ESTIMATION OF VOLUME CHANGE FUNCTIONS FOR UNSATURATED SOILS

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Abstract

The estimation of unsaturated soil property functions has followed the pattern of using the saturated soil properties along with the soil-water characteristic curve to predict the behavior of the unsaturated soil. This has been the procedure for the permeability function and the shear strength functions for unsaturated soils. It has been more difficult to develop procedures for the volume change functions for an unsaturated soil; however, it is suggested that a similar procedure can be adopted. This paper suggests a series of postulates that can be used in estimating the volume change functions (during monotonic loading) for an unsaturated soil.

INTRODUCTION

Procedures to estimate the unsaturated soil property functions with respect to permeability and shear strength have been developed based upon the saturated soil properties and the soil-water characteristic curve. These procedures have proven to be of great value in the implementation of unsaturated soil mechanics into standard geotechnical engineering practice.

Estimation of the Seepage Function

The constitutive relationship to describe flow through a saturated or unsaturated soil is Darcy's law. The coefficient of permeability for an unsaturated soils is a function of the amount of water in the soil and can be written in terms of the stress state.

[1] $k_w = func\ [\ k_s, (\sigma - u_a), (u_a - u_w)\ ]$

It is generally considered sufficient to quantify the amount of water in the soil as a function of soil suction, $(u_a - u_w)$. The unsaturated coefficient of permeability can then be written as a function of the saturated coefficient of permeability and the normalized water content in addition to an additional fitting parameter, q, to complete the functional relationship.

[2] $k_w(u_a - u_w) = func\ [(w(u_a - u_w)/w_s), q, k_s]$

Numerous analyses have been proposed for the estimation of the permeability function for unsaturated soils (Fredlund et al, 1994; Leong and Rahardjo, 1997b). Common to all methods is the existence of a mathematical function relating the saturated coefficient of permeability and the soil-water characteristic curve.

Estimation of the Shear Strength Function

The shear strength equation is a constitutive relationship defining the shear strength of a soil in terms of the stress state variables and soil properties. Laboratory studies over a wide range of soil suctions, have shown that the friction angle with respect to soil suction can be written as a function of the effective angle of internal friction and the stress state (Fredlund et al, 1987).

[3] $\tan \varphi^b = func[\tan \varphi', (\sigma_n - u_a), (u_a - u_w)]$

The normalized water content of an unsaturated soil, (*i.e.*, $w(u_a - u_w)/w_s$), is a function of the complete stress state but is generally simplified as a function of soil suction. Therefore, the increase in strength with respect to soil suction can be written in terms of the normalized water content used to describe the soil-water characteristic curve. It is also necessary to include an additional fitting parameter, p, to account for deficiencies in the one-to-one fit between water content and shear strength.

[4] $\tan \varphi^b = func[\tan \varphi', (w(u_a - u_w)/w_s), p]$

where:

$w(u_a - u_w)$ = water content at a particular soil suction,

w_s = water content under saturated conditions.

Mathematical equations that have been proposed and verified for the soil-water characteristic curve and as a result, it is possible to write a closed-form equation for the shear strength constitutive surface for an unsaturated soil. In each case, the shear strength of the unsaturated soil becomes a function of the saturated shear strength parameters and the soil-water characteristic curve.

Estimation of the Volume-Mass Functions

Over the past several decades, the volume change behavior of an unsaturated soil has been linked to two independent stress state variables; namely, $(\sigma - u_a)$ and $(u_a - u_w)$. This constitutive formulation forms the basis for modeling the volume change of an unsaturated soil. Modeling of such soil processes as stress/deformation, shrink/heave, and consolidation require an adequate description of the constitutive volume change behavior of a soil.

The overall volume change of an unsaturated soil can be defined as a change in void ratio in response to a change in the stress state.

[5] $$de = \frac{\partial e}{\partial(\sigma - u_a)} d(\sigma - u_a) + \frac{\partial e}{\partial(u_a - u_a)} d(u_a - u_a)$$

where:

e = void ratio, and

σ = total normal confining stress (e.g., isotropic confining pressure).

Equation [5] can be viewed as having two parts; namely, a part that is the designation of the stress state (i.e., $(\sigma - u_a)$ and $(u_a - u_w)$) and a part that is a designation of the soil properties (i.e., $(\partial e / \partial(\sigma - u_a))$ and $(\partial e / \partial(u_a - u_w))$). The soil properties can be viewed as the slope of the void ratio constitutive surfaces as shown in Fig. 1. The soil properties are moduli that vary as a function of the stress state. The soil moduli associated with the net normal stress, $(\sigma - u_a)$, can be written in a general functional form.

[6] $$\partial e / \partial(\sigma - u_a) = func[(\sigma - u_a),(u_a - u_w)]$$

The term *func* means that the soil property is a function of the stress state. At a particular stress state, the compressibility modulus for the void ratio constitutive surface with respect to $(\sigma - u_a)$, can be designated as a constant.

[7] $$\partial e / \partial(\sigma - u_a) = m_1^s$$

Similarly, the soil moduli associated with soil suction, $(u_a - u_w)$, can be written in a general functional form.

[8] $$\partial e / \partial(u_a - u_w) = func[(\sigma - u_a),(u_a - u_w)]$$

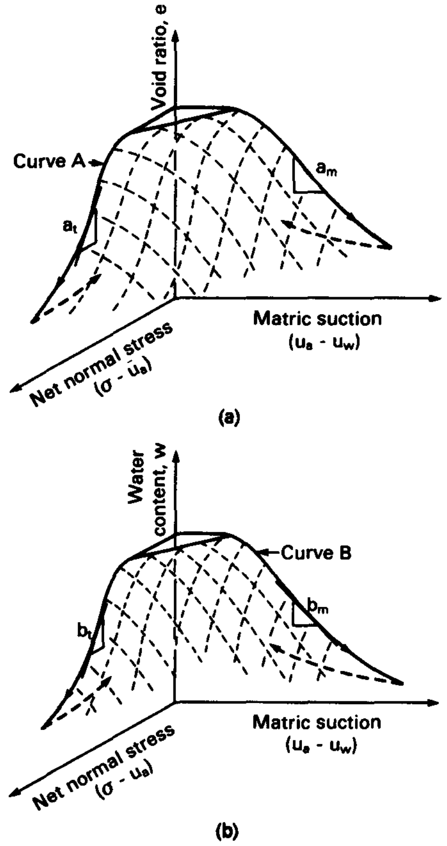

Figure 1 Three-dimensional void ratio and water content constitutive surfaces for an unsaturated soil. (a) Void ratio constitutive surface; (b) water content constitutive surface. (Fredlund, 1993)

At a particular stress state, the compressibility modulus for the void ratio constitutive surface with respect to (u_a - u_w), can be designated as a constant.

[9] $$\partial e / \partial(u_a - u_w) = m_2^s$$

Each of the soil moduli is function of both stress state variables. In order to define the magnitude of the soil moduli corresponding to any stress state, there needs to be a constitutive equation describing the entire void ratio constitutive surface. The equation then needs to be differentiated with respect to each of the stress state variables in order to obtain the compressibility moduli. At present, no equations have been published to represent the entire void ratio constitutive surface in terms of the stress state variables.

Two constitutive relationships are required to define the volume-mass variables in terms of the stress state variables. The need for two independent constitutive relations for an unsaturated soil can be demonstrated through the differentiation of the basic volume-mass relationship (i.e., $Se = wD_r$).

[10] $$\int_{e_o}^{e_f} S de + \int_{S_o}^{S_f} e dS = D_r \int_{w_o}^{w_f} dw$$

where:

w = water content, with the subscript, *o*, and *f*, representing the initial and final states, respectively,

S = degree of saturation, and

D_r = relative density of the soil solids.

The water content constitutive surface can be used as a second relationship for defining the volume-mass behavior of an unsaturated soil (Fig. 1). The water content constitutive relationship can be written the following general form.

[11] $$dw = \frac{\partial w}{\partial(\sigma - u_a)} d(\sigma - u_a) + \frac{\partial w}{\partial(u_a - u_w)} d(u_a - u_w)$$

Once again, Eq. [11] has a part that designates the stress state and a part that designates an unsaturated soil property that is a function of the stress state. The soil moduli associated with the net normal stress variable, $(\sigma - u_a)$, can be written as a general function.

[12] $$\partial w / \partial(\sigma - u_a) = func[(\sigma - u_a), (u_a - u_w)]$$

At a particular stress state, the compressibility modulus for the water content constitutive surface, with respect to $(\sigma - u_a)$, can be designated as a constant.

[13] $$\partial w / \partial(u_a - u_w) = m_1^w$$

Similarly, the soil moduli associated with the soil suction, $(u_a - u_w)$, can be written as a general function of the stress state.

[14] $$\partial w / \partial(u_a - u_w) = func[(\sigma - u_a), (u_a - u_w)]$$

At a particular stress state, the compressibility modulus for the water content constitutive surface, with respect to $(u_a - u_w)$, can be designated as a constant.

[15] $$\partial w / \partial(u_a - u_w) = m_2^w$$

At present, there is no published equation to represent the entire water content constitutive surface. Once an appropriate equation is formulated, the derivatives will provide the soil moduli values corresponding to any stress state.

FORMULATION FOR THE ESTIMATION OF THE VOLUME-MASS CONSTITUTIVE SURFACES (LOADING)

The formulation of the constitutive surfaces is based on common laboratory compression, shrinkage and soil-water characteristic curve tests. Methods for the mathematical representation of these soil-property functions are presented. Mathematical representation are proposed for the compression, shrinkage, and soil-water characteristic curve soil property functions. The proposed equations include two new equations to represent the compression curve, and one new equations to represent the shrinkage curve.

A series of assumptions are then presented to form a guide for the combining of the independent mathematical representations and the subsequent formulation of void ratio and water content constitutive surfaces. The end result is a mathematical representation of two independent constitutive surfaces. The mathematical representations can then be used to compute the compressibility (and subsequently the elastic moduli) of the soil corresponding to any stress state in the soil. These formulations provide the necessary information for the generation of elastic soil property functions that can be used for numerical modeling of soil behavior.

Postulates for Volume Change Constitutive Surface (Loading)

The first constitutive surface selected is related to overall volume change and can be defined in terms of void ratio, e, or specific volume, υ, (i.e., 1+e). The void ratio is used herein to define the first constitutive surface.

A series of "postulates" are proposed for the prediction of the volume-mass constitutive relations. The postulates establish a series of priorities that must be adhered to when attempting to estimate the volume-mass relationships. Certain information has become well established in the research literature and this information forms a series of hierarchical priorities when predicting the constitutive surfaces.

Postulates for the Volume Change Constitutive Surface

The soil structure constitutive surface can be defined as the relationship between two independent stress state variables and a deformation state variable. The independent stress state variables are:

$(\sigma - u_a)$ = net normal (isotropic) confining pressure, and
ψ = soil suction.

The deformation state can be defined in terms of void ratio, e. The proposed "postulates" for the soil structure (i.e. void ratio) constitutive surface are as follows for the case of an increase in both of the stress state variables (i.e., a monotonic decrease in volume). In addition, it is assumed that the testing of the soil starts with the specimen being in a saturated state. There are a number of loading stress paths as well as wetting and drying paths that could be analyzed; however, it is important to start by developing constitutive surfaces for the conditions on which the most information is available.

ψ *Postulate 1*

The primary reference condition for the volume change (overall) constitutive relationship is determined by applying a net (isotropic) total stress loading of the soil with the pore-water and pore-air pressures maintained at zero, while measuring the change in void ratio.

This relationship is commonly referred to as the drained, effective stress loading path for a saturated specimen (Figure 2). The term "isotropic" is placed in brackets to suggest that isotropic loading is the preferred form of loading. However, it is also possible for K_o or other forms of net total stress loading to be considered. Isotropic stress loading is preferable because: a) the stress path is the same as that used for critical state (or elasto-plastic) models, and, b) the matric suction stress state variable is also isotropic in character.

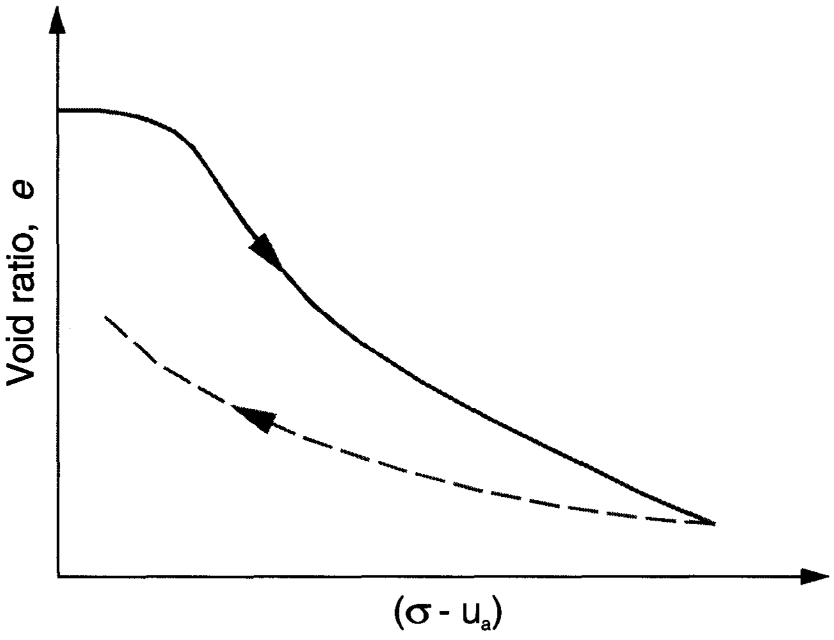

Figure 2 Typical loading and unloading curves of void ratio versus the applied load.

ψ *Postulate 2*

The <u>*secondary reference condition*</u> *for the volume change (overall) constitutive relationship is determined by applying various soil suctions to the soil with the net isotropic stress equal to zero, while measuring the change in void ratio (Figure 3).*

There is a practical difficulty associated with directly measuring the volume change versus soil suction relationship. The difficulty is related to measuring volume change in three directions while changing soil suction. As a result of the above difficulty, "Postulate 2a" suggests an alternate means to indirectly provide the necessary secondary reference condition. An alternate procedure makes use of a combination of data from a shrinkage test and a soil-water characteristic curve test.

ψ *Postulate 2a*

The void ratio versus soil suction relationship can also be computed using the soil-water characteristic curve for the soil along with the shrinkage *curve, both sets of data are measured under condition of zero net isotropic stress.*

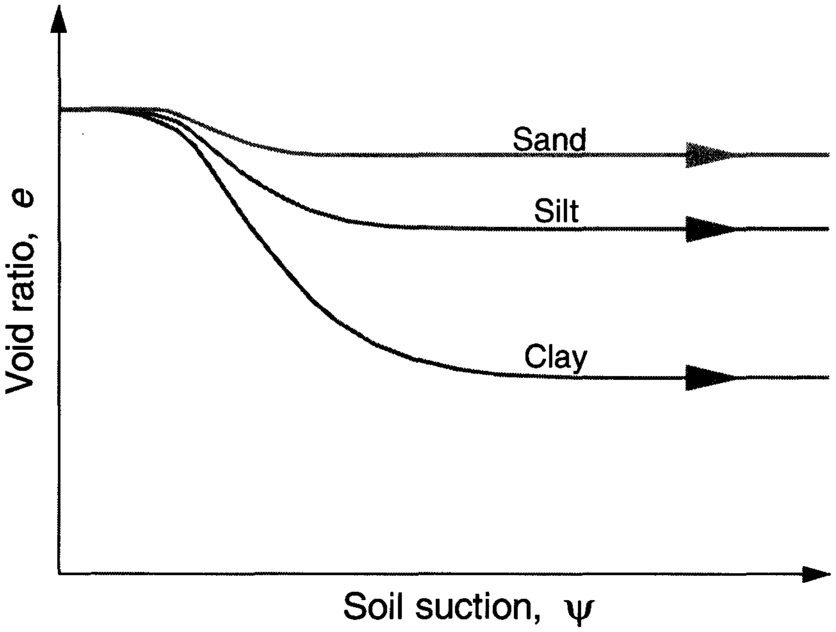

Figure 3 Typical void ratio versus soil suction plot for three soils (suction increase)

Figure 4 shows three typical soil-water characteristic curves under drying conditions (or conditions of an increase in suction. Figure 4 shows a typical shrinkage curve associated with the drying of a clay soil.

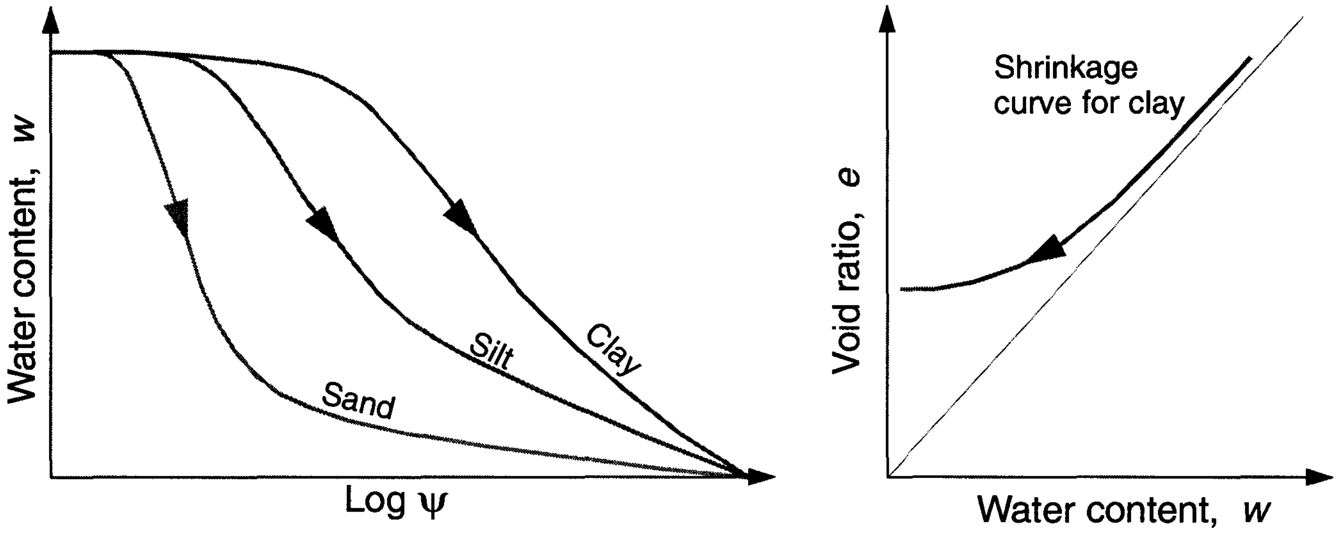

Figure 4 a) Typical soil-water characteristic curves for three soil types. b) Typical shrinkage curve for a clay soil.

It is possible to combine the results of a pressure plate test (i.e., soil-water characteristic curve data) and a shrinkage test to obtain a void ratio versus suction plot. The shrinkage test defines a curve that gives the ratio of change in volume of water to overall volume, for a change in soil suction. Mathematically, the slope of the shrinkage curve can be written as follows.

[16] $$\frac{de}{dw} = \frac{de/d\psi}{dw/d\psi} = \frac{a_m}{b_m}$$

Combining the two sets of information makes it possible to compute the void ratio versus soil suction relationship. This forms the second reference (or limiting) condition for the soil structure constitutive surface.

ψ *Postulate 3*

There is a unique volume change constitutive surface defined for conditions of monotonic deformation.

The surface for a decrease in volume under an increase in stresses is considered herein and shown in Figure 5.

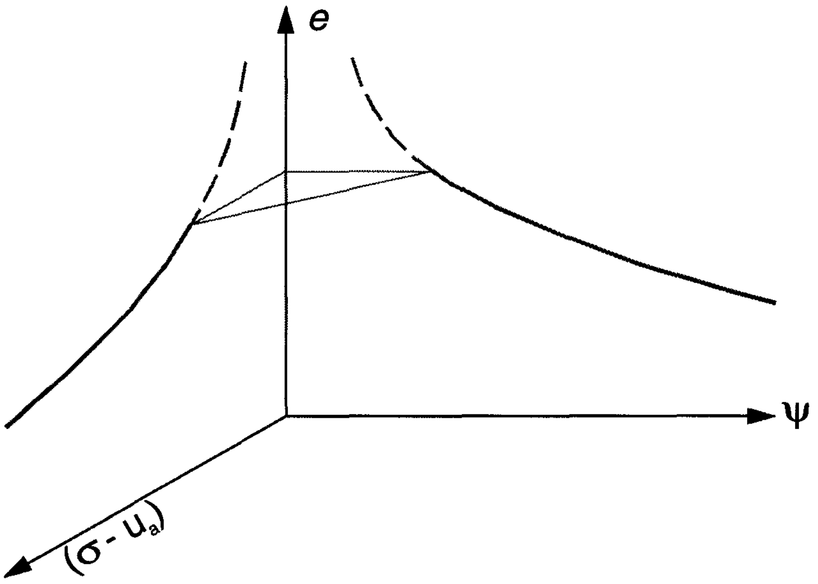

Figure 5 Three-dimensional plot showing the primary and secondary reference conditions for the void ratio constitutive surface.

The limiting (or reference) conditions associated with the void ratio constitutive surface have now been defined. The next step is to define the character of the constitutive surface between the limiting reference conditions. The remaining postulates pertain to establishing intermediate stress state conditions on the constitutive surface.

ψ *Postulate 4*

> *The slope along any constant net total stress plane on the volume change constitutive surface is a function of the void ratio, as defined on the zero soil suction plane.*

This postulate comes about as a result of Postulate 1 where it is stated that the void ratio versus net total stress is the primary and most fundamental relationship between void ratio and the stress state. As a consequence of Postulate 4, the slope of any line emanating from the soil suction versus void ratio curve, in a constant suction plane, must be equal to the compressibility defined on the primary reference curve at a corresponding void ratio. Appropriate slopes for the constitutive surface can be determined by constructing a triangle in the horizontal plane, between the reference conditions.

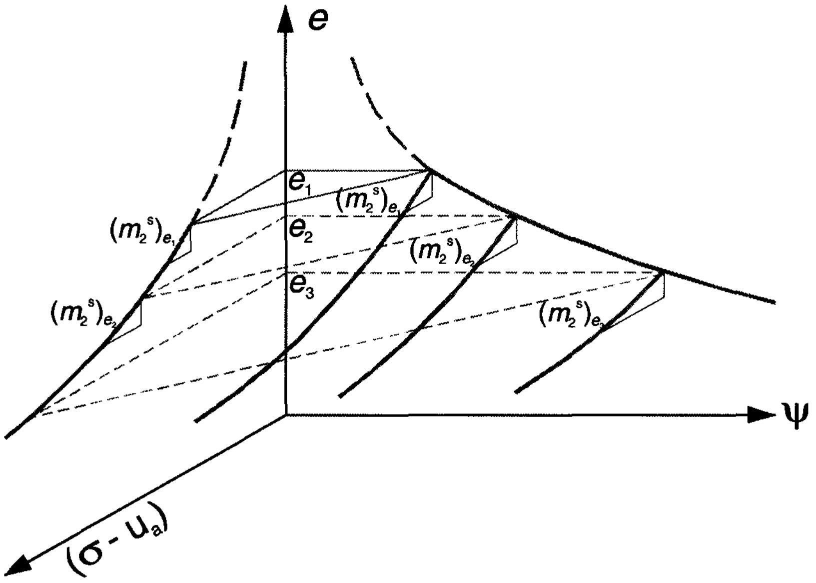

Figure 6 Illustration of the definition of the void ratio constitutive surface based on the slopes of the primary reference curve.

ψ *Postulate 5*

There is a one-to-one relationship between the effects of changes in net total stress and a change in soil suction, when the soil suction is less than the air entry value of the soil (Figure 7).

This mean that a 45 degree relationship will be defined between the two stress state variables when the void ratio constitutive surface is viewed along the void ratio axis.

The straight line contour across the constitutive surface should be theoretically correct as long as the soil is saturated. This is in accordance with the effective stress concept for a saturated soil. It should be noted that the dashed lines drawn in Figure 7 may not intersect the secondary reference condition along the plane of net total stress equal to zero. It is necessary to comment further on the air entry value of the soil before suggesting a further refinement on void ratio contours.

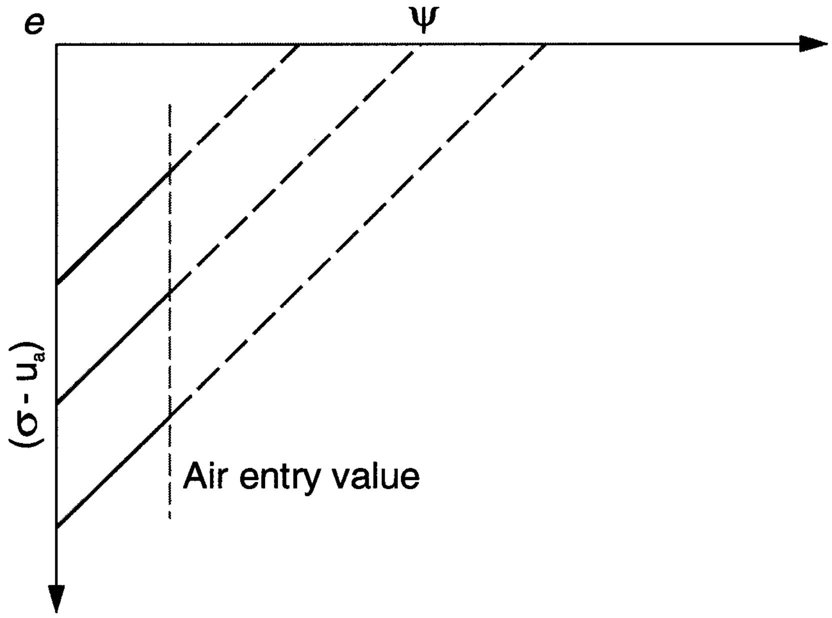

Figure 7 Variation of constant void ratio contours when the surface is viewed along the void ratio axis.

ψ *Postulate 5a*

As a first approximation, the air entry value of the soil can be assumed to be a constant, but for greater refinement, the air entry value may need to be defined as a function of void ratio or the net isotropic stress (Figure 8).

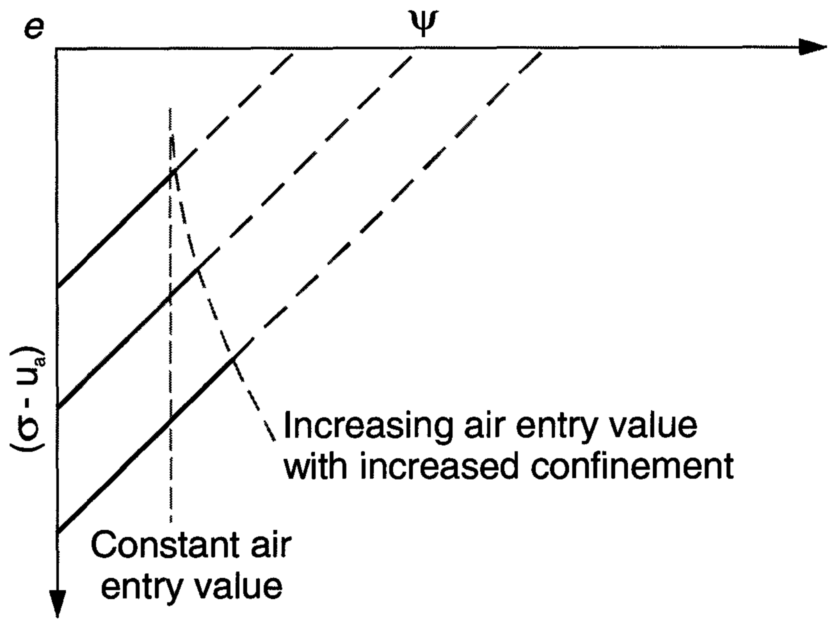

Figure 8 Effect of a variation in the air entry value on the void ratio contours.

The air entry value would be anticipated to increase with a decrease in void ratio. This means that the 45 degree contour would be adhered to for a greater distance from the net total stress reference plane. No attempt is made at this time to define the air entry value of the soil as a function of void ratio (or stress state).

ψ *Postulate 6*

A gradual curve forms from the air entry value to the secondary reference condition, corresponding to a particular void ratio on the soil structure constitutive surface (Figure 9).

The curve must be tangent at the air entry value and increase in curvature as the secondary reference condition is approached. This means that it should always be possible to join the secondary reference curve provided it is positioned further from void ratio than is the primary reference curve. In other words, at a particular void ratio, the soil suction value should exceed the net total stress value.

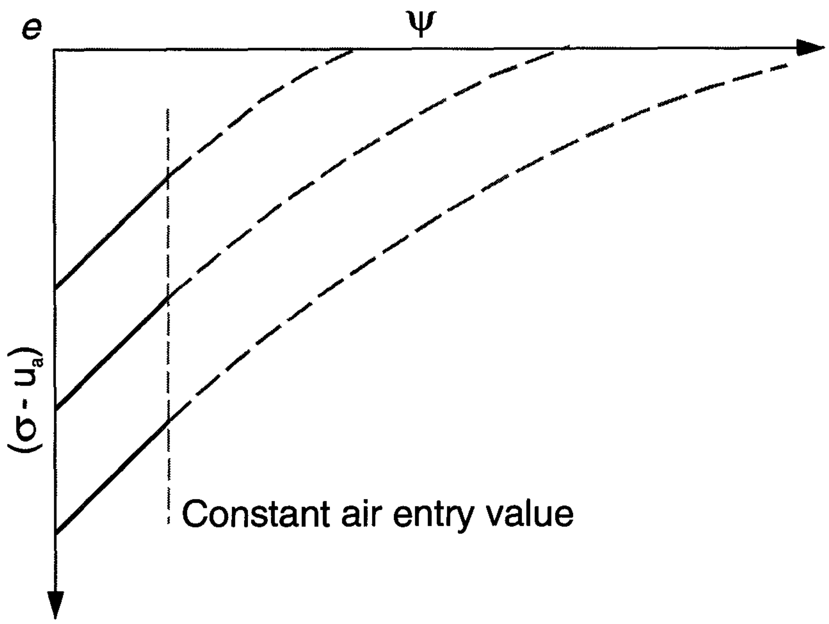

Figure 9 Variation in the constant void ratio contours as the soil becomes increasingly desaturated.

The curves should always bend in the direction of the soil suction axis for a clayey, stable-structured soil. For a sandy soil, the curves will bend even more rapidly and may never reach the reference soil suction axis.

The loading portion of the void ratio constitutive surface can be approximated using the steps outlined above. The general character of the void ratio constitutive surface should apply for sands, silts and clays. The greatest difficulty should be observed in defining the constitutive surface near to initial conditions. This is due to the fact that not all of the tests are started from precisely the same stress state and volume-mass state. As well, different tests may follow different stress paths particularly near the start of the test. It is therefore necessary to take into consideration the initial state of the soil. For example, the soil could be initially slurried, compacted or be in an undisturbed state.

The above postulates do not cover all aspects of defining the void ratio constitutive surface. The postulates pertain to the loading (by net total stress or soil suction) constitutive surface of an initially saturated soil. Other postulates will be required to define the unloading void ratio constitutive surface. Still other postulates are required for the case where one state variable is increased while the other one may be decreased. The scope of this thesis is limited to monotonic loading of an initially saturated soil.

Postulates for the Water Content Constitutive Surface

The water content constitutive surface can be defined as the relationship between two independent stress state variables and a deformation state variable. The water content constitutive surface is defined loading conditions with a corresponding, monotonic decrease in water content. The independent stress state variables are:

$(\sigma-u_a)$ = net normal (isotropic) confining pressure, and

(ψ) = soil suction.

The deformation state can be defined in terms of gravimetric water content, w. The proposed "postulates" for the establishment of the water content constitutive surface are as follows. The constitutive surface related to an increase in each of the stress state variables is defined. The soil is assumed to start from a saturated condition.

ψ *Postulate 7*

The primary reference condition for the water content constitutive relationship is determined by applying a suction to the soil with the net isotropic stress maintained at zero, while measuring the change in water content.

This relationship is known as the soil-water characteristic curve and is usually plotted with soil suction on a log scale. An example, along with the definition of the air entry value and residual conditions, is shown in Figure 10.

ψ *Postulate 8*

The secondary reference condition for the water content change constitutive relationship is computed from the void ratio constitutive surface for the condition of zero soil suction where the void ratio is multiplied by the specific gravity of the soil (i.e., $w = e/G_s$*).*

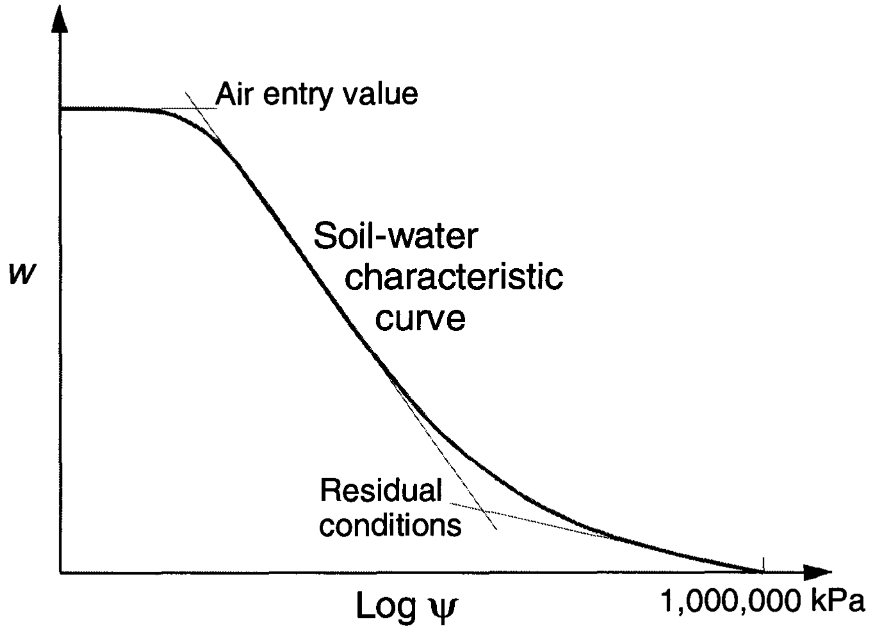

Figure 10 The water content versus soil suction relationship forms the primary reference condition for the water content versus soil suction constitutive relationship.

The water content versus net normal stress has the same character as the compression curve as shown in Figure 11

The water content versus net loading curve is secondary in its role in defining the intermediate conditions of the water content constitutive surface.

ψ *Postulate 9*

There is a unique water content constitutive surface defined for conditions of monotonic water volume change.

The surface for a decrease in water volume under an increase in stresses is considered herein and is shown in Figure 12.

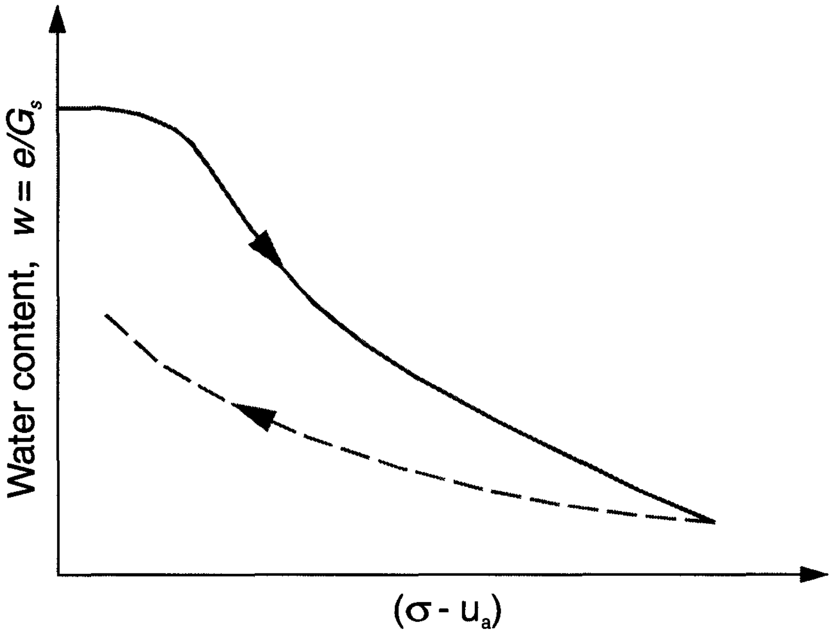

Figure 11 The secondary reference condition for the water content constitutive surface is equal to the primary reference condition for the void ratio constitutive surface divided by the specific gravity of the soil, G_s.

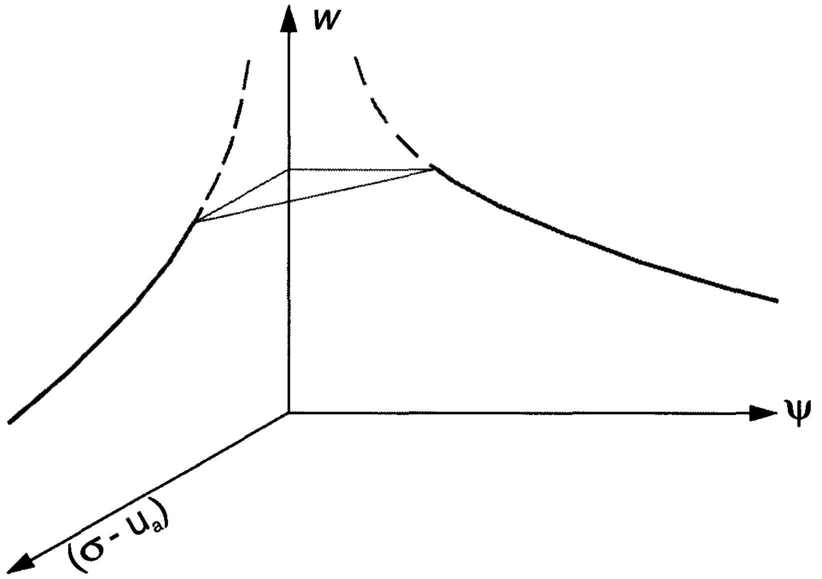

Figure 12 Three-dimensional plot showing the primary and secondary reference conditions for the water content constitutive surface.

ψ *Postulate 10*

The slope along any constant soil suction plane on the water content constitutive surface is a function of the water content, as defined on the zero net total stress plane.

Figure 13 illustrates how the water content constitutive surface is strongly influenced by the soil-water characteristic curve.

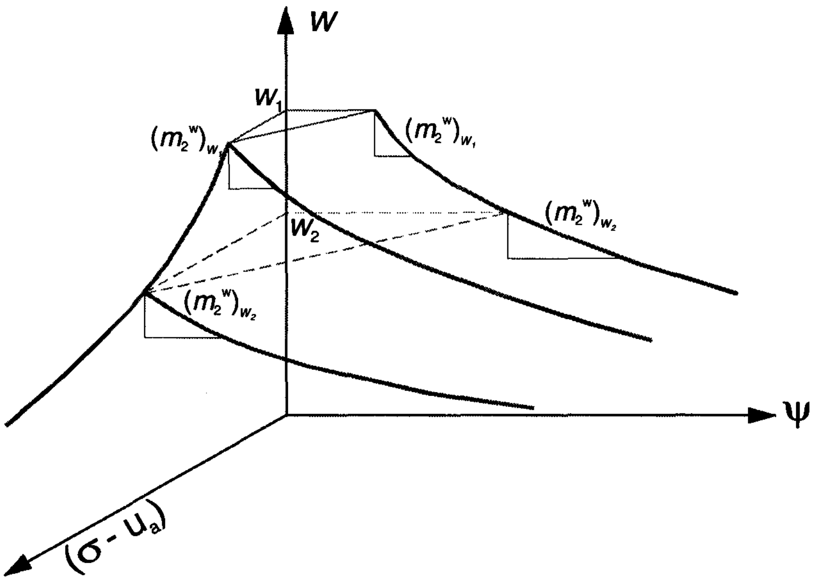

Figure 13 Illustration of the definition of the water content constitutive surface based on the slopes of the primary reference curve.

ψ *Postulate 11*

There is a one-to-one relationship between the effects of a change in net total stress and a change in soil suction, when the soil suction is less than the air entry value of the soil.

This means that a 45 degree relationship will be defined between the two stress state variables when the water content constitutive surface is viewed along the water content axis as shown in Figure 14. Once again the air entry value can be made a function of the water content (or stress state) as described for Postulate 5 of the void ratio constitutive surface.

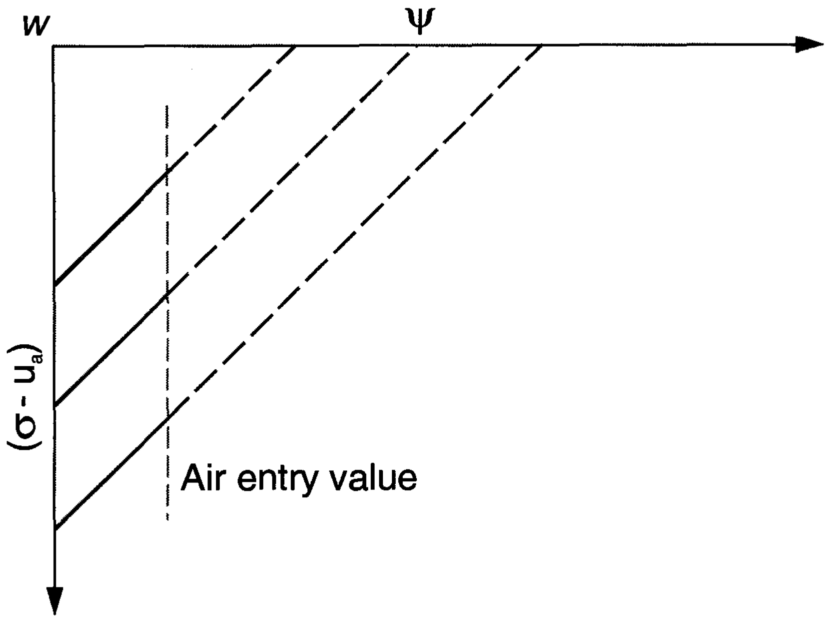

Figure 14 Variation of the constant water content contours when the constitutive surface is viewed along the water content axis.

ψ *Postulate 12*

A gradual curve forms from the air entry value to the primary water content reference condition, corresponding to a particular water content on the water content constitutive surface.

The curve must be tangent at the air entry value and increase in curvature as the primary reference condition is approached as shown in Figure 15. In other words, the character of the water content constitutive surface is different from that of the void ratio constitutive surface, as the soil becomes unsaturated.

It is possible that the air entry value of the soil will change with decreasing initial water contents. However, there does not appear to have been sufficient experimental evidence to confirm this variation in air entry value at this point.

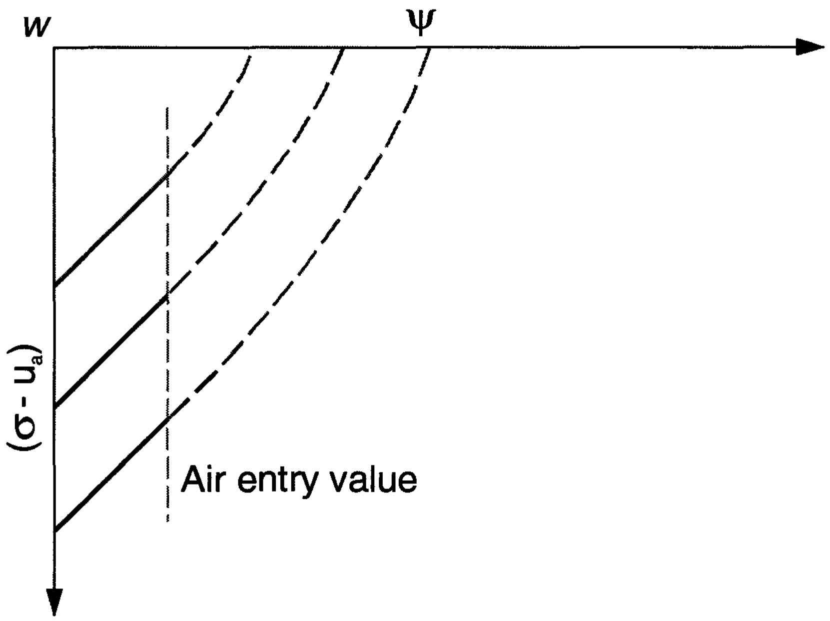

Figure 15 Variation in the constant water content contours as the soil becomes increasingly desaturated.

The proposed character of the water content constitutive surface should apply for sands, silts and clays. It is necessary to take into consideration the initial state of the soil. For example, the soil could be initially slurried, compacted or be in an undisturbed state.

Summary

A series of postulates have been provided for the estimation of the volume change and water content constitutive surfaces for an unsaturated soil (for the case of monotonic loading). Once again, the estimation procedures makes use of the saturated volume change properties of the soil along with the soil-water characteristic curve information. The suggested postulates need to be tested using experimental data on volume change and water content change.

List of References