EVALUATION OF PHYSICAL BASED HYSTERSIS MODELS OF SOIL-WATER CHARACTERISTIC CURVE

ĐÁNH GIÁ CÁC MÔ HÌNH TRỄ VẬT LÝ CỦA ĐƯỜNG ĐẶC TÍNH ĐẤT-NƯỚC

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ABSTRACT: An evaluation of five physical based soil-water hysteresis models is presented. The choice of these five models is based on their physical meaning and previous studies. The study showed that, Mualem's model-II (1974) is the simplest model that can be used to obtain the best results in predicting the scanning curves in most cases. Taking into account the effect of pore blockage in Mualem's dependent model (1984b) improves the accuracy of the model in few cases. Mualem's independent model (1984a) and Hogarth et al.'s model (1988) are good for predicting the main wetting curve in some cases. Because of its simplicity, Hogarth et al.'s model is recommended for predicting the main wetting curve.

TÓM TẮT: Báo cáo trình bày đánh giá 5 mô hình trễ vật lý của đất-nước.Việc chọn 5 mô hình này dựa trên các nghiên cứu trước đây và ý nghĩa vật lý của chung. Nghiên cứu cho thấy mô hình II của Mualem (1974) là mô hình đơn giản nhất có thể sử dụng để tra kết quả trong phần lớn các trường hợp dựa trên đường đặc tính. Mô hình phụ thuộc Mualem (1984b) có xét đến ảnh hưởng của lỗ rỗng làm tăng tính chính xác của mô hình trong một số trường hợp. Mô hình độc lập Mualem (1984a) và mô hình Hogarth (1988) ứng dụng tốt để dự đoán đường cong ướt trong một số trường hợp. Mô hình Hogarth được kiến nghị để xác lập đường cong ướt do tính đơn giản của mô hình.

Key words: *hysteresis model, soil-water characteristic curve, soil suction, water content*

1. INTRODUCTION

The relationship between the water content and the suction of an unsaturated soil is called the soil-water characteristic curve. Soil-water characteristic curves play an important role in understanding the behavior of unsaturated soils. The soil-water characteristic curve is hysteretic, (i.e., the water content at a given suction for a wetting path is less than that for a drying path). The difference in water content of the wetting and drying processes is caused by the following reasons (Klausner, 1991):

1- Irregularities in the cross-sections of the void passages or "ink-bottle" effect.

2- The effect of the contact angle, which is greater in an advancing meniscus then in a receding one.

3- Entrapped air or a vacuum condition, which advances or recedes meniscus, respectively, along with the corresponding pressure or suction.

4- Thixotropic regain or aging due to wetting and drying history of the soil.

The soil-water characteristic curve has been used in the modelling of unsaturated soil behaviour such as seepage modelling and in predicting other unsaturated soil property functions such as permeability function and shear strength function. In these applications, the soil-water characteristic curve is often assumed to be a unique curve because of the difficulty in characterizing hysteresis. A potential hysteresis model is needed for its possible application in unsaturated soils.

There are different names have been given to the various parts of the soil-water hysteresis curves. The names applied to the hysteresis curves in Fig. 1 are as follows:

1 - Initial drying curve

2 - Main drying curve

3 - Main wetting curve

4 - Primary wetting scanning curve (wetting scanning curve)

5 - Primary drying scanning curve (drying scanning curve)

There are infinite number of higher order scanning curves (i.e., scanning curves issuing from the scanning curves).

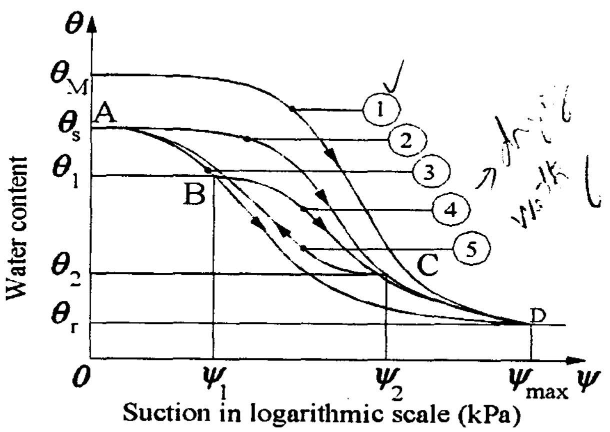

Fig.1 Schematic Illustration of soil-water hysteresis curves of a soils

Several studies have been made on the evaluation of hysteresis models (i.e., Jaynes, 1985; Viaene et al., 1995). In the each study, only a few sets of data were used for evaluation.

This paper presents an evaluation of the primary hysteresis models presented in literature. These models include 'independent' and 'dependent' models as presented by Mualem (1974, 1984a, 1984b), Mualem and Miller (1979), and Hogarth et al. (1988). All five models can predict the scanning curve at any order. However, a comparison of only primary scanning curves was implemented because of the insufficiency of data for higher order scanning curves. The calculation of primary scanning curves and higher order scanning curves is based on the same assumptions. Hence, the model that gives good results for the primary scanning curves should also give good results for the higher order scanning curves. A brief summary of five models will be presented in the next section.

2. THEORY

All five models are physical based soil-water hysteresis models. The wetting and drying processes of the soils are based on the domain theory.

There are two diagrams describing the distribution of water in a soil during the wetting and drying processes: 1. Nộel diagram and 2. Mualem's diagram. In the five models, only Hogarth et al.'s model (1988) uses the Nộel diagram and the rest use the Mualem's diagram. These five hysteresis models apply the "similarity hypothesis" to reduce required measured data for calibration.

a. Mualem's model-II (1974)

Mualem's model-II allows the prediction of the scanning curves from the main curves. The drying scanning curve starting at a suction ψ_1 on the main wetting curve can be calculated as:

$$\theta_d(\psi_1,\psi) = \theta_w(\psi) + \frac{[\theta_w(\psi_1) - \theta_w(\psi)]}{[\theta_s - \theta_w(\psi)]}[\theta_d(\psi) - \theta_w(\psi)] \qquad (1)$$

where,

$\theta_w(\psi)$ = water content on the main wetting curve at suction ψ

$\theta_d(\psi)$ = water content on the main drying curve at suction ψ

θ_s = water content at zero suction

The wetting scanning curve starting at suction ψ_2 on the main drying curve can be calculated as:

$$\theta_w(\psi_2,\psi) = \theta_w(\psi) + \frac{[\theta_s - \theta_w(\psi)]}{[\theta_s - \theta_w(\psi_2)]}[\theta_d(\psi_2) - \theta_w(\psi_2)] \qquad (2)$$

b. Mualem and Miller's model-III$_{expl}$ (1979)

Model-III$_{expl}$ is an improvement of the Mualem and Dagan's model-III (1975). This model is a dependent domain model, that means the effects of pore blockage has been taken into account. However, in this model the effect of pore blockage was taken into account for the drying process only.

Theoretically, this model is similar to Mualem's model-II (1974). However, it takes into account the effects of pore blockage and brings in one more unknown variable (P_d^*). An additional drying scanning curve is required to calibrate the model. The pore blockage function can be expressed as:

$$P_d^*(\theta_0) = \frac{\text{Change of actual water content}}{\text{Change of water content (no blockage)}} \qquad (3)$$

The drying scanning curve starting at a suction ψ_1 on the main wetting curve can be calculated by:

$$\theta_d(\psi_1,\psi) = \theta_w(\psi_1) - P_d^*(\theta_0)[1 - H(\psi)]\cdot[\theta_w(\psi_1) - \theta_w(\psi)] \qquad (4)$$

The wetting scanning curve starting at suction ψ_2 on the main drying curve can be calculated by:

$$\theta_w(\psi_2,\psi) = \theta_d(\psi_2) + P_d^*(\theta_{02})[1 - H(\psi_2)]\cdot[\theta_w(\psi) - \theta_w(\psi_2)] \qquad (5)$$

where, P_d^* is a function of water content calculated by the Mualem's model-II (i.e., no effect of pore blockage).

$$H(\psi)=\frac{\theta(\psi_1,\psi)-\theta_w(\psi)}{\theta_w(\psi_1)-\theta_w(\psi)};\ \text{for } \theta(\psi_1,\psi)\le\theta_{0L} \quad (6)$$

θ_0 = water content at calculated suction on the main drying curve (i.e., without pore blockage effect)

θ_{02} = water content at the suction ψ_2 on the main drying curve (i.e., without pore blockage effect)

θ_{0L} = limit water content that pore blockage takes effect (i.e., $P_d^*=1$ for $\theta_0>\theta_{0L}$)

This model is quite sophisticated and more details can be found in the original paper (Mualem and Miller, 1979).

c. Mualem's independent model (1984a)

Mualem's independent model (1984a) has the same assumptions as Mualem's universal model (1977) (i.e., $h(\psi) = l(\psi)$). With the aid of an additional wetting scanning curve, better results can be obtained. An advantage of this model is its ability to predict the main wetting curve. The main wetting curve can be calculated by:

$$S_w(\psi)=1-\frac{1-S_w(\psi_{max},\psi)}{[1-S_d(\psi_{max})]^{1/2}};\ \text{for } \psi_{min}\le\psi\le\psi_{max} \quad (7)$$

and

$$S_w(\psi)=1-[1-S_d(\psi)]^{1/2};\ \text{for } \psi_{max}\le\psi \quad (8)$$

Where, ψ_{max} = starting suction of the additional wetting scanning curve on the main drying curve,

S = degree of saturation

The scanning curves can be calculated by applying Mualem's model-II (1974).

d. Mualem's independent model (1984b)

Mualem's dependent model (1984b) requires the main hysteresis curves to predict the scanning curves. Similar to Mualem and Miller's model-III$_{expl}$ (1979) model, this model takes into account the effects of pore blockage by adding a pore blockage function (P_d). In this model, P_d is a function of actual water content. In order to solve the problem without using an additional drying scanning curve, an assumption was added:

$$H(\psi)=\theta_w(\psi)\sqrt{\theta_s} \quad (9)$$

The pore blockage function P_d is a function of actual water content on the main drying curve and can be expressed by:

$$P_d(\theta)=\frac{\theta_s[\theta_s-\theta_d(\psi)]}{[\theta_s-\theta_w(\psi)]^2} \quad (10)$$

The drying scanning curve starting at suction ψ_l on the main wetting curve can be calculated by:

$$\theta_d(\psi_1,\psi)=\theta_w(\psi_1)-P_d(\theta)[\theta_s-\theta_w(\psi)][\theta_w(\psi_1)-\theta_w(\psi)]/\theta_s \quad (11)$$

The wetting scanning curve starting at a suction ψ_2 on the main drying curve can be calculated as:

$$\theta_w(\psi_1,\psi)=\theta_d(\psi_1)-P_d(\theta_1)[\theta_u-\theta_w(\psi_1)][\theta_w(\psi)-\theta_w(\psi_1)]/\theta_s \quad (12)$$

e. Hogarth et al.'s model (1988)

Hogarth et al.'s model (1988) is an improvement of Parlange's model (1976, 1980). This model requires the main drying curve and two meeting points in the high and low suction ranges of the main wetting and the main drying curves.

The main drying curve can be expressed by:

$$\theta_d(\psi)=\theta_s\cdot\left(\frac{\psi_{ae}}{\psi}\right)^{\lambda}\frac{1+\lambda-\lambda\frac{\psi_{we}}{\psi}}{1+\lambda-\lambda\frac{\psi_{we}}{\psi_{ae}}};\ \text{for } \psi>\psi_{ae} \quad (13)$$

$$\theta_d(\psi)=\theta_s;\ \text{for } \psi\le\psi_{ae}$$

By using a curve fitting procedure for the main drying curve, two fitting parameters: ψ_{ae} and λ can be obtained. Two other parameters can be obtained from experiments: ψ_{we} (water entry value) and ψ_{res} (suction at the residual water content). The main wetting curve can be expressed by:

$$\theta_w(\psi)=\frac{\theta_s}{1+\lambda-\lambda\frac{\psi_{we}}{\psi_{ae}}}\cdot\left[\left(\frac{\psi_{ae}}{\psi}\right)^{\lambda}+\frac{\lambda}{\psi_{res}}\left(\frac{\psi_{ae}}{\psi_{res}}\right)^{\lambda}(\psi-\psi_{we})\right]\text{for } \psi\ge\psi_{ae} \quad (15)$$

and

$$\theta_w(\psi)=\frac{\theta_s}{1+\lambda-\lambda\frac{\psi_{we}}{\psi_{ae}}}\cdot\left[\left(1+\lambda-\lambda\frac{\psi}{\psi_{ae}}\right)+\frac{\lambda}{\psi_{res}}\left(\frac{\psi_{ae}}{\psi_{res}}\right)^{\lambda}(\psi-\psi_{we})\right],\ \text{for } \psi_{ae}\ge\psi\ge\psi_{we} \quad (16)$$

and

$$\theta_w(\psi)=\theta_s \quad \text{for } \psi_{we}\ge\psi \quad (17)$$

Where, ψ_{res} is the suction at the residual water content.

The drying scanning curves can be expressed by:

$$\theta_d(\psi_1,\psi)=\frac{\theta_s}{1+\lambda-\lambda\frac{\psi_{we}}{\psi_{ae}}}\cdot\left[\left(1+\lambda-\lambda\frac{\psi_1}{\psi}\right)+\frac{\lambda}{\psi_{res}}\left(\frac{\psi_{ae}}{\psi_{res}}\right)^{\lambda}(\psi_1-\psi_{we})\right],$$

$$\text{for } \psi_{ae}\ge\psi\ge\psi_{we} \qquad (18)$$

and

$$\theta_d(\psi_1,\psi)=\theta_w(\psi_1), \qquad \text{for } \psi\ge\psi_{ae} \qquad (19)$$

Replacing ψ_{res} with ψ_2 in equations (15), (16), and (17) yields a set of equations for calculating the wetting scanning curve starting at a suction ψ_2 on the main drying curve.

3. METHOD AND MATERIALS

The evaluation of the models was based on 28 different sets of soil data collected from the literature. The various models are compared in terms of the 'fit' they provide to the measured data. These soil-water hysteresis curves were collected by using the "Un-Scan-It" computer software. This program allows picking the data points on a graph with user-defined coordinates. Soil data were stored in Spreadsheets as a Microsoft Excel file.

To avoid confusion between of swelling and shrinking soils, the soil-data collected was presented in the form of volumetric water content and converted to the form of gravimetric water content. The names of 28 soils are presented in the Table 1.

Table 1. List of soil-data

ID	Soil name	Experimenter	No. Scannin g curves
1	Adelaide Dune Sand	Talsma (1970)	11
2	Caribou Silt Loam	Topp (1969)	6
3	Ceramic (No. 1)	Man Feng (1999)	11
4	Coarse Sand	Viaene et al. (1995)	1
5	Dune Sand	Grillham, Klute and Heermann (1976)	7
6, 7, 8, 9,	Glass Bead (4, 20, 35, 50 degrees)	Nimmo and Miller (1986)	2
10	Glass Beads	Bomba and Miller (1967)	2
11	Glass Beads Ballotini	Poulovassili (1962)	8
12	Mixed Sand Fraction	Poulovassili and Ghamry (1978)	11

Table 1. List of soil-data (continued)

13	Molongo Sand	Talsma (1970)	8
14	Packed Sand	Vachaud and Thony (1971)	6
15, 16, 17	Plainfield Sand Loam (20, 35, 50 degree)	Nimmo and Miller (1986)	2
18, 19, 20	Plano Silt Loam (4, 35, 50 degree)	Nimmo and Miller (1986)	2
21, 22	Pouros body I, II (sand)	Poulovassilis (1970)	6
23	Rideau Clay Loam	Topp (1971)	6
24	Rubicon Sandy Loam	Topp (1971)	9
25	Sand	Poulovassilis and Childs (1971)	11
26	Sand (No.17)	Perrens and Watson (1977)	10
27	Sand (R8)	Perrens and Watson (1977)	11
28	Wray Dune Sand	Mualem and Klute (1984)	1

The comparison of the five models is based on the following criteria: R squared values, percentage deviation, and absolute percentage deviation.

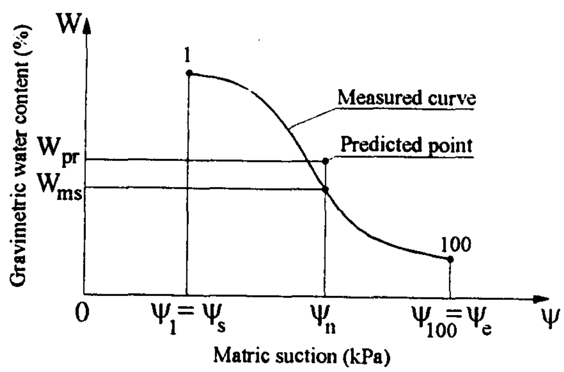

Fig.2 Schematic illustration of dividing a hysteresis curve according to the suction range.

Three criteria were applied on all the predicted and calculated hysteresis curves at 100 points on each curve. These 100 points are determined by dividing each hysteresis curve into 99 equally spaced suction value (see Fig. 2). Three criteria can be calculated as shown below:

Percentage deviation (%):

$$PD(\%)=\sum_{i=1}^{100}\left(\frac{W_{pr}(i)-W_{ms}(i)}{W_{ms}(i)}\right) \qquad (20)$$

Absolute percentage deviation (%):

$$APD(\%) = \sum_{i=1}^{100} Abs\left(\frac{W_{pr}(i) - W_{ms}(i)}{W_{ms}(i)}\right) \qquad (21)$$

R square:

$$R^2 = \frac{\left(\sum_{i=1}^{100} W_{ms}(i).W_{pr}(i) - \frac{\sum_{i=1}^{100} W_{ms}(i)\sum_{i=1}^{100} W_{pr}(i)}{100}\right)^2}{\left(\sum_{i=1}^{100} W_{pr}^2(i) - \frac{\left(\sum_{i=1}^{100} W_{pr}(i)\right)^2}{100}\right)\left(\sum_{i=1}^{100} W_{ms}^2(i) - \frac{\left(\sum_{i=1}^{100} W_{ms}(i)\right)^2}{100}\right)} \qquad (22)$$

where, W_{pr} is the predicted water content and W_{ms} is the measured water content at a suction.

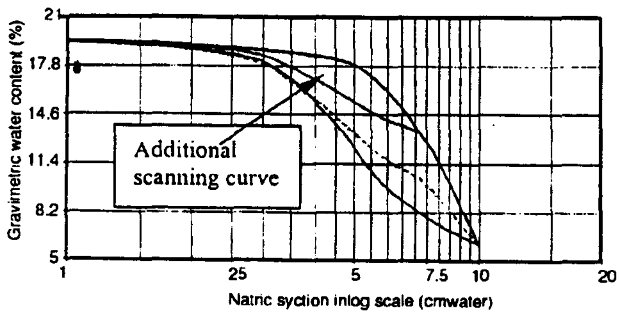

Fig. 3 Main wetting curve calculated by Mualem's independent model (1984a) for Adelaide dune sand (Talsma,1970) with the aid of an additional wetting scanning curves starting at a low suction. The solid lines are measured curves, dashed line is the predicted curve.

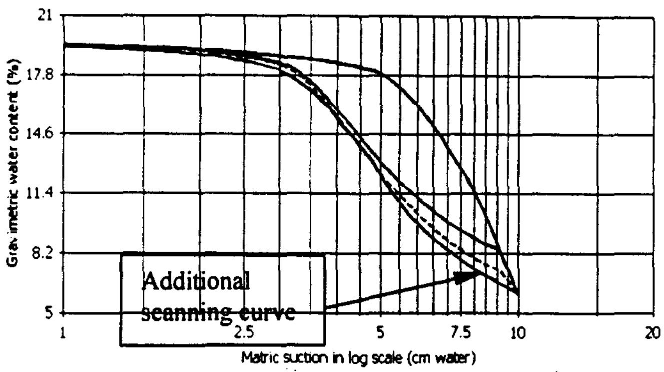

Fig. 4 Main wetting curve calculated by Mualem's independent model (1984a) for Adelaide dune sand (Talsma, 1970) with the aid of an additional wetting scanning curve starting at a high suction.

An additional wetting scanning curve starting at a high suction was used and Mualem's independent model (1984a) gave better results than Hogarth et al.'s model (1988) in predicting the main wetting curve for most soils (Fig. 3 and Fig. 5). On the other hand, when an additional wetting scanning curve starting at a low suction was used, the Hogarth et al.'s model (1988) gives the better results (Fig. 4). The Hogarth et al.'s model (1988) requires fewer measured data and therefore, is recommended for engineering applications.

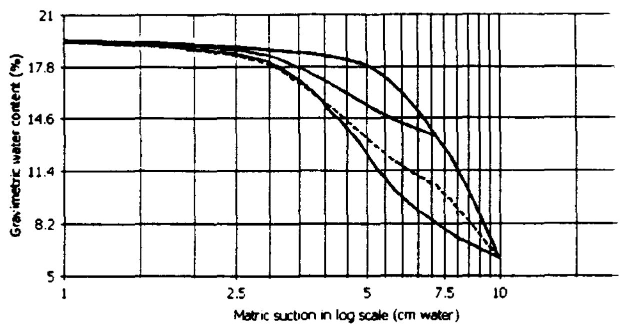

Fig. 5 Calculation result of Hogarth et al.'s model for Adelaide dune sand (Talsma, 1970)

The average values of two criteria: *R square* and *Absolute percentage deviation* of five models were calculated on scanning curves of 28 soils. The raking of the models based on these two criteria are presented in the Table 2 and Table 3 below:

Table 2. Ranking based on Absolute Percentage Deviation

Pos	M-1	M-2	M-3	M-4	M-5
1st	12	5	6	5	0
2nd	11	14	0	3	0
3rd	5	4	7	10	2
4th	0	0	7	9	9
5th	0	2	5	1	16
Total	28	25	25	28	27

Table 3. Ranking based on *R square*

Pos	M-1	M-2	M-3	M-4	M-5
1st	9	8	5	3	3
2nd	9	8	4	3	4
3rd	7	4	9	5	3
4th	3	3	7	7	5
5th	0	2	0	10	12
Total	28	25	25	28	27

Notations: *M-1* to *M-5* in the Table 2 and 3 represent five hysteresis models: M-1 is the Mualem's model-II (1974), M-2 is the Mualem and Miller's model-III_{expl} (1979), M-3 is the Mualem's indepdent model (1984a), M-4 is the Mualem's dependent model (1984b), and M-5 is the Hogarth et al. model (1988) respectively. *Pos* is the position in the rank of five models. The word, *Total* means the number of soils that were studied for the model.

The results show that there is no agreement between *R square* values and *Absolute percentage deviation* values. The evaluations based *R square* values may give the wrong results.

Mualem's model-II (1974) is the best model for predicting the scanning curves. Mualem's dependent model (1984b) can predict only the drying scanning curve starting at high water content for the non-uniform soils better than Mualem's model-II (Fig. 6 and Fig. 7). Pore blockage function was taken into account for drying processes, therefore the calculation results of the following models: Mualem's model-II (1979), Mualem and Miller's model-III$_{expl}$ (1979) and Mualem's dependent model (1984b) for the wetting scanning curves should be the same. The differences between the calculated wetting curves of these models are caused by the calculation procedures.

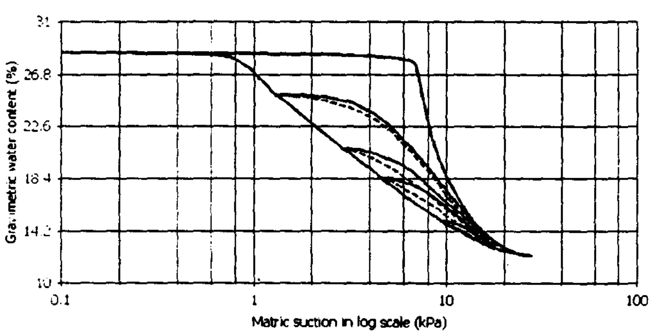

Fig. 6 The drying scanning curve predicted by Mualem's dependent model (1984b) for Rubicon Sandy Loam soil (Topp, 1971)

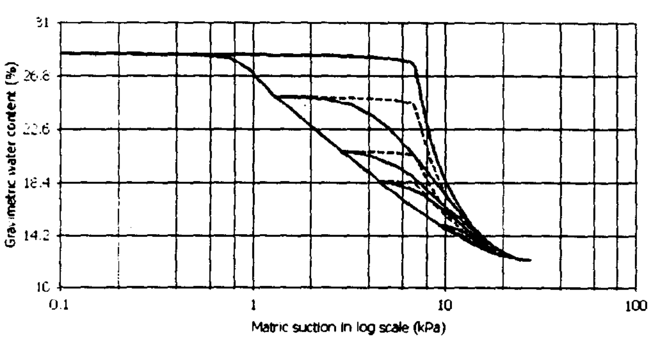

Fig. 7 The drying scanning curve predicted by Mualem's model-II (1974) for Rubicon Sandy Loam soil (Topp, 1971)

Mualem and Miller's model-III$_{expl}$ (1979) can predict the drying scanning curve well with the aid of an additional drying scanning curve. For some soils, few mathematical difficulties were encountered. Pore blockage function (P_d) cannot be determined properly.

When the additional wetting scanning curve starting at high suction was used for calibration, Mualem's independent model (1984a) could accurately predict the wetting scanning curve starting at lower suction. However, the reversal is not always correct (see Fig. 8 and Fig. 9).

Hogarth et al.'s model (1988) could not to predict the scanning curve well because the basic assumptions of this model is simpler than that of Mualem model-II (i.e., $f(\psi_d, \psi_w) = h(\psi_d)$). The aid of Brooks and Corey's equation does not improve the accuracy of the model for predicting the scanning curves (Fig. 10).

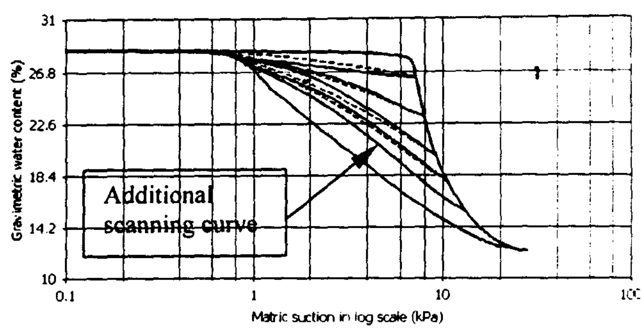

Fig. 8 The wetting scanning curves predicted by Mualem's independent model (1984a) with the aid of a wetting scanning curves starting at higher suction for Rubicon Sandy Loam soil (Topp, 1971)

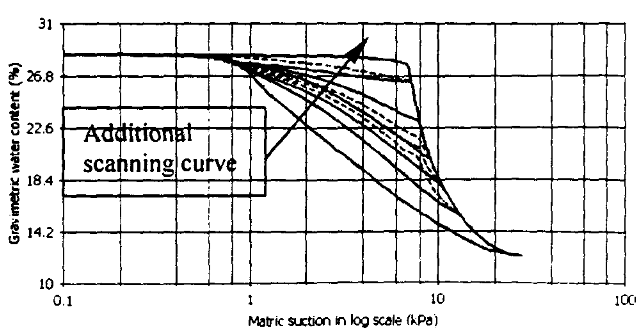

Fig. 9 The wetting scanning curves predicted by Mualem's independent model (1984a) with the aid of a wetting scanning curves starting at lower suction for Rubicon Sandy Loam soil (Topp, 1971)

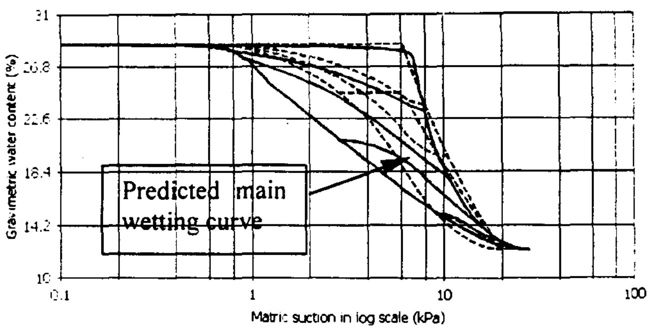

Fig. 10 The scanning curves predicted by Hogarth et al.'s model (1988) for Rubicon Sandy Loam soil (Topp, 1971)

5. CONCLUSION

The conclusions based on the calculation results of five physical based hysteresis models applied to 28 soil datasets are as follows:

1. Mualem's model-II (1974) is the simplest model and also the best model for predicting scanning curves. It is recommended for predicting the scanning curves.

2. Hogarth et al.'s model (1988) is not always the best model for predicting main wetting curve. However, it requires simple measured data for calibration, therefore it is recommended for the applications.

3. Taking into account the effect of pore blockage will not always improve the accuracies of the hysteresis models (i.e., Mualem's model-II (1974) and Mualem's dependent model (1984b)).

4. Mualem and Miller's model-III_{expl} is the most complex model. Some mathematical problems were noticed during the study. It is not recommended for the applications.

5. Using R square values to evaluate soil-water characteristic curves may not be accurate.

REFERENCES

1) Ayers NR. N. and K.K. Watson, 1977. Experimental and Numerical studies of soil-water movement to a water table. *6th Australasian Conference on Hydraulics and Fluid Mechanics Adelaide, The Institution of Engineers Australia, National Conference Publication* No. 77/12: 72-75

2) Grillham R. W., A Klute and D. F. Heermann, 1976. Hydraulic properties of a porous medium: Measurement and empirical representation. *Soil Sci. Soc Am. J.*, Vol. 40, pp. 203-207

3) Hogarth W. L., J. Hopmans, J. Y. Parlange and R. Haverkamp, 1988. " Application of a simple soil-water hysteresis model," *Journal of Hydrology*, vol. 98, pp. 21-29

4) Mualem Y. and A. Klute, 1984. Predictor-Corrector Method for Measurement of Hydraulic Conductivity. *Soil Sci., Soc. Am. J.*, vol 48, pp. 992-1000

5) Mualem Y. and E. E. Miller, 1979. A hysteresis Model Based on an Explicit Domain-Dependence Function. *Soil Sci., Soc. Am. J.*, vol 43, pp. 1067-1073

6) Mualem Y. and G Dagan, 1975. A Dependent Domain Model of Capillary Hysteresis. *Water Resour. Res.*, vol. 11, no. 3, pp. 452-460.

7) Mualem Y., 1974. A Conceptual Model of Hysteresis. *Water Resour. Res.*, vol. 10, no. 3, pp. 514-520.

8) Mualem Y., 1977. Extension of the Similarity Hypothesis Used for Modeling the Soil Water Characteristics. *Water Resour. Res.*, vol. 13, no. 4, pp. 773-780.

9) Mualem Y., 1984a. A Modified Dependent Domain Theory of Hysteresis. *Soil Sci.*, vol. 137, no. 5, pp. 283-291.

10) Mualem Y., 1984b. Prediction of the Soil Boundary wetting Curve. *Soil Sci.*, vol. 137, no. 6, pp.379-390.

11) Nimmo J. R. and E. E. Miller, 1986. The Temperature of Isothermal Moisture vs. Potential characteristic of soils. *Soil Sci. Soc. Am. J. vol.* 50, pp. 1105-1113

12) Parlange Jean-Yves, 1976. Capillary Hysteresis and the Relationship Between Drying and Wetting Curves. *Water Resour. Res.*, vol. 12, no. 2, pp. 224-228.

13) Perrens S. J. and K. K. Watson. 1977. Numerical Analysis of Two-Dimensional Infiltration and Redistribution. *Water Resour. Res.*, vol. 13, pp. 781-790

14) Poulovassili A., 1962. Hysteresis of pore water, an application of the concept of independent domains. *Soil Science*. Vol. 93, pp. 405-412

15) Poulovassilis A. 1970. Hysteresis of Pore Water in Granular Porous Bodies. *Soil Science*, vol 109: 5-12

16) Poulovassilis A., 1970. Hysteresis of Pore Water in Granular Porous Bodies. Soil Science, vol 109, pp. 5-12

17) Poulovassilis and Childs, 1971. The hysteresis of pore water: The non-independence of Domains. *Soil Sci.*, Vol. 112, no. 5, pp. 301-312

18) Talsma T., 1970. Hysteresis in two sands and the Independent Domain Model. *Water Resour. Res.* 6, pp. 965-970

19) Topp, G. C. 1971. Soil water hysteresis in silt loam and clay loam soils. *Water Resour. Res.* 7: 914-920

20) Vachaud G., J. L. Thony, 1971. Hysteresis during infiltration and redistribution in a soil column at different initial Water content. *Water Resour. Res.* Vol. 7, pp. 914-920

21) Vieane P., H. Vereecken, J. Diels, and J. Feyen, 1995. A statistical analysis of six hysteresis models for the moisture retention characteristic. *Soil Sci.* vol. 157, no. 6, pp. 345-355

22) Watson K. K., Reginato R. J. and Jackson R. D., 1975. Soil Water Hysteresis in a Field Soil. *Soil Sci. Soc. Am. proc.*, vol. 39, no. 2, pp. 242-246

23) Jaynes D. B., 1985. Comparison of soil-water hysteresis models. *Journal of Hydrology*, vol. 75, pp. 287-299