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A COMPONENT SYSTEM APPROACH TO SOLVING SATURATED/UNSATURATED SOILS PROBLEMS

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ABSTRACT: The solution of highly non-linear, partial differential equations for saturated/unsaturated soil systems is proposed with assistance from mathematical partial differential equation solvers. The solving of highly non-linear partial differential equations can benefit from mathematicians who have been specially trained to solve these problems. Therefore, the solution algorithms are separated from the applications problem being solved and the engineer is left to focus on the engineering aspects of the problem. The solution of saturated/unsaturated seepage problems is used to illustrate the use of general partial differential equation solvers.

1. INTRODUCTION

Computer programs were first written for solving slope stability and other geotechnical engineering problems during the 1960's. These programs were written by engineers who had learned the Fortran (or comparable) programming language. Since this time, the computer industry has developed extremely rapidly, resulting in greatly increased computing power being available for solving engineering problems. In order to take advantage of recent developments in the computing industry, it has become necessary to work with persons who are specially trained in computing. The main persons involved in developing geotechnical engineering software have increasingly come from the computing software field and the engineer has taken on a new role with respect to software development.

This paper illustrates how the changing computing paradigm is affecting software development in the geotechnical area. Solutions to saturated-unsaturated seepage problems are used to illustrate the changing role of the engineer. The engineer's responsibility involves the characterization of the soil properties and soil property functions that may take on a variety of forms, in addition in other aspects related to the characterization of the problem. The partial differential equations to be solved become highly non-linear and require the input from persons trained in this area of mathematics. This has given rise to the use of general partial differential equation solvers that are designed to solve equations from many areas of engineering. The use of partial differential equation solvers will be illustrated in the paper. Their advantages and disadvantages will also be described.

The assemblage of a computer software package can be compared to the assemblage of a stereo system where the various functions are compartmentalized and interchangeable. Problems related to seepage through saturated/unsaturated soil systems will be used to illustrate the component systems approach through the use of general partial differential equation solvers.

2. COMPARISON TO A STEREO COMPONENT SYSTEM

Typical components of the stereo sound system are shown in Fig. 1 with the amplifier being at the heart of the system. Inputs to the amplifier can be in the form of a tape drive, a CD drive or a mini disk drive. Outputs from the amplifier can be fed to speakers or to a recording system (e.g., tape recorder). The inputs and outputs have standard specifications that are compatible to all amplifiers.

Computer programs have like-wise been developed along the line of a series of possibilities for input and output of information from the main computational algorithm as shown in Fig. 2. While this parallel has been true since the early developments of computer software packages in the 1960's, it has not been possible to interchange the input, output and main algorithms from various application software engineering packages. However, standardization for the transfer of information from one computer program to another has now been achieved through the use of C^{++}, Visual Basic and operating environments such as Windows.

There is an emerging desire to numerically model saturated/unsaturated soil systems through the use of estimated unsaturated soil property functions. One of the primary "hurdles" to overcome has been the ability to provide computer software that will ensure convergence when solving the non-linear partial differential equations. This has become a specialized area of mathematics that goes beyond the training of most engineers. On the other hand, the solution of non-linear partial differential equations has become a specialty area of mathematics and computer science programmers. The solution of these non-linear engineering problems needs to be placed in the hands of specialists.

In commenting on recent developments and advancements in computing capability relative to slope stability analysis, Duncan (1996) stated, "This generation of computer programs, now on the horizon, will permit geotechnical engineers to concentrate on engineering and to take the computations for granted". He goes on to say, "... the engineer will still have to understand soil mechanics and the computer program thoroughly to avoid misuse".

3. DEVELOPMENT OF PARTIAL DIFFERENTIAL EQUATION SOLVERS

A number of mathematical computer programs have emerged over the past couple of decades for the solution of math related problems. One of the common packages is MathCad[1]. Software such as MathCad can be used to solve a wide range of calculus and mathematics equation. It also performs numerical solutions for simple partial differential equations. However, the solution of differential equations is an extensive area that requires the development of specialized software packages.

PDEase[2] is one of the first general-purpose partial differential equation solvers. This computer package is marketed by Macsyma Inc. Another similar software package is FlexPDE[3] marketed by PDE Solutions Inc. PDE Solutions has been extended to solu-

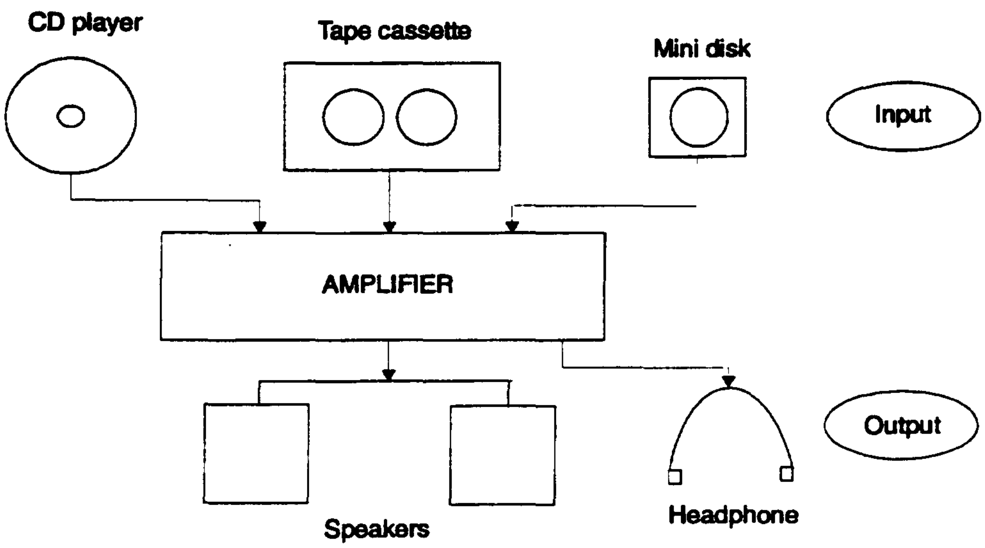

Figure 1. Basic components of a stereo sound system

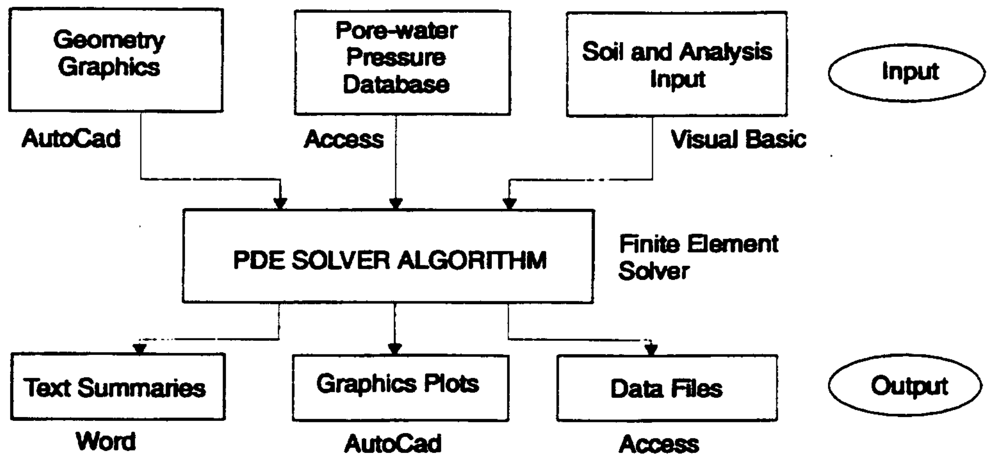

Figure 2. Potential system components for a numerical solver

tions in three-dimensions. It is fully maintained and undergoing ongoing development and enhancements. Other partial differential equation solvers have also been developed in other countries of the world (e.g., U.K.).

There are several characteristics of these software packages that are of interest to geotechnical engineers. First, these software packages can readily interface with other input and output software packages (e.g., AutoCAD[4]). Second, the focus is on ensuring proper converging solutions where highly non-linear equations are involved.

The user of the general partial differential equation software must simply type in the partial differential equation to be solved. The equation contains the variables to be solved along with a series of material properties. The boundary conditions around a designated region must also be specified.

4. CHARACTERISTICS OF A SATURATED/UNSATURATED SOIL SEEPAGE, PARTIAL DIFFERENTIAL EQUATION

One general partial differential equation can be written for both saturated and unsaturated soils, as well as transient and steady state seepage conditions. However, for steady state seepage the storage variable, m_2^w, can be set to zero. The initial differential equation can be written as follows for a two-dimensional case.

$$[1] \qquad k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} + \frac{\partial k_x}{\partial x}\frac{\partial h}{\partial x} + \frac{\partial k_y}{\partial y}\frac{\partial h}{\partial y} = \frac{m_2^w}{\rho_w g}\frac{\partial h}{\partial t}$$

where: h is hydraulic head to be solved as various points in a finite element mesh, x and y are cartesian coordinate directions, k_x and k_y are coefficients of permeability in the x and y directions, ρ_w is density of water, g is acceleration due to gravity, t is time, and m_2^w is the water storage modulus for the soil.

The hydraulic head, h, consists of two components; namely, a pressure head and an elevation head.

$$[2] \qquad h = \frac{u_w}{\rho_w g} + Y$$

where: u_w is pore-water pressure and Y is elevation above an arbitrary datum.

The coefficients of permeability are not constants for unsaturated soils but rather functions of the stress state in the soil and can be written as follows:

$$[3] \qquad k_w = func[k_s, (\sigma - u_a), (u_a - u_w)]$$

where: k_s is saturated coefficient of permeability, $(\sigma - u_a)$ is net normal stress, and $(u_a - u_w)$ is matric suction.

The coefficient of permeability of the unsaturated soil is predominantly a function of the matric suction and has the form shown in Fig. 3. The coefficient of permeability function can be indirectly computed or estimated from the soil-water characteristic curve. The relationship between the two curves is shown in Fig. 3. Mathematically the relationship can be expressed as follows:

$$[4] \qquad k(u_a - u_w) = k_s \Theta^q$$

where: q is a fitting parameter necessary to ensure a one-to-one fit between the predicted and the ex-

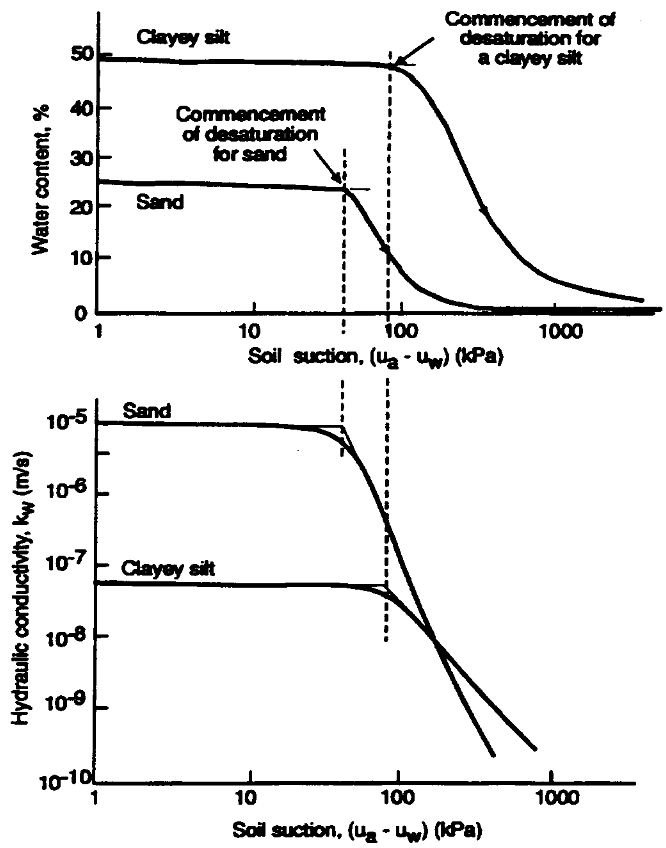

Figure 3. Illustration of the relationship between the soil-water characteristic curve and the permeability function for two soil types

perimental function, Θ is dimensionless water content defined as the water content at any soil suction, $w(u_a-u_w)$ divided by the saturated water content w_s (i.e., $\Theta = w(u_a-u_w)/w_s$).

The water content at any soil suction can be defined from the predicted, estimated or measured soil-water characteristic curve. Any one of several proposed equations could be used but the Fredlund and Xing (1993) equation will be used for illustration purposes in this paper.

$$[5] \qquad w(u_a - u_w) = C(u_a - u_w)\frac{w_s}{\left[ln\left(e + \frac{(u_a - u_w)}{a}\right)^n\right]^m}$$

where: a, n and m are soil parameters required to best-fit the soil-water characteristic curve and e is base of the natural logarithm.

The coefficient of permeability function can now be written as follows:

$$[6] \qquad k(u_a - u_w) = C(u_a - u_w)\frac{k_s}{\left[ln\left(e + \left(\frac{u_a - u_w}{a}\right)^n\right)\right]^{mq}}$$

A second mathematical, unsaturated soil property function is required when solving transient seepage problems. This is the storage function, m_2^w, which is actually the slope of the soil-water characteristic curve. Therefore, the soil-water characteristic curve must be differentiated with respect to soil suction and used when numerically solving the partial differential seepage equation.

$$[7] \qquad m_2^w(u_a - u_w) = \frac{G_s}{(1+e)}\frac{dw}{d(u_a - u_w)}$$

5. FORMATS FOR INPUT OF SOIL PROPERTY FUNCTIONS

Any one of several forms could be used for inputting the soil property functions when solving the partial differential equation of seepage for a saturated/unsaturated soil system. These forms can be broadly categorized as i) a mathematical equation or ii) a series of data points as shown in Fig. 4. The series of data points could possibly be handled in a number of different ways of input when solving the partial differential equation. Some systems for handling the data points are given in Fig. 4 and each system is further illustrated in Fig. 5.

Equation 6 is an example of a closed-form equation for the coefficient of permeability function. The techniques for handling a series of points apply equally for the permeability function or the water storage function.

The closed-form mathematical equations are always the superior technique to use for data input. Other forms are acceptable but generally have inherent errors and limitations associated with each procedure. For example, the use of a spline function can produce convergence problems as well as other errors.

6. EXAMPLE PROBLEM INVOLVING SEEPAGE THROUGH A SATURATED/UNSATURATED SOIL SYSTEM

A large number of example problems as well as a parametric study have been performed using partial differential equation solvers (Thieu, 1999). However, as part of this paper, only one example problem involving transient, saturated/unsaturated seepage is presented. The example problem involves the raising

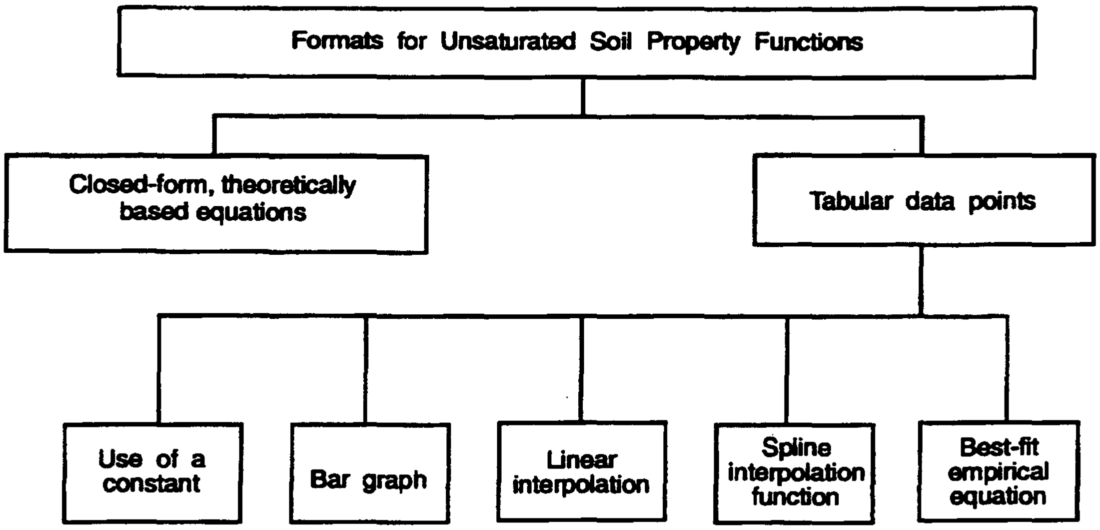

Figure 4. Summary of possible formats for inputting unsaturated soil property functions

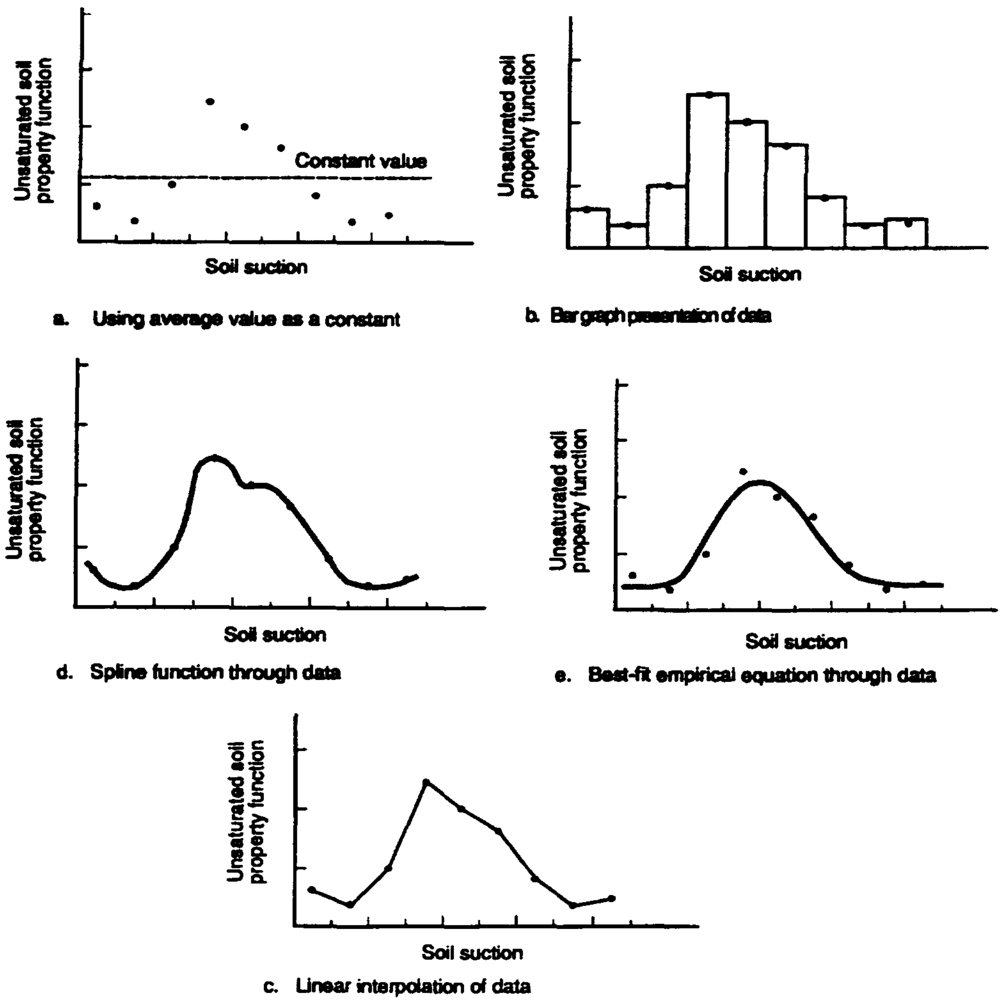

Figure 5. Illustration of formats for inputting unsaturated soil properties as a function of soil suction

of the reservoi behind a dam from 4m to 10m. A cross-section of the dam along with the boundary conditions is shown in Fig. 6.

Several procedures were used to input the coefficient of permeability and the water storage functions. Each soil property function was written with respect to soil suction. Figure 7 graphically shows two mathematical and one linear interpolation function that was used to define the coefficient of permeability of the unsaturated soil. Figure 8 shows three mathematical and one linear interpolation function that were used to define the water storage modules, m_2^w, for the unsaturated soil.

The reservoir was instantaneously raised from 4m to 10m and the transient seepage process was began. Figure 9 shows the pore-water pressure contours after an elapsed time of 15 hours. The four sets of results are for different ways of inputting the water storage modulus. Convergence was achieved, and reasonable results obtained for all four procedures for inputting the unsaturated soil property functions. Different permeability functions were also used but only the case for the Fredlund and Xing (1993) function is shown. Similar results are shown in Fig. 10 for an elapsed time of 255 hours.

7. DISCUSSION OF THE RESULTS

Some effort is required in setting up the general partial differential equation solvers for the saturated/unsaturated seepage analysis. Once this is done one time, it is possible to solve a wide variety of seepage problems.

The general-purpose, partial differential equation solvers are certainly versatile for handling a wide range of possibilities for inputting unsaturated soil

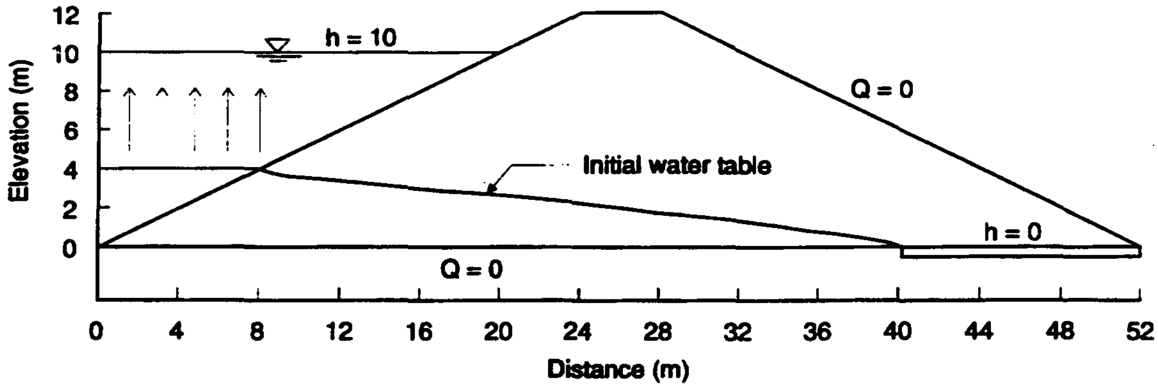

Figure 6. Cross-section of the dam, along with boundary conditions, used to illustrate the use of general partial differential equation solvers and unsaturated soil property functions

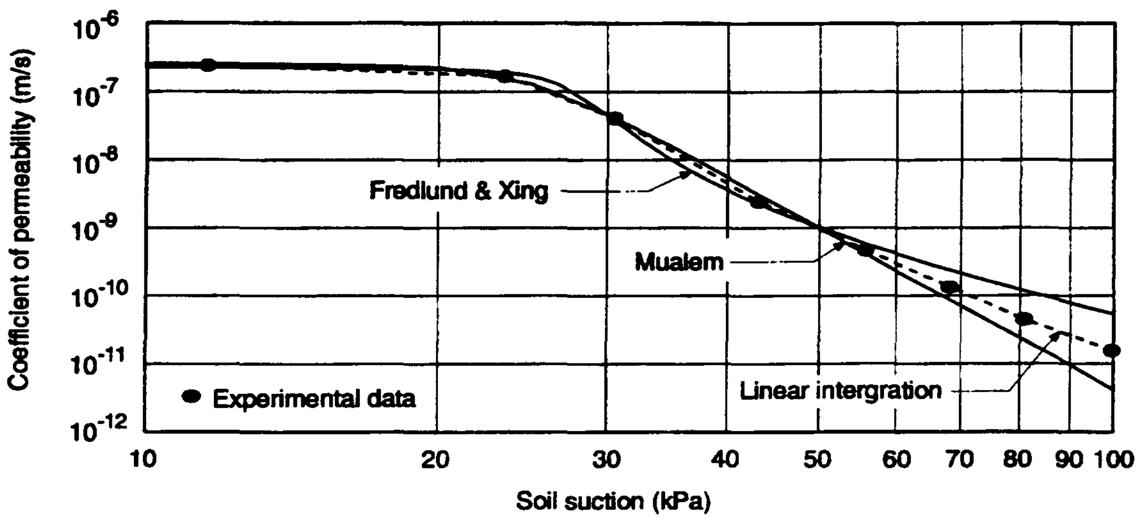

Figure 7. Three permeability functions selected to solve the example problem

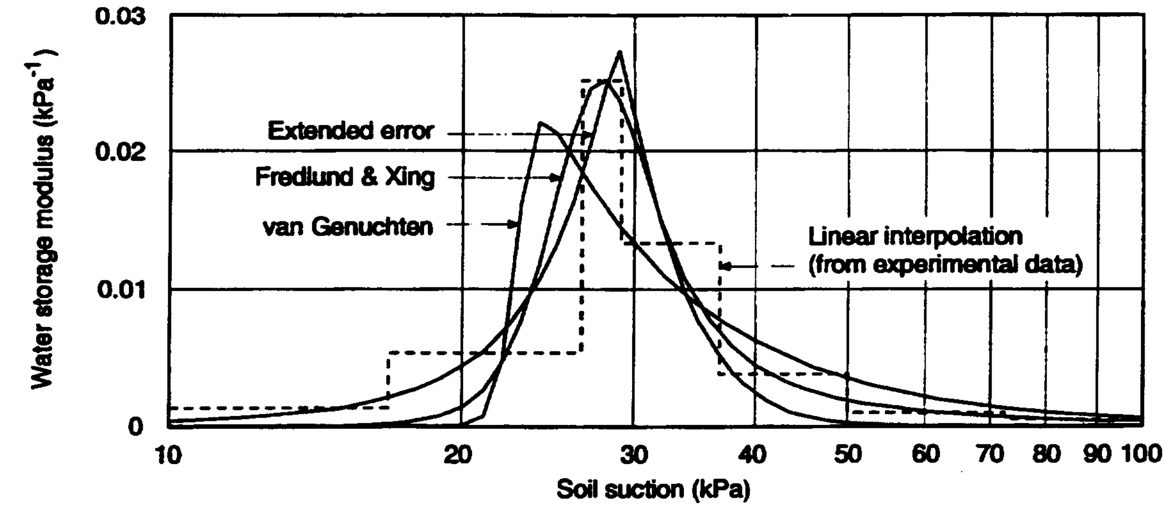

Figure 8. Three storage functions used to specify the relationship between m_2^w and soil suction

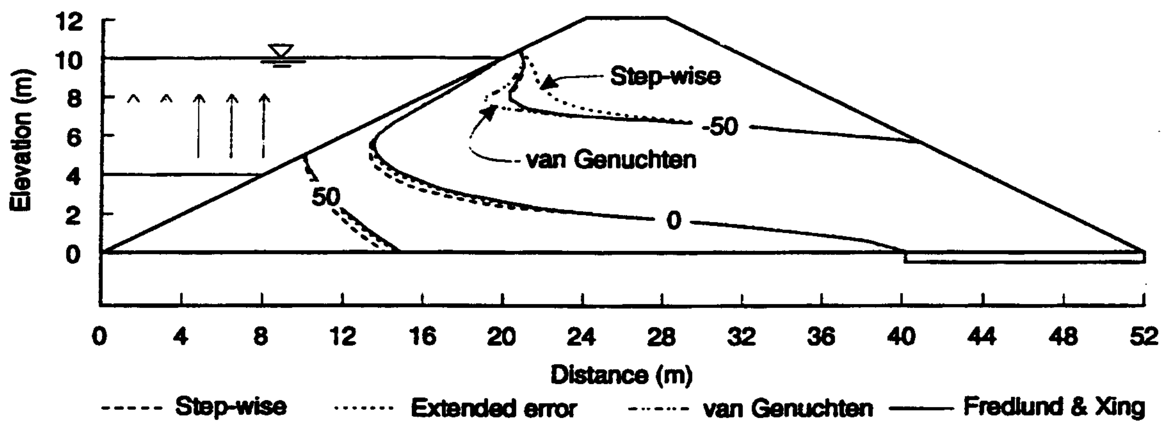

Figure 9. Constant pressure isopachs after 15 hours when using different procedures for inputting the water storage modulus function, m_2^w

property functions. A primary concern when solving saturated/unsaturated seepage problems, is whether or not the solution will converge. Experience to-date would indicate that there are benefits in having problems related to mathematical non-linearity handled by well-trained experts in this complex area.

It may once again be best for geotechnical engineers to focus on the assessment of the soil properties and the boundary conditions, while leaving complex mathematical solutions to mathematicians and software specialists.

8. SUMMARY AND CONCLUSIONS

Following is a summary of some of the conclusions that can be made regarding the use of a component systems approach (i.e., with partial differential equation solvers) for solving saturated/unsaturated seepage problems.

1. General partial differential equation solvers such as PDEase, show great potential for solving saturated/unsaturated seepage problems.
2. General partial differential equation solvers provide a number of features important to solving unsaturated soils problems:
   i. Ensuring convergence when solving non-linear equations,
   ii. Allowing material properties to be input in a variety of forms,
   iii. Allowing material properties to be non-linear in character.
3. PDEase can readily be used to solve two-dimensional steady state and transient, saturated/unsaturated seepage problems. The per-

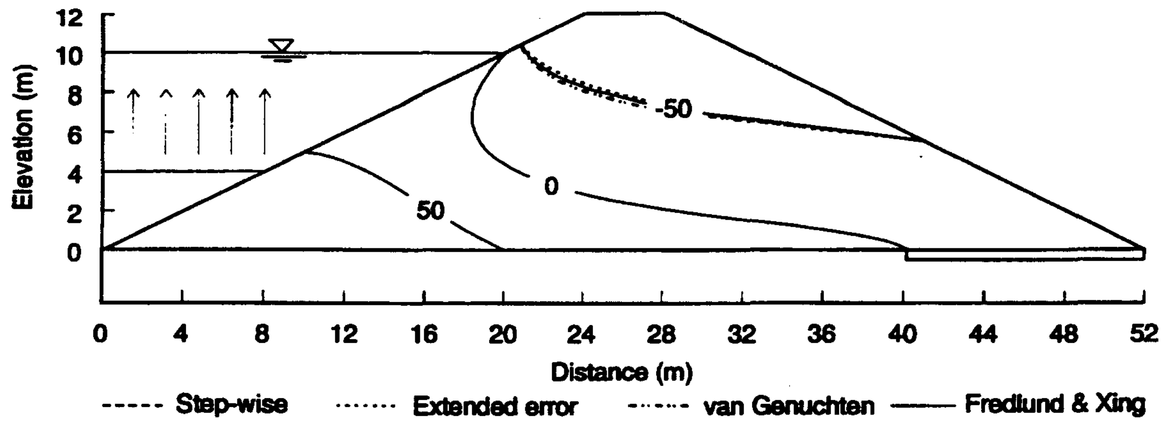

Figure 10. Constant pressure isopachs after 255 hours when using different procedures for inputting the water storage modulus function, m_2^w

formance of the PDEase software has been verified through comparative studies with Seep/W[5] as well as other published results.

4. It is possible to use a variety of formats for the input of unsaturated soil property functions into a general partial differential equation solver. The formats for data input can vary from being a series of data points to a closed-form mathematical equation.
5. The PDEase software has a number of features that are of particular value when solving the unsaturated seepage problems.
   i. Ease to control the accuracy of the solution because of the automated grid generation and automated error analysis,
   ii. Allows the user to specify various boundary conditions and material properties, either using mathematical expressions or numerical look-up tables.
   iii. Non-linear problems can be described and solved through internal hierarchy rules defined within the general partial differential equation solver. PDEase appears to have powerful convergence capabilities for non-linear analyses.
6. The partial differential equation solver, PDEase, has the potential to interface with many other software packages, such as MathCad, AutoCAD and other graphics packages, as well as databases, such as Access and Visual Basic.

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1 MathCad is a proprietary product of MathSoft Inc., Dept. 60M40, 101 Main Street, Cambridge, MA 02142, USA

2 PDEase is a proprietary product of Macsyma Inc., Arlington, MA 02174, USA.

3 FlexPDE is a proprietary product of PDE Solutions Inc., 2120 Spruce Way, Antioch, CA 94509, USA.

4 AutoCAD is a proprietary product of Autodesk Inc., 20400 Stevens Creek Boulevard, Cupertino, CA 95014-2217, USA.

5 Seep/W is a proprietary product of Geo-Slope International, #1830, 633 –6th Avenue S.W., Calgary, Alberta, T2P 2Y5, Canada.