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Relationship between shear strength and matric suction in an unsaturated silty soil

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ABSTRACT: Unsaturated soil regions with high soil suctions are generally located near ground surface. This study reports the results of triaxial tests on a silty soil. The modified triaxial cell apparatus applied a constant net normal stress and varying soil suctions as well as having a constant high total suction with the net normal stress varying. Matric suctions were applied to soil specimens in the triaxial cell, through a ceramic base plate for suctions up to 450 kPa. Higher total suctions were applied to soil specimens in a glass desiccator where salt solutions were used to create a constant relative humidity chamber. Total suction values up to 292,400kPa were used in this test program. This study showed that the failure envelope with respect to suction was composed of two segments; namely, a curvilinear segment at low suctions and a horizontal segment at high suctions.

1 INTRODUCTION

An unsaturated soil profile can be classified as follows; namely, a dry soil zone, a two-phase zone and a capillary fringe zone. A soil is classified as dry when its water content is in the residual stage of unsaturation. This is often the situation near to the ground surface. Geotechnical engineers need to be able to assess the shear strength of unsaturated soils in the dry or high suction range. Dry soils have a behavior somewhat different in character from that of saturated soils.

A theoretical framework for volume change, shear strength and permeability for unsaturated soils has become generally accepted in geotechnical engineering. Unsaturated soil property functions for an unsaturated soil are required for numerical modeling. Examples of unsaturated soil property functions are the coefficient of permeability functions, the water storage functions, the shear strength functions and the volume change functions. One of several procedures can be used to obtain unsaturated soil property functions. These are direct measurements, the use of the soil-water characteristic curve and the use of classification tests such as the grain size test. The mathematical modeling of unsaturated soils through the use of soil-water characteristic curves is becoming more generally accepted. Several proposed mathematical procedures appear to provide a reasonable estimate of the non-linear character of unsaturated soil property functions.

A conventional triaxial test or a modified direct shear test can be used to measure the relationship between soil suction and shear strength. Its weakness is that it has proven to be costly and time-consuming. Unsaturated soil mechanics has a lack of experimental data on the shear strength of a soil in the residual stage of unsaturation. It is important to quantify the contribution to shear strength over the entire suction range (i.e., 0-1,000,000 kPa).

1.1 *Purpose of this study*

This study reports the results obtained from (i) triaxial tests at a constant net confining pressure (i.e., 25 kPa) and a varying matric suction (from the low suction range to the high suction range) and (ii) triaxial tests at a constant high suction (i.e., 217,000 kPa) and varying net confining pressure. The triaxial tests, using a modified, conventional cell, were performed on a statically compacted silty soil. Matric suctions were applied to the soil specimens placed on a ceramic plate for suctions up to 450 kPa. Soil specimens were brought to equilibrium with high matric suctions through the vapor pressure technique using a salt solution and the relative humidity control technique.

This study investigates the relationship between the shear strength and matric suction over the entire range of suction. This study determines a fitting parameter, κ, associated with the empirical equation proposed by Fredlund et al. (1995) in order to estimate the shear strength of an unsaturated soil. The shear strength parameter, ϕ', with respect to net confining pressure is assumed to be coincident with an effective angle of internal friction for the saturated soil.

1.2 *Review of shear strength of an unsaturated soil*

The shear strength of unsaturated soils been formulated in terms of two independent stress state variables (Fredlund et al. 1978). The linear form of the equation for the shear strength of an unsaturated soil is commonly expressed as follows:

$$\tau = c' + (\sigma_n - u_a)\cdot \tan\phi' + (u_a - u_w)\cdot \tan\phi^b \quad (1)$$

where: τ = shear stress on the failure plane at failure, c' = intercept of the "extended" Mohr-Coulomb failure envelope on the shear stress axis where the net normal stress and the matric suction are equal to zero, $(\sigma_n - u_a)$ = net normal stress on the failure plane, ϕ' = angle of internal friction with respect to the net normal stress, $(u_a - u_w)$ = matric suction, ϕ^b = angle indicating the rate of increase in shear strength relative to matric suction.

The shear strength of unsaturated soils can be determined in the laboratory using a modified, conventional triaxial cell and modified direct shear apparatus (Gan and Fredlund 1988). The shear strength versus matric suction envelope has been found to be non-linear (Gan et al. 1988). Donald (1956) shows that the slope of the shear strength versus matric suction envelope can even be negative.

Gan and Fredlund (1996) studied the saturated and unsaturated shear strength behavior of two saprolitic soils using direct shear tests and triaxial tests. Results show that the relationship between shear strength and matric suction takes on a non-linear form. The extent of the increase in the shear strength with matric suction is related to the soil-water characteristic curve for the soil and to the amount of dilation occurring during shear.

Mahalinga-Iyer and Williams (1985) conducted unconsolidated undrained triaxial tests on two lateritic soils. High suctions on the standard compacted soil specimens were measured using the filter paper method. The results show that the slope of the failure surface with respect to the matric suction decreases sharply in the suction range up to 1,000 kPa. When the matric suction exceeds 1000 kPa, the shear strength decreases slightly with an increase in matric suction.

Fredlund et al. (1995) proposed a model for predicting the shear strength of unsaturated soils using the soil-water characteristic curve and saturated shear strength parameters, c', ϕ'. The proposed equation is as follow:

$$\tau = c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)\Theta^{\kappa}\tan\phi' \quad (2)$$

where: Θ = dimensionless volumetric water content, κ = is the fitting parameter.

The dimensionless water content is expressed as:

$$\Theta = \frac{\theta_w}{\theta_s} \quad (3)$$

where: θ_w = volumetric water content at any suction, and θ_s = volumetric water content at saturation. Fredlund et al. (1995) indicate that satisfactory predictions of the shear strength of an unsaturated soil with measured data. The fitting parameter, κ, depends on soil properties such as the plasticity of the soil; taken to be one or greater for highly plastic soils.

Vanapalli et al. (1996) emphasize that the soil-water characteristic curve is closely related to the shear strength of an unsaturated soil and suggested a second procedure for its estimation. The formulation for the estimation of shear strength was based on same concept as used in the empirical equation proposed by Fredlund et al. (1995).

$$\tau = c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)\left[\tan\phi'\left(\frac{\theta_w - \theta_r}{\theta_s - \theta_r}\right)\right] \quad (4)$$

where: θ_w = volumetric water content at any matric suction, θ_s = saturated volumetric water content, θ_r = residual volumetric water content.

The relative water content term (i.e., $(\theta_w - \theta_r)/(\theta_s - \theta_r)$) is called the normalized water content. In recent years several investigators have proposed empirical procedures to predict the shear strength function for unsaturated soil by using the soil-water characteristic curve and the saturated shear strength parameters. The soil-water characteristic curve provides a conceptual understanding for the behavior of unsaturated soils. In 1999 Vanapalli and Fredlund summarized various mathematical prediction model for the shear strength of an unsaturated soil.

2 TEST PROCEDURE

A fine-grained silty soil with no cohesion was used for the shear strength tests. The statically compacted specimens had a height of 100 mm and a diameter of 50 mm. The initial water content was 9.6 %. The laboratory testing program consisted of Test Program 1 and Test Program 2. The two testing programs involved the use of a conventional triaxial cell modified for both air-pressure and pore-water pressure control. A five bar high air entry disk was sealed onto the bottom pedestal of the modified triaxial cell and was used to facilitate the separate control of the pore-air pressure and pore-water pressure.

2.1 *Test program 1*

Test Program 1 involved the consolidation stage and shearing stage under a constant net confining pressure but with varying matric suctions. The net confining pressure was 25 kPa. The matric suction applied to the soil specimen varied up to 450 kPa. The constant net confining pressure and matric suction were maintained during the shearing process.

Table 1. Salt solutions recommended for relative humidity control.

Salt Solution	Relative humidity %	Total suction kPa
Sodium Choride	75.5	38,600
Magnesium Nitrate	54.3	84,000
Magnesium Choride	33	152,400
Lithium Choride	11.5	292,400

High suction values were applied to the specimen using the vapor equilibrium technique and the relative humidity control technique. Salt solutions were used in the glass desiccator along with the soil specimens, in the vapor equilibrium technique. Four different salt solutions were used in these tests to obtain four relative humidities and soil suctions as summarized in Table 1 (Yong 1967). After the air voids in the soil were approaching equilibrium with each relative humidity applied by the salt solutions, the soil specimens was assumed to be desiccated. The desiccated soil specimens were consolidated under an isotropic confining pressure of 25 kPa in the triaxial cell. A low shearing rate of 0.014 mm per minute was adopted for strain. The deviator stress was applied until the axial strain exceeded 20%. In Test Program 1, the relationship between maximum deviator stress (i.e., shear strength) and matric suction was observed over a wide range of soil suctions.

2.2 *Test program 2*

All soil specimens in Test Program 2, were placed directly into a temperature and relative humidity controlled chamber in order to apply a high total suction. A relative humidity of 20% was selected for Test Program 2. A relative humidity of 20% corresponds to a total suction of 217,000 kPa. The soil specimens subjected to the high suctions was sheared under varying net confining pressures. The net confining pressures ranged from 24.5 kPa to 73.5 kPa. The rate of axial strain was the same as in Test Program 1. The shear strength parameter with respect to net confining pressure was measured.

The soil-water characteristic curve for the silty soil was also measured. It is generally accepted in geotechnical engineering that the estimation of the unsaturated soil property functions are closely related to the soil-water characteristic curve. The change in water content of the soil with increasing soil suction was measured in terms of gravimetric water content, w, using the pressure plate technique, the vapor pressure technique and the relative humidity control technique.

3 TEST RESULTS AND DISCUSSIONS

3.1 *Soil-water characteristic curve*

Figure 1 shows the soil-water characteristic curve for the silty soil. The soil-water characteristic curve shows the desaturation characteristics of the silty soil in the drying process. The gravimetric water content as the ordinate axis shows that the amount of water in the soil remains constant for suctions up to about 10 kPa. The key features of the soil-water characteristic curve are the air-entry value, the slope of the straight line portion of desorption branch, the residual water content and the residual suction. The air-entry value of the soil can be identified as the pressure at which air first enters the pores of the soil. The air-entry value for the silty soil was obtained by extending the constant slope portion of the soil-water characteristic curve to intersect the horizontal suction axis at saturated conditions. The air-entry value for the silty soil is about 10 kPa.

Figure 1 can be visualized as an integrated frequency distribution curve of the pore sizes in the soil. The curve has been defined using a numerical prediction model equation proposed by Fredlund and Xing (1994). All the soil pores are filled with water until the soil suction reaches the air-entry value. After the matric suction reaches the air-entry value, desaturation starts in the transition stage. The amount of water in the soil decreases significantly with increasing soil suction in the transition stage. A coarse-grain soil such as a gravel or sand shows a tendency to desaturate at a fast rate with increasing

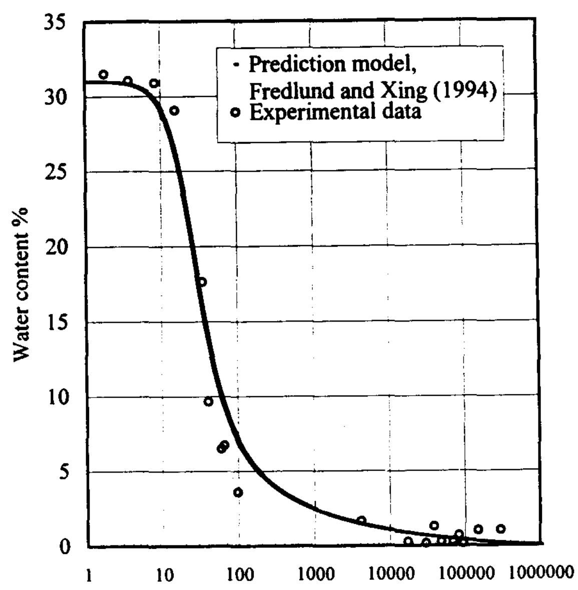

Figure 1. Soil-water characteristic curve for the silty soil.

Table 2. Summary of stress conditions during triaxial tests.

Net normal stress kPa	Soil suction kPa	Shear strength kPa
25	68	77
25	120	89
25	140	115
25	160	111
25	220	143
25	300	148
25	450	220
25	38600	204
25	84000	188
25	152400	166
25	292400	212
25	217000	212
49	217000	242
74	217000	272

suctions. The rate of desaturation, however, decreases with an increase in fine content. The slope of branch in the transition stage for the silty soil is steeper than that of kaolin.

Eventually the desaturation stage reaches the residual stage of unsaturation along the desorption portion of a soil-water characteristic curve. Large increases in soil suction lead to a relatively small change in water content in the residual stage of unsaturation. At suctions greater than the residual suction, flow occurs through the pores in the vapor phase. The wetted contact area has reduced significantly and the volume change in the unsaturated soil becomes negligible. The residual state of unsaturation can be defined using an empirical, graphical procedure. The residual suction was obtained as the point where the line extending from 1,000,000 kPa along the curve intersects the line tangent to the steeper portion of the soil-water characteristic curve. The residual water content and residual suction of the silty soil are 2.5 % and 200 kPa, respectively.

3.2 *Relationship between shear strength and matric suction*

The relevant stress conditions (i.e., shear strength) during the triaxial tests, either under a constant net normal stress and varying matric suction or under a constant suction and varying net normal stress, are summarized in Table 2. Figure 2 shows the relationship between shear strength and matric suction up to 450 kPa with the triaxial tests under a constant net normal stress. Shear strength increases with increasing matric suction. An "extended" form of the Mohr-Coulomb equation (i.e., Eq.1), is used to interpret the test results. When matric suction is zero (i.e., saturated conditions), the shear strength is calculated as 23 kPa with c' equal to 0 kPa, ϕ' equal to 43 degrees under a constant net normal stress of 25 kPa. That value shows the point where the failure envelope intersects the shear strength versus matric suction plane. The entire failure envelope is defined using the equation (2). The computed failure envelope represents the best-fit line with fitting parameter, κ, of 0.38. The failure envelope indicated a linear relationship until matric suction up to the air-entry value of 10 kPa. The initial slope of the shear strength versus matric suction envelope, ϕ^b, is equal to the effective friction angle, ϕ' at suctions below the air entry value. At matric suctions exceeding the air-entry value, the measured shear strength increases. The value of the friction angle, ϕ^b, however, decreases when the desaturation approaches residual conditions. The shear strength versus matric suction envelope was found to be non-linear. The peak strength envelope is composed of two segments, a linear segment and a curvilinear segment. The transition from the linear segment to the curvilinear segment occurs at approximately the air-entry value of soil. At or near a matric suction of 400 kPa, the slope of failure envelope, ϕ^b, approaches an angle of approximately zero. An increase in matric suction produces no further increase in shear strength. The effect of matric suction on the shear strength is clearly related to the soil-water characteristic curve of the silty soil.

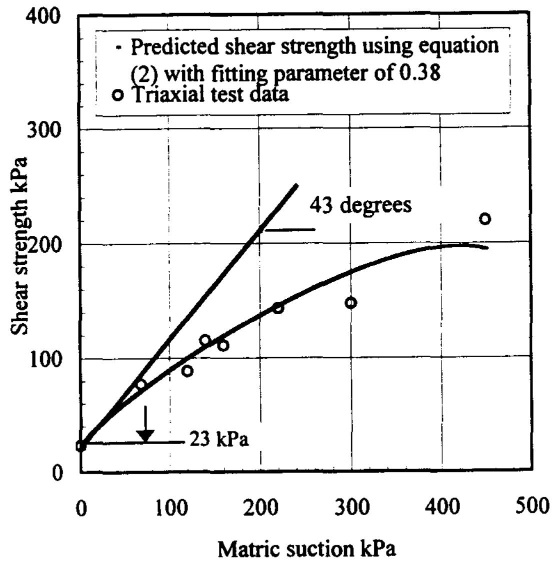

Figure 2. Relationship between shear strength and matric suction at constant net normal stress of 25 kPa.

3.3 *Relationship between shear strength and total suction*

The shear strength data from triaxial tests on a silty soil subjected to high soil suctions, using the salt solution desiccator technique, are presented in Fig. 3. A constant net normal stress of 25 kPa was applied and various total suctions were used. The soil

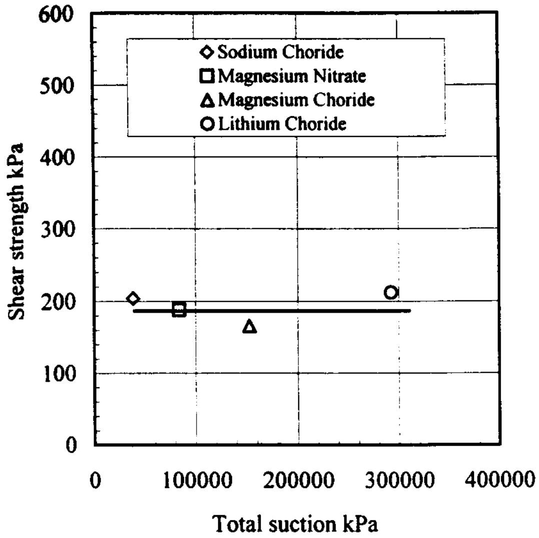

Figure 3. Relationship between shear strength and total suction at a net normal stress of 25 kPa.

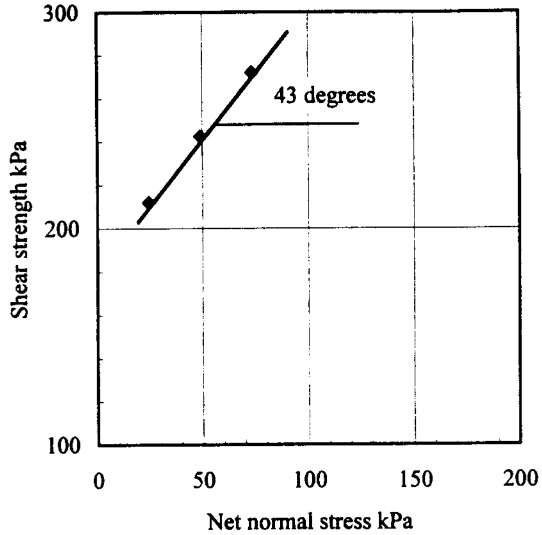

Figure 4. Relationship between shear strengrth and net normal stress at a constant suction of 217,000 kPa.

suctions applied were always larger than the residual suction of 200 kPa. Figure 3 shows that a considerable increase in soil suction produces no increase in shear strength. At suctions in excess of residual suction, the shear strength envelope is essentially a horizontal line. The suction in the residual stage makes less of a contribution to increasing the shear strength as compared to the suction in the transition stage. Fredlund (1998) commented that the shear strength of an unsaturated soil appears to remain essentially constant beyond residual state suction from a theoretical and empirical model standpoint without undertaking laboratory tests.

3.4 *Relationship between shear strength and net normal stress for unsaturated soil subjected to high total suction*

Figure 4 shows the relationship between shear strength and net normal stress under a constant total suction of 217,000 kPa. High total suctions were applied to the silty soil equilibrated at a relative humidity of 20%. The failure envelope with respect to the net normal stress was studied using the triaxial test results. The slope of that failure envelope (i.e., the angle, ϕ', in Eq.1) was computed to be 43 degrees. The computed, ϕ', values are quite consistent with the effective internal friction angle for the saturated silty soil. The effective angle of internal friction for the saturated soil appears to equally contribute to the shear strength of unsaturated soil.

4 CONCULUSIONS

This study conducted triaxial compression tests for an unsaturated silty soil over a wide range of soil suctions. Suctions were applied using the pressure plate apparatus, salt solution desiccators and a relative humidity control chamber. The compacted, unsaturated silty soil was desaturated to the residual state of unsaturation through the use of high suctions. The relationship between shear strength and soil suction is non-linear as it goes into the transition stage. After the suction exceeds the residual suction, the shear strength remains a constant (i.e., essentially horizontal failure envelope with respect to soil suction). The angle, ϕ^{b}, with respect to suction varied from the angle of internal friction of the saturated silty soil to zero at high suctions.

This study also used the triaxial test to study the effect of net normal stress on shear strength of a soil in the residual state of unsaturation. The shear strength increases with the net normal stress for the unsaturated silty soil with a suction of 217,000 kPa. The ratio of shear strength to net normal stress is coincident with the effective angle of internal friction of the saturated silty soil.

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