{sh} = the minimum void ratio, $e_{\min}$; b_{sh} = slope of the line of tangency; c_{sh} = curvature of the shrinkage curve; and $a_{sh}/b_{sh} = G_s/S$ = constant for a specific soil.

A non-linear, least-squares fitting algorithm was used to fit the equation to various data sets. A typical fit of the data for an initially saturated soil is shown in Figure 6.

4 STUDY OF THE PARAMETERS ASSOCIATED WITH THE SHRINKAGE EQUATION

The shrinkage curve equation is sufficiently general to fit a variety of measured shrinkage curves. The effects of soil structure and initial degree of saturation can be accommodated using the proposed equation and the soil parameters have physical meaning.

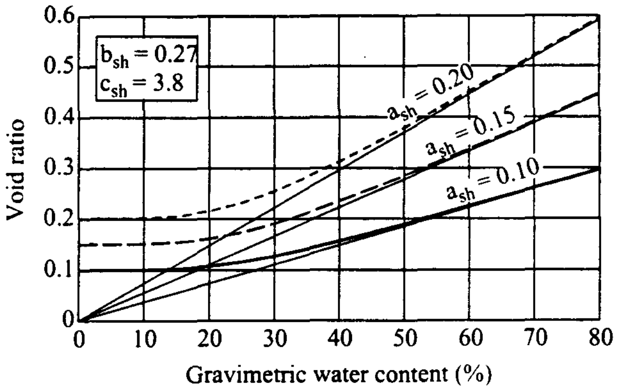

Figure 7. Effect of varying the a_{sh} parameter on the shrinkage equation.

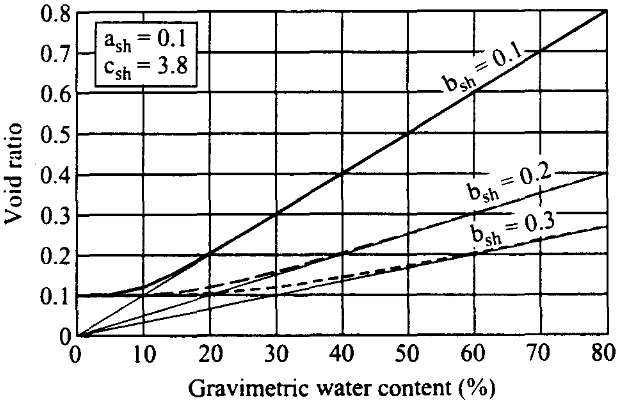

Figure 8. Effect of varying the b_{sh} parameter on the shrinkage equation.

The a_{sh} parameter represents the minimum void ratio possible from the shrinkage of the soil. If degree of saturation, S, is held constant, the relationship between void ratio and gravimetric water content is linear. The slope of the line between void ratio and water content is related to the initial volume–mass properties of the soil specimen as shown in Equation (2). The slope of the tangent line must be equal to a_{sh}/b_{sh}. The effect of varying the a_{sh} parameter can be seen in Figure 7:

$$\frac{a_{sh}}{b_{sh}} = \frac{G_s}{S} \tag{2}$$

The influence of changing the b_{sh} parameter is shown in Figure 8. The b_{sh} parameter is equal to the slope of the degree of saturation line on the shrinkage plot. Equation (2) dictates that once the minimum void ratio of a soil is known, the b_{sh} parameter can be calculated. The b_{sh} parameter can then be considered a fixed parameter when fitting the equation to experimental data by a least-squares analysis.

The c_{sh} equation parameter controls the rate of curvature of the shrinkage curve as the soil begins to

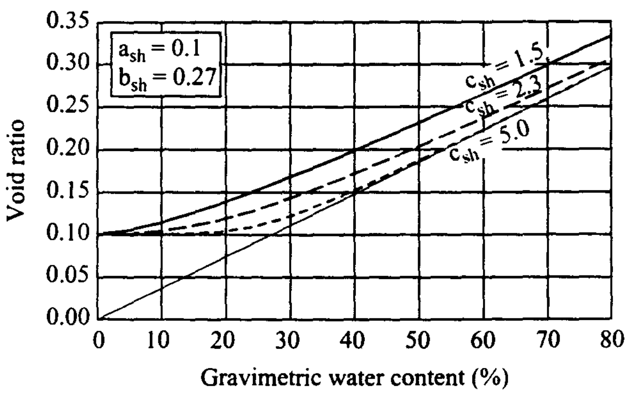

Figure 9. Effect of varying the c_{sh} parameter on the shrinkage equation.

desaturate as shown in Figure 9. The variation in the c_{sh} parameter allows the three basic types of drying behavior to be best fit.

5 ESTIMATION OF THE SHRINKAGE CURVE

The volume change in a soil specimen is generally not measured when performing a laboratory test for the soil–water characteristic curve. The experimental data associated with the soil–water characteristic curve is generally the gravimetric water content versus soil suction. The shrinkage curve can be used to estimate the void ratio versus soil suction relationship.

A dataset of shrinkage curves published in journal papers was selected and best fit with the proposed equation for the shrinkage curve. Since the ratio between the a_{sh} and b_{sh} equation parameters is fixed by the degree of saturation, S, and the specific gravity, G_s, the only two parameters that can vary are a_{sh} and c_{sh}. When the shrinkage curves were grouped according to their initial states (i.e. *undisturbed* or *slurried*), a consistency in the c_{sh} parameter was noted. The average value of the c_{sh} parameter for *undisturbed* soil specimens was 9.57 with a standard deviation of ±9.33. The average value of the c_{sh} parameter for initially *slurried* soil specimens was 25.31 with a standard deviation of ±25.41. Finally, the average value for the c_{sh} parameter for *compacted* soil specimens was 8.47 with a standard deviation of ±3.56.

The minimum void ratio or a_{sh} equation parameter can be estimated from data presented by Holtz & Kovacs (1981) as shown in Figure 10 or from other available information. The data allows the shrinkage limit to be estimated once the plastic limit is known. The linear regressions on the data can be used as estimators for the a_{sh} curve parameter. The results of the above estimation technique were applied to a randomly selected group of the soils in the dataset. One example is shown in Figure 11 for a dataset produced by Nascimento (1961).

In summary, the estimation techniques worked well to estimate the shape of the shrinkage curve but not as well to estimate the minimum void ratio for most soils. The most critical parameter in the estimation of the shrinkage curve is the minimum void ratio, a_{sh}. It is therefore suggested that the minimum void ratio be experimentally measured, if possible. The c_{sh} provides the shape of the shrinkage curve. The physical difference in the shrinkage properties produced by varying the c_{sh} parameter was found to be minimal and as such the estimation is considered reasonable.

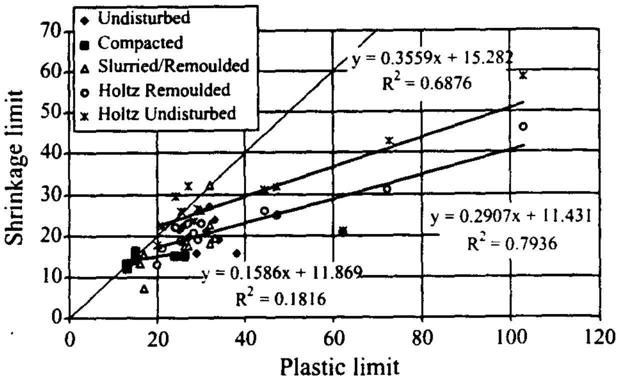

Figure 10. Relationship between the shrinkage limit and the plastic limit for soils in the current dataset and soils from Holtz & Kovacs (1981).

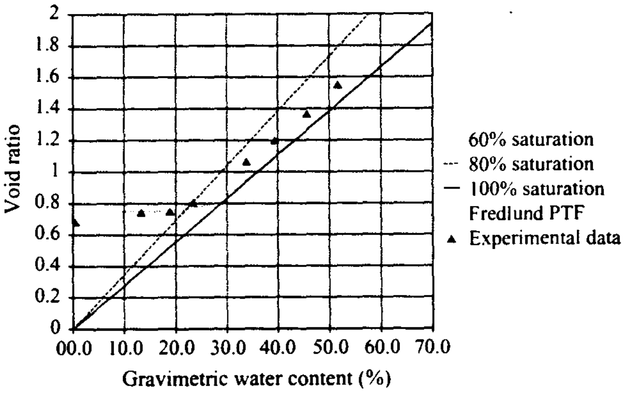

Figure 11. Results of shrinkage pedo-transfer function for a alluvial clay originally published by Nascimento (1961), $a_{sh} = 0.749$, $b_{sh} = 0.27$, $c_{sh} = 9.57$, $R^2 = 0.933$.

6 CALCULATION OF VOID RATIO VERSUS SOIL SUCTION

The soil–water characteristic curve describes the relationship between the water content of a soil and soil suction. The volume changes occurring when drying a plastic soil may be substantial and are of relevance in the interpretation of the data. An example of soil–water characteristic curve data for a highly plastic black clay (Dagg et al. 1966) is shown in Figure 12.

If the change in volume is measured during the drying process, it is possible to determine the void ratio and plot the shrinkage curve. However, the void ratio versus soil suction relationship can also be estimated through the use of the shrinkage curve.

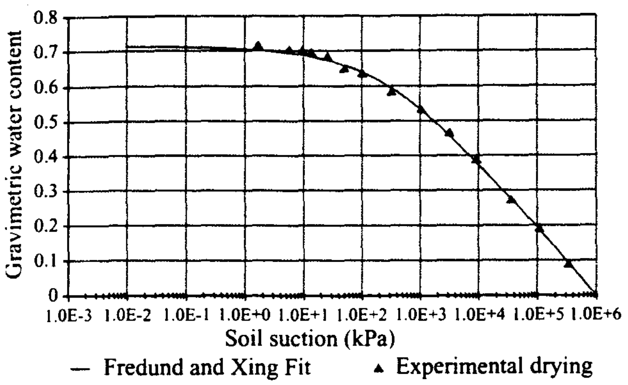

Figure 12. Experimental data for black clay originally presented by Dagg et al. (1966).

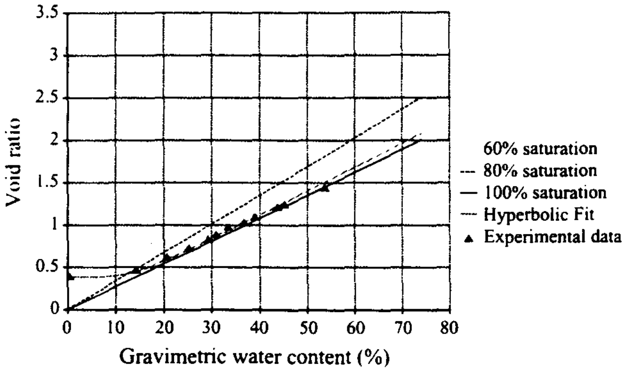

Figure 13. Experimental shrinkage data for black clay originally presented by Dagg et al. (1966), a_{sh} = 0.386, $b_{sh} = 0.14$, $c_{sh} = 5.04$, $R^2 = 0.993$.

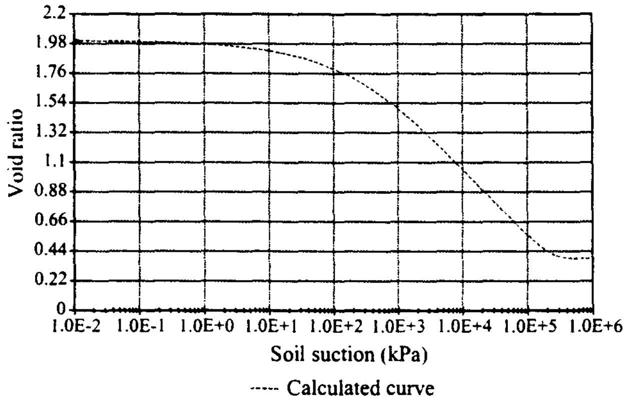

Figure 14. Calculated volume change curve for a black clay originally presented by Dagg et al. (1966).

Typical measurements for the shrinkage curve of a soil are performed at a net normal stress of zero. The shrinkage curve shown in Figure 13 for the black clay, represents the measurement of state along the limiting boundary of the constitutive surface. Figure 14 shows the void ratio versus soil suction constitutive relationship computed from the soil–water characteristic curve and the shrinkage curve. The curve for void ratio versus soil suction was calculated by combining the Fredlund & Xing (1994) equation.

Equation (3) with the proposed equation for the shrinkage curve (Eq. (1)) is represented as follows:

$$w_w(\psi) = w_s\left[1 - \frac{\ln\left(1+\frac{\psi}{h_r}\right)}{\ln\left(1+\frac{10^6}{h_r}\right)}\right] \times \left[\frac{1}{\left[\ln\left[\exp(1)+\left(\frac{\psi}{a_f}\right)^{n_f}\right]\right]^{m_f}}\right] \qquad (3)$$

The relationship between void ratio and soil suction represents the limiting boundary condition of the void ratio constitutive surface.

7 CONCLUSIONS

The paper presents information on the general character of the shrinkage curve relationship and its relevance to unsaturated soil mechanics. A mathematical equation is proposed to describe the shrinkage curve. Each of the soil parameters used in the equation has a physical meaning. The proposed equation can be best fit to a dataset of volume and water content measurements on a soil. Also, information is provided regarding how best to estimate the shrinkage curve for a particular soil.

The shrinkage curve can be combined with the soil–water characteristic curve for a soil to compute the volume change function between void ratio and soil suction for an unsaturated soil.

REFERENCES

Croney, D. & Coleman, J.D. 1953. Soil moisture suction properties and their bearing on the moisture distribution in soils. *Proceedings of the 3rd international conference on soil mechanics and foundation engineering, 1 S. 1–4, 13–18.*

Fredlund, M.D. 2000. *Unsaturated soil property functions.* Ph.D. Dissertation. University of Saskatchewan, Saskatoon, 292 pp.

Fredlund, D.G. & Xing, A. 1994. Equations for the soil-water characteristic curve. *Canadian Geotechnical Journal* 31(3): 521–532.

Haines, W.B. 1923. The volume-changes associated with variations of water content in soil. *Journal of Agricultural Science* 13: 296–310.

Holtz, R.D. & Kovacs, W.D. 1981. *An introduction to geotechnical engineering.* Englewood Cliffs, NJ: Prentice-Hall.

Marinho, F. 1994. *Shrinkage behaviour of some plastic soils.* Ph.D. Dissertation. Imperial College, London, UK.

Nascimento, U. 1961. Contribution on the physical interpretation of soil properties.

Sillers, W.S. 1996. *The mathematical representation of the soil-water characteristic curve.* M.Sc. Thesis. University of Saskatchewan, Saskatoon.