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Predicting the permeability function for unsaturated soils

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ABSTRACT: The permeability function for an unsaturated soil can be estimated by raising the equation for the soil-water characteristic curve to a power and then multiplying the result by the saturated coefficient of permeability. However, when using this procedure there is one variable that must be assumed; namely, the power, q. This paper presents the analysis of several hundred experimental data sets and provides guidance with regard to the most suitable values to use for the exponent, q. The resulting values of q are obtained from best-fit analyses according to various soil types. The overall average q value for all soils analyzed was 3.3 and the standard deviation was 1.4. The results also showed that sandy soils had a lower q value than clayey soils.

1 INTRODUCTION

Several earth structures such as soil covers and soil liners are commonly constructed for the management of various types of wastes. It is important to know the water storage and hydraulic conductivity properties in order to perform an adequate design of these structures.

There are several empirical, but theoretically-based models proposed in the literature for the prediction of the seepage through unsaturated soils. These models are useful for the estimation of the coefficient of permeability with respect to suction. The predictions are commonly made with the use of the soil-water characteristic curve and the saturated coefficient of permeability. The soil-water characteristic curve used in these models is commonly measured in the laboratory using a Pressure Plate apparatus. An estimation of the soil-water characteristic curve can also be obtained through the use of a knowledge-based database system such as SoilVision (Fredlund, 1997).

Fredlund and Xing (1994) proposed an equation for the soil-water characteristic curve that can be used to best-fit data over the entire range of soil suctions from 0 to 1,000,000 kPa. Fredlund (1995) also showed how it was possible to use the saturated coefficient of permeability and the soil-water characteristic curve to obtain a permeability function. The procedure involved integration over the range of the soil-water characteristic curve, starting from saturated soil conditions. The coefficient of permeability over the entire range of soil suctions is defined as the permeability function.

In 1997, Leong and Rahardjo suggested that the permeability function could also be approximated through the use of a single additional parameter applied to the soil-water characteristic curve. The variable was an exponent applied to the equation for the soil-water characteristic curve. When using this procedure, a new soil parameter, q, must be estimated in some manner. The objective of this research paper is to study the nature of the power variable, q, required in order to estimate the permeability function for unsaturated soils.

2 CONCEPTS OF SEEPAGE FOR UNSATURATED SOILS

The constitutive relationship to describe flow through a saturated or unsaturated soil is Darcy's law.

$$v = -k_w \frac{dh}{dy} \tag{1}$$

where v = flow velocity over the discharge area; and k_w = coefficient of permeability.

The proportionality variable between velocity and hydraulic gradient is assumed to be a constant for saturated soils, k_s, but becomes a permeability function for an unsaturated soil. The coefficient of permeability of an unsaturated soil is a function of the amount of water in the soil which, in turn, can be written in terms of the stress state of the soil (Huang et al. 1998).

$$k_w = func\,[k_s,\ (\sigma - u_a),\ (u_a - u_w)] \tag{2}$$

It is generally considered sufficient to quantify the amount of water in the soil as a function of soil suction, $(u_a - u_w)$ or ψ. The unsaturated coefficient of permeability can then be written as a function of the saturated coefficient of permeability and the dimensionless water content, Θ, that is equal to $w(\psi)/w_s$. The variable $w(\psi)$ is equal to the water content at any soil suction, ψ, and w_s is the saturated soil water content. It is possible to include an additional fitting parameter, q, to complete the functional relationship.

$$k_w(u_a - u_w) = func\,[(w(\psi)/w_s),\ q,\ k_s] \tag{3}$$

Numerous analyses have been proposed for the estimation of the permeability function for unsaturated soils (Fredlund et al. 1994; Leong and Rahardjo, 1997). Common to all methods is the existence of a mathematical relationship between the coefficient of permeability and the soil-water characteristic curve.

3 THE SOIL-WATER CHARACTERISTIC CURVE

The soil-water characteristic curve has played a dominant role in understanding the behavior of unsaturated soils in disciplines such as soil science, soil physics, agronomy and agriculture. As a consequence of the long history associated with the use of the soil-water characteristic curve, large amounts of information and experimental data are available from these disciplines. The soil-water characteristic curve is now recognized as one part of the overall water phase constitutive relationship in geotechnical engineering. The soil-water characteristic curve is of great value in predicting unsaturated soil property functions.

The proposed equation defining the soil-water characteristic curve, by Fredlund and Xing (1994) provides a mathematically based function over the entire range of suctions from zero to 1,000,000 kPa. The relationship is empirical, being derived using the assumption that the soil consists of a set of interconnected pores that are randomly distributed. The equation, written in terms of gravimetric water content, w, is as follows:

$$w = \left[1 - \frac{ln\left(1 + \frac{\psi}{\psi_r}\right)}{ln\left(1 + \frac{1{,}000{,}000}{\psi_r}\right)}\right]\left[\frac{w_s}{\left(ln\left(e + \left(\frac{\psi}{a}\right)^n\right)\right)^m}\right] \tag{4}$$

where a = a suction value corresponding to the inflection point on the curve that has physical meaning in its relationship to the air-entry value of the soil; n = soil parameter related to the slope of the soil-water characteristic curve in the transition stage; ψ = soil suction (i.e., matric suction, $(u_a - u_w)$, at low suctions and total suction at high suctions); m = parameter related to the residual water content; w_r = water content at residual conditions; e = natural number, 2.71828.

Equation 4 can be written in a dimensionless water content form, Θ, by dividing both sides of the equation by the saturated water content.

$$\Theta = \left[1 - \frac{ln\left(1 + \frac{\psi}{\psi_r}\right)}{ln\left(1 + \frac{1{,}000{,}000}{\psi_r}\right)}\right]\frac{1}{\left(ln\left(e + \left(\frac{\psi}{a}\right)^n\right)\right)^m} \tag{5}$$

The first term in brockets is a correction factor that ensures that the function goes through zero (at a suction of 1,000,000 kPa). Equation 5 can be used to best-fit the desorption (or adsorption) branches of soil-water characteristic curve data. The fitting parameters (i.e., a, n and m values) can be determined using a non-linear regression procedure such as the one proposed by Fredlund and Xing (1994).

4 COEFFICIENT OF PERMEABILITY (OR HYDRAULIC CONDUCTIVITY) FUNCTION

The shape of the hydraulic conductivity function (or permeability function) bears a relationship to the shape of the soil-water characteristic curve. Figure 1 compares the soil-water characteristic curves and hydraulic conductivity functions for sand and clayey silt. The hydraulic conductivity for both soils remains relatively constant from zero suction up to the air entry value of the soil. The change in the hydraulic conductivity of a soil occurs at approximately the air entry value of the soil. The hydraulic conductivity decreases rapidly beyond the air entry value, for both soils. The decrease in the hydraulic conductivity is due to the reduction in the cross sectional area of flow. The initial hydraulic conductivity, or saturated hydraulic conductivity, k_s, of the sand can be two or more orders of magnitude greater than that of the clayey silt. As the suction increases, it is possible that the hydraulic conductivity of the sand will decrease by more than two orders of magnitude. Under certain conditions, it is possible for the clayey silt to be more permeable than the sand.

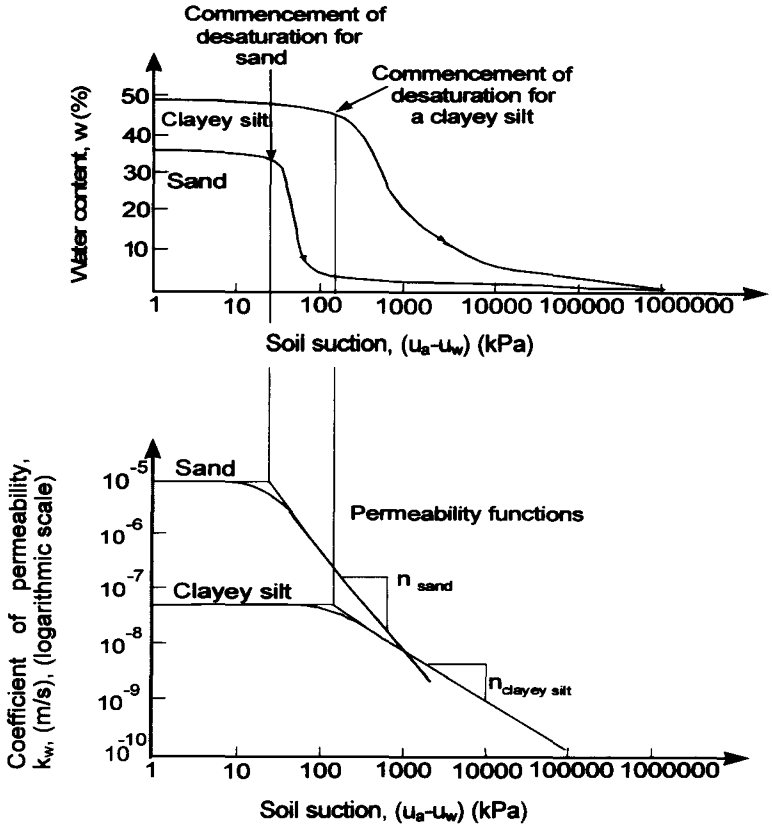

Figure 1. Water content and coefficient of permeability versus soil suction.

5 ESTIMATION OF THE PERMEABILITY FUNCTION FROM THE SOIL-WATER CHARACTERISTIC CURVE

The permeability function of an unsaturated soil can be predicted with sufficient accuracy for many engineering applications with a knowledge of the saturated coefficient of permeability and the soil-water characteristic curve. Several investigators have proposed empirical functions for predicting the permeability function (Huang et al. 1998). The soil-water characteristic curve equation developed by Fredlund et al. (1994) along with the saturated coefficient of permeability, can be used to compute the relationship between hydraulic conductivity and soil suction. Previous studies (Leong and Rahardjo 1997; Benson and Gribb 1997) have shown that proposed integration procedures involving the use of the Fredlund and Xing (1994) soil-water characteristic curve, provide a good estimate of the permeability function.

The calculation of the permeability function is performed by dividing the water content versus suction relationship into several water content increments. This is equivalent to integrating along the water content axis. The numerical integration procedure can be used to compute data points for a permeability function for the unsaturated soil.

$$k_r(w) = \int_{w_r}^{w} \frac{w-x}{\psi^2(x)}dx \Big/ \int_{w_r}^{w_s} \frac{w_s-x}{\psi^2(x)}dx \qquad (6)$$

where k_r = relative coefficient of permeability; and x = a variable of integration representing water content.

The accuracy of the prediction of the permeability function was shown to improve when the complete soil-water characteristic curve was used. Although the permeability function can be computed down to zero water content, it should be noted that the function may be more indicative of vapor flow in the region beyond the residual stress state. As a result, it may be more reasonable to leave the hydraulic conductivity as a constant, k_{res}, beyond the residual state. When calculating the permeability function, it is convenient to perform the integration along the soil suction axis as shown in the following equation.

$$k_r(\psi)=\int_{\psi}^{\psi_r}\frac{w(y)-w(\psi)}{y^2}w'(y)dy\bigg/\int_{\psi_{aev}}^{\psi_r}\frac{w(y)-w_s}{y^2}w'(y)dy \tag{7}$$

where ψ_{aev} = air-entry value of the soil under consideration; ψ_r = suction corresponding to the residual water content w_r; ψ = a variable of integration representing suction; y = a variable of integration representing the logarithm of suction; and w' = the derivative of the soil-water characteristic curve. Leong and Rahardjo (1997) reported that the "computed coefficient of permeability for this statistical model (i.e., equations 6 and 7 in this paper) showed good agreement with the measured coefficient of permeability."

To avoid the numerical difficulties associated with performing the integration over the entire soil suction range, it is more convenient to perform the integration on a logarithm scale. The proposed models have been found to be most satisfactory for sandy soils whereas agreement with experimental data may prove to be less satisfactory for clayey soils. Equation 8 can be multiplied by the dimensionless water content raised to a power (i.e., Θ^p) in order to provide greater flexibility in computing the permeability function. The additional parameter, p, is assumed to account for tortuosity in the soil pores (Maulem 1986) but it also a parameter whose magnitude must be assumed.

$$k_r(\psi)=\Theta^p(\psi)\frac{\displaystyle\int_{ln(\psi)}^{b}\frac{w(e^y)-w(\psi)}{e^y}w'(e^y)dy}{\displaystyle\int_{ln(\psi_{aev})}^{b}\frac{w(e^y)-w_s}{e^y}w'(e^y)dy} \tag{8}$$

Based on research work by Kunze et al. (1968), the value of the power, p, can be assumed to be 1 unless there is reason to assume otherwise.

Leong and Rahardjo (1997) suggested that rather than performing the above integration, the dimensionless equation for the soil-water characteristic curve, simply be raised to a power, q. Therefore, the permeability function can be written in the following form.

$$k_r(\psi)=k_s[\Theta(\psi)]^q \tag{9}$$

where $\Theta(\psi)$ = dimensionless form of the soil-water characteristic curve; and q = a new soil parameter.

This form for the permeability function is obviously attractive due to its simplicity. The equation is simple to use and clearly illustrated the relationship between the soil-water characteristic curve and the permeability function.

In order to use Equation 9, it is necessary to know what value to use for the q soil fitting parameter. The primary purpose of the present study is to determine typical values for the new soil parameter, q. The procedure used to evaluate the q parameter, along with the results obtained, are presented below.

In the original study undertaken by Leong and Rahardjo (1997), the data from six soils was used to assess the magnitude of the q soil fitting parameter. The results of their study are shown in Table 1.

Table 1. Summary of typical q parameters from the study by Leong and Rahardjo (1997).

Soil type	a kPa	n	m	q
Beit Netofa clay	389	0.69	1.176	52.12
Rehovot sand	2.25	4.32	1.235	6.04
Touchet silt loam	7.64	7.05	0.506	4.55
Columbia sandy loam	5.81	10.59	0.381	5.79
Superstition sand	2.66	6.86	0.525	6.21
Yolo light clay	2.93	2.11	0.379	9.57

Table 2. Summary of soil types analyzed for permeability function.

USDA Textural Classification	Number of samples
Clay	21
Clay loam	18
Loam	12
Loamy sand	29
Sand	49
Sandy clay loam	17
Sandy loam	30
Silt loam	74
Silty clay	34
Silty clay loam	18

6 PRESENT RESEARCH PROGRAM

The present research study involved the analysis of approximately 300 sets of permeability data. Each data set consisted of: i.) experimental results from the measurement of the soil-water characteristic curve, ii.) experimental results on the measurement of the coefficient of permeability under various applied soil suctions, and iii) the measured saturated coefficient of permeability. All experimental results were extracted from the SoilVision (Software the proprietary property of SoilVision system Ltd.)

knowledge-based system. The soils were divided into a number of categories depending upon their USDA textural classification. The categories, along with the number of data sets in each category, are shown in Table 2.

In addition to analyzing each soil type independently, the combination of all soil types was also analyzed.

Each of the soil-water characteristic curve data sets was first best-fit to determine the *a, n,* and *m* parameters associated with the Fredlund and Xing (1994) equation. Then the soil-water characteristic curve and the saturated coefficient of permeability were used to best-fit the permeability data in accordance with the equation proposed by Leong and Rahardjo (1997). The best-fit analysis of the permeability data gave rise to a calculation of the *q* soil parameter. All of the above analyses were conducted using the SoilVision software program.

The *q* soil parameter for all soil types have been statisically analyzed. There is no analysis of the soil parameters associated with the best-fit of the soil-water characteristic curves.

7 PRESENTATION AND ANALYSIS OF THE DATA

A typical best-fit of the Leong and Rahardjo (1997) procedure for a sand soil is shown in Figure 2. A value of *q* parameter was 2.54. A similar best-fit for a silt loam is shown in Figure 3 and the *q* parameter was 3.62. Both plots illustrate that the predicted permeability function quite closely fits the experimental data when the *q* parameter is known.

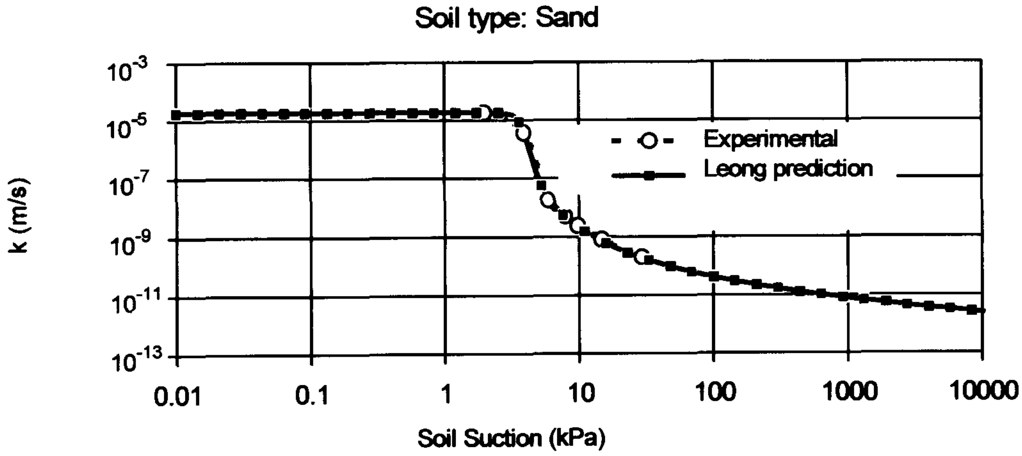

Figure 2. Comparison of permeability function predicted using Leong and Rahardjo (1997) with experimental data for sand.

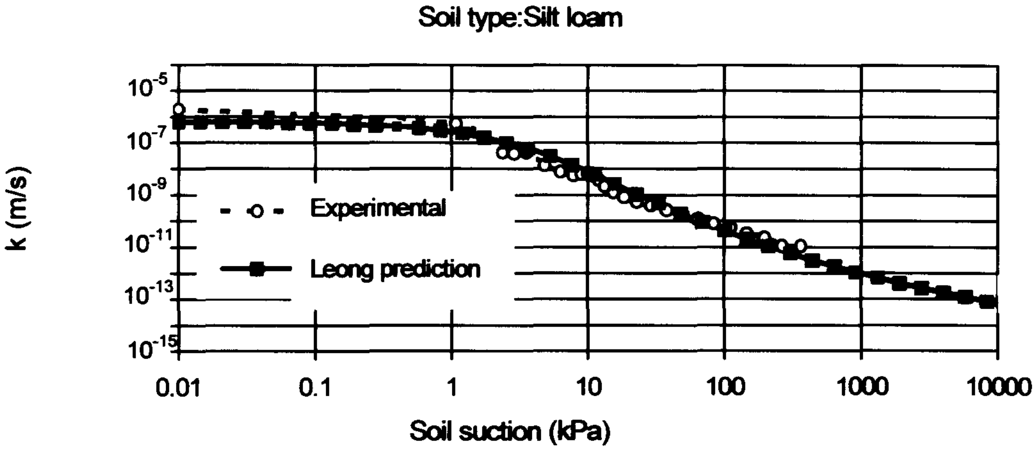

Figure 3. Comparison of permeability function predicted using Leong and Rahardjo (1997) with experimental data for silt loam.

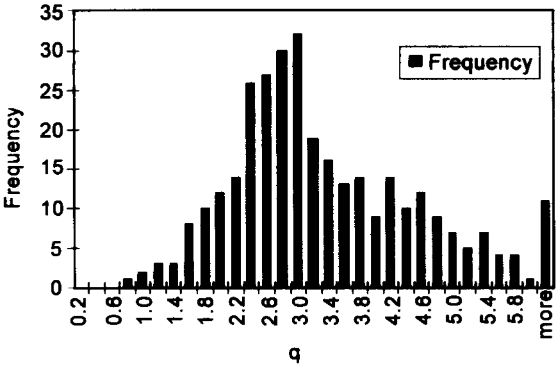

Figure 4. Frequency distribution associated with the q parameter when all soil types are combined.

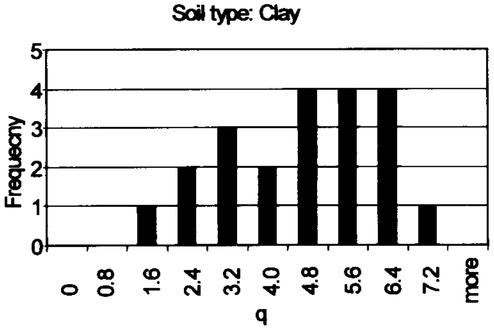

Figure 5. Frequency distribution for clay.

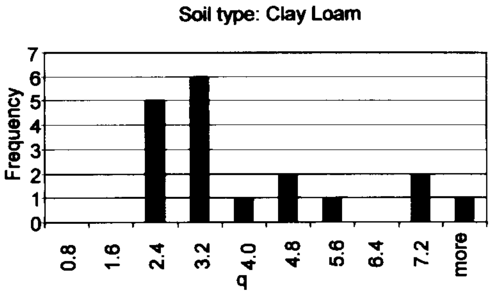

Figure 6. Frequency distribution for clay loam.

Figure 4 shows the frequency distribution associated with the q parameter when all soil types are combined. The statistical analysis of the q parameter is presented in Table 3.

The results show that the overall mean value for q is 3.29 and the standard deviation of q is 1.40. These values yield an overall coefficient of variation of 43%, which is quite high. The overall median value is 2.96 and the mode is 5.61.

Frequency distributions have also been drawn for each of the soil types and are presented in Figures 5 to 14. Some of the frequency distributions are not close to a normal distribution and this is most likely due to the limited number of samples in each category. Table 4 summarizes the statistical properties associated with the computed q parameters.

The sand showed a mean q value of 2.37 while the clay soils showed a mean q value of 4.34. The sandy loam showed a mean q value of 2.86 while the clay loam showed a mean q value of 3.58. The results indicate that there is a definite trend towards a larger q value for soils with higher plasticity. This trend is quite consistent for all soils categories.

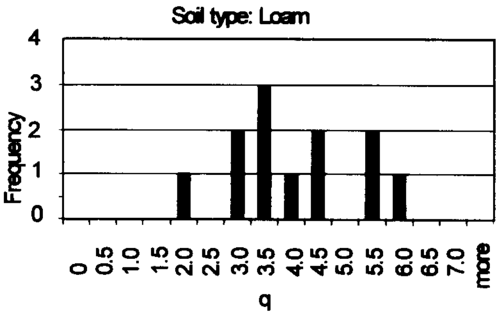

Figure 7. Frequency distribution for loam

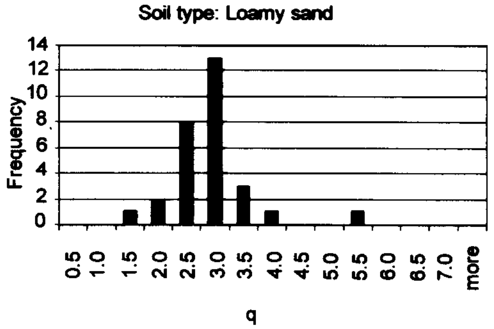

Figure 8. Frequency distribution for loamy sand

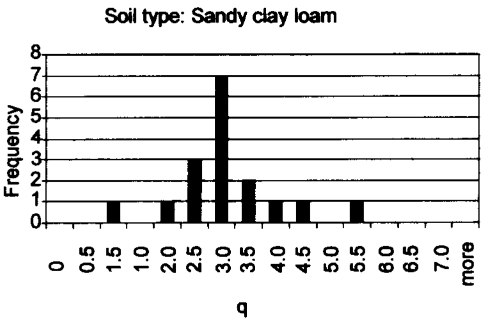

Figure 9. Frequency distribution for sandy clay loam

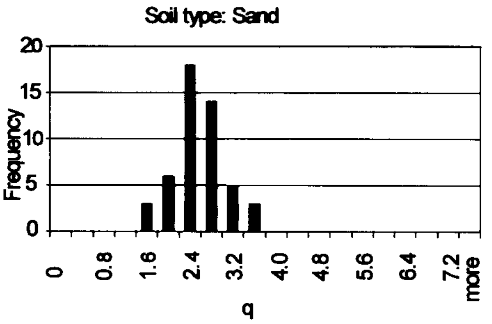

Figure 10. Frequency distribution for sand

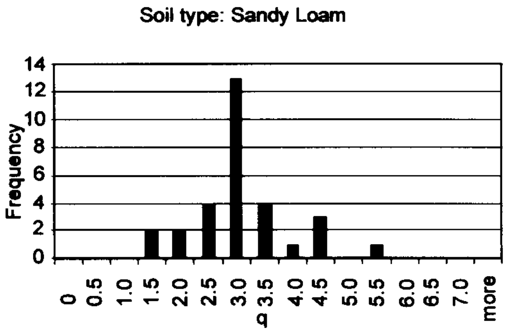

Figure 11. Frequency distribution for sandy loam

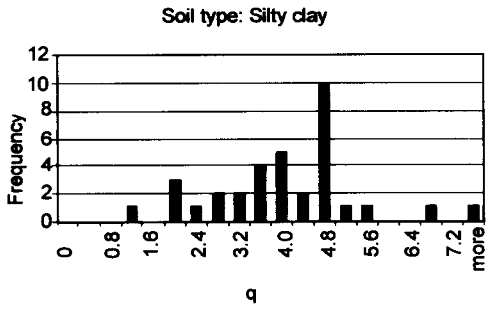

Figure 12. Frequency distribution for silty clay

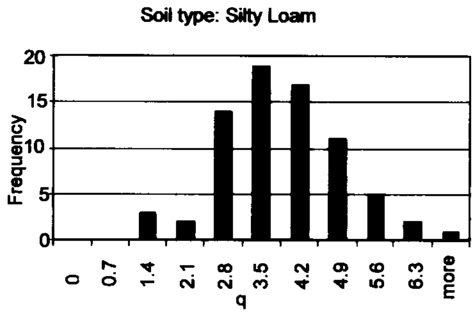

Figure 13. Frequency distribution for silty loam

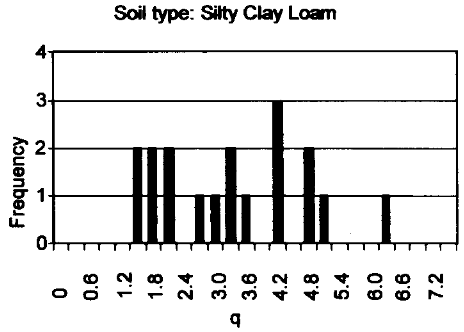

Figure 14. Frequency distribution for silty clay loam

Table 3. Statistical analysis of the q soil parameter

Statistic variable	q soil parameter
Mean	3.29
Standard error	0.08
Median	2.96
Mode	5.61
Standard deviation	1.40
Sample variance	1.96
Kurtosis	14.80
Skewness	2.35
Range	14.39
Minimum	0.64
Maximum	15.03
Number of soils	323

The standard deviation was 0.49 for the sand soil and 1.50 for the clay soil. The standard deviation was 0.84 for the sandy loam and 1.81 for the clay loam. The results indicate that there is less scatter in the fitting parameter q as the soil becomes closer to a sandy material. Also, the fitting parameter, q, moves closer towards 1.0 as the soil becomes sandy.

8 SUMMARY OF FINDINGS

The mean fitting parameters for all soil types ranged from 2.4 to 5.6. The statistical results provide a gen-

Table 4. Statistics of permeability according to Leong function for various soil types

Statistics	Clay	Clay loam	Loam	Sand	Sandy clay Loam	Silty clay	Silty clay loam	Silt loam	Sandy loam	Loamy sand
Mean	4.34	3.58	3.78	2.37	2.80	5.59	3.22	3.52	2.86	2.67
Median	4.71	2.62	3.56	2.36	2.62	4.77	3.18	3.46	2.85	2.59
Mode	3.00	2.80	3.25	2.20	2.75	4.60	4.05	3.15	2.75	4.63
Standard Deviation	1.50	1.81	1.16	0.49	1.00	1.31	1.36	1.09	0.84	0.68
Sample Variance	2.25	3.29	1.34	0.24	0.99	1.73	1.86	1.19	0.71	0.46
Range	5.42	5.92	4.14	2.25	4.71	6.33	4.97	5.99	4.20	4.01
Minimum	1.52	1.84	1.63	1.25	0.64	1.11	1.28	0.83	1.02	1.41
Maximum	6.94	7.76	5.76	3.49	5.35	7.44	6.25	6.82	5.22	5.42
Number of sets	21	18	12	49	17	34	18	74	30	29

eral indication of the range and scatter that can be anticipated when using the Leong and Rahardjo (1997) equation for the estimation of the permeability function for an unsaturated soil.

It would be of value to have a larger data base for analyzing the q parameter. At the same time, it must be recognized that there is always considerable scatter in the results. Certainly, the q parameter will tend to be greater than 1.0 and could vary over a considerable range. The estimation procedure proposed by Leong and Rahardjo (1997) is of value but there does not appear to be unique values for the q parameter.

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