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Anisotropic stress–strain law for wetting-induced soil collapse

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ABSTRACT: This paper addresses the problem of formulating appropriate stress–strain laws for wetting-induced soil collapse. A series of available data for three collapsible soils, two compacted and one at natural conditions, have been analyzed. Laboratory tests comprised triaxial tests, triaxial permeameter tests under isotropic stress conditions, and oedometer tests wherein horizontal net normal stresses were measured and not measured. These data indicate that collapsible soils present a stress–strain anisotropy along wetting paths that is induced by anisotropic stress states. Based on that data, an anisotropic non-linear elastic stress–strain law is proposed. Stress paths and laboratory tests required to determine the soil anisotropy properties are discussed. A coupled numerical simulation of oedometer wetting tests using the anisotropic model is presented. The simulations demonstrate how the proposed anisotropic stress–strain law can be used in numerical analyses and shows that the proposed law performs significantly better than a conventional isotropic relationship.

1 INTRODUCTION

There is available data in the research literature indicating that collapsible soils exhibit anisotropy, in terms of strains, in response to changes in matric suction along wetting paths (Lawton et al. 1991, Pereira & Fredlund 2000, Peixoto 1999). This anisotropy is observed when the soil is subjected to wetting paths from anisotropic stress states. The stress-induced anisotropic collapse under wetting paths has not been fully considered in current stress–strain relationships.

In order to incorporate this feature of soil behavior into numerical models a more general stress–strain relationship is required. Laboratory tests that allow the determination of the anisotropy properties of a soil are discussed. Significant improvements in the prediction of soil collapse behavior are obtained when the stress-induced anisotropy is taken into account.

2 EXPERIMENTAL EVIDENCES

The one-dimensional collapse of soils has been extensively studied using conventional oedometer tests (Jennings & Knight 1957). Stress–strain laws formulated based on one-dimensional tests are applicable to one-dimensional problems. However, the use of deformation properties obtained from one-dimensional tests in the simulation of a two or three-dimensional problem requires assumptions regarding the soil properties in the other directions. The assumption that a soil mass collapses uniformly in all directions, independent of the complete stress state applied, has lead to errors in the prediction of the performance of earth structures (Miranda 1988).

Lawton et al. (1991) presented an important contribution for understanding soil deformation due to wetting under general stress state conditions. The authors studied the volumetric deformation, either swell or collapse, of a clayey sand. This soil is characterized by liquid and plastic limits of 34% and 19% respectively, and a specific gravity of 2.73. The laboratory tests were conducted on specimens compacted at a dry unit weight of 16.8 kN/m^3 (85% of modified Proctor maximum dry density) and a water content of about 10% (optimum). A "double-triaxial" test procedure was performed. Each as-compacted specimen was first consolidated and then soaked under a constant stress ratio (i.e. constant axial–radial stress ratio). Both axial and radial strains were monitored in all phases of the laboratory tests. The authors reported that the laboratory test results showed a strong relationship between the magnitude of the volumetric strain (either collapse or swell) and the mean net normal stress applied. In addition, the volumetric strain due to wetting appeared to be independent of the principal total stress ratio applied. It was found, however, that the individual components of axial and radial strain depend significantly on the applied stress ratio.

The stress-induced anisotropy was characterized, for soil collapse due to wetting, by a strain ratio different than one for the triaxial tests under an anisotropic stress state. Dakshanamurthy (1979) had found similar results previously for a swelling soil.

Pereira & Fredlund (2000) studied the behavior of a silty sand derived from a residual granitic gneiss soil (w_L = 29%, w_P = 17%, G_s = 2.64) compacted at metastable conditions. The specimens were prepared at a water content of 10.5% (4% dry of optimum) and at a dry unit weight of about 14.75 kN/m^3, which corresponds to void ratio equal to 0.76. These authors carried out a laboratory testing program using an oedometer and isotropic compression tests on as-compacted soil specimens subjected to different wetting stress paths. The horizontal stress after wetting in the oedometer test was determined by assuming that the volumetric collapse is a function of the mean stress (Lawton et al. 1991 findings). The mean stress on the oedometer test for a given void ratio after wetting was assumed as the same mean stress corresponding to the same void ratio after wetting in the isotropic compression test (Fig. 1).

Assuming an initial k_0 equal to 0.5 for the as-compacted condition (Miranda 1988), and taking into account the procedure illustrated in Figure 1, it was found that the horizontal stress in the oedometer test

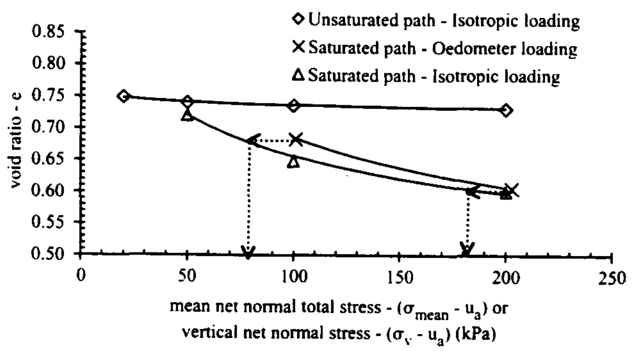

Figure 1. Void ratio versus confining or vertical net normal stress at initial (as-compacted) and final (saturated) conditions (modified from Pereira & Fredlund 2000).

increased towards complete saturation. During wetting, as the soil suffered collapse in the direction of the major net normal stress (vertical), the soil tried to expand in the direction of the minor net normal stress (horizontal). These results are physically similar to those presented by Lawton et al. (1991).

Pereira (1996) emphasized that the combination of oedometer and isotropic loading tests form an effective way to better characterize the soil collapsing behavior under general stress state conditions. However, these two tests do not provide as much information as triaxial tests.

Peixoto (1999) presented another alternative for the study of soil collapse taking into account the more general stress states. The collapsing behavior of a porous and metastable clayey soil was studied. The soil covers a large portion of the city of Brasilia, Brazil. Laboratory tests were carried out on natural specimens using an oedometer cell instrumented to measure lateral stresses. The soil was a natural sandy clay, with w_L = 47%, w_P = 32%, G_s = 2.77, and an average initial void ratio of about 1.44.

Figures 2 and 3 present Peixoto's (1999) results. In these tests each soil specimen was consolidated under constant matric suctions. The results are illustrated in terms of void ratio and horizontal net normal stress data (and fitted functions) versus both mean net normal stress and matric suction. A dependency between the soil collapsing deformation and the net normal horizontal stress increase was verified. Figures 2b and 3b illustrate that the soil experiences, along wetting stress paths, an increase in compressibility beyond a critical matric suction, which is a function of the applied mean stress.

Wetting oedometer tests under constant vertical stress, without the control of soil suction, were also performed. Since soil suction was not controlled, a void ratio state surface could not be drawn. However, the wetting tests showed that both total volumetric collapse and the horizontal stress after saturation were close to those obtained by the saturated compression tests (Figs 2 and 3).

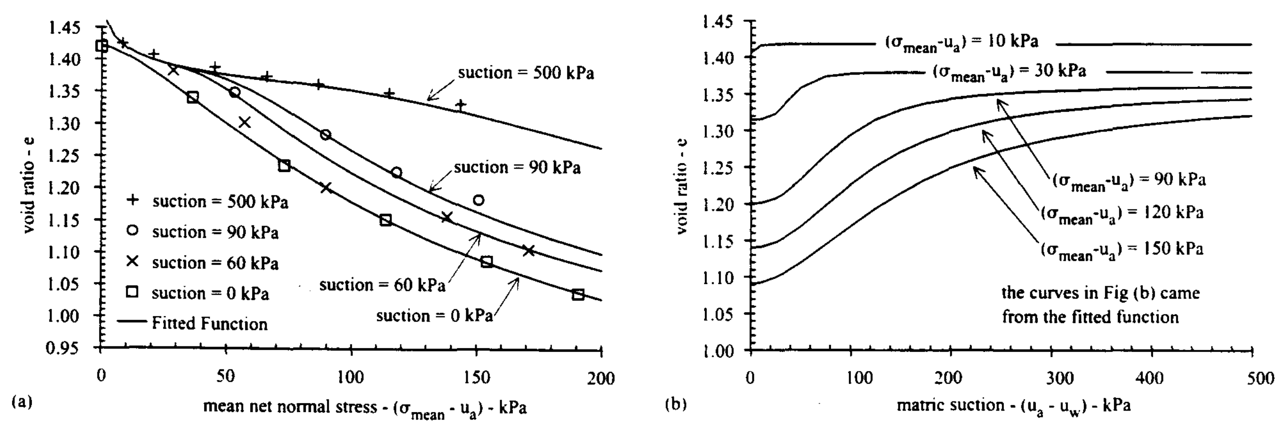

Figure 2. Void ratio state surface: (a) test results and fitted function – void ratio versus mean net normal stress; (b) void ratio versus matric suction.

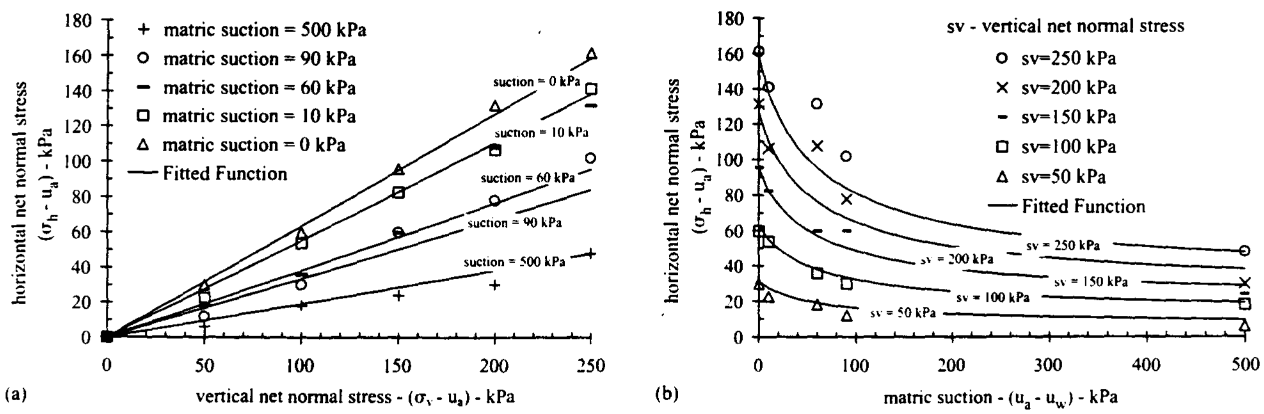

Figure 3. Measured horizontal net normal stresses and fitted function: (a) horizontal net normal stress versus vertical net normal stress; (b) horizontal net normal stress versus matric suction.

Therefore, the data from the compression tests were considered representative of wetting-induced collapse.

The results from Peixoto (1999) showed that oedometer tests with measurement of horizontal stress are an effective way of studying soil collapse taking into account the mean stress applied.

3 NON-LINEAR ELASTIC STRESS–STRAIN CONSTITUTIVE MODELING

The features of soil collapse observed in previous research studies cannot be properly reproduced using an isotropic non-linear elastic model. This is particularly true in terms of changes in matric suction. A stress-induced anisotropic constitutive model for soil collapse is required. The basis for the proposed anisotropic law is the non-linear incremental elastic isotropic relationships proposed by Fredlund (1979).

3.1 *Isotropic stress–strain law*

Fredlund (1979) extended the generalized Hooke's stress–strain equations to unsaturated soils by using two stress state variables; namely, net total stress $(\sigma - u_a)$, and matric suction $(u_a - u_w)$:

$$d\varepsilon_x = \left(\frac{1}{E}\right) d(\sigma_x - u_a) - \left(\frac{\mu}{E}\right) d(\sigma_y + \sigma_z - 2u_a) + \left(\frac{1}{H}\right) d(u_a - u_w) \tag{1}$$

$$d\varepsilon_y = \left(\frac{1}{E}\right) d(\sigma_y - u_a) - \left(\frac{\mu}{E}\right) d(\sigma_x + \sigma_z - 2u_a) + \left(\frac{1}{H}\right) d(u_a - u_w) \tag{2}$$

$$d\varepsilon_z = \left(\frac{1}{E}\right) d(\sigma_z - u_a) - \left(\frac{\mu}{E}\right) d(\sigma_x + \sigma_y - 2u_a) + \left(\frac{1}{H}\right) d(u_a - u_w) \tag{3}$$

$$d\gamma_{xy} = \left(\frac{1}{G}\right) d\tau_{xy} \quad d\gamma_{xz} = \left(\frac{1}{G}\right) d\tau_{xz} \quad d\gamma_{yz} = \left(\frac{1}{G}\right) d\tau_{yz} \tag{4}$$

where $d\varepsilon_i$ and $d\sigma_i$ are the normal strain and stress increments in i directions; E is the Young's modulus; μ is the Poisson ratio; H is the elastic modulus for soil structure with respect to suction change; $d\gamma_{ij}$ and $d\tau_{ij}$ are the shear strain and stress increments respectively, in i planes and j directions; and G is the shear modulus, $G = E/2(1-\mu)$.

The modulus E and H can be obtained from the void ratio state surface. The Poisson ratio requires additional experimental information, as explained latter. Using Equations (1)–(4) to reproduce oedometer test condition would show the soil specimen losing contact with the oedometer ring as matric suction decreases. However, the test data presented in the previous section showed that the horizontal stress increases during wetting-induced collapse.

3.2 *Anisotropic stress–strain law*

A material is anisotropic if the properties are not the same in all directions. To understand the physical meaning of the proposed anisotropic law, it is important to distinguish between intrinsic and stress-induced anisotropy.

Intrinsic anisotropy is a function of the internal constitutive nature of the material. Dispersed-structured clays, for example, behave anisotropically in terms of deformability and shear strength because the soil particles are oriented in a preferential direction. On the other hand, stress-induced anisotropy can be defined as an anisotropic behavior based on the applied anisotropic total stress state. In other words, there is an anisotropy when the soil is under anisotropic stress state during wetting.

Peixoto (1999) presented oedometer test results for one example of what is here called stress-induced collapse. In those tests, the initial horizontal stress was lower than the vertical stress and increased due to the gradual reduction in matric suction upon wetting. There was compression in the direction of higher net normal stress (vertical) and the soil specimen had the tendency to expand in the direction of the lower net normal stress (horizontal). The triaxial tests performed by Lawton et al. (1991) showed similar behavior.

In order to properly reproduce soil collapse, an incremental anisotropic stress-induced non-linear elastic model based on a macroscopic and phenomenological approach is herein proposed. The anisotropic behavior can be reproduced through the following modification of Equations (1)–(3):

$$d\varepsilon_x = \left(\frac{1}{E}\right) d(\sigma_x - u_a) - \left(\frac{\mu}{E}\right) d(\sigma_y + \sigma_z - 2u_a) + \left[\frac{(1+\xi_x)}{H}\right] d(u_a - u_w) \quad (5)$$

$$d\varepsilon_y = \left(\frac{1}{E}\right) d(\sigma_y - u_a) - \left(\frac{\mu}{E}\right) d(\sigma_x + \sigma_z - 2u_a) + \left[\frac{(1+\xi_y)}{H}\right] d(u_a - u_w) \quad (6)$$

$$d\varepsilon_z = \left(\frac{1}{E}\right) d(\sigma_z - u_a) - \left(\frac{\mu}{E}\right) d(\sigma_x + \sigma_y - 2u_a) + \left[\frac{(1+\xi_z)}{H}\right] d(u_a - u_w) \quad (7)$$

where ξ_i are coefficients of anisotropy.

The relationships between shear stresses and shear strains remain as in Equation (4). In order to keep the volumetric strain due to wetting solely as a function of H, the summation of the ξ_i coefficients must be null. Only the individual components of normal strain are affected by ξ_i.

Depending on the values of ξ_i, different situations can be reproduced. For the sake of simplicity, let us assume an axisymmetric condition (ξ_x, $\xi_z = \xi_h$ and $\xi_y = \xi_v$), defining $k_0 = (\sigma_h - u_a)/(\sigma_v - u_a)$ and assuming $k_0 \leq 1$. The following values of ξ_i and corresponding behavior are produced:

(1) $\xi_h < -1$ and $\xi_v > 2$. These are parameters anticipated in the cases where $k_0 < 1$. This situation indicates that the soil specimen undergoes expansion in the horizontal direction and compression in the vertical direction during wetting, indicating anisotropic collapse.

(2) $\xi_h = -1$ and $\xi_v = 2$. These are parameters anticipated in cases where $k_0 < 1$. This leads to compression in the vertical direction and zero horizontal strain for the soil specimen under wetting paths, indicating a less pronounced anisotropy than in the previous case. Though this condition is physically possible, this leads to a model singularity (leads to division by zero) and must be avoided by assuming, for example, $\xi_h = -0.999$.

(3) $-1 < \xi_h < 0$ and $0 < \xi_v < 2$. These are the parameters anticipated in cases where k_0 is smaller but close to 1. This leads the soil specimen undergoing compression in both the horizontal and vertical directions under wetting paths, but there is still some anisotropy.

(4) $\xi_h = 0$ and $\xi_v = 0$. These are the parameters anticipated in cases where $k_0 = 1$. This leads to isotropic horizontal and vertical compression for wetting paths.

The k_0 value is expected to be smaller or equal to 1.0 for a collapsible soil. The tests done by Lawton et al. (1991), Peixoto (1999), and Pereira & Fredlund (2000) show values of k_0 smaller than unity. Values of k_0 higher than 1.0 are typically, but not absolutely, related to expansive soils. Although there are results in the literature showing that some expansive soils present anisotropic swelling, this is not discussed herein.

3.3 *Determination of the soil properties*

There are several ways to determine the stress–strain constitutive parameters; E, H, μ, and ξ_i. For a collapsible soil, regardless of the laboratory test available, it is necessary to measure the complete stress and strain states because the k_0 value is not constant and cannot be guessed.

The values of E and H can be related to the coefficients of volume change, m_1^s and m_2^s, given by the void ratio state surface (Fredlund 1979). According to the findings of Lawton et al. (1991) amongst others, volume change is function of the mean net normal stress. Therefore, the void ratio state surface has to be written as a function of matric suction and mean net normal stress. Equations (8)–(11) can be used for obtaining m_1^s, m_2^s, E, and H:

$$m_1^s = \frac{d\varepsilon_v}{d(\sigma_{mean} - u_a)} = \frac{1}{1+e_0}\frac{de}{d(\sigma_{mean} - u_a)} \quad (8)$$

$$m_2^s = \frac{d\varepsilon_v}{d(u_a - u_w)} = \frac{1}{1+e_0}\frac{de}{d(u_a - u_w)} \quad (9)$$

$$m_1^s = \frac{3(1-2\mu)}{E} \quad (10)$$

$$m_2^s = \frac{3}{H} \quad (11)$$

where $d\varepsilon_v$ is the change in volumetric strain, $\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z$; σ_{mean} is the mean normal stress, $\sigma_{mean} = (\sigma_x + \sigma_y + \sigma_z)/3$; e_0 is initial void ratio in each increment; and de is change in void ratio.

Equations (8)–(11) show that the void ratio state surface allows the determination of the coefficients of volume change, E and H, given that μ has been determined. Collapsible soils suffer a change in structure and changes in μ are expected during wetting. There is a need for a constitutive surface for μ. The Poisson ratio can be indirectly determined from oedometer tests by loading the soil specimen in the vertical direction and measuring the lateral stress along paths under constant matric suction. Triaxial tests can be used when measurements of both vertical and horizontal deformations are determined.

Pereira & Fredlund (2000) assumed a constant ξ_i coefficient for a metastable soil. The ξ_i values were obtained by back-calculating oedometer tests. In their analysis, due to the lack of measurement of horizontal stresses along wetting paths in the oedometer tests, the void ratio was the target variable for the definition of the ξ_i coefficients.

Measurements of horizontal stresses in an oedometer test allow the definition of the ξ_i coefficients as a function of the stress state variables. A test is required where a soil specimen undergoes a gradually controlled wetting process under k_0-oedometer conditions and under a constant net vertical stress. The horizontal coefficient of anisotropy, ξ_h, can be determined according to Equation (12), which is derived from Equation (1) or (3) by setting ε_h (ε_x or ε_z) equal to zero (k_0-oedometer conditions):

$$\xi_h = -\left(\frac{1-\mu}{E}\right) H \frac{d(\sigma_h - u_a)}{d(u_a - u_w)} - 1 \tag{12}$$

Equation (12) shows that the ξ_h values (i.e. ξ_x and ξ_z) are function of the soil properties; E, H, μ, as well as being a function of the incremental ratio involving horizontal stress and matric suction acting under k_0-oedometer conditions. The ξ_v value (ξ_y) is obtained from the relationship $\xi_x + \xi_y + \xi_z = 0$. The values for ξ in the horizontal directions (i.e. x and z) were assumed to be equal since the oedometer test reproduces an axisymmetric situation. To determine the influence of the intermediate principal stress on the anisotropic strain a true triaxial test is required.

The numerical simulations presented in the next section compares the performance of the three distinct approaches. The first approach makes use of the herein proposed stress-induced anisotropic model, which uses the ξ function. The second approach utilizes the anisotropic model using a constant ξ value. The third approach presents the results obtained when using the isotropic model.

4 VERIFICATION OF THE STRESS-INDUCED ANISOTROPIC MODEL

The void ratio state surface and the horizontal stress and Poisson ratio surfaces, presented in Figures 2–4 respectively, were used in the numerical simulations of a wetting oedometer test (Gitirana Jr. 1999). The function for μ was obtained using the $(\sigma_h - u_a)/(\sigma_h + \sigma_v - 2u_a)$ ratio. The degree of saturation S_r state surface and the hydraulic conductivity function k_w used in the coupled numerical analysis are from Pereira (1996). The functions for S_r and k do not present a major interference on the results in terms of stress–strain behavior.

All the functions used, including the ξ function, have continuous first derivatives. Singularities could pose

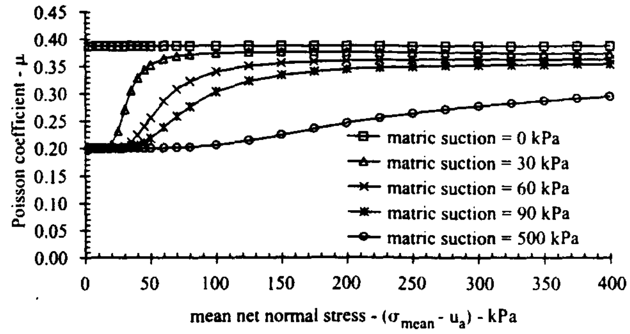

Figure 4. Poisson ratio function.

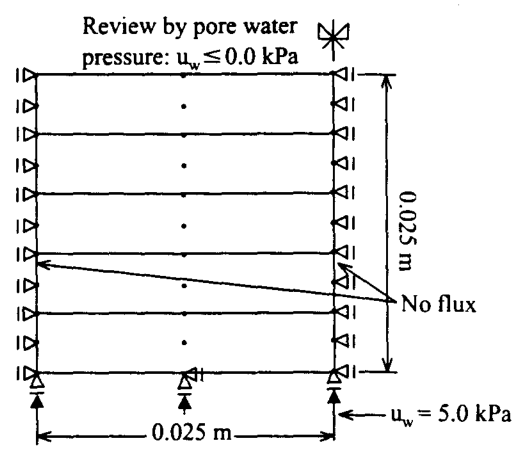

Figure 5. Finite element mesh and boundary conditions.

difficulties to the numerical solution. The numerical code used was COUPSO, written by Pereira (1996) and modified by Gitirana Jr. (1999) to solve axisymmetric problems and use the ξ function.

The geometry and boundary conditions for the oedometer test are presented in Figure 5. Due to existing symmetry, only half of the soil specimen is modeled. The initial stress state adopted was: $(u_a - u_w) = 500$ kPa; $(\sigma_v - u_a) = 250$ kPa; and $(\sigma_h - u_a) = 47.5$ kPa. The initial horizontal net normal stress was obtained from the data presented in Figure 3. The wetting test was simulated by imposing $u_w = 5$ kPa at the bottom nodes of the mesh. The wetting front advances upwards. The upper boundary is considered as a free drainage surface, $u_w \leq 0$ kPa.

4.1 *Numerical simulation results and discussion*

All the nodes behave in a similar manner. Figure 3 shows that for an adopted vertical net stress of 250 kPa, the final horizontal stress after saturation is 158.1 kPa, which corresponds to a mean net normal stress equal to 188.7 kPa. According to Figure 2 the final void ratio is 1.05 for that final stress state.

A constant ξ value of −4.7 was obtained by using a trial and error back-analysis. The known final void ratio of 1.05 was the final condition to be achieved.

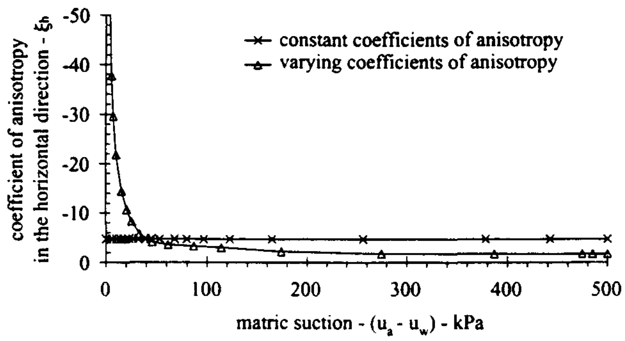

Figure 6. Evolution of the ξ_h values during wetting as a function of $(u_a - u_w)$.

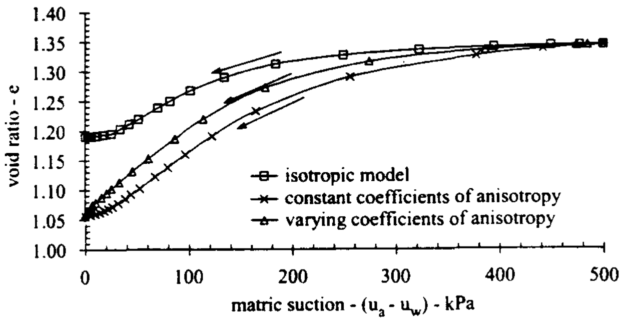

Figure 8. Evolution of void ratio during wetting as a function of $(u_a - u_w)$.

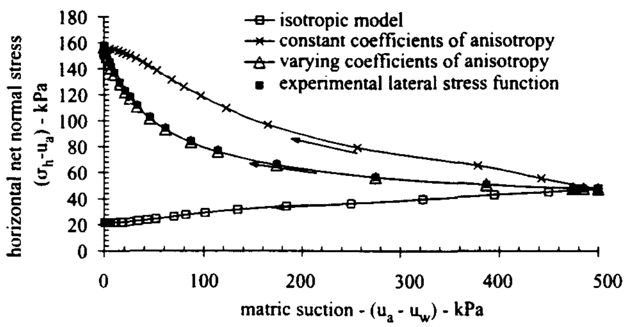

Figure 7. Evolution of horizontal net normal stress during wetting as function of $(u_a - u_w)$.

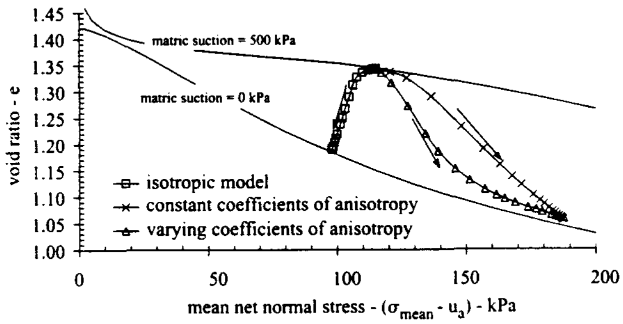

Figure 9. Evolution of void ratio during wetting as a function of $(\sigma_{\text{mean}} - u_a)$.

Figure 6 shows the comparison between values for ξ_h obtained from the back-analysis and actual values calculated from Equation (12). The results show that the measured ξ_h values suffer a pronounced increase during the latter stages of saturation as a result of soil collapse, as previously illustrated in Figure 2.

Figure 7 shows the evolution of the computed horizontal net normal stresses along the wetting path. Using the isotropic model, a reduction in net normal lateral stresses is obtained. In this case, the nodes at the lateral face of the mesh try to contract, but the boundary condition applied does not allow any horizontal deformation. This results in a reduction in the net normal horizontal stress.

The black dots in Figure 7 are the experimental measured values presented previously in Figure 3. As anticipated, varying the ξ_h values reproduces the exact evolution of lateral stresses. The use of a constant ξ_h value gives a correct final net normal lateral stress. However, a constant ξ_h does not predict an accurate evolution of this horizontal stress. The constant ξ_h value was back-calculated to give the correct final net normal horizontal stress, which is related to the final mean net normal stress.

It can be seen that the isotropic model shows the deviator stress increasing during wetting. Therefore, the isotropic model results in a soil condition closer to failure during a wetting process than it is in reality. The use of a constant ξ value produced deviator stresses higher than the actual stresses in the intermediate stages of saturation. The use of varying ξ values furnishes better estimations of stress states. This procedure is valuable when analyzing earth structures where there is a risk of failure problems.

Figures 8 and 9 show the evolution of the void ratio during wetting, as a function of matric suction and mean net normal stress, respectively. Since the isotropic model predicts a reduction in the lateral net stress and thus a reduction in the mean net normal stress, the numerically computed collapse is lower than shown by the actual results. When using constant ξ values, the correct volumetric collapse is obtained. However, void ratio values at intermediate wetting stages are not as accurate as the void ratios obtained when using the variable ξ values.

5 CONCLUSIONS

The incrementally non-linear elastic constitutive relationships for the consolidation of an unsaturated soil, as proposed by Fredlund (1979), requires the inclusion of stress induced anisotropic factors in order to properly reproduce the soil collapse along wetting paths under k_0-oedometer conditions.

Numerical simulations of wetting-induced soil collapse under k_0-oedometer conditions showed an adequate performance of the proposed anisotropic constitutive relationship.

Research on general triaxial stress conditions is necessary to produce a better understanding in terms of the stress-induced anisotropic parameters for unsaturated soils.

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