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Prediction of Volume Change in an Expansive Soil as a Result of Vegetation and Environmental Changes

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Abstract

This paper presents a numerical model to predict the volume change in an expansive soil as a result of vegetation and environmental changes. The model formulation is based on the general theory of unsaturated soil behavior. Three typical volume change situations associated with water uptake by tree roots and water infiltration into soils due to watering at surface are presented.

Introduction

Small buildings constructed in expansive soils are often subjected to severe distress subsequent to construction, as a result of changes in the surrounding environment. Changes in the environment may occur as a result of water uptake by vegetation, removal of vegetation and the excessive watering of a lawn. The magnitude of damage to an engineering structure caused by vegetation depends on a series of factors, which includes type of vegetation, soil and groundwater conditions, climate, foundation types and the distance from vegetation.

Many methods of volume change prediction have been proposed. The methods of volume change prediction are based either on soil suction measurements (or estimations) or on one-dimensional oedometer tests. Problems in engineering practice related to volume change and heave, are generally two-dimensional or three-dimensional in nature and are generally due to environmental changes or the effect of vegetation.

This paper presents a finite element, numerical model for the simulation of typical two-dimensional volume change problems that are often encountered in

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engineering practice. The model formulation is based on the general theory of unsaturated soil behavior. The soil suction conditions in a soil profile are computed through the use of a steady state or a transient unsaturated soil water flow analysis. The volume change constitutive relations for the unsaturated soil are formulated using two stress state variables; namely, $(\sigma-u_a)$ and (u_a-u_w), where σ is total normal stress, u_a is pore-air pressure and u_w is pore-water pressure. Darcy's law is applied to the flow of water, where the coefficient of permeability is written as a function of soil suction. The volume change model is based on two sets of non-linear constitutive relations, (i.e., one for changes in void ratio and the other for changes in water content). While the analysis and example problems are two-dimensional, the theory can readily be extended for 3-dimensional analyses.

The mechanism is explained whereby distress is caused to an engineered structure. The distress is generally related to changes in the ground surface flux boundary condition or due to the water demands of vegetation such as the root system of large trees. The root system of large trees is simulated as a flux boundary condition applied to the finite element mesh. In this way, the volume changes in the surrounding soil mass can be computed.

The engineering problems associated with unsaturated soils have proven to be complex since the soil properties are non-linear functions of stress state variables. The coefficient of permeability and the water storage term for the soil are assumed to be functions of soil suction in the analysis of water seepage. The unsaturated soil volume change functions are determined from typical one-dimensional oedometer results. The elastic modulus functions, E, with respect to changes in net normal stress, and H, with respect to changes in matric suction, are calculated from the swelling indices, C_t, with respect to changes in net normal stress, and C_m, with respect to changes in matric suction, respectively. The Poisson's ratio is assumed to be a constant.

Solutions for the water flow part of the analysis, as well as the stress-deformation part of the analysis, are obtained through the use of a general, partial differential equation solver, PDEase2D (1996). Three typical two-dimensional volume change problems associated with expansive soils are used to demonstrate the applicability of the proposed model. The first situation is associated with deformations in a soil profile due to water being extracted from a root zone (e.g., the roots of trees) within a soil mass. The root zone is simulated as a flux applied to the soil mass. The second situation is associated with the settlement of a house induced by a line of trees growing close to the house. The third situation is associated with the heave of a concrete slab on grade due to loading and water infiltration into the soil.

Theory of Volume Change of Unsaturated Soils

Continuity Requirements

The continuity requirement for an unsaturated soil, assuming the soil particles to be incompressible, can be stated as follows (Fredlund and Rahardjo, 1993):

$$\frac{\Delta V_v}{V_0} = \frac{\Delta V_w}{V_0} + \frac{\Delta V_a}{V_0} \quad (1)$$

where: V_0 = initial overall volume of soil element, V_v = volume of soil voids, V_w = volume of water, and V_a = volume of air.

By using a Cartesian coordinate system and referencing deformation to an elemental volume, the total volumetric deformation of an unsaturated soil element, $d\varepsilon_v$, can be written as the sum of the normal strains:

$$d\varepsilon_v = \frac{dV_v}{V_0} = d\varepsilon_x + d\varepsilon_y + d\varepsilon_z \quad (2)$$

where: $d\varepsilon_x$, $d\varepsilon_y$, $d\varepsilon_z$ = normal strain components in x, y, and z-direction, respectively.

Constitutive Relations

Two stress state variables are needed to describe volume change behavior of an unsaturated soil (Fredlund and Morgenstern, 1977). These stress state variables are net normal stress, $(\sigma - u_a)$, and matric suction, $(u_a - u_w)$, where σ is total normal stress, u_a is pore-air pressure and u_w is pore-water pressure. With the use of these two stress state variables, volume changes in the soil due to externally applied loads and the environmental changes (i.e., change in groundwater table, water uptake by a tree or infiltration) can be considered separately.

Assuming the soil behaves in an incrementally isotropic, linear elastic material, the soil structure constitutive relations can be written as follows (Fredlund and Rahardjo, 1993):

$$d\varepsilon_{ij} = \frac{1+\mu}{E} d(\sigma_{ij} - u_a) - \frac{\mu}{E} d(\sigma_{kk} - 3u_a)\delta_{ij} + \frac{d(u_a - u_w)}{H}\delta_{ij} \quad (3)$$

where: ε_{ij} = components of the strain tensor for the soil structure, σ_{ij} = components of the total stress tensor for the soil structure, $\sigma_{kk} = (\sigma_{11} + \sigma_{22} + \sigma_{33})$, δ_{ij} = the Kronecker delta, μ = Poisson's ratio, E = modulus of elasticity for the soil structure with respect to a change in net normal stress, and H = modulus of elasticity for the soil structure with respect to a change in matric suction.

Assuming water is incompressible, the water phase constitutive relations can be formulated in a semi-empirical approach as follows (Fredlund and Rahardjo, 1993):

$$\frac{dV_w}{V_0} = \frac{1}{E_w} d(\sigma_{ii} - 3u_a) + \frac{1}{H_w} d(u_a - u_w) \quad (4)$$

where: E_w = water volumetric modulus associated with a change in net normal stress and H_w = water volumetric modulus associated with a change in matric suction.

Equations 3 and 4 present the constitutive relationships in elasticity forms. These elasticity forms can be used to solve non-linear elastic volume change problems associated with unsaturated soils in two- or three-dimensions. Fredlund and Rahardjo (1993) also presented the constitutive relationships for soil structure and water phase in compressibility forms as follows:

$$\frac{dV_v}{V_0} = m_1^s d(\sigma_{mean} - u_a) + m_2^s d(u_a - u_w) \tag{5}$$

$$\frac{dV_w}{V_0} = m_1^w d(\sigma_{mean} - u_a) + m_2^w d(u_a - u_w) \tag{6}$$

where:

m_1^s = coefficient of volume change with respect to net normal stress [i.e., $3(1-2\mu)/E$];
m_2^s = coefficient of volume change with respect to matric suction [i.e., $3/H$];

m_1^w = coefficient of water volume change with respect to net normal stress [i.e., $3/E_w$];
m_2^w = coefficient of water volume change with respect to matric suction [i.e., $1/H_w$]; and
σ_{mean} = mean net normal stress [i.e., $(\sigma_x+\sigma_y+\sigma_z)/3$].

The constitutive relationships for the soil structure and water phase of an unsaturated soil can be presented graphically (Fig. 1) by plotting void ratio and volumetric water content against the independent stress state variables, $(\sigma-u_a)$ and (u_a-u_w). Coefficients of volume change corresponding to the unloading surface can be subscripted with an "*s*" to represent the word "*swelling*" (i.e., m_{1s}^s and m_{2s}^s).

The volumetric water content versus matric suction relation is called a soil-water characteristic curve (SWCC), and the coefficient of water volume change with respect to matric suction (i.e., m_2^w) is also called a water storage coefficient.

The constitutive surface for the soil structure can also be obtained when void ratio is plotted with respect to the logarithms of the stress state variables (Fig. 2). The

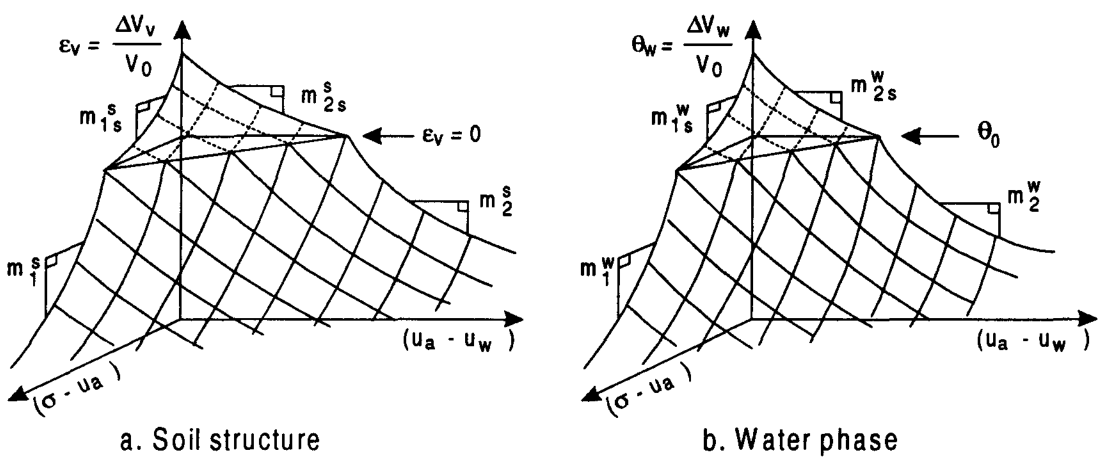

Figure 1. Three-dimensional constitutive surfaces for soil structure of an unsaturated soil (Fredlund and Rahardjo, 1993)

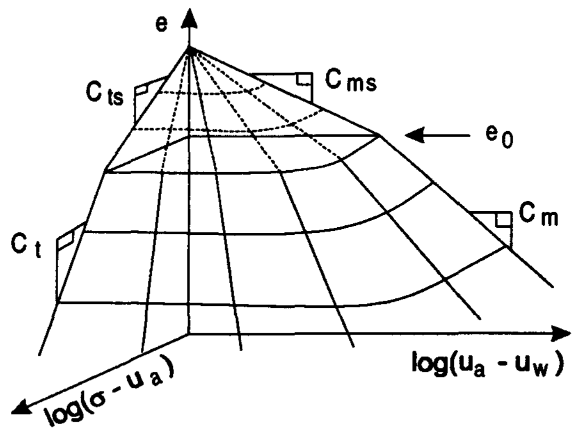

Figure 2. Semi-logarithmic plot of void ratio versus net normal stress and matric suction (Fredlund and Rahardjo, 1993)

logarithmic plots are essentially linear over a relatively large stress range on the extreme planes (i.e., the $log(\sigma - u_a) = 0$ plane and $log(u_a - u_w) = 0$ plane) (Fredlund and Rahardjo, 1993). Slopes of the void ratio versus logarithm of net normal stress or matric suction lines are called volumetric change indices, C_t or C_m (Fig. 2). These indices are used in many methods of one-dimensional heave prediction. The elastic modulus function, E is calculated from the volume change index C_t, with respect to changes in net normal stress. The elastic modulus function, H is calculated from the volume change index C_m, with respect to changes in matric suction.

Saturated/Unsaturated Water Flow

In stress-deformation analyses, displacements are calculated from changes in stress states (i.e., changes in net normal stress and matric suction) and the elastic moduli. Matric suction profile is necessary to describe the initial and final stress conditions in soils. If the pore-air pressure is assumed to be atmospheric, the distribution of pore-water pressure is equivalent to the matric suction distribution. The prediction of pore-water pressure must take into consideration in response to changes in the surface flux boundary conditions (i.e., infiltration, evaporation, and evapotranspiration) and the fluctuation of the ground-water tables. The pore-water pressure distribution within the soil can be estimated by performing a saturated/unsaturated seepage analysis.

The governing partial differential equation for water flow through a heterogeneous, anisotropic, saturated/unsaturated soil can be derived by satisfying conservation of mass for a representative elemental volume, assuming that flow follows Darcy's law with a non-linear coefficient of permeability. If it is assumed that the total stress remains constant during a transient process and that pore-air pressure is atmospheric, the differential equation can be written as follows for the two-dimensional transient case:

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y \frac{\partial h}{\partial y}\right)=m_2^w \gamma_w \frac{\partial h}{\partial t} \qquad (7)$$

where: h = total head (i.e., pore-water pressure head plus elevation head); k_x and k_y = coefficient of permeability of the soil in the x- and y-direction, respectively; γ_w = the unit weight of water (i.e., 9.81 kN/m^3), m_2^w = the slope of the soil water characteristic curve. Both the coefficient of permeability and coefficient of water storage are dependent on stress states in soils (i.e., net normal stress and matric suction). However, these coefficients of an unsaturated soil are predominantly a function of the matric suction.

The water storage indicates the amount of water taken or released by the soil as a result of a change in the pore-water pressure and can be represented by the slope of the soil-water characteristic curve. Therefore, the water storage function is obtained by differentiating the soil-water characteristic curve with respect to matric suction. Numerous equations have been proposed to simulate the soil-water characteristic curve (Gardner, 1958; van Genuchten, 1980; Fredlund and Xing, 1994). The soil-water characteristic curve described in the present study is limited to the Fredlund and Xing (1994) equation. The Fredlund and Xing (1994) equation is shown below:

$$\theta = \theta_s C(\psi)\left[\frac{1}{ln\left(e + (\psi/a)^n\right)}\right]^m \tag{8}$$

where: $C(\psi)$ = a correction factor for high soil suctions, defined as:

$$C(\psi) = \left(1 - \frac{ln(1 + \psi/\psi_r)}{ln(1 + 1000000/\psi_r)}\right)$$

where: ψ = soil suction (kPa), e = natural log base, 2.71828..., θ_s = volumetric water content at saturation, ψ_r = total suction corresponding to the residual water content θ_r, a = a soil parameter which is related to the air entry value of the soil (kPa), n = a soil parameter which controls the slope at the inflection point in the soil-water characteristic curve, and m = a soil parameter which is related to the residual water content of the soil.

There are many permeability functions that have been proposed to represent the permeability function of an unsaturated soil (e.g., Gardner, 1958; Phillip, 1986; Fredlund and Xing, 1994; Leong and Rahardjo, 1997). These equations involve finding best-fit parameters, which produces a curve that fits the measured data. The equation proposed by Leong and Rahardjo (1997) is used to describe the permeability function for transient water flow analysis in this paper. Leong and Rahardjo (1997) illustrated that the coefficient of permeability is a power function of volumetric water content. Using the Fredlund and Xing (1994) equation with $C(\psi)$ = 1, the permeability function was shown to take the following form:

$$k = k_s\left[\frac{1}{ln\left(e + (\psi/a)^n\right)}\right]^{mp} \tag{9}$$

The parameter p can be determined by using a curve fitting of the coefficient of permeability data. The slope of soil-water characteristic curve (i.e., coefficient of water storage) is obtained by differentiating the Fredlund and Xing (1994) equation (Fredlund, 1995). The water storage function is shown below for correction factor $C(\psi) = 1$:

$$m_2^w = -mn \frac{\theta_s}{\left[\dfrac{1}{\ln\left(e + (\psi/a)^n\right)}\right]^{m+1}} \frac{\psi^{n-1}}{ea^n + \psi^n} \tag{10}$$

where: $e = 2.718$ (i.e., natural log base).

The transient water flow equation (Eq. 7) along with the equation of a soil-water characteristic curve (Eq. 8) and a permeability function (Eq. 9), can be used to predict pore-water pressure profiles (i.e., suction profiles) at different times during a seepage process. The suction profiles can then be used to compute the suction change for the stress-deformation analysis. The deformations due to changes in suction during any time period can then be predicted by specifying the initial and final soil suction profile.

For steady state seepage, only the coefficient of permeability is required because the time dependent term in Eq. (7) disappears and the storage function disappears. Gardner's equation(1958) is used for steady-state flow analyses.

$$k = \frac{k_s}{1 + a\left(\dfrac{\psi}{\rho_w g}\right)^n} \tag{11}$$

where: ψ = soil suction (kPa), k_s = coefficient of permeability at saturation, a = constant inversely proportional to the breaking point of the function, and n = constant related to the slope of the function.

The permeability function (Eq. 11) can be used to predict the pore-water pressure distribution (i.e., suction distribution) at equilibrium in the soil under specified boundary conditions. The suction profile then can be used to compute suction changes for the stress-deformation analysis to predict deformations.

Stress-Deformation

Equations of equilibrium for the soil structure of an unsaturated soil are:

$$\sigma_{ij,j} + b_i = 0 \tag{12}$$

where: σ_{ij} = components of the net total stress tensor, and b_i = components of the body force vector.

The partial differential equations for the soil structure can be derived from constitutive equations (Eq. 3) and the force equilibrium equations (Eq. 12). The governing partial differential equations in term of displacements in x- and y-direction (i.e., u and v) for plane strain loading ($d\varepsilon_z = 0$) are as follows:

$$\frac{\partial}{\partial x}\left\{c\left[(1-\mu)\frac{\partial u}{\partial x}+\mu\frac{\partial v}{\partial y}-\frac{(1+\mu)}{H}(u_a-u_w)\right]\right\}+\frac{\partial}{\partial y}\left\{G\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)\right\}=0 \quad (13)$$

$$\frac{\partial}{\partial x}\left\{G\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)\right\}+\frac{\partial}{\partial y}\left\{c\left[\mu\frac{\partial u}{\partial x}+(1-\mu)\frac{\partial v}{\partial y}-\frac{(1+\mu)}{H}(u_a-u_w)\right]\right\}+\rho g=0 \quad (14)$$

where: $c=\frac{E}{(1-2\mu)(1+\mu)}$, $G=\frac{E}{2(1+\mu)}$, ρ = density of the soil, and g = acceleration due to gravity.

Equations 13 and 14 are non-linear since the elastic moduli are functions of the stress state variables. The elastic moduli, E and H, can be calculated from the volume change indices, initial void ratio and Poisson's ratio as follows (Hung and Fredlund, 2000):

$$E=n_t(\sigma_{ave}-u_a)_{ave} \quad (15)$$

$$n_t=\frac{4.605(1+\mu)(1-2\mu)(1+e_0)}{C_t}$$

$$H=n_m(u_a-u_w)_{ave} \quad (16)$$

$$n_m=\frac{4.605(1+\mu)(1+e_0)}{C_m}$$

where: n_t = coefficient that relates net normal stress with elastic modulus E, n_m = coefficient that relate matric suction with elastic modulus H, $\sigma_{ave}=(\sigma_x+\sigma_y)/2$, $(s_{ave}-u_a)_{ave}$ = average of the initial and final net normal stress for an increment, and $(u_a-u_w)_{ave}$ = average of the initial and final matric suction for an increment.

Equations 13 and 14 can be used to compute the displacements in horizontal and vertical directions under an applied load or due to changes in matric suction. These equations can also be used to compute the induced stresses in the soil under an applied load. Because of soil property non-linearity, an incremental procedure is used to obtain the solution of these equations. In the incremental procedure, the values of elastic moduli E and H are assumed to be unchanged within each stress and strain increment, but are changed from one loading increment to another.

Example Problems

Three typical volume change problems that are often encountered in engineering practice are analyzed. The matric suction conditions in the soil mass were first predicted by performing a saturated/unsaturated water flow analysis. Then the deformations in soils caused by changing matric suction were predicted by performing a stress-deformation analysis. Both seepage and stress-deformation analyses are performed using a Partial Differential Equation Solver, PDEase2D.

Example 1: Influence of Trees to Surrounding Soil

The first example problem is associated with the deformation in soil near a line of trees. It is assumed that the trees are planted in a line at every 5 m. The example considers a 10 m thick layer of clay soil. The coefficient of permeability of the soil is described using Gardner's (1958) equation with a saturated coefficient of permeability equal to 5.79×10^{-8} m/s (i.e., 5 mm/day), parameters a and n equal to 0.001 and 2, respectively. The initial void ratio of the soil is equal to 1.0, and the volume change index with respect to matric suction, C_m is equal to 0.2.

The initial matric suction is taken to be hydrostatic with an unchanged ground water table at the 15 m depth. This represent the water content conditions in soils in the winter when water uptake by the trees is low. It is then assumed that one tree will extract 0.3 m^3 of water per day in summer and the steady state condition is attained. The value of 0.25 to 0.5 m^3/day was suggested by Perpich et. al. (1965). The water uptake zone for the trees is from the 1 m to 3 m depth, with uptake rate decreasing linearly with depth. This pattern of tree root water uptake was suggested by de Jong (2000).

Deformations in the soil profile due to water uptake by trees (from initial to final matric suction state) were predicted. The elastic modulus function (Fig. 3) with respect to matric suction for the soil was computed using a given initial void ratio, volume change index, and assumed Poisson's ratio equal to 0.3. The function can be calculated from Eq. 16 as follows:

$$H = 59.9(u_a - u_w)_{ave} \tag{17}$$

The initial and final matric suction conditions used to predict deformations in the stress-deformation analysis are obtained from the steady state seepage analysis

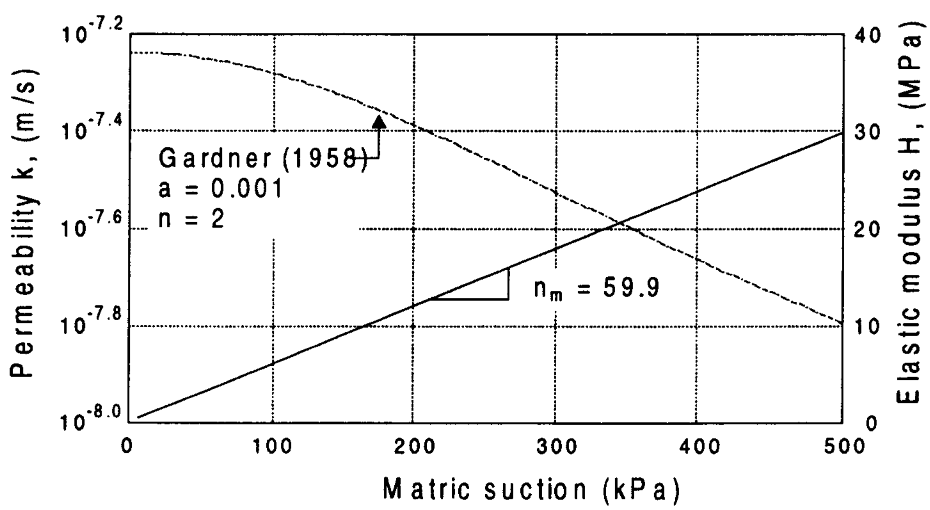

Figure 3. Permeability function, k and elastic modulus function, *H*, Example 1

using the coefficient of permeability function shown in Fig. 3. For boundary conditions for initial matric suction condition, a -15 m total head was specified at the lower boundary and a zero total head was specified at other boundaries. For final matric suction conditions, a -15 m total head was specified at the lower boundary, a boundary outflow value was specified along the left side of the soil domain at a depth from 1 m to 3 m and zero total head was specified at other boundaries. The outflow value decrease linearly from 15 mm/day at the 1 m depth to zero mm/day at the 3 m depth. This boundary condition represents 0.3 m^3/day of water being extracted by one tree from the soil (i.e., 2 sides x 2 m depth x 5 m wide x 15 mm/day = 0.3 m^3/day).

The matric suction distributions in the soil at equilibrium are shown in Figs. 4 and 5 for initial and final conditions, respectively. The initial matric suction varied from 147 kPa at ground surface to 49 kPa at 10 m depth. The final matric suction varied from 260 kPa at tree root to 49 kPa at 10 m depth. The matric suction change was a maximum at the tree root level and decreased with distance from the tree.

The deformations in the soil profile due to water uptake by tree roots was then predicted through the use of the stress-deformation analysis. The boundary condition for the stress-deformation analysis involved having the soil free to move in the vertical direction and fixed in horizontal direction at the left and right sides of the domain. The lower boundary was fixed in both directions.

Figure 6 presents contours of vertical displacement in the soil for this

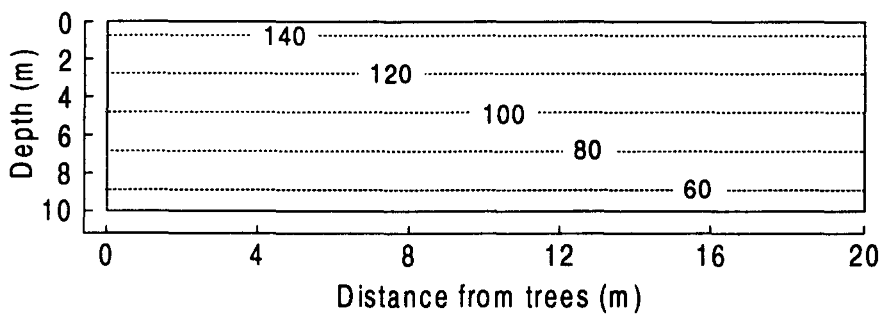

Figure 4. Initial matric suction (kPa) profile, Example 1

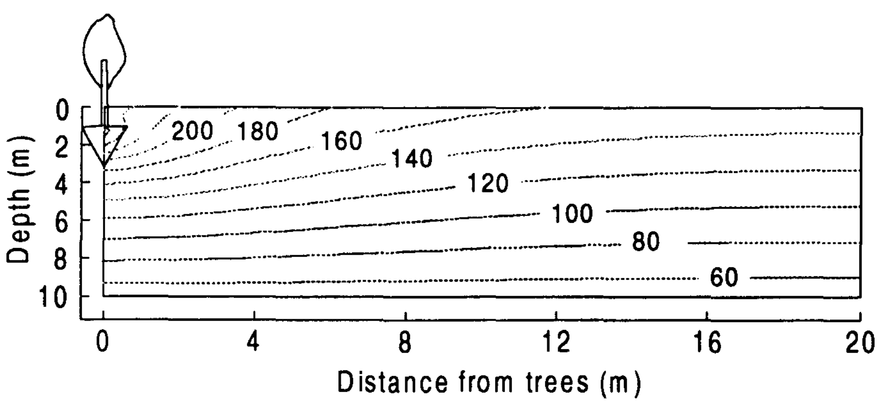

Figure 5. Final matric suction (kPa) profile, Example 1

example, with the volume change index with respect to matric suction equal to 0.2 and the tree-root uptake rate equal to 0.3 m^3/day per tree. The ground movements at various depths are shown in Fig. 7. It can be seen that the movements near the ground surface were quite large within a horizontal distance of about 4 m from the trees. The movements decreased rapidly with distance until at 12 m. The displacements also decreased rapidly with depth. The predicted displacements shown in Fig. 7 have the same pattern of displacements as these monitored by Bozozuk and Burn (1960). At ground surface, a value of settlement of 85 mm at tree location decreased to 40 mm at 8 m from the trees. At 4 m from the trees, a settlement of 65 mm at ground surface decreased to 40 mm at the 3 m depth, and about 20 mm at the 5 m depth.

The example was also analyzed for the case where the water uptake rate was 0.3 m^3/day and the volume change index varied with overburden pressure in soils (i.e., volume change index decreases linearly from 0.2 at ground surface to 0.05 at 10

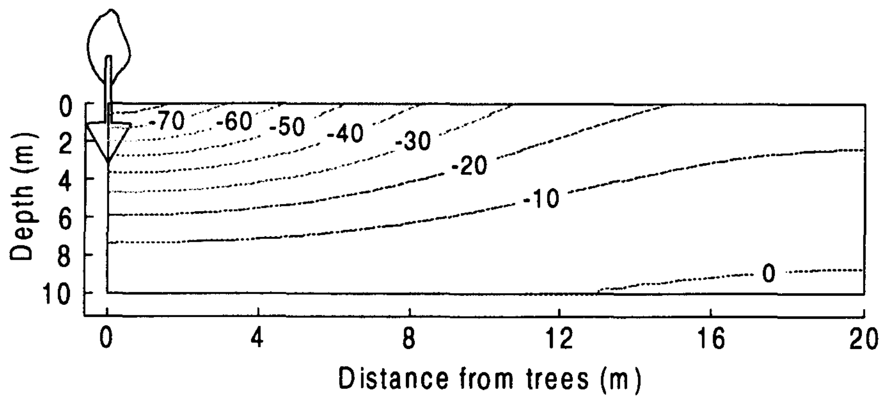

Figure 6. Contours of vertical displacement (mm), Example 1

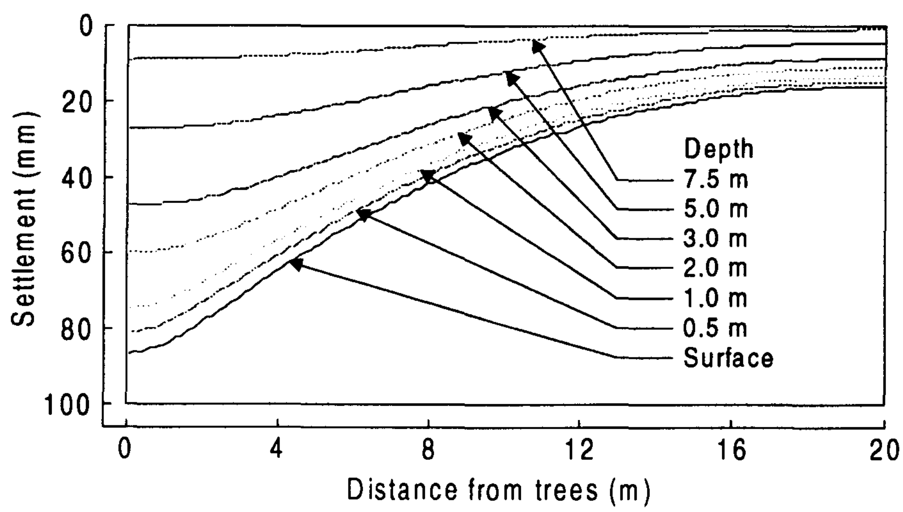

Figure 7. Variation with depth of ground movements near a line of trees, Example 1

m depth). Contours of vertical displacement are presented in Fig. 8 for this case. Most of displacements take place near ground surface.

Figure 9 presents contours of settlement for the case with a volume change index equal to 0.2 and a water uptake rate equal to 0.5 m^3/day. About 12% more settlement is obtained at tree location in comparison to the case when the water uptake rate was 0.3 m^3/day (i.e., (102 - 85)/85 = 12%).

Example 2: Influence of a Line of Trees to a House Footing

Example 2 simulates the settlement of a house induced by a line of trees growing close to the house. This example is illustrated in Fig. 10. The house foundation is placed at a 2 m depth in a clay soil layer that is 10 m thick. The line of trees is 4 m from the house. Water uptake rate of the trees is 0.5 m^3/day per tree and the water uptake zone is from 1 m to 3 m depth. It is assumed that the ground water table is unchanged at 15 m below ground surface. A layer of 0.3 m thick of concrete with elastic modulus of 100,000 kPa is used to describe the foundation and basement walls of the house. Soil properties are the same as those used in Example 1 (Fig. 3).

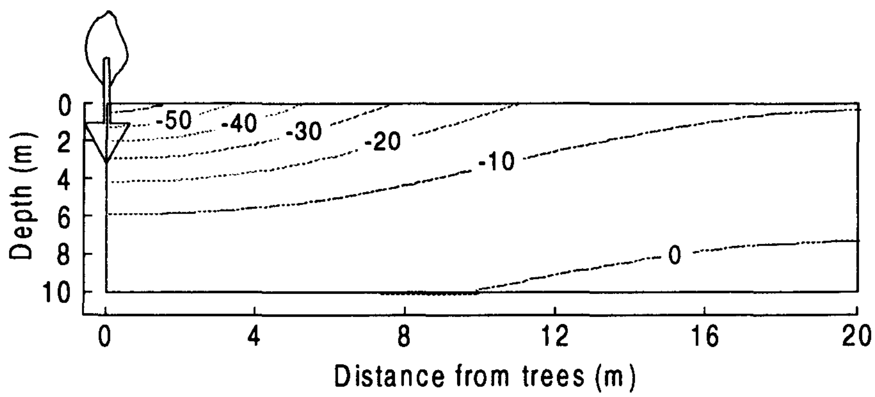

Figure 8. Contours of vertical displacement (mm) when volume change index varied from 0.2 at surface to 0.05 at the 10 m depth, Example 1

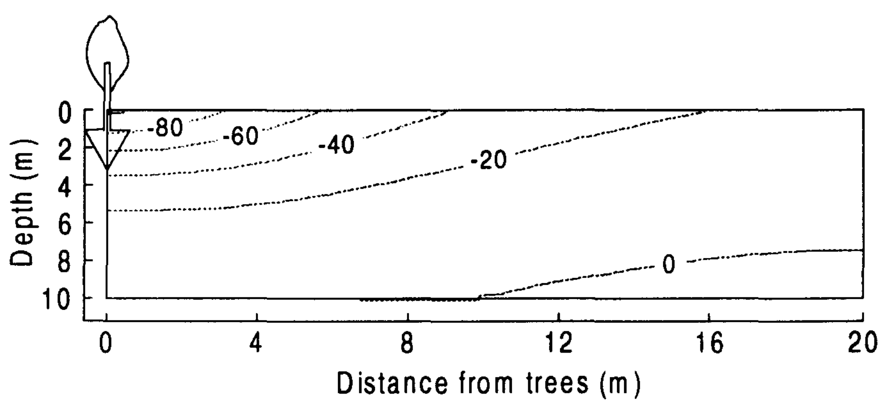

Figure 9. Contours of vertical displacement (mm) when water uptake rate was 0.5 m^3/day and volume change index equaled to 0.2, Example 1

Matric suction conditions in the soil profile were obtained through steady state unsaturated seepage analyses. The initial matric suction profile is the same as that shown in Fig. 4. The final matric suction profile is shown in Fig. 11. Final matric suction varied from 409 kPa at tree root to 49 kPa at lower boundary of the soil domain. The contours of changes in matric suction are presented in Fig. 12. The closer to the tree, the more change in suction is observed. Figure 12 also showed a similar pattern of moisture deficit near trees presented in Biddle (1983). The results of stress-deformation analysis are shown in Figs. 13 and 14 as contours of horizontal displacement and contours of vertical displacement. A maximum foundation settlement of 80 mm and minimum settlement of 25 mm was observed. A maximum settlement in the soil profile took place at tree location and decreased with horizontal distance and depth.

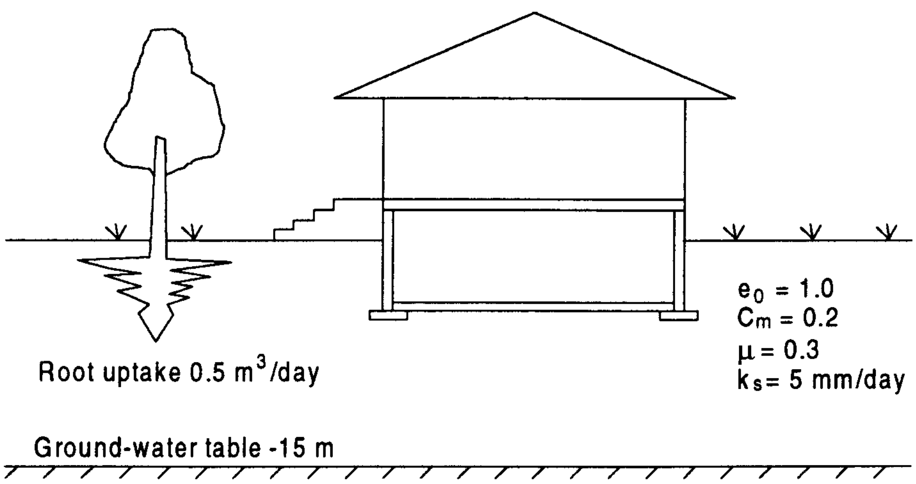

Figure 10. Illustration of Example 2

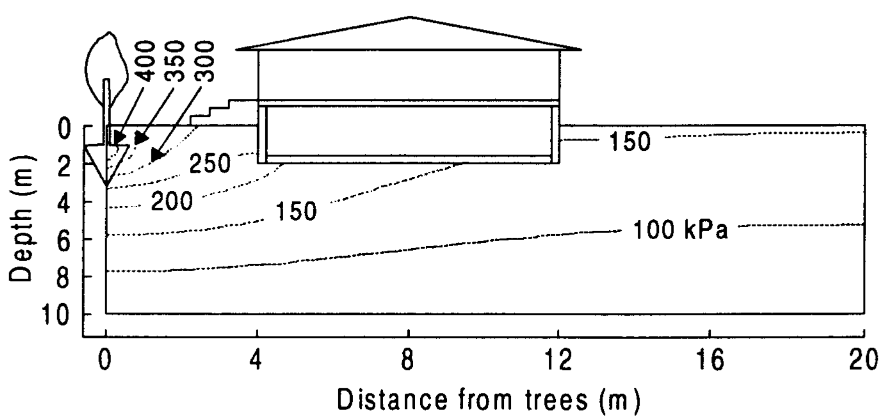

Figure 11. Contours of final matric suction (kPa), Example 2

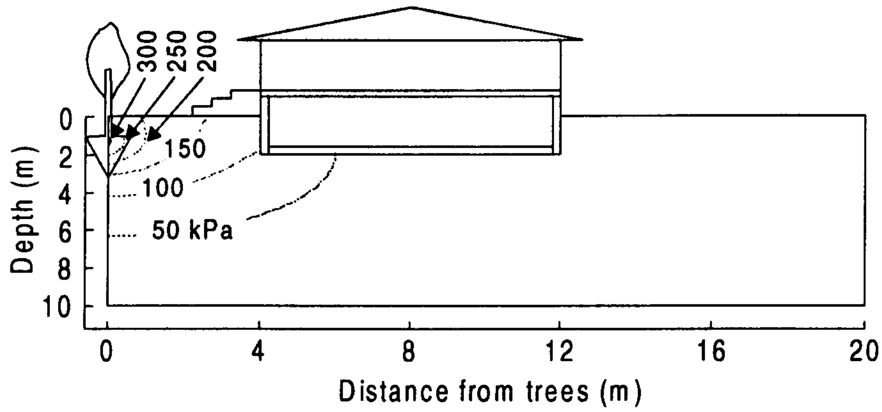

Figure 12. Contours of changes in matric suction (kPa), Example 2

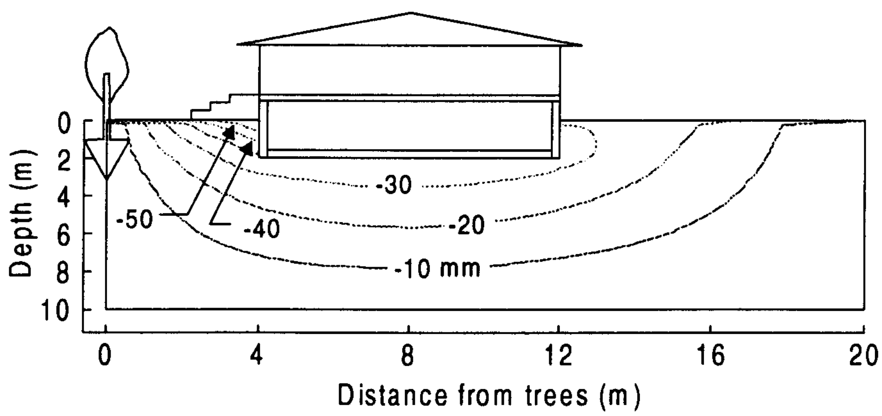

Figure 13. Contours of horizontal displacement (mm), Example 2

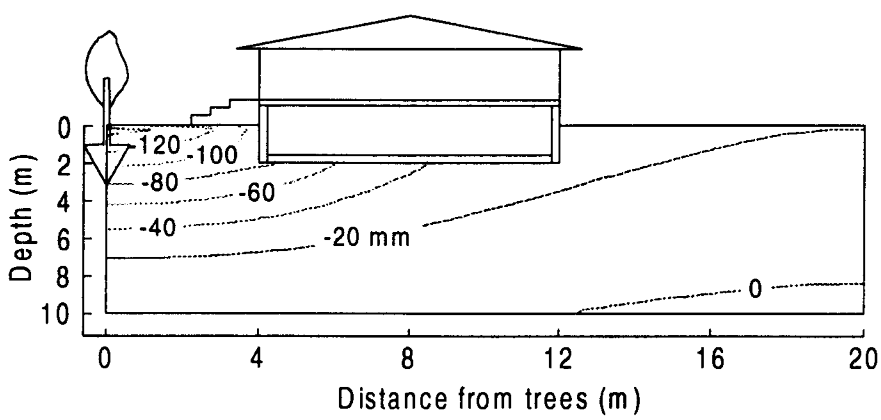

Figure 14. Contours of vertical displacement (mm), Example 2

Example 3: Influence of Ground Surface Flux to a Concrete Floor Slab

This example is presented to show a comprehensive volume change problem in an unsaturated soil. This example considers the deformations in a soil profile under a concrete floor slab due to applied load and changes in matric suction (i.e., water infiltration into the soil profile from surface). The deformations will be predicted for various predetermined elapsed times. The problem to be analyzed is illustrated in Fig. 15. A concrete slab, 0.3 m thick and 8 m wide is placed on a 5 m thick layer of swelling clay. All the soil properties to be used in the analysis are presented in Figs. 15 and 16. The soil has an initial suction of 700 kPa throughout the profile. The applied load on the slab, including its weight is 10 kPa. The right side of the slab is covered with an impermeable flexible layer. The grass at the left side is watered with the flux of 5.79×10^{-9} m/s.

The soil-water characteristic curve is described using the Fredlund and Xing (1994) equation, with the parameters *a* equal to 100 kPa, *n* equals 1.5 and *m* equals 1. The permeability function is described using the equation proposed by Leong and Rahardjo (1997) based on the Fredlund and Xing (1994) function for the soil-water characteristic curve with *p* equal to 1. Because the parameter *p* is equal to 1, the soil-water characteristic curve and the permeability function have the same shape and are shown in Fig. 16. The concrete slab has elastic modulus with respect to net normal stress equal to 1 MPa.

The elastic modulus function with respect to net normal stress, E, can be calculated from the given compressive index with respect to net normal stress, initial void ratio and assumed Poisson's ratio using Eq. 15 and written as follows:

$$E = 12.89(\sigma_{ave} - u_a)_{ave} \quad (18)$$

The elastic modulus function with respect to matric suction, *H*, is calculated using Eq. 16 for the soil with given initial void ratio, swelling index with respect to matric suction, and assumed Poisson's ratio. This function can be written as follows:

$$H = 184.20(u_a - u_w)_{ave} \quad (19)$$

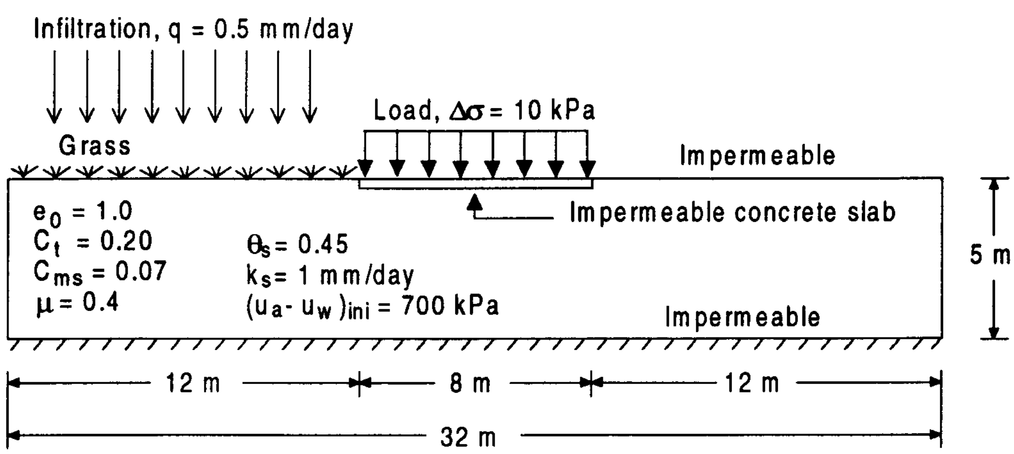

Figure 15. Illustration of Example 3

The matric suction conditions in the soil at various elapsed times can be predicted by performing an unsaturated transient seepage analysis. For the transient seepage analysis, a boundary flow value of 5.79×10^{-9} m/s is specified along the uncovered surface and zero flux is specified at the other boundary sides. The initial matric suction is equal to 700 kPa. Matric suction profile at 400 days of constant infiltration is presented in Fig. 17 as a typical matric suction distribution in soils.

Deformation in the soil mass due to loading can be assumed to respond immediately, while the deformation due to wetting is a time dependent process. Therefore, the stress-deformations due to loading and due to wetting need to be analyzed independently. A stress-strain analysis is performed in six stages. One stage is for changes in net normal stress and the other five are for changes in suction. In the first stage, the deformation due to loading (i.e., change in net normal stress) is modeled. The second stage is carried out to determine the deformation due to changes in matric suction after 100 days of watering the grass. In this stage, the initial matric suction profile has a constant value of 700 kPa, and the final matric suction profile is computed from the water flow analysis, at the first elapsed time of 100 days. The third stage gives the deformation due to changes in suction in the next 100 days (i.e., from 100th to 200th day). The fourth, fifth and sixth stages are for the

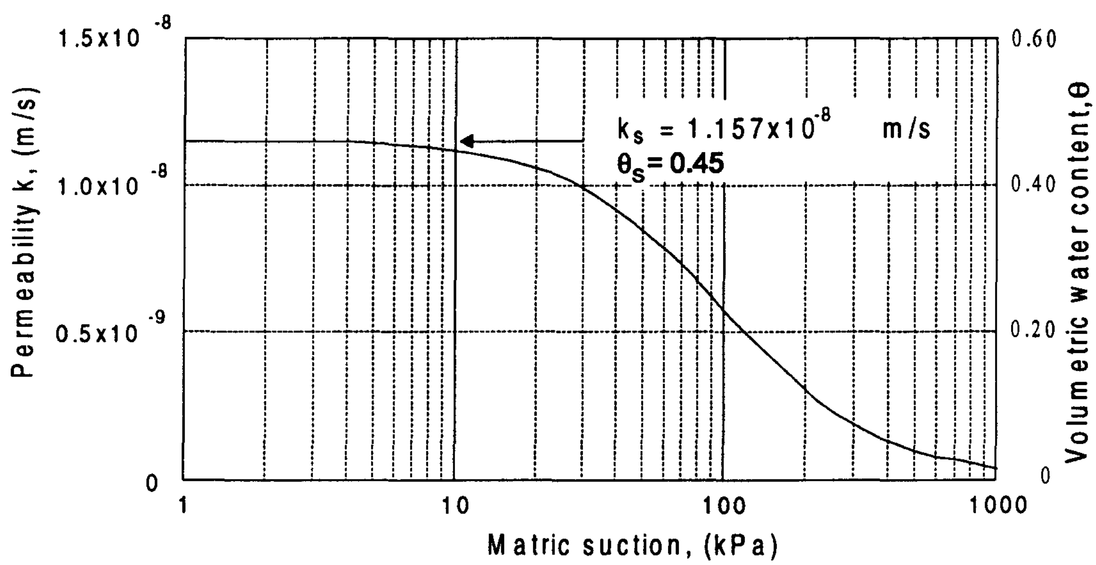

Figure 16. Permeability function and SWCC, Example 3

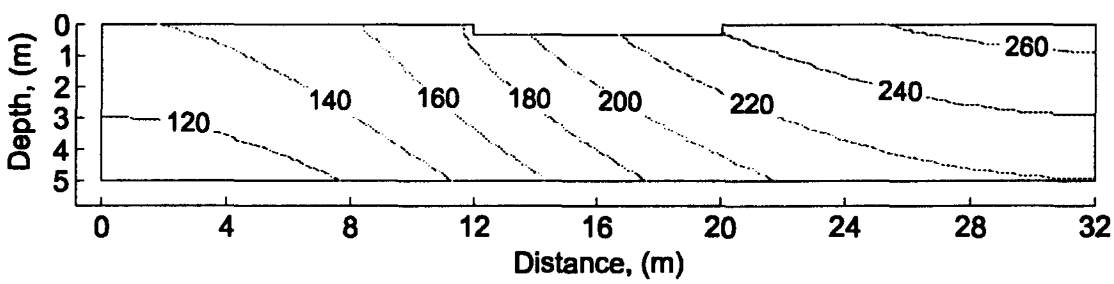

Figure 17. Matric suction (kPa) profile at day 400, Example 3

time periods from 200th to 300th day, 300th to 400th day, and 400th to 450th day, respectively.

The boundary conditions are specified for the stress-deformation analysis as follows. At the left and right sides of the domain, the soil is free to move in the vertical direction while it is fixed in the horizontal direction. The lower boundary is fixed in both directions.

The initial vertical total stress is calculated from total stress theory and the initial horizontal stress is calculated using a coefficient of earth pressure at rest, K_0, equal to 0.67.

The vertical displacement and horizontal displacement due to wetting for each stage of the analysis are predicted. The cumulative vertical displacements at the surface are presented in Fig. 18 for each stage. Figures showing distribution of horizontal displacements are not presented due to lack of space in the paper. The maximum calculated heave increases from 20 mm during the first 100 days to 45 mm in the stage from "Day 300" to "Day 400" of infiltration, and in the last stage (i.e., in the last 50 days), a maximum heave of 46 mm was predicted. Small horizontal displacements were observed during the infiltration (i.e., maximum of 1.5 mm for the first 100 days and 5 mm for the last 50 days).

Figure 19 presents the cumulative vertical displacements below the concrete slab. The vertical displacements caused by loading are presented by the contours in Fig. 20. A maximum settlement of 65 mm was predicted for the concrete slab.

Total heave due to wetting for 450 days is presented in Fig. 21. As much as 155 mm of heave was predicted at the surface of the exposed area (grass portion), and 90 mm of heave was predicted at the covered portion. The differential heave of the concrete slab was about 30 mm. The horizontal displacements take place only in the grassed portion, where 10 mm of horizontal displacements was predicted. The

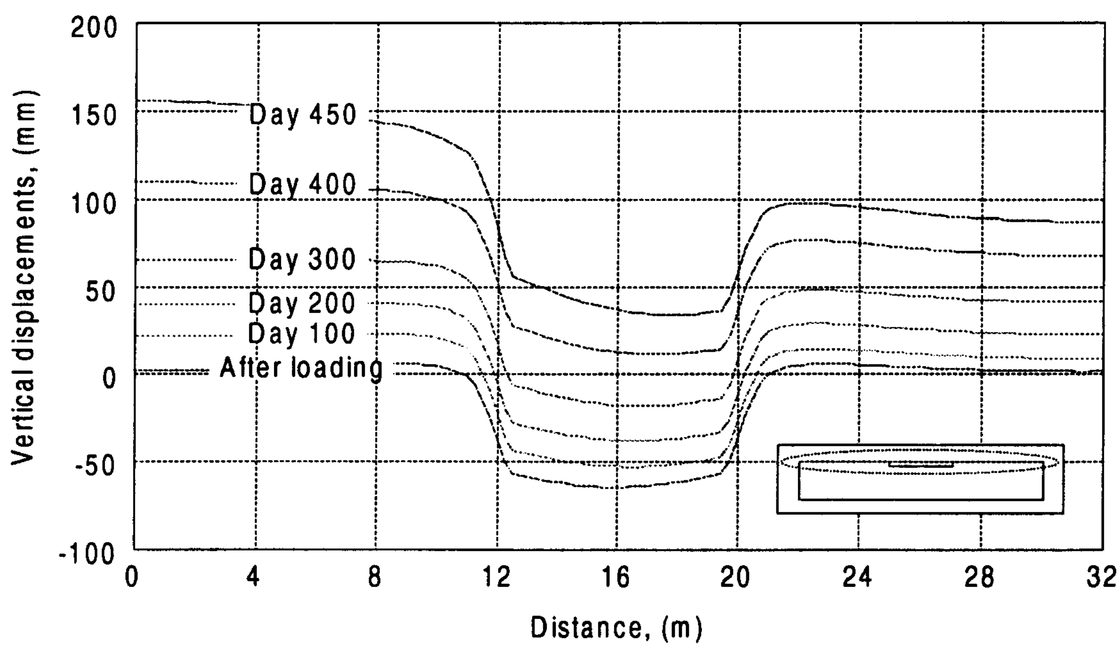

Figure 18. Cumulative vertical displacement at surface at various times, Example 3

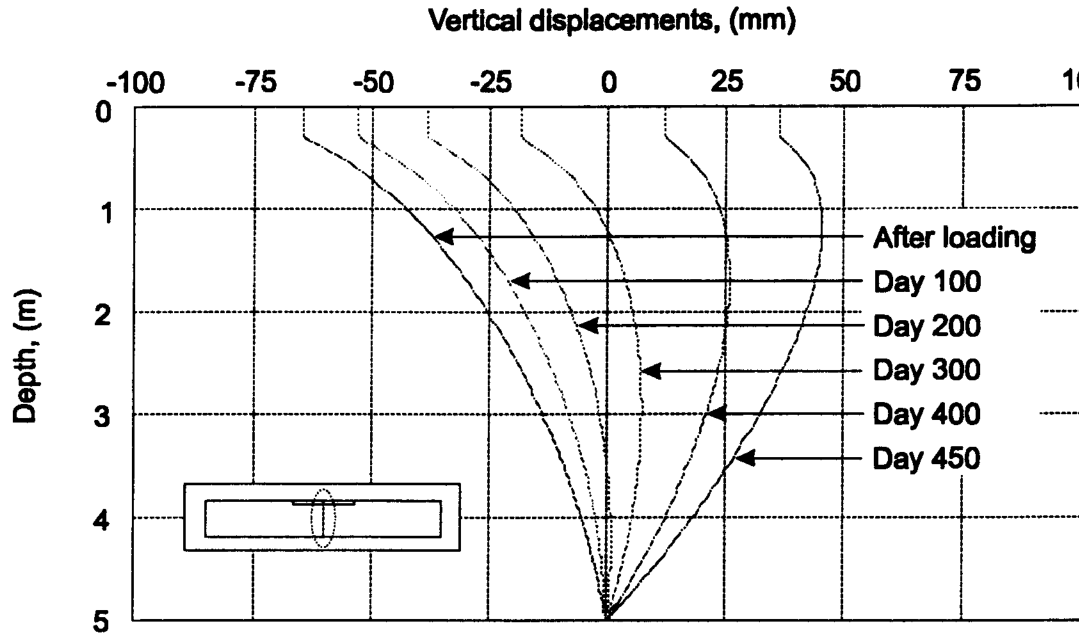

Figure 19. Cumulative vertical displacements versus depth below the slab at various times, Example 3

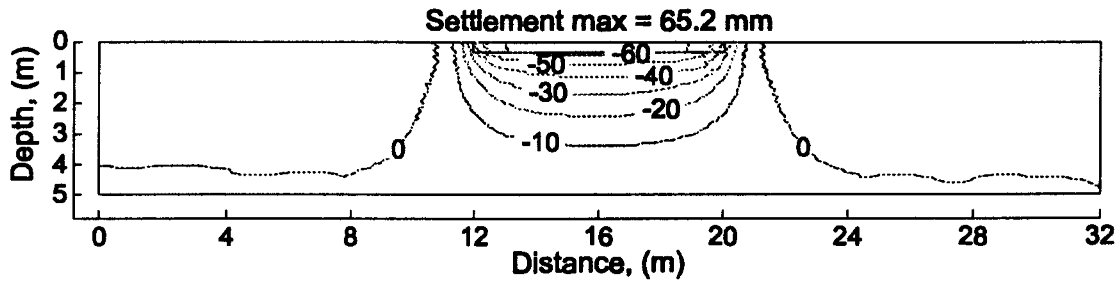

Figure 20. Contours of vertical displacement due to loading (mm), Example 3

differential heaves predicted depend on the rate of infiltration; a lower rates of infiltration results in lower differential heave.

Figure 22 presents total vertical displacements due to the combined effects of loading and wetting in 450 days. There was about 155 mm of heave in the grassed portion, about 40 mm of heave below the concrete slab and about 90 mm of heave in the cover area. Maximum horizontal displacements of 20 mm took place at about 1.5 m depth below the end of the concrete slab.

Conclusion

A method of volume change prediction based on the general theory of unsaturated soil behavior have been used to study the influence of water uptake by tree roots and water infiltration to surrounding soils and nearby structures. Results of the analyses appear to be reasonable and consistent with measured values. The results show that magnitude of deformation of structures and soils close to the trees decreased with vertical and horizontal distance from trees, and with decreasing of water uptake rates. It is possible to combine unsaturated soil theory for seepage and stress analysis to study the influence of vegetation and environmental conditions on light engineered structures.

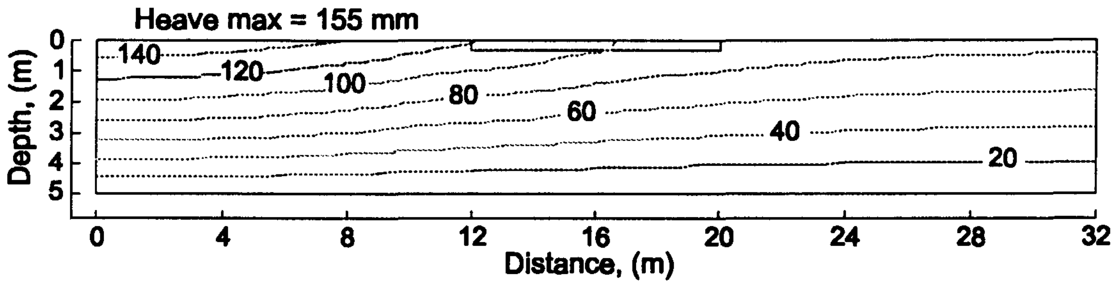

Figure 21. Contours of vertical displacement due to wetting after 450 days (mm), Example 3

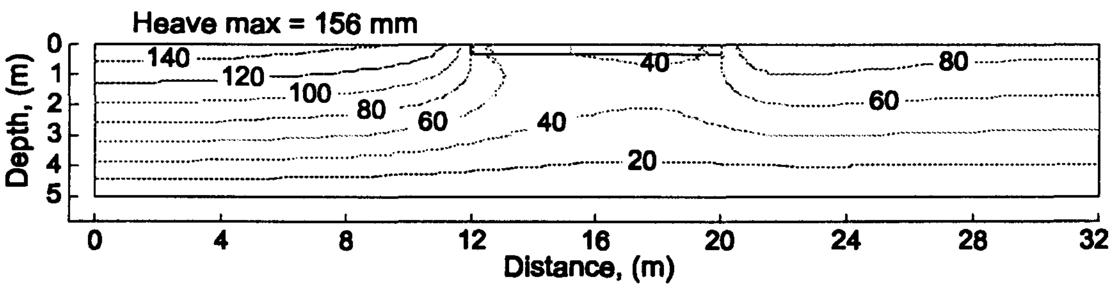

Figure 22. Contours of vertical displacement due to loading and wetting after 450 days (mm), Example 3

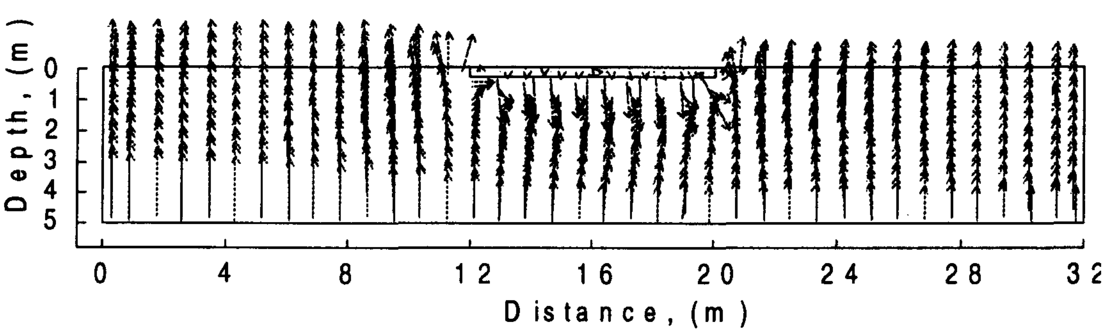

Figure 23. Distribution of deformation vectors due to loading and wetting after 450 days (mm), Example 3

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