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Coupled solution for the prediction of volume change in expansive soils

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ABSTRACT: This paper deals with the swelling behavior of an expansive soil under transient wetting process. The swelling behavior of soils is predicted by using a coupled procedure that involves the equilibrium of the soil structure and the continuity of the water seepage. The coupled equations are formulated based on the general theory of consolidation for unsaturated soils. The finite element method is used for the solution of the coupled equations. The constitutive relationships for both the soil structure and the pore-water phase are formulated by using an incrementally non-linear elastic model. The phenomenological model approach is based on the concept of state surfaces that are obtained from a laboratory-testing program. The hydraulic conductivity of the swelling soil is estimated from the saturated hydraulic conductivity and soil–water characteristic curve. A two-dimensional example problem is analyzed to illustrate the applicability of the numerical modeling for the prediction of the behavior of an expansive soil mass under a transient wetting process.

1 INTRODUCTION

Swelling is the response of an expansive soil to a transient wetting process. The swelling behavior depends on the soil properties and the boundary loading conditions imposed on the soil mass during a wetting process. The swelling of a soil in response to transient seepage requires a coupled solution involving the soil mass equilibrium and the continuity of the pore-water.

A numerical solution that makes use of a theoretical two-dimensional non-linear elastic model is presented to simulate of the volume change and suction changes associated to the wetting process in an expansive soil mass. The model is formulated based on the general theory of consolidation for unsaturated soils as proposed by Fredlund & Rahardjo (1993). The volume change constitutive relationships for the soil under unsaturated soil conditions are formulated using the two stress state variables, namely, net normal stress, $(\sigma - u_a)$, and matric suction, $(u_a - u_w)$, where σ is the total normal stress, u_a is the pore-air pressure and u_w is the pore-water pressure. Darcy's law is applied to the flow of water through the soil under both saturated and unsaturated conditions. The following assumptions have been made for the model formulation: (1) the soil is isotropic, (2) the air phase is continuous, (3) the soil follows an incrementally non-linear elastic constitutive relationship, (5) soil particles and the pore-water are incompressible.

The elastic moduli, the hydraulic conductivity and water storage are functions of the stress state variables. The elastic moduli are calculated from the constitutive surfaces for both void ratio and degree of saturation. These constitutive surfaces can be obtained from a conventional oedometer testing program. The measured constitutive surfaces allow the definition of the non-linear elastic moduli functions (Fredlund & Rahardjo 1993). The Poisson's ratio for the soil must be either assumed to be a function of the stress state or a constant value.

This paper presents a numerical solution for the prediction of volume change due to the wetting of an expansive soil mass that rests under a flexible cover. The solution results are illustrated through the use of a coupled seepage–stress equilibrium analysis. A finite element program, called COUPSO (Pereira 1996) was used for the analysis of the example problem.

2 THEORY OF VOLUME CHANGE FOR UNSATURATED SOILS

The consolidation of an unsaturated soil under isothermal conditions, requires simultaneously the solution of the governing equations describing equilibrium for the soil structure and mass flow of the air and water phases. These equations require constitutive relationships for the constituent phases as well as flow laws for the fluid phases. Continuity requirement must be satisfied in order to ensure the consistency in the solution.

The consolidation of an unsaturated soil can occur by assuming a continuous air phase that drains immediately in response to environmental changes. In this case only one equilibrium condition and seepage condition need to be considered in the coupled solution.

2.1 *Continuity requirements*

The continuity requirement for an unsaturated soil can be stated as follows (Fredlund & Rahardjo 1993):

$$\frac{\Delta V_v}{V_0} = \frac{\Delta V_w}{V_0} + \frac{\Delta V_a}{V_0} \tag{1}$$

where V_0 = initial overall volume of the referential soil element, V_v = volume of soil voids, V_w = volume of water, and V_a = volume of air.

Using a Cartesian coordinate system and referencing deformation to an elemental volume, the total volumetric deformation of an unsaturated soil element, $d\varepsilon_v$, can be written as the sum of the normal strains:

$$d\varepsilon_v = \frac{dV_v}{V_0} = d\varepsilon_x + d\varepsilon_y + d\varepsilon_z \tag{2}$$

where $d\varepsilon_x$, $d\varepsilon_y$, $d\varepsilon_z$ = normal strain components in x-, y-, and z-direction, respectively.

2.2 *Constitutive relationships*

Two stress state variables are used to describe volume change behavior of an unsaturated soil (Fredlund & Morgenstern 1977). The use of these stress state variables allows the combination of volume changes of both soil structure and water phase to be predicted using constitutive soil properties.

Fredlund & Rahardjo (1993) presented the following constitutive relationships for the soil structure and the water phase in compressibility forms:

$$\frac{dV_v}{V_0} = m_1^s\, d(\sigma_{\text{mean}} - u_a) + m_2^s\, d(u_a - u_w) \tag{3}$$

$$\frac{dV_w}{V_0} = m_1^w\, d(\sigma_{\text{mean}} - u_a) + m_2^w\, d(u_a - u_w) \tag{4}$$

where m_1^s = coefficient of total volume change with respect to net normal stress, m_2^s = coefficient of total volume change with respect to changes in matric suction, m_1^w = coefficient of the pore-water volume change with respect to changes in net normal stress, m_2^w = coefficient of the pore-water volume change with respect to changes in matric suction, and σ_{mean} = the mean net normal stress [i.e. $(\sigma_x + \sigma_y + \sigma_z)/3$].

The coefficients of total volume changes can be calculated from constitutive surfaces for void ratio and degree of saturation of the soil (Fredlund & Rahardjo 1993):

$$m_1^s = \frac{1}{1+e_0}\frac{de}{d(\sigma_{\text{mean}} - u_a)} \tag{5}$$

$$m_1^s = \frac{1}{1+e_0}\frac{de}{d(u_a - u_w)} \tag{6}$$

$$m_1^w = \frac{S}{1+e_0}\frac{de}{d(\sigma_{\text{mean}} - u_a)} + \frac{e}{1+e_0}\frac{dS}{d(\sigma_{\text{mean}} - u_a)} \tag{7}$$

$$m_2^w = \frac{S}{1+e_0}\frac{de}{d(u_a - u_w)} + \frac{e}{1+e_0}\frac{dS}{d(u_a - u_w)} \tag{8}$$

where e = void ratio, e_0 = initial void ratio, and S = degree of saturation.

Assuming that the soil behaves as an incrementally isotropic and linear elastic material, the soil structure stress–strain constitutive relationships can also be written as follows (Fredlund & Rahardjo 1993):

$$d\varepsilon_{ij} = \frac{1+\mu}{E}d(\sigma_{ij} - u_a) - \frac{\mu}{E}d(\sigma_{kk} - 3u_a)\delta_{ij} + \frac{d(u_a - u_w)}{H}\delta_{ij} \tag{9}$$

where ε_{ij} = components of the strain tensor for the soil structure, σ_{ij} = components of the total stress tensor for the soil structure, δ_{ij} = the Kronecker delta, μ = Poisson's ratio, $E = 3(1-2\mu)/m_1^s$, elastic modulus for the soil structure with respect to a change in net normal stress, and $H = 3/m_2^s$, elastic modulus for the soil structure with respect to a change in matric suction.

Assuming that the pore-water phase is incompressible, the volumetric water phase constitutive relationship can also be formulated in a semi-empirical approach as follows (Fredlund & Rahardjo 1993):

$$\frac{dV_w}{V_0} = \frac{1}{E_w}d(\sigma_{ii} - 3u_a) + \frac{1}{H_w}d(u_a - u_w) \tag{10}$$

where $E_w = 3/m_1^w$, volumetric water modulus associated to a change in net normal stress and $H_w = 1/m_2^w$, volumetric water modulus associated to a change in matric suction.

The constitutive relationships expressed by Equations (9) and (10), can be used in coupled solutions of problems associated to unsaturated soils in two- or three-dimensions. Such solutions require experimental results to properly define the phenomenological soil constitutive relationships, including the Poisson's ratio for the soil.

2.3 *Flow law*

Darcy's law can be used to describe water flow through soils in both saturated and unsaturated condition (Freeze & Cherry 1979). A non-linear relationship can be used to take into account the dependency between the hydraulic conductivity and the pore-water pressure in the unsaturated soil mass. For the case where the Cartesian coordinates are the same as the direction of the major and minor hydraulic conductivity values, Darcy's law is written as follows:

$$q_i = -k_{wi}\frac{\partial}{\partial x_i}\left(\frac{u_w}{\gamma_w} + Y\right) \tag{11}$$

where q_i = Darcy's flux in i-direction; k_{wi} = hydraulic conductivity in i-directions: γ_w = unit weight of water and Y = elevation.

2.4 *Equilibrium equations*

Equations of equilibrium for the soil structure of an unsaturated soil are

$$\sigma_{ij,j} + b_i = 0 \tag{12}$$

where σ_{ij} = components of the net total stress tensor and b_i = components of the body force vector.

2.5 *Water continuity equation*

The water continuity equation in an unsaturated soil is written as follows (Fredlund & Rahardjo 1993):

$$\frac{\partial(\rho_w V_w/V_o)}{\partial t} + \nabla\cdot(\rho_w q) = 0 \tag{13}$$

where ρ_w = water density, $\nabla = (\partial/\partial x)i + (\partial/\partial y)j + (\partial/\partial z)k$, the divergence operator, $q = q_x i + q_y j + q_z k$, the Darcy's flux.

From a practical point of view, the water phase is incompressible (i.e. ρ_w is constant) and by combining Equations (4) and (13), the water continuity equation can be expressed as follows:

$$m_1^w\frac{\partial(\sigma_{\text{mean}} - u_a)}{\partial t} + m_2^w\frac{\partial(u_a - u_w)}{\partial t} = -\left(\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z}\right) \tag{14}$$

3 COUPLED EQUATIONS FOR CONSOLIDATION OF UNSATURATED SOILS

The solution of coupled seepage-deformation problems requires the formulation of the equations in terms of primary unknowns, usually displacements and pore-water pressure. The equilibrium equations can be derived in terms of those primary unknowns by using the equilibrium equation (Eq. (12)) and the constitutive relationships (Eqs (9), (10)). In turn, the water continuity equation (Eq. (14)) also requires Darcy's law (Eq. (11)). Pereira (1996) presented the coupled equations for plane strain conditions (Eqs (15), (16) and (17)). The equations were expressed using three primary unknowns (i.e. the horizontal displacement, u; the vertical displacement, v; the pore-water pressure, u_w) for the case of a continuously atmospheric air phase:

$$\frac{\partial}{\partial x}\left(c_{11}\frac{\partial u}{\partial x} + c_{12}\frac{\partial v}{\partial y}\right) + c_{33}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) - d_s\frac{\partial(u_a - u_w)}{\partial x} + b_x = 0 \tag{15}$$

$$c_{33}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial y}\left(c_{12}\frac{\partial u}{\partial x} + c_{22}\frac{\partial v}{\partial y}\right) - d_s\frac{\partial(u_a - u_w)}{\partial y} + b_y = 0 \tag{16}$$

$$\beta_{w1}\frac{\partial\varepsilon_v}{\partial t} + \beta_{w2}\frac{\partial(u_a - u_w)}{\partial t} = \frac{\partial}{\partial x}\left(k_x\frac{\partial u_w}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y\frac{\partial u_w}{\partial y}\right) \tag{17}$$

where b_x, b_y = body forces in x- and y-direction,

$$c_{11} = c_{22} = \frac{(1-\mu)E}{(1+\mu)(1-2\mu)}$$

$$c_{12} = \frac{\mu E}{(1+\mu)(1-2\mu)}$$

$$c_{33} = \frac{E}{2(1+\mu)}$$

$$d_s = \frac{E}{(1-2\mu)H}$$

$$\beta_{w1} = \frac{E}{E_w(1-2\mu)}$$

$$\beta_{w2} = \frac{1}{H_w} - \frac{3E/H}{(1-2\mu)E_w}$$

Assuming a constant value of Poisson's ratio for the soil, the solution of the system of Equations (15), (16) and (17) requires the definition of the elastic moduli E, H, E_w and H_w for the soil structure and the pore-water phase as well as the hydraulic conductivity function, k_w. The moduli E, H, E_w and H_w are obtained by using Equations (9) and (10) when the compressibility parameters (i.e. m_1^s, m_2^s, m_1^w and m_2^w) are defined. These compressibility parameters are obtained from Equations (5)–(8) when the constitutive surfaces for void ratio and degree of saturation are known. The complexity of the transient system of coupled equations requires the use of numerical procedures for its solution. Finite element methods have commonly been used in geotechnical engineering practice for the solution of coupled phenomena.

4 NUMERICAL SOLUTION

The finite element computer program, named COUPSO was developed by Pereira (1996) to analyze coupled problems involving equilibrium and transient seepage in unsaturated soils. COUPSO presents as main characteristic the use of nine-node quadrilateral Lagrangian elements.

COUPSO has been utilized for the solution of coupled problems in one- and two-dimensional consolidation of soils at both saturated and unsaturated conditions (Pereira 1996). In particular, COUPSO has been used to simulate the behavior of collapsing soil structures under wetting stress paths.

5 EXAMPLE PROBLEM

The COUPSO program is herein utilized in the study of the volume change behavior of a swelling soil mass under a transient wetting. The soil mass is a 5 m deep deposit of unsaturated swelling clay that rests underneath a flexible cover (Fig. 1). The initial matric suction in the soil mass was assumed to be constant and with a value equal to 300 kPa.

The initial geo-static stress state condition in the soil deposit was calculated by assuming the soil mass at rest conditions and with a K_0 value equal to 0.67. This corresponds to an initial Poisson ratio equal to 0.4. The transient wetting process was induced by imposing a pore-water pressure equal to 0 kPa at the ground surface of the soil deposit under the flexible cover, as illustrated in Figure 1. Such a boundary condition simulates the water infiltration into the soil mass due to leakage through the flexible cover. The analysis is conducted by tracking both the swelling soil behavior and matric suction changes as a function of time as the wetting front advances into the soil mass.

The soil deposit is assumed to be Regina clay. Regina clay is a highly swelling, post-glacial lake deposit in Saskatchewan, Canada. According to the Soil Unified Classification System this soil is an inorganic clay of high plasticity. Table 1 presents index properties and mineralogical composition for this soil. Shuai (1996) conducted a laboratory-testing program, which consisted of a series of K_0-oedometer tests to study the volume change behavior of Regina clay. These tests were conducted on compacted specimens and were used to define the void ratio constitutive surface required for the numerical model, COUPSO.

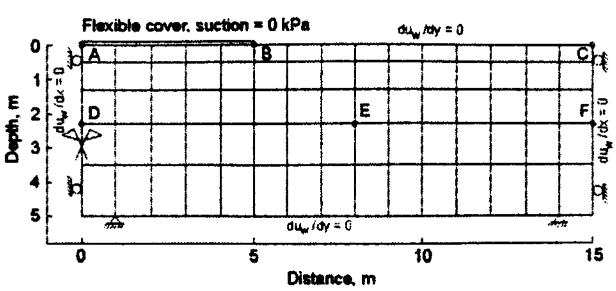

Figure 1. Mesh and boundary conditions of the domain.

The experimental data presented by Shuai (1996) was used to compute the void ratio constitutive surface as a function of the net mean stress and the matric suction. The net triaxial stress state for the oedometer tests was defined by calculating the net horizontal stress after assuming a constant K_0 value equal to 0.67. The void ratio constitutive surface was defined using a logistic function (Pereira 1996), best-fit using the software SIGMAPLOT, 1994. Equation (18) presents the best-fitted analytical function obtained. The best-fitted function and experimental data for the void ratio constitutive surface are superimposed in Figure 2:

$$e = e_u + \frac{e_f - e_u}{1 + \left(\frac{u_a - u_w}{c}\right)^b} \tag{18}$$

where $e_u = 0.9067 + 0.0001 \ln(\sigma_{mean} - u_a)$; $e_f = 1.1634 - 0.3289/\{1 + [(\sigma_{mean} - u_a)/178.8306]^{-1.20}\}$; $b = 0.0044\,(\sigma_{mean} - u_a) + 0.7864$; and $c = -0.0002\,(\sigma_{mean} - u_a)^2 + 0.6584\,(\sigma_{mean} - u_a) + 55.8482$.

Table 1. Index properties of the testing soil (Shuai 1996).

Soil	Regina clay
Location	Regina, Sask., Canada
Atterberg limits	LL = 69.9%, PL = 31.9%, PI = 38.0%
Grain size distribution (based on ASTM D422, 1988)	Sand: 2.2%, silt: 32.9%, clay: 64.9%
Unified Soil Classification System	CH, Inorganic clay of high plasticity
Standard compaction	Maximum dry density: 14.01 kN/m^3
	Optimum water content: 28.5%
Mineralogical composition	Montmorillonite: 20%, illite: 42%, kaolinite: 14%, mixed (12 Å) mineral layer: 24%

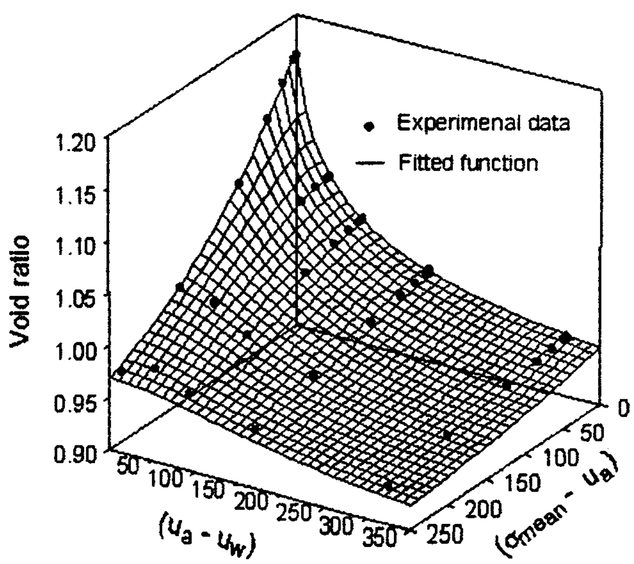

Figure 2. Void ratio constitutive surface.

Shuai (1996) also presented experimental data to represent the changes in the volumetric water content along the stress paths followed. Equation (19) presents the best-fitted function for the degree of saturation surface based on the experimental data. The best-fitted function and experimental data for the degree of saturation surface are presented in Figure 3:

$$S = S_0 + \frac{1 - S_0}{1 + \left(\frac{u_a - u_w}{c}\right)} \tag{19}$$

where $S_0 = 0.00007(\sigma_{mean} - u_a) + 0.7714$; $c = 0.0440 (\sigma_{mean} - u_a) + 10.3760$.

Shuai (1996) defined the hydraulic conductivity function for the swelling soil by combining the soil-water characteristic curve and Gardner's equation

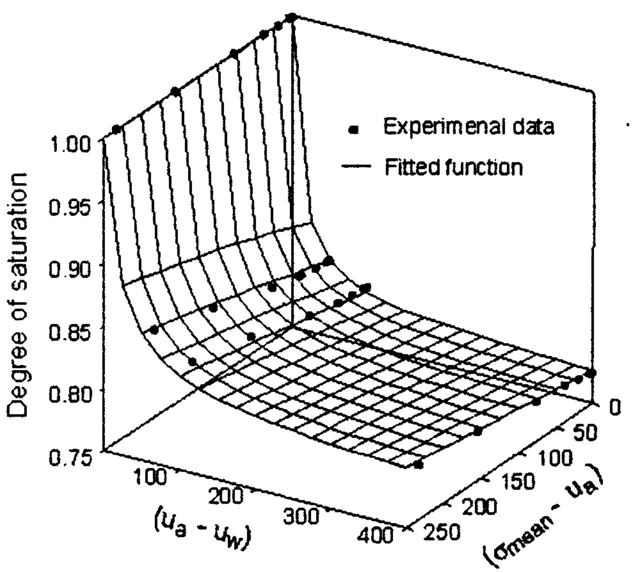

Figure 3. Degree of saturation constitutive surface.

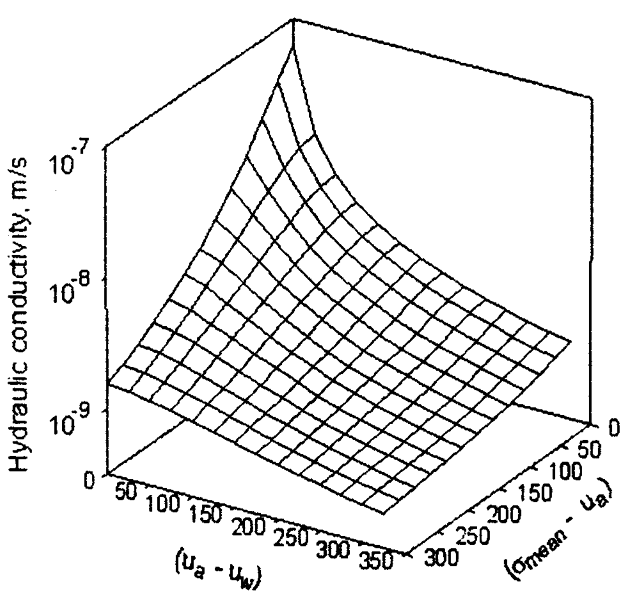

Figure 4. Hydraulic conductivity constitutive surface.

(Gardner 1958). The hydraulic conductivity for the soil at saturated conditions was expressed as a function of the void ratio. Figure 4 presents the hydraulic conductivity function surface obtained. Equation (20) presents the analytical model for the hydraulic conductivity:

$$k_w = \frac{k_{w0} e^b}{1 + a\left(\frac{u_a - u_w}{\rho_w g}\right)^n} \tag{20}$$

where $k_{w0} = 0.4 \times 10^{-8}$ m/s; e = void ratio, $b = 18.5$, $a = 0.01$; $n = 1.1$.

6 COMPUTER RESULTS

The finite element mesh and boundary conditions (Fig. 1) show that the mesh has 75 nine-node quadrilateral elements and 341 nodes. Boundary conditions are specified for both displacements and seepage. The surface of the domain has no restriction in terms of displacements. The domain is prevented from movement in the horizontal direction at both left (by symmetry) and along the right limit. The bottom limit is prevented from moving in both the horizontal and vertical directions. Zero pore-water pressure is specified under the flexible cover and zero flux condition is specified elsewhere.

Accumulated heave versus time for locations A, B, C in the domain (Fig. 1) is shown in Figure 5. Heave patterns in Figure 5 indicate that most of the heave below the flexible cover occurs in the first 100 days after wetting commences. The evolution of matric suction versus time for points D, E, F are presented in Figure 6. The

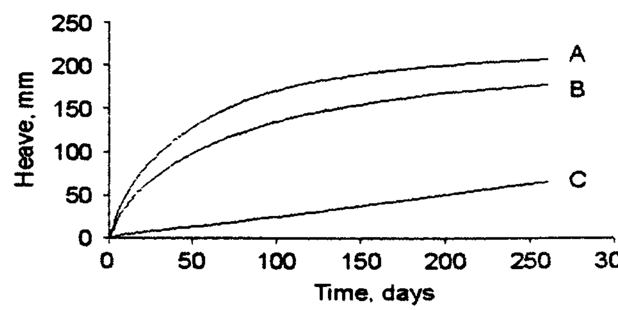

Figure 5. Heave versus time for points A, B and C.

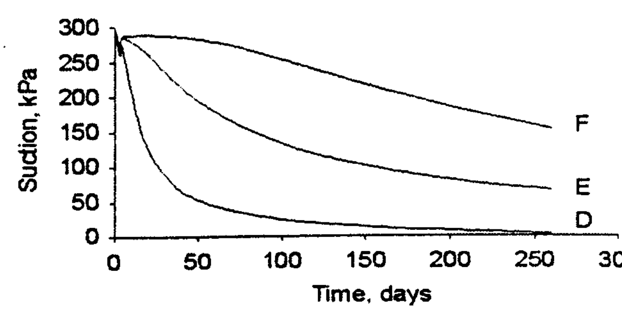

Figure 6. Development of matric suction for points D, E and F.

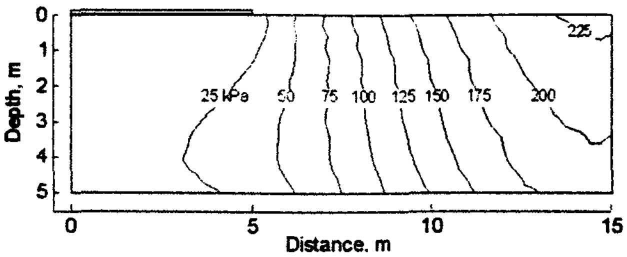

Figure 7. Suction distribution after 168 days.

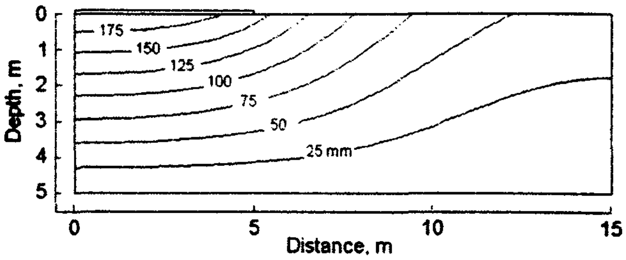

Figure 8. Heave distribution after 168 days.

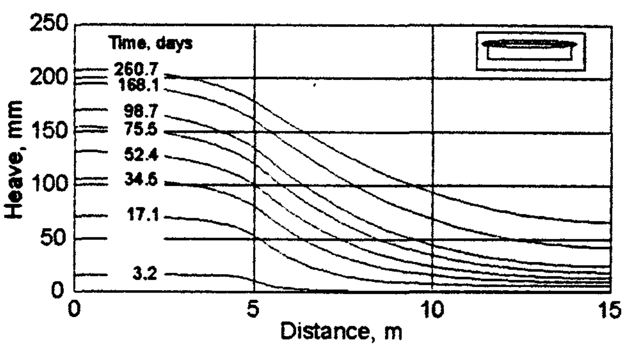

Figure 9. Development of heave with time at the surface.

results show that the soil under the cover reaches complete saturation, (i.e. $u_w = 0$ kPa), in about 250 days. The distribution of suction and vertical displacement in the soil after 168 days of infiltration is presented in Figures 7 and 8. It is shown that at that time the matric suction below the cover has been reduced to a value less than 25 kPa, and that a heave of about 195 mm has been computed immediately below the flexible cover.

The cumulative heave at surface points is presented in Figure 9. This figure shows that there is a maximum differential heave of about 150 mm, between points A and C, and it occurred in the first 100 days. Results of the analysis provide an indication of suction distribution, displacements and stresses in soil profile at any elapsed time as the transient wetting process occurs.

7 CONCLUSION

The following conclusions can be drawn from this study:

(1) The general theory of consolidation for unsaturated soils can be applied to the analysis of volume change behavior associated with expansive soils along wetting stress paths.
(2) The numerical model COUPSO is a potentially useful tool that can be used to perform coupled analysis associated with expansive soil behavior.
(3) The coupled seepage-deformation model provides a better understanding of the swelling behavior of expansive soils.

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