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APPLICATION OF UNSATURATED SOIL MECHANICS IN GEOTECHNICAL ENGINEERING

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ABSTRACT: In the tropics, high temperature and rainfall are conducive to the weathering of rock formations and the formation of residual soils. Most residual soils are in an unsaturated condition and it is now possible to reliably measure negative pore-water pressures in the field. The behaviour of residual soils falls into the realm of unsaturated soil mechanics. The active development of unsaturated soil mechanics began two to three decades after the commencement of "saturated" soil mechanics which is still the main stay in geotechnical engineering practice. In the last two decades, the identification of the stress state variables governing unsaturated soil behaviour has led to a major advancement in the theoretical framework of unsaturated soil mechanics. In the last decade or so, laboratory testing of unsaturated soils has also seen major development with the advancement of data acquisition systems and instrumentation. The greatest obstacle to the application of unsaturated soil mechanics in geotechnical engineering practice is the long duration involved in unsaturated soil testing. In the last five years, there is an increased awareness that the key to the estimation of unsaturated soil behaviour is the soil-water characteristic curve. The test duration for obtaining a soil-water characteristic curve is considerably much shorter than that to obtain other unsaturated soil properties. In this paper, the role of the soil-water characteristic curve in estimating unsaturated soil properties such as permeability and shear strength is examined. The application of unsaturated soil mechanics in design is illustrated using examples on bearing capacity and slope stability.

KEYWORDS: Residual soils. suction, unsaturated, soil-water characteristic curve, geotechnical engineering.

1. INTRODUCTION

Water plays an important role in all human activities. The Euphrates and the River Nile played an important role in early civilisation. This trend continues in the development of many cities. Singapore's downtown, for example, developed around the Singapore River and her port. The geological deposit associated with coastal land is usually sediment of soft clays. It is thus not surprising to find geotechnical engineering practice dealing mainly with soft clays. As a city grows, it is inevitable that areas further inland is being developed to accommodate the needs of a growing population. The geological deposit in areas further inland is no longer soft clay but stiffer materials. The soils in these areas, unlike the coastal soft sediments, are unsaturated but current geotechnical engineering practice is still deeply ingrained in "saturated" soil mechanics or conventional soil mechanics. In fact, many parts of the earth's land surface are arid or semi-arid. There is evidence that more of the earth's land surface will become increasing more arid (Figure 1).

The past two decades has seen a rapid development of unsaturated soil mechanics with the identification of the two stress state variables, matric suction (u_a-u_w) and net normal stress $(\sigma-u_a)$, governing unsaturated soil behaviour [1]. It can be easily illustrated using these stress state variables that unsaturated soil mechanics is more general and encompasses all the principles of saturated soil mechanics. For example, the commonly used Mohr-Coulomb shear strength criterion in practise can be extended for unsaturated soil [2]:

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$$\tau = c' + (\sigma - u_a)\tan\phi' + (u_a - u_w)\tan\phi^b \tag{1}$$

where τ = shear strength, c' = effective cohesion, ϕ' = effective angle of friction and ϕ^b = angle describing change in shear strength due to change in matric suction. The representation of Equation 1 is given in Figure 2. As the soil becomes saturated, u_w approaches u_a and Equation 1 becomes:

$$\tau = c' + (\sigma - u_w)\tan\phi' \tag{2}$$

which is the familiar Mohr-Coulomb shear strength criterion for saturated soils.

In saturated soil mechanics though many laboratory tests are available to obtain soil properties for different stress conditions, the unconfined compression (UC) and the unconsolidated undrained (UU) tests are still widely used in practice due to time and cost. There are now many equivalent tests for unsaturated soils. However, the duration of these tests is always much longer and more complicated than the tests for saturated soils. Using the same reasoning for choice of saturated soil tests, unsaturated soil mechanics will only gain favour with the practitioners if a more expedient method of obtaining the unsaturated soil properties is available. Recent research [3,4,5,6,] indicates that many unsaturated soil properties can be obtained indirectly using the soil-water characteristic curve. The objective of this paper is to examine the role of the soil-water characteristic curve in determining permeability and shear strength. The application of unsaturated soil mechanics to geotechnical design of bearing capacity and slope stability is also examined.

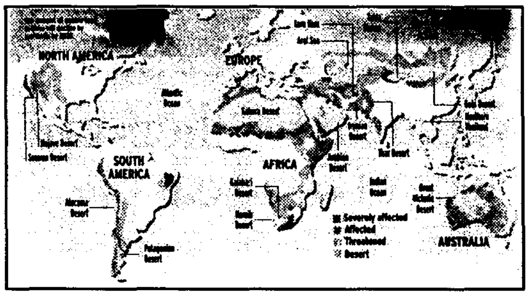


Figure 1. Arid regions in the world

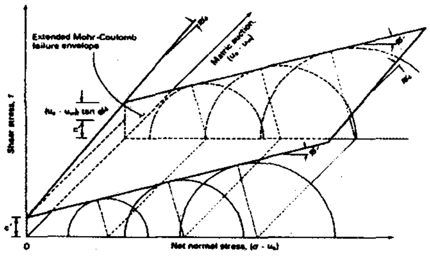


Figure 2. Extended Mohr-Coulomb criterion for unsaturated soil

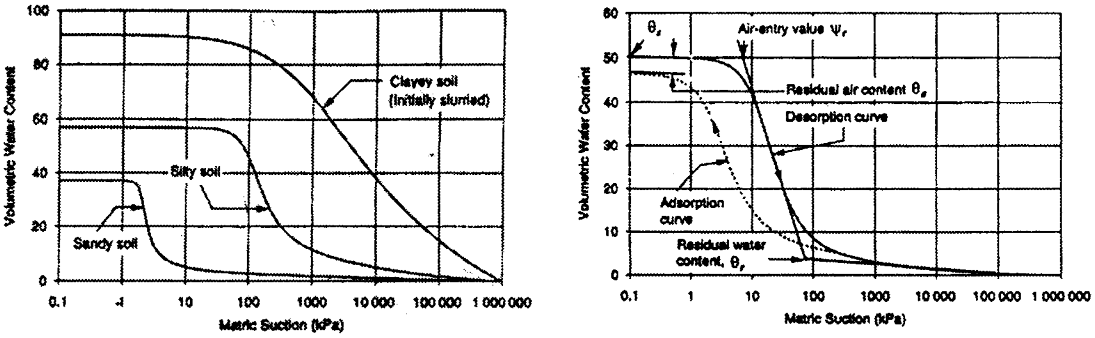
2. SOIL-WATER CHARACTERISTIC CURVE

The soil-water characteristic curve expresses the relationship of volumetric water content with matric suction for a soil. In place of volumetric water content, gravimetric water content or degree of saturation can also be used. Typical soil-water characteristic curves for sandy, silty and clayey soils are illustrated in Figure 3a. Important parameters of the soil-water characteristic curve are indicated in Figure 3b. There are many equations suggested for the soil-water characteristic curve [7]. However, it was found that the following equation by [8] gave the best fit to the soil-water characteristic data:

$$\theta_w = \frac{\theta_s}{\left\{\ln\left[e + \left(\frac{u_a - u_w}{a}\right)^n\right]\right\}^m} \tag{3}$$

where θ_w = volumetric water content, θ_s = saturated volumetric water content, e = natural base of logarithms, $(u_a - u_w)$ = matric suction, u_a = pore-air pressure, u_w = pore-water pressure, a, n and m = constants.

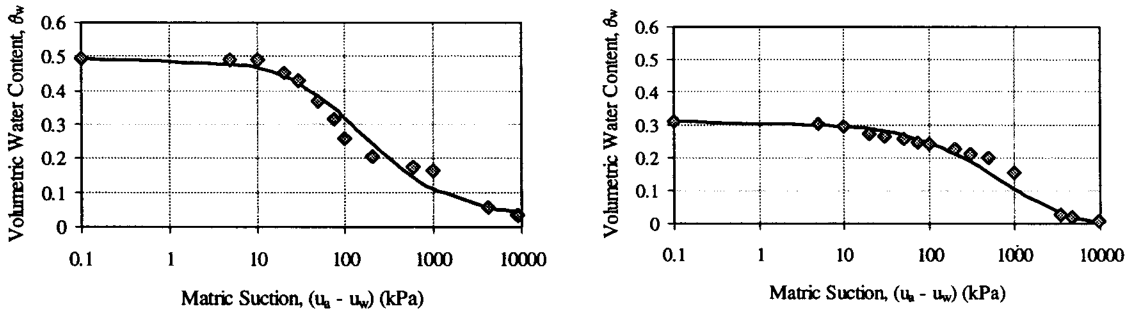
The method of determining the soil-water characteristic curve for coarse or medium textured soils and fine textured soils can be found in ASTM D2325-68 and ASTM D3152-72 [9], respectively. These tests allow soil-water characteristic curve of up to 1520 kPa matric suction to be developed. Other methods such as the salt solution method [10] can be used to develop the soil-water characteristic curve for the higher suction range. The soil-water characteristic curves for two Singapore residual soils determined in this manner are shown in Figure 4.



(a) Soil-water characteristic curves for different soil types [8]

(b) Parameters of soil-water characteristic curve [8]

Figure 3. Soil-water characteristic curve



(a) Bukit Timah residual soil

(b) Jurong Formation residual soil

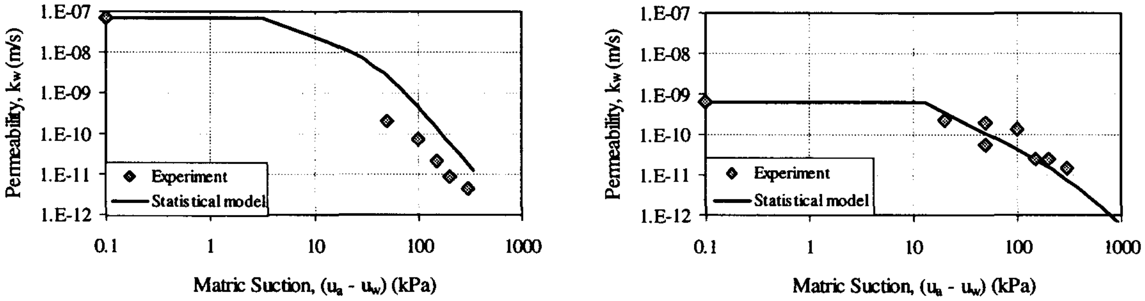
Figure 4. Soil-water characteristic curves of two Singapore residual soils

3. PERMEABILITY

Darcy's law which governs water flow through saturated soils is also valid for unsaturated soils [11]. In unsaturated soils, water flows through the water phase and thus there exists an intimate relationship between permeability and soil-water characteristic curve. Several approaches can be used to obtain the permeability function [$k_w(\theta_w)$] of an unsaturated soil from its soil-water characteristic curve [6]. The most rigorous approach is through the use of a statistical model given by the following expression [16]:

$$k_w(\theta_w) = \frac{T_s^2}{2\rho_w g \mu} \frac{n^2}{m^2} \sum_{i=1}^{l} \frac{2(l-i)-1}{(u_a - u_w)_i^2} \tag{4}$$

where T_s = surface tension of water; ρ_w = density of water; μ = dynamic viscosity of water; n = porosity of soil; and m (= $\theta_s/\Delta\theta_w$) = total number of intervals; l (=$\theta_w/\Delta\theta_w$) = number of intervals corresponding to θ_w; and $(u_a - u_w)_i$ = matric suction corresponding to the midpoint of the ith interval of the soil-water characteristic curve. The permeability functions for two Singapore residual soils obtained from the soil-water characteristic curve using the statistical model are shown in Figure 5 together with the permeability obtained by direct measurements.



(a) Bukit Timah residual soil

(b) Jurong Formation residual soil

Figure 5. Permeability functions of two Singapore residual soils

4. SHEAR STRENGTH

As illustrated in Equation 1 and Figure 2, the shear strength of an unsaturated soil can be represented by an extended Mohr-Coulomb criterion. It has been proposed that the shear strength of an unsaturated soil can be obtained from the saturated soil shear strength parameters and the soil-water characteristic curve [5,6]. In this context, it is useful to use the concept of a fourth phase in unsaturated soil, i.e., the contractile skin as suggested by [1]. The stress state variables, $(\sigma - u_a)$ and $(u_a - u_w)$, are the surface tractions controlling the soil structure and contractile skin. Therefore, the contribution of matric suction to shear strength can be represented by an increase in contact surface given by the normalised area of water, a_w [12]. The normalised area of water a_w is defined as:

$$a_w = \left(\frac{\theta_w}{\theta_s}\right)^{\kappa} \tag{5}$$

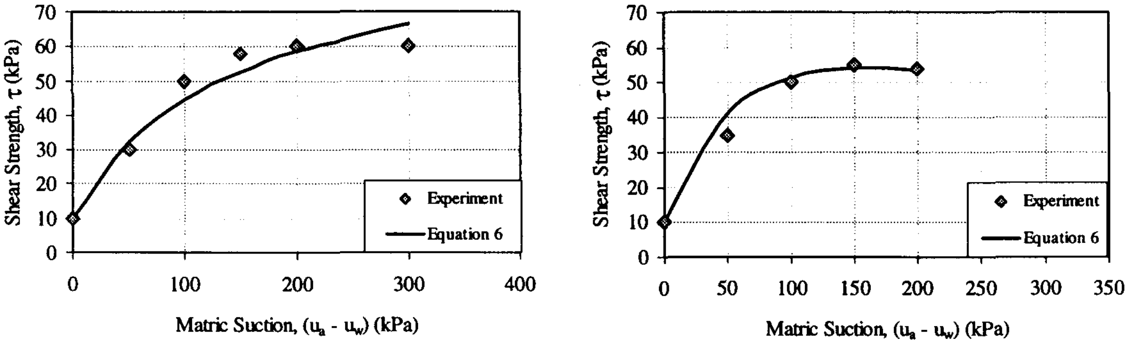
where κ = fitting parameter. The increase in shear strength due to matric suction as given by $\tan \phi^b$ can then be treated as equal to $(a_w \tan \phi')$ and therefore, Equation 1 can be rewritten as:

$$\tau = c' + (\sigma - u_a)\tan\phi' + (u_a - u_w)a_w \tan\phi' \tag{6}$$

The shear strengths given by Equation 6 for two Singapore residual soils with $(\sigma - u_a) = 0$ are shown in Figure 6 together with the experimental values from [13]. The values of κ are 1.35 and 4.6 for the Bukit Timah and Jurong Formation residual soils, respectively. The effective angles of friction, ϕ', are 32^O and 51^O for Bukit Timah and Jurong Formation residual soils, respectively. With sufficient test data, it is possible to establish the range of values for κ for different soil types.

5. APPLICATION OF UNSATURATED SOIL MECHANICS

Examples of application of unsaturated soil mechanics to the bearing capacity and slope stability problems are presented below.



(a) Bukit Timah residual soil (b) Jurong Formation residual soil

Figure 6. Shear strength of two Singapore residual soils

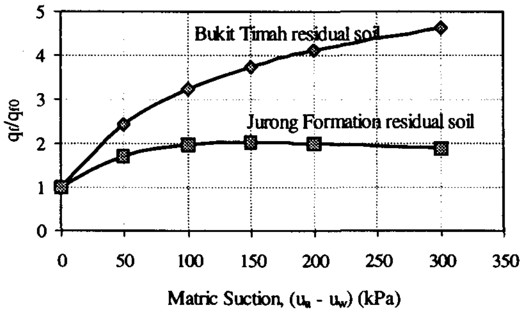


Figure 7. Effect of suction on bearing capacity stability

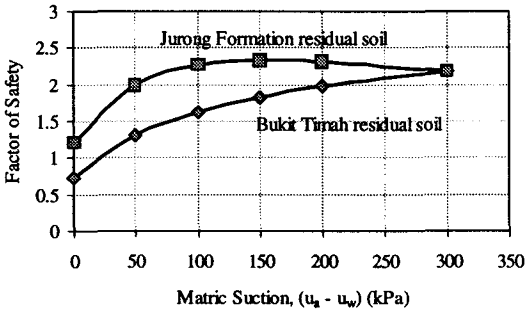


Figure 8. Effect of suction on slope

5.1 Bearing Capacity

The ultimate bearing capacity q_f of a footing on soil is given as

$$q_f = cN_c + \gamma DN_q + 0.5\gamma BN_\gamma \tag{7}$$

where c = cohesion, γ = unit weight of soil, D = foundation depth of soil, B = footing width, and N_c, N_q, N_γ = bearing capacity factors. For a surface strip footing of width B = 1m, the normalised ultimate bearing capacity (q_f/q_{f0}) for different matric suctions in Bukit Timah and Jurong Formation residual soils are shown in Figure 7. The ultimate bearing capacity q_{f0} refers to the case where matric suction $(u_a - u_w) = 0$. It can be observed that the increase in ultimate bearing capacity due to matric suction in the Bukit Timah residual soil is more effective than that in the Jurong Formation residual soil which stagnates after a matric suction of 100 kPa.

5.2 Slope Stability

Consideration of slope stability in unsaturated soils can be at different levels of sophistication. In this paper, the simplest case using a total stress approach [14] is illustrated. The factor of safety of a slope F is given by the following equation:

$$F = N_0 \frac{s_u}{\gamma H} \tag{8}$$

where N_0 = stability number which is a function of slope angle and the depth to the rigid layer, s_u = undrained shear strength (estimated using Equation 6) and H = height of slope. For a slope of height = 12m, slope angle = 45° and a rigid layer at infinite depth, $N_0 = 5.53$. Assuming a nominal value of 30 kPa for $(\sigma - u_a)$, the factor of safety for slopes in Bukit Timah and Jurong Formation residual soils can be computed as shown in Figure 8. It is interesting to note that the slope in Bukit Timah residual soil is less than unity for $(u_a - u_w) = 0$ and the factor of safety becomes greater than unity for a matric suction of 25 kPa. Similar to the observation for bearing capacity, the factor of safety for the slope in the Jurong residual soils does not show any further improvement for a matric suction of greater than 100 kPa.

6. CONCLUSION

The role of the soil-water characteristic curve in determining permeability and shear strength of an unsaturated soil has been illustrated for two Singapore residual soils. The use of the soil-water characteristic curve in providing estimates of unsaturated soil properties will be attractive to practitioners who intend to incorporate matric suction in geotechnical engineering problems. The application of unsaturated soil mechanics for two common geotechnical engineering problems, bearing capacity and slope stability, is illustrated with reference to two Singapore residual soils. Consideration of suction in unsaturated soils will lead to more realistic geotechnical engineering design.

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