Ground and Water: Theory to Practice
Proceedings of the 55th Canadian Geotechnical and 3rd Joint IAH-CNC and CGS Groundwater Specialty Conferences. Niagara Falls. Ontario, October 20-23. 2002
Edited by D. Stolle. A.R. Piggott and J.J. Crowder and published by the Southern Ontario Section of the Canadian Geotechnical Society

A SIMPLE SOIL-WATER HYSTERESIS MODEL FOR PREDICTING THE BOUNDARY WETTING CURVE

Hung Q. Pham, Graduate Student, University of Saskatchewan, Saskatoon, Canada
Delwyn G. Fredlund, Professor Emeritus, University of Saskatchewan, Saskatoon, Canada
S. Lee Barbour, Professor, University of Saskatchewan, Saskatoon, Canada

ABSTRACT

The hysteretic nature of the soil-water characteristic curve has been known for a long time; however, in most routine engineering and agriculture applications, it has been ignored due to the effort required to fully characterize hysteresis during laboratory testing. The soil-water hysteresis model evaluated in this paper is the Feng and Fredlund (1999) model. An experimental testing program was implemented to evaluate the accuracy of the model for two soils; namely, Beaver Creek sand and processed silt. The results illustrate that the predicted boundary curves of the two soils are quite close to the experimental data points. This model is quite simple to use and readily applicable in engineering practice.

RÉSUMÉ

L'hystérésis associée à la courbe caractéristique eau-sol est connue depuis longtemps. Cependant, dans la plupart des applications en ingénierie et agriculture, l'hytérésis a été ignorée dû à l'effort requis pour caractériser entièrement l'hystérésis en laboratoire. Le modèle d'hystérésis eau-sol évalué dans cet article est celui de Feng and Fredlund (1999). Un programme expérimental a été développé pour évaluer la précision du modèle pour deux types de sol: soient le sol sableux de Beaver Creek et le "silt" fabriqué. Les résultats ont démontré que les courbes de frontières prédites pour les deux types de sol sont semblables aux données expérimentales. Ce modèle s'avère simple à utiliser et immédiatement applicable en ingénierie.

1. INTRODUCTION

The Soil-Water Characteristic Curve (SWCC), plays an important role in understanding the behavior of unsaturated soils. Soil-water characteristic curves can be used to estimate the hydraulic conductivity and shear strength functions that are subsequently used to model unsaturated soil behavior (Barbour, 1999; Fredlund and Rahardjo, 1993). Some of the common names given to the hysteretic branches of the soil-water characteristic curve are shown in Fig. 1.

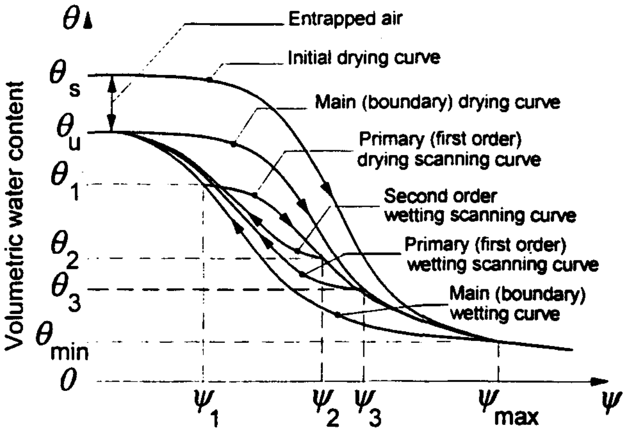


Figure 1. Commonly used definitions for hysteretic soil-water hysteresis curves.

A number of hysteresis models have been proposed, however only a few can predict the boundary wetting curve or the boundary drying curve (Mualem, 1977, 1984a; Parlange, 1976; Hogarth et al., 1988; Nimmo, 1992). Moreover, most models require knowledge of the main boundary curves in order to predict scanning curves (i.e., Philip, 1964; Hank *et al.*, 1964; Mualem, 1973, 1974, 1984b; Dane and Wierenga, 1975; Hogarth *et al.*, 1988; Nimmo, 1992; Scott *et al.*, 1983; Jaynes, 1985; Kawai *et al.*, 2000; Karube and Kawai, 2001). The models for predicting the boundary wetting curve are complex and the data required for calibration of the predictive method is considerable since it involves the determination of specific meeting points on the boundary hysteresis curves at high and low values of soil suction. In this paper, a simple model for predicting the boundary wetting soil-water characteristic curve is presented.

2. THEORY OF THE FENG AND FREDLUND (1999) MODEL

The Feng and Fredlund (1999) hysteresis model uses the Feng and Fredlund (1999) soil-water characteristic curve-fitting equation to present both the boundary drying curve and the boundary wetting curve. This equation can be written as follows:

$$w(\psi) = \frac{w_u b + c\psi^d}{b + \psi^d} \qquad [1]$$

where:

w_u = water content on the boundary drying curve at zero soil suction, and

b, c, d = curve-fitting parameters.

The proposed equation was developed based on measurements of the hysteresis associated with the water content versus suction relationship for variable pore size ceramics (Feng, 1999). In equation 1, the parameter c represents the water content at a relatively high soil suction. When w_u and c are constant, the parameter d represents the slope of the curve and both parameters d and b control the *air entry value* along the curve. Typical curves obtained from equation 1 are presented in Figs. 2 and 3.

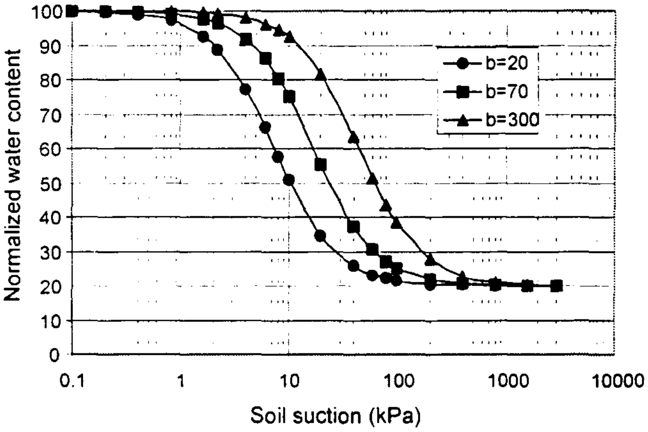


Figure 2. Sample plots of Equation 1 with $w_u = 100$, $c = 20$ and $d = 1.5$ (b varies)

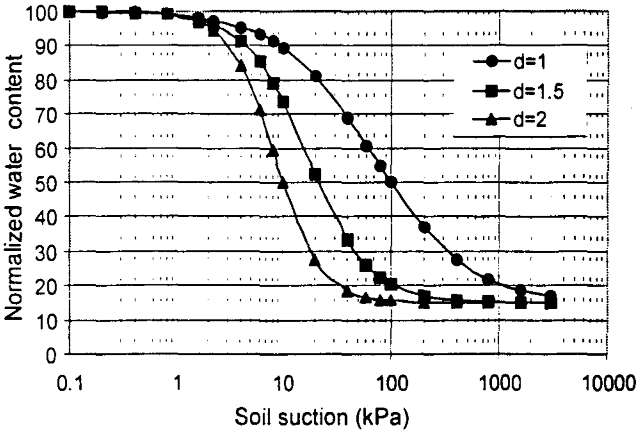


Figure 3. Sample plots of Equation 1 with $w_u = 100$, $c = 20$ and $b = 70$ (d varies).

Feng and Fredlund (1999) showed that the entire boundary wetting curve could be predicted with only two measured points on the boundary wetting curve; however, the authors did not specify the locations of these. Suggestions for the location of these two points on the boundary wetting curve are presented in this paper.

The curve-fitting parameters for the boundary drying curve are obtained using a best-fit analysis of equation 1 to the measured data. The water content at zero soil suction of the boundary drying curve is equal to that of the boundary wetting curve and the residual water content of the boundary wetting curve is close that of the boundary drying curve. Once the boundary drying curve is measured, the two parameters of the best-fit curve for the boundary wetting curve can be approximated (i.e., w_u and c). There are only two additional curve-fitting parameters required for the boundary wetting curve; namely, b and d. In order to find these two curve-fitting parameters, two additional points on the boundary wetting curve are required. Different boundary wetting curves will be obtained depending on the selection of various points. To obtain the appropriate soil parameters, two points need to be measured along the sloping line on the best-fit curve. The location of two points is based on the observations of the boundary hysteresis curves for 34 soils.

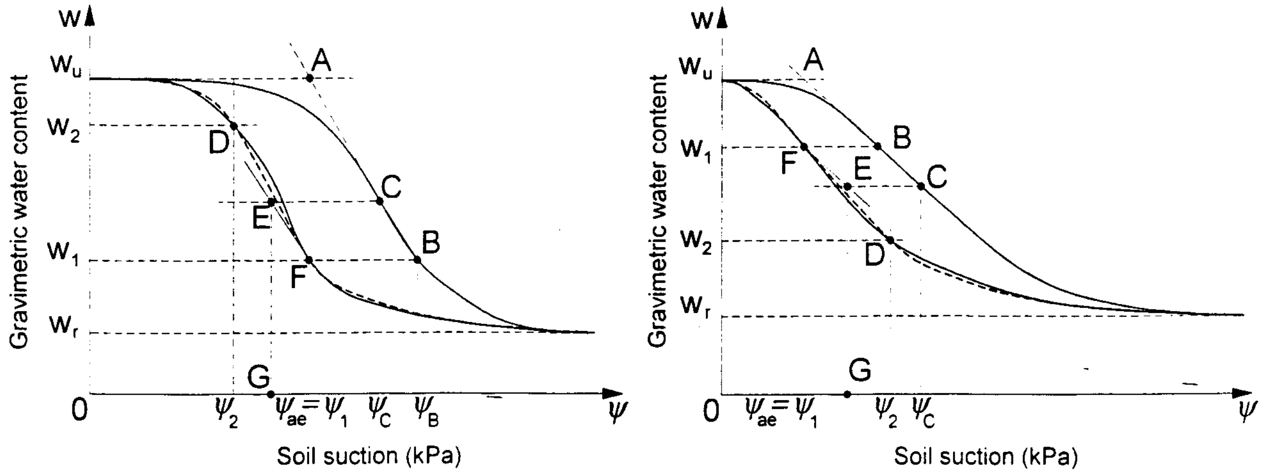
The location of the first point on the boundary wetting curve can be the water content at a value of soil suction close to the air entry value of the boundary drying curve. It is suggested that, the soil suction for the first point be approximated using equation 2.

$$\psi_1 \approx \left(\frac{b}{10}\right)^{\frac{1}{d}} \qquad [2]$$

where:

b, d are best-fit parameters of the boundary drying curve (equation 1).

The soil suction of the second point on the boundary wetting curve can be chosen based on the location of the first point. The second point on the boundary wetting curve should be a point having a suction value that is symmetrical to the suction value of the first point about a vertical line passing through the inflection point of the boundary wetting curve. In order to locate the second point, the location of the inflection point of the boundary wetting curve needs to be determined. By observing the hysteretic SWCCs for 34 soils, it was observed that the slope of the boundary drying curves is generally similar to the slope of the boundary wetting curves. The inflection point of the boundary wetting curve can be estimated using the slope of the boundary drying curve and the first point on the boundary wetting curve. It is important to note that using the slope of the boundary drying curve to estimate the inflection point of the boundary wetting curve does not mean the results of the model gives a boundary wetting curve parallel to the boundary drying curve.



a.) Clayey soil

b.) Coarse soil

Figure 4. Schematic illustrations of procedures for predicting the boundary wetting curve; (a) for a coarse soil and (b) for a clayey soil using the Feng and Fredlund (1999) model. The continuous lines are measured curves, the dashed lines are predicted curves

The soil suction values for the first and second additional points, ψ_1 and ψ_2 are now defined. The water content on the boundary wetting curve at a soil suction of ψ_1 is w_1 (i.e., point F in Figs. 4a and 4b). The soil suction value of the second additional point on the boundary wetting curve (i.e., point D Figs. 4a and 4b) can be determined as follows:

- Determine the inflection point of the boundary wetting curve, E by assuming the two boundary curves are parallel.
- The soil suction value at the second point on the boundary wetting curve, ψ_2, can be calculated using equation 3.

$$|\psi_2 - \psi_E| = |\psi_1 - \psi_E| \qquad [3]$$

where:

ψ_E = soil suction at point E.

Values of $|\psi_1 - \psi_E|$ can be calculated by considering the inflection point on the boundary drying curve (i.e., point C). EF is parallel BC; therefore,

$$|\psi_1 - \psi_E| = |\psi_B - \psi_C| \qquad [4]$$

The value of soil suction at point B, ψ_B, can be calculated by solving equation 5:

$$w_1 = \frac{w_u b + c\psi_B^d}{b + \psi_B^d} \qquad [5]$$

Solving equation 5, yields,

$$\psi_B = \left(\frac{b(w_u - w_1)}{w_1 - c}\right)^{\frac{1}{d}} \qquad [6]$$

Soil suction values, ψ_C, at the inflection point of the boundary drying curve can be calculated as follows:

$$\psi_C = b^{\frac{1}{d}} \qquad [7]$$

The soil suction at the second additional point required for the boundary wetting curve, ψ_2, can be determined from equation 8:

$$\psi_2 = \psi_1 - 2\left(\left(\frac{b(w_u - w_1)}{w_1 - c}\right)^{\frac{1}{d}} - b^{\frac{1}{d}}\right) \qquad [8]$$

where:

w_u = water content on the boundary drying curve at zero soil suction,

b, c, d = curve-fitting parameters of the boundary drying curve.

ψ_1 = soil suction at the first additional point, and

w_1 = water content at the first additional point.

Once two points on the boundary wetting curve are known, two unknown curve-fitting parameters b_w and d_w for the boundary wetting curve can be calculated by solving equations 9 and 10:

$$w_1 = \frac{w_u b_w + c\psi_1^{d_w}}{b_w + \psi_1^{d_w}} \qquad [9]$$

$$w_2 = \frac{w_u b_w + c\psi_2^{d_w}}{b_w + \psi_2^{d_w}} \qquad [10]$$

where:

w_u = water content on the boundary drying curve at zero soil suction,

b_w, c, d_w = curve-fitting parameters of the boundary wetting curve,

ψ_1, w_1 = soil suction and gravimetric water content of the first point, respectively, and

ψ_2, w_2 = soil suction and gravimetric water content of the second point, respectively.

Solving equations 9 and 10, gives,

$$d_w = \frac{\log\left(\frac{(w_1 - c)(w_u - w_2)}{(w_u - w_1)(w_2 - c)}\right)}{\log(\psi_2/\psi_1)} \qquad [11]$$

$$b_w = \frac{(w_1 - c)\psi_1^{d_w}}{w_u - w_1} \qquad [12]$$

3. EXPERIMENTAL PROGRAM

The two soils used in the laboratory test program were Beaver Creek Sand and Processed Silt from Saskatchewan, Canada. The grain size distributions of the two soils are shown in Fig. 5. Specimens of the two soils were prepared as compacted specimens at water contents slightly lower than the optimum water content (i.e., dry optimum at standard ASTM D-698).

Three specimens were prepared from each soil and the following hysteretic SWCCs were measured: initial drying curve, boundary wetting curve, and boundary drying curve. All the tests were performed using a pressure plate apparatus constructed at the University of Saskatchewan, Saskatoon, Canada (Fig. 6). The testing procedures are the same as those used by Feng (1999).

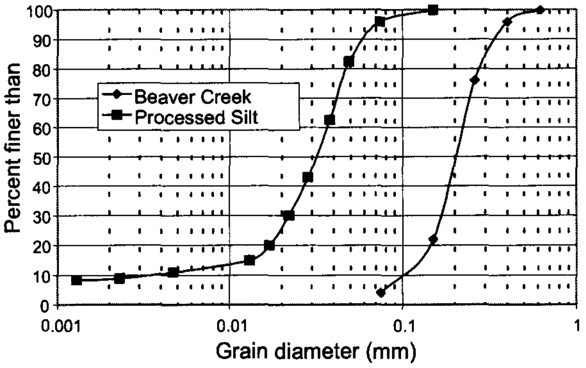


Figure 5. Grain size distribution curves of the Beaver Creek Sand and the Processed Silt (after Bruch, 1993).

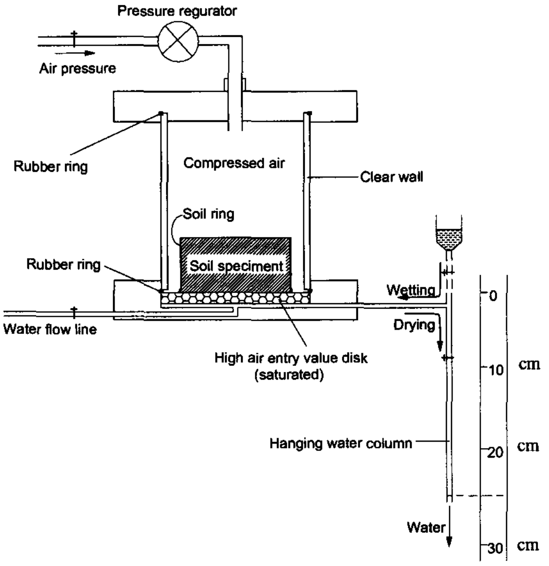


Figure 6. Schematic illustration of the pressure plate produced at the University of Saskatchewan, Canada.

The test results show good agreement between the three identical specimens of the two soils. The average results of the three specimens were used to represent the data points from the laboratory test program. The actual soil data points were then best-fit using the following soil-water characteristic curve-fitting equations: Brooks and Corey (1964), van Genuchten (1984), Fredlund and Xing (1994), and Feng and Fredlund (1999). The results show that the Feng and Fredlund (1999) equation appears to be the most appropriate equation to fit the actual data (equation 1). The actual soil data points and the best-fit curves using the Feng and Fredlund (1999) equation are shown in Figs. 7 and 8. The Processed Silt contains about 7% clay particles; therefore, a small amount of volume change may have occurred during the test.

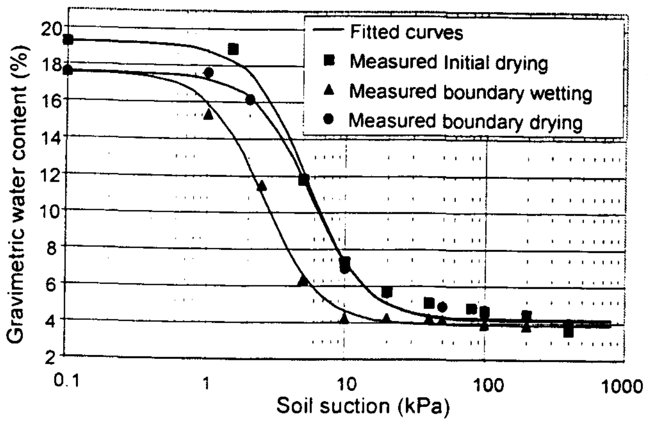


Figure 7. Hysteretic soil-water characteristic curves for the Beaver Creek Sand: the continuous lines are best-fit using the Feng and Fredlund (1999) equation and the data points are the average of 3 tests.

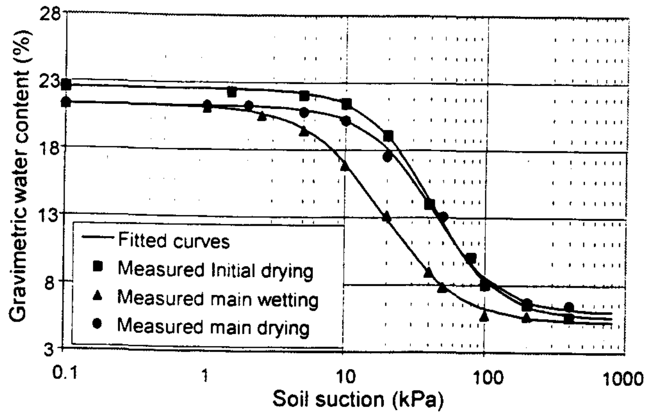


Figure 8. Hysteretic soil-water characteristic curves of the Processed Silt: the continuous lines are best-fit using the Feng and Fredlund (1999) equation and the data points are the average of 3 tests.

4. COMPARISON WITH EXPERIMENTAL RESULTS

The proposed model was used to predict the measured hysteretic soil-water characteristic curves obtained from the above experimental. The Feng and Fredlund (1999) curve-fitting equation was used to best-fit the measured data points on the boundary drying curve. The first additional points on the boundary wetting curve were calculated using equation 2. The soil suction at the second point on the boundary wetting curve were calculated using equation 8. The best-fit parameters for the boundary wetting curve were calculated using equations 11 and 12. The calculated results are presented in Table 1 and Figs. 9 and 10.

Table 1. Calculated best-fit parameters for the two soils.

Boundary drying curve			Boundary wetting curve		
	Sand	Silt		Sand	Silt
w'_u	17.59	21.29	w'_u	17.59	21.29
b	38.28	850.05	b	7.44	110.43
c	4.12	5.96	c	3.89	5.24
d	2.09	1.81	d	2.10	1.61
Positions of two additional points			Predicted boundary wetting curve		
	Sand	Silt		Sand	Silt
ψ_1	1.90	11.67	w'_u	17.59	21.29
w_1	12.92	16.81	b	7.54	137.23
ψ_2	4.87	38.20	c	4.12	5.96
w_2	6.77	9.12	d	2.17	1.72

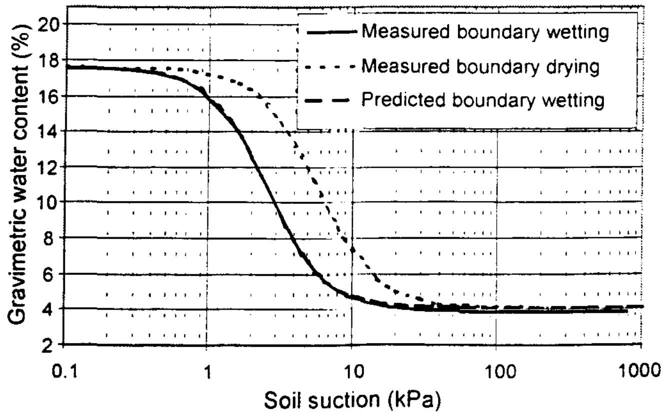


Figure 9. Predicted and measured boundary wetting curves for Beaver Creek sand using the Feng and Fredlund (1999) model.

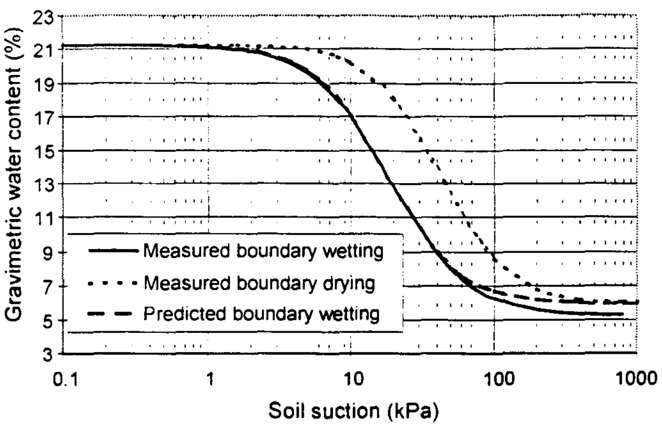


Figure 10. Predicted and measured boundary wetting curves for the processed silt using the Feng and Fredlund (1999) model.

The results show that the boundary wetting curve predicted using Feng and Fredlund (1999) model is quite close to the measured curve. The water content on the predicted boundary wetting curve is slightly higher than that of the measured boundary wetting curve. The error is caused by an insufficient number of measured data points on the boundary drying curve at high soil suctions. In order to increase the accuracy of the model, one of the following suggestions should be followed: 1.) increase the number of measured data points on the boundary drying curve at high soil suction; 2.) the residual soil suction of the boundary wetting curve, c_w, be chosen equal to the water content of the point having the highest soil suction on the measured boundary drying curve.

5. CONCLUSION

The Feng and Fredlund (1999) model requires a relatively simple dataset for calibration. The two additional points on the main wetting curve were clearly defined by the experimental program. The model predicts the boundary wetting curve quite well. The Feng and Fredlund (1999) model can be expanded to use more general curve fitting equations to describe both boundary curves. For example, when the Fredlund and Xing (1999) or van Genuchten (1980) curve fitting equation is used, three additional points along the boundary wetting curve need to be measured for calibration. The proposed model should be easy to use in engineering practice because of its simplicity.

REFERENCES

Barbour S. Lee. 1998. Nineteenth Canadian Geotechnical Colloquium; the soil-water characteristic curve; a historical perspective. *Canadian Geotechnical Joumal*, 35(5): 873-894.

Brooks, R. H., and Corey, A. T. 1964. Hydraulic properties of porous media. *Hydrology paper number 3*. Colorado state University, Fort Collins.

Bruch, P. G. 1993. A laboratory study of evaporative fluxes in homogeneous and layered Soils. *Master of science thesis*, University of Saskatchewan, Saskatoon, Canada.

Dane, J. H. and Wierenga, P. J. 1975. Effect of hysteresis on the prediction of infiltration, redistribution and drainage of water in layered soil. *Joumal of Hydrology*, 25: 229-242.

Feng, M. 1999. The effects of capillary hysteresis on the measurement of matric suction using thermal conductivity sensors. *Master of Science thesis*, University of Saskatchewan, Saskatoon, Canada.

Fredlund, D. G., and Rahardjo, H. 1993. *Soil mechanics for unsaturated soils*. John Wiley and Sons, New York, NY.

Fredlund, D. G., and Xing, A. 1994. Equations for the Soil-Water Characteristic Curve. *Canadian Geotechnical Joumal*, 31: 521-532.

Hank, R. J., Klute, A., and Bresler, E. 1969. A numerical method for estimating infiltration redistribution, drainage, and evaporation of water from soil. *Water Resources Research*, 13: 992-998.

Hogarth, W. L. Hopmans, J., Parlange, J. Y., and Haverkamp, R. 1988. Application of a simple soil-water hysteresis model. *Joumal of Hydrology*, 98: 21-29.

Jaynes, D. B. 1985. Comparison of soil-water hysteresis models. *Joumal of Hydrology*, 75: 287-299.

Karube, D., and Kawai, K. 2001. The role of pore water in the mechanical behaviour of unsaturated soils. *To be published in the Joumal of Geotechnical Engineering division, ASCE.*

Kawai, K., Karube, D., and Kato, S. 2000. The model of water retention curve considering effects of void ratio. *Proceeding Asian Conference on Unsaturated Soils,* pp. 329-334.

Mualem, Y. 1974. A conceptual model of hysteresis. *Water Resources Research*, 10(3): 514-520.

Mualem, Y. 1977. Extension of the similarity hypothesis used for modeling the soil water characteristics. *Water Resources Research,* 13(4): 773-780.

Mualem, Y. 1984a. Prediction of the soil boundary wetting curve. *Journal of Soil Science*, 137(6): 379-390.

Mualem, Y. 1984b. A modified dependent domain theory of hysteresis. *Journal of Soil Science*, 137(5): 283-291.

Mualem, Y., and Dangan, G. 1975. A dependent domain model of capillary hysteresis. *Water Resources Research*, 11(3): 452-460.

Mualem, Y., and Miller, E. E. 1979. A hysteresis model based on an explicit domain-dependence function. *Soil Science Society of America Journal*, 43: 1067-1073.