Interslice force functions for computing active and passive earth force

Noshin Zakerzadeh, D.G. Fredlund, and D.E. Pufahl

Abstract: Recent methods to calculate the lateral earth force on a retaining wall have involved the method of slices and limit equilibrium concepts. An important issue in formulating the solution is the selection of an appropriate interslice force function (i.e., the ratio of the shear force to the normal force of vertical slices along the slip surface). This paper proposes interslice force functions that can be used to compute the active and passive earth forces in conjunction with the limit equilibrium method. An example problem involving a vertical wall with a horizontal backslope is analyzed using the general limit equilibrium (GLE) method and the proposed interslice force functions. The procedure recommended to compute the lateral earth force and the point of application of the force is outlined. The computed lateral earth forces, and the point of application, are compared with those from the classical solutions. For the active case, reasonable results were obtained when using an interslice force function that varies linearly from the starting point of the slip surface (at some distance from the wall) to the end point of the slip surface (adjacent to the wall). For the passive case, reasonable results were obtained when using an interslice force function that remains at zero from the starting point of the slip surface (at some distance from the wall) to the midpoint of the slip surface and then varies linearly from the midpoint of the slip surface to the end point of the slip surface (adjacent to the wall).

Key words: lateral earth force, limit equilibrium method, interslice force function, active earth force, passive earth force.

Résumé : Les méthodes récentes pour calculer la force des terres latérale sur un mur de soutènement ont fait appel à la méthode des tranches et aux concepts d'équilibre limite. Un point important dans la formulation de la solution est la sélection d'une fonction appropriée de forces inter-tranches (i.e., le rapport de la force de cisaillement sur la force normale aux tranches verticales le long de la surface de glissement). Cet article propose des fonctions de forces inter-tranches qui peuvent être utilisées pour calculer la force de poussée et de butée des terres avec la méthode d'équilibre limite. Comme exemple, un problème impliquant un mur vertical avec un talus au sommet horizontal est analysé au moyen de la méthode générale d'équilibre limite (GLE) et des fonctions de forces inter-tranches proposées. L'on décrit la procédure recommandée pour calculer la force latérale des terres et le point d'application de la force. Les forces latérales et le point d'application sont comparés avec les solutions classiques. Pour le cas de la poussée, des résultats raisonnables ont été obtenus en utilisant une fonction de forces inter-tranches qui varie linéairement à partir du début de la surface de glissement (à une certaine distance du mur), jusqu'au bout de la surface de glissement (adjacent au mur). Pour le cas de la butée, des résultats raisonnables ont été obtenus en utilisant une fonction de forces inter-tranches qui reste à zéro à partir du début de la surface de glissement (à une certaine distance du mur), jusqu'au point milieu de la surface de glissement, et qui par la suite varie linéairement de ce point milieu jusqu'au bout de la surface de glissement (adjacent au mur).

Mots clés : force latérale des terres, méthode d'équilibre limite, fonction de forces inter-tranches, force de poussée, force de butée.

[Traduit par la Rédaction]

Introduction

Various methods have been developed to calculate the lateral earth forces on retaining walls and the passive resistance of anchors. Some of the classical methods include those of Coulomb (1776), Rankine (1857), and Terzaghi (1941), to cite a few. More recent developments involve the use of methods of slices and the concept of limit equilibrium when calculating the lateral earth force.

The general limit equilibrium (GLE) method proposed for slope stability analysis (Fredlund et al. 1981) satisfies both moment and force equilibrium and can be applied to a variety of slip surfaces, different soil layers with independent parameters, and different conditions of loading. Pore-water pressures in the soil can also be accommodated. The GLE method can also be applied to either a circular or a noncircular (e.g., composite) slip surface. In this paper, the force equilibrium solution of GLE is used to compute the active and the passive earth forces. The moment equilibrium solution of GLE is used to compute the point of application of the active and passive earth forces. An important issue in

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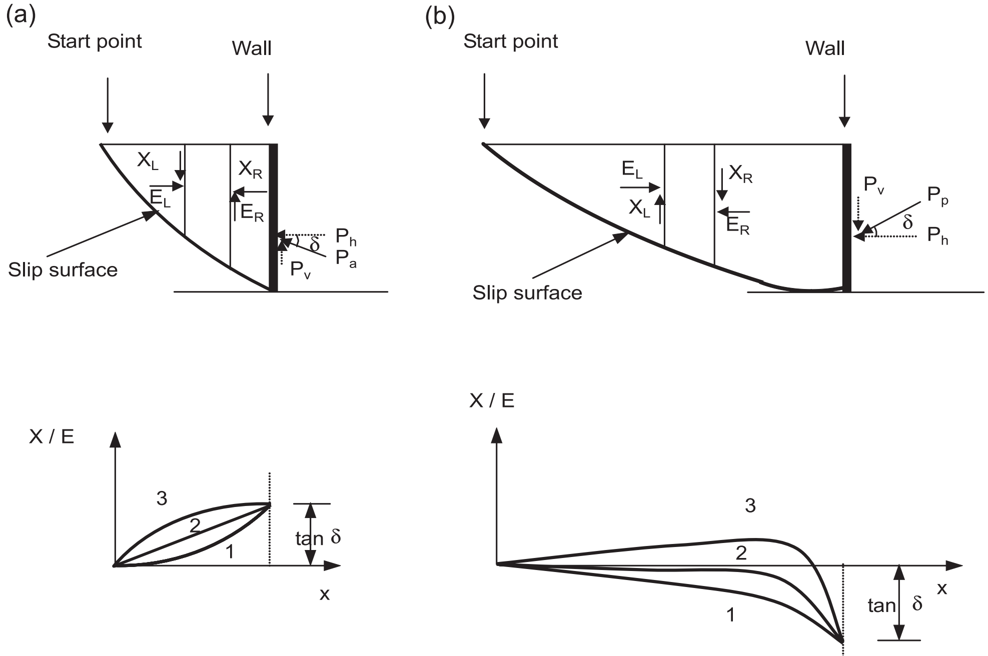
Fig. 1. Possible interslice force functions for a horizontal backslope: (*a*) active case, (*b*) passive case (from Rahardjo and Fredlund 1984).



formulating the solution involves the selection of an appropriate and reasonably accurate interslice force function that relates the normal and shear stresses between the slices of the sliding mass. The conventional λ value (defined as the coefficient representing the fraction of the function used to calculate lateral earth force) in a slope stability analysis can be set to a known value while the point of application of the earth force becomes an unknown. The net result is a deterministic solution satisfying force and moment equilibrium.

Morgenstern and Price (1965) described a variety of interslice force functions that could be used for slope stability analyses. Fan (1983) used an elasticity solution to compute an approximate interslice force function for the lateral earth force problems. Rahardjo and Fredlund (1984) used a variety of plausible interslice force functions for computing the active and passive lateral earth forces.

In this paper, appropriate interslice force functions, based on elasticity calculations, are proposed for solving lateral earth force problems. The proposed interslice earth force functions are based on the results of Fan (1983), Rahardjo and Fredlund (1984), and Morgenstern and Price (1965). An example problem, along with the proposed interslice force functions, is used to study the magnitude and point of application of active and passive earth forces. The procedure to compute the lateral earth force and the point of application is explained for the active and passive force cases, and the results are compared with those from classical solutions. Conclusions and a summary of findings related to the interslice force functions are presented. Mathematical forms are recommended for the general form of the interslice force functions for both the active and passive cases. The study has been limited to the case of simple, horizontal backslope and one soil type. The computational procedure is based on the assumption that the interslice force function need only be approximate to compute the active and passive earth forces. A second assumption is that an elasticity solution can provide a sufficiently accurate interslice force function.

Background on lateral earth force calculations

Different methods have been used to solve lateral earth force problems. The Rankine (1857) solution was presented for a smooth wall and a planar slip surface. The Coulomb (1776) method has been shown to be adequate for problems that involve a modest amount of wall friction in the active and passive cases. However, the Coulomb solution is not suitable when computing the passive resistance of a wall with large values of wall friction due to the assumption of a planar slip surface (Terzaghi and Peck 1967).

Janbu (1957) suggested the use of the generalized procedure of slices and a composite slip surface to solve lateral earth force problems. The active and passive earth pressures were calculated for a dry sand. Shields and Tolunay (1972) computed values for the passive earth pressure coefficient, K_p, based on the logarithmic spiral method proposed by Terzaghi and Peck (1967). Solutions were presented for a dry, cohesionless sand with a horizontal surface behind a vertical, rough wall.

Shields and Tolunay (1973) have calculated passive earth pressure coefficients, K_p, using a method of slices and a composite slip surface. The results from the method of slices produced passive earth pressure coefficients that were slightly lower than the results previously computed with the logarithmic spiral method (i.e., an average of 4.5% lower for values of wall friction angle δ ranging between 0 and 20°).

Fredlund et al. (1981) proposed the use of the GLE method of slices for slope stability problems. Rahardjo (1982) extended the GLE formulation to lateral earth pressure problems and illustrated the influence of the interslice

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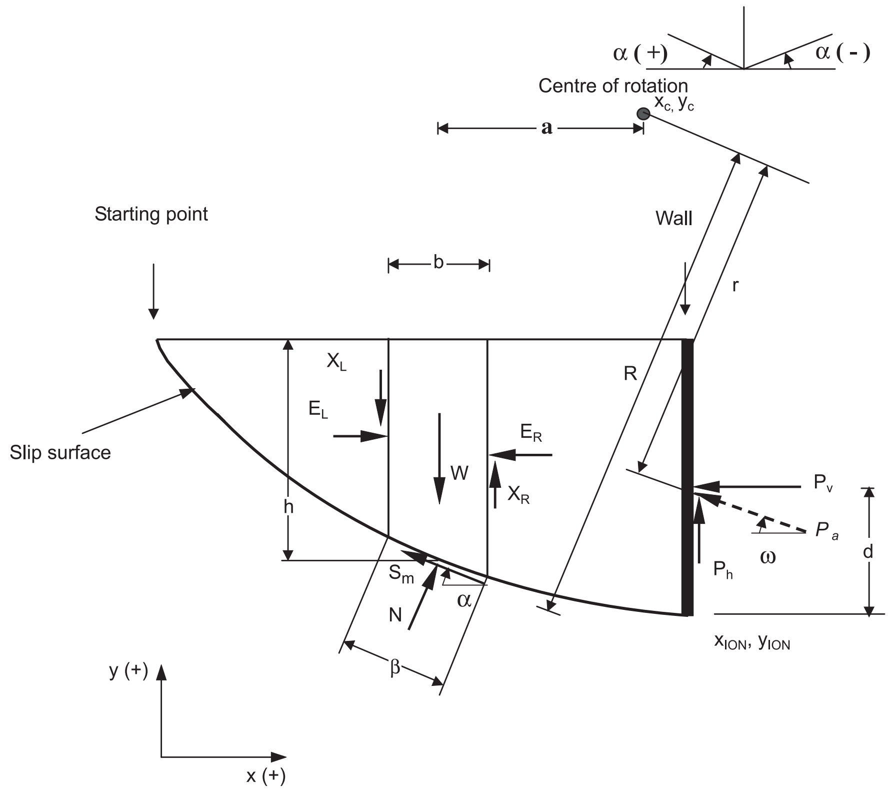
Fig. 2. Forces acting on a slice of a slip surface and a vertical wall in the lateral earth force problems (active case).



force function on the values of the lateral force and the point of application. Fan (1983) used the finite element stress analyses with a linear elastic soil model to compute an approximate function for the direction of the resultant interslice forces associated with the active and the passive cases.

Rahardjo and Fredlund (1984) illustrated the influence of a variety of possible interslice force functions on the magnitude of the lateral earth force. Figure 1 shows three interslice force functions that were used for the active case and three functions that were used for the passive case (Rahardjo and Fredlund 1984).

Chen and Li (1998) used a method of slices to calculate the magnitude of the active force. Different slip surfaces were assumed and the slip surface associated with the maximum active force was taken to be the critical slip surface. The point of application of the active force was assumed and the magnitude of the active earth force satisfying force and moment equilibrium was calculated. The magnitude of the active force was computed for various assumptions of point of application. The effect of wall friction angle was not considered.

In this paper interslice force functions are used along with the GLE method to calculate the lateral earth force and the point of application for the active and passive cases. The proposed interslice force functions are based on the study of Fan (1983) and suggestions of Rahardjo and Fredlund (1984). A vertical wall with a horizontal backfill composed of a granular material (without pore-water pressure) was used as an example problem. The lateral earth force satisfying force equilibrium was computed and moment equilibrium was used to compute the point of application. Various values of wall friction angle were used in the calculations. The calculated results (i.e., magnitude of the force and point of application) were compared with those obtained from classical solutions such as Rankine (1857), Coulomb (1776), and Shields and Tolunay (1972).

Formulation of the theory to compute the lateral earth force

An arbitrary slip surface is assumed and the soil mass above the slip surface is divided into slices. Force and moment equilibrium equations can be applied to the soil mass that is at the point of incipient failure (i.e., factor of safety is at 1.0). The forces acting on each slice are identified and the magnitude of the shear force mobilized on the base of each slice is computed in accordance with the Mohr-Coulomb failure criterion:

$$[1] \qquad \tau = c' + \sigma_n \tan \phi'$$

where τ is the shear strength at failure, c' is the effective cohesion at failure, σ_n is the normal stress at failure, and ϕ' is the effective angle of shearing resistance of the soil.

Figure 2 shows the forces acting on one slice of an assumed slip surface for the active case. For the passive case, the direction of some of the forces must be reversed, otherwise the general approach is the same as that for the active case.

The normal force, *N*, and the weight of each slice, *W*, are assumed to act through the centre of the base of each slice. The lateral earth force, P_a, acts on the last slice at an inclination angle, ω, to the horizontal. The normal force on the base of the slice is computed from force equilibrium in

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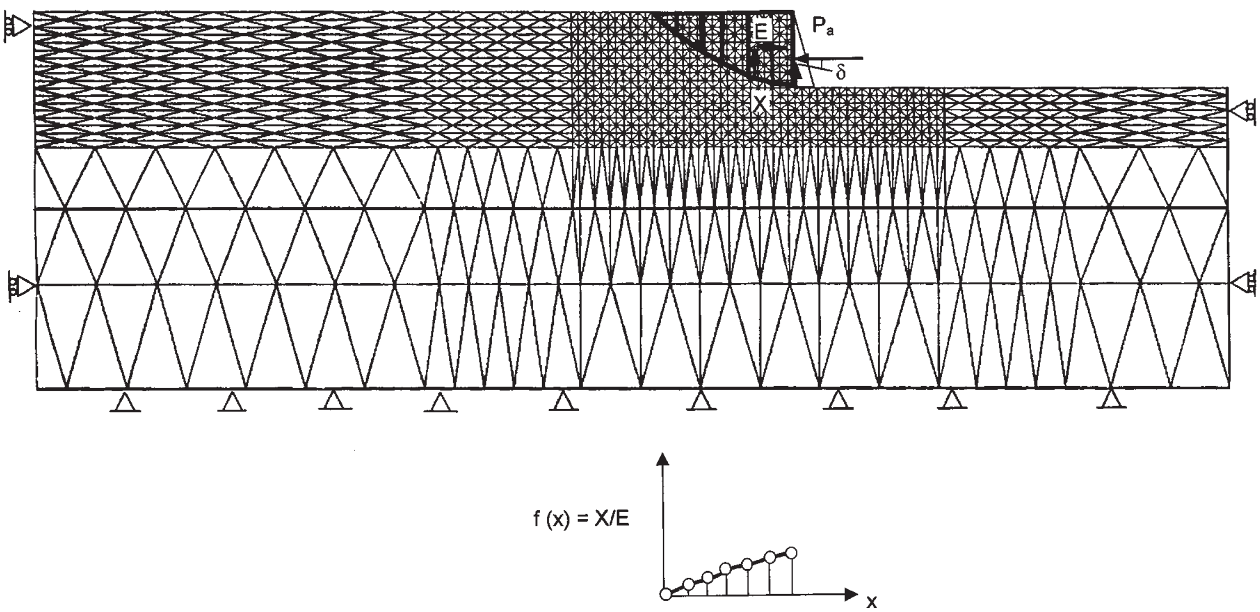
Fig. 3. Finite element mesh and the geometry of an example problem used by Fan (1983).



the vertical direction. The summation of forces in the vertical direction for each slice gives

$$[2] \qquad W + (X_L - X_R) - S_m \sin \alpha - N \cos \alpha - [P_a \sin \omega] = 0$$

where X_L and X_R are the vertical shear interslice forces on the left and right sides of the slice, respectively; S_m is the shear force mobilized on the base of the slice; and α is the angle between the tangent to the centre of the base of the slice and the horizontal.

The term $[P_a \sin \omega]$ is relevant only for the slice where P_a acts (i.e., the interface between the soil and the wall). The inclination angle ω depends on the wall friction angle δ and the inclination angle of the wall. When the wall is vertical, $\omega = \delta$.

The magnitude of the shear force mobilized on the base of each slice is computed using the Mohr-Coulomb failure criterion:

$$[3] \qquad S_m = \frac{\beta(c' + \sigma_n \tan \phi')}{F_s}$$

where β is the length of the slice at the base, and F_s is the factor of safety.

Substituting eq. [3] into eq. [2] and solving for N, the total normal force on the base of the slice, gives

$$[4] \qquad N = \frac{W + (X_L - X_R) - \dfrac{c'\beta \sin \alpha}{F_s} - [P_a \sin \omega]}{m_\alpha}$$

where

$$[5] \qquad m_\alpha = \cos \alpha + \frac{\sin \alpha \tan \phi'}{F_s}$$

The horizontal equilibrium of forces on each slice is used to compute the interslice normal forces. The summation of forces in the horizontal direction for each slice gives

$$[6] \qquad E_R = E_L + N \sin \alpha - S_m \cos \alpha - [P_a \cos \omega]$$

where E_R and E_L are the horizontal normal interslice forces on the right and left sides of the slice, respectively.

The interslice shear forces are obtained using an assumption regarding the direction of the interslice forces. The interslice force direction can be described as follows using an arbitrary interslice force function, $f(x)$, of the form used for slope stability problems (Morgenstern and Price 1965):

$$[7] \qquad \frac{X}{E} = \lambda f(x)$$

where $f(x)$ is a mathematical function that describes the relationship between the shear force X and normal force E on the slides of each vertical slice of the sliding mass; x is the length from the starting point of the slip surface to a desired point on the slip surface (Fig. 2); and λ is a coefficient representing the fraction of the function used in the calculation of the lateral earth force.

A λ variable is necessary to balance the number of known and unknown variables when solving slope stability problems. However, in the case of a lateral earth force problem, it is necessary that the interslice force function (i.e., X/E) be fixed to balance the number of known and unknown variables. In other words, λ is taken to be a fixed value. Later it is shown that the fixed value is equal to the coefficient of wall friction. This means that the elasticity-based interslice force function is fixed when computing the lateral earth force.

The magnitude of the lateral earth force, P_a, can be calculated from the overall equilibrium of forces in the horizontal direction:

$$[8] \qquad P_a = \frac{\sum N \sin \alpha - \sum S_m \cos \alpha}{\cos \omega}$$

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Fig. 4. Comparison between interslice force functions suggested by Zakerzadeh (1998) and those used by Fan (1983) for the active case when $\delta = 10°$.

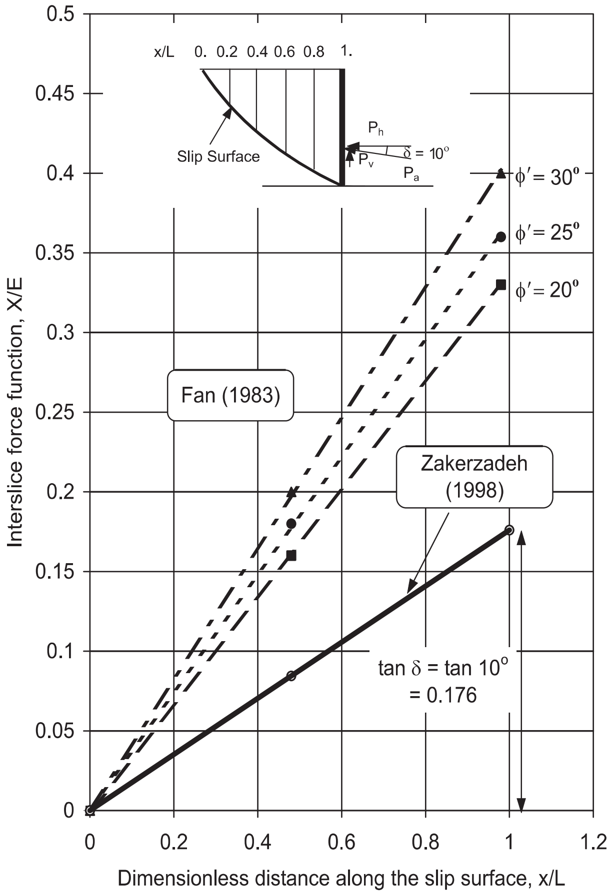
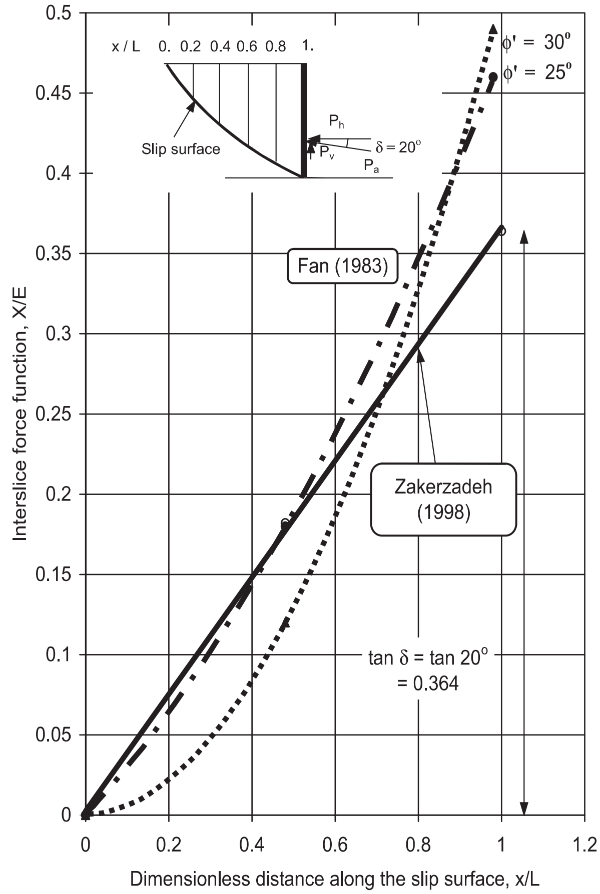


Fig. 5. Comparison between interslice force functions suggested by Zakerzadeh (1998) and those used by Fan (1983) for the active case when $\delta = 20°$.



Substituting the mobilized shear force equation (i.e., eq. [3]) into eq. [8] and solving for the force equilibrium factor of safety, F_{sf}, gives

$$[9] \qquad F_{sf} = \frac{\sum(\beta c' + N \tan\phi')\cos\alpha}{\sum N \sin\alpha - [P_a \cos\omega]}$$

Assuming a factor of safety, F_{sf}, equal to 1.0, the magnitude of the lateral earth force, P_a, can be calculated as follows:

$$[10] \qquad P_a = \frac{\sum N \sin\alpha - \sum(\beta c' + N \tan\phi')\cos\alpha}{\cos\omega}$$

It is desirable to directly solve for the lateral force, P_a; however, the Slope/W computer software (Geo-Slope International Ltd. 1996) used in this study only solves for the factor of safety, F_{sf} (i.e., eq. [9]). Therefore, it is necessary to use a trial and error procedure to compute the lateral earth force corresponding to a designated factor of safety (e.g., F_{sf} = 1.0).

The summation of moments for all slices about the centre of rotation (x_c, y_c in Fig. 2) can be used to compute the point of application for the lateral earth force:

$$[11] \qquad \sum Wa - \sum S_m R - [P_a r] = 0$$

where a is the horizontal distance from the centreline of each slice to the centre of rotation, R is the moment arm associated with the mobilized shear force S_m for any shape of slip surface, and r is the moment arm associated with the lateral earth force. The critical lateral active earth force is designated using the subscript a (i.e., P_a). The subscript p can be used to designate the lateral passive earth force (i.e., P_p). Substituting the mobilized shear force equation (i.e., eq. [3]) into eq. [11] and solving for the factor of safety with respect to moment equilibrium, F_{sm}, gives

$$[12] \qquad F_{sm} = \frac{\sum c'\beta R + \sum NR \tan\phi'}{\sum Wa - [P_a r]}$$

The moment equilibrium factor of safety, F_{sm}, must be equal to 1.0 and eq. [12] can be solved for the distance r:

$$[13] \qquad r = \frac{\sum Wa - \sum c'\beta R - \sum NR \tan\phi'}{P_a}$$

The point of application from the base of the wall, d, can be computed using the following equation:

$$[14] \qquad d = (y_c - y_{ion}) - \frac{r}{\cos\omega} - (x_{ion} - x_c)\tan\omega$$

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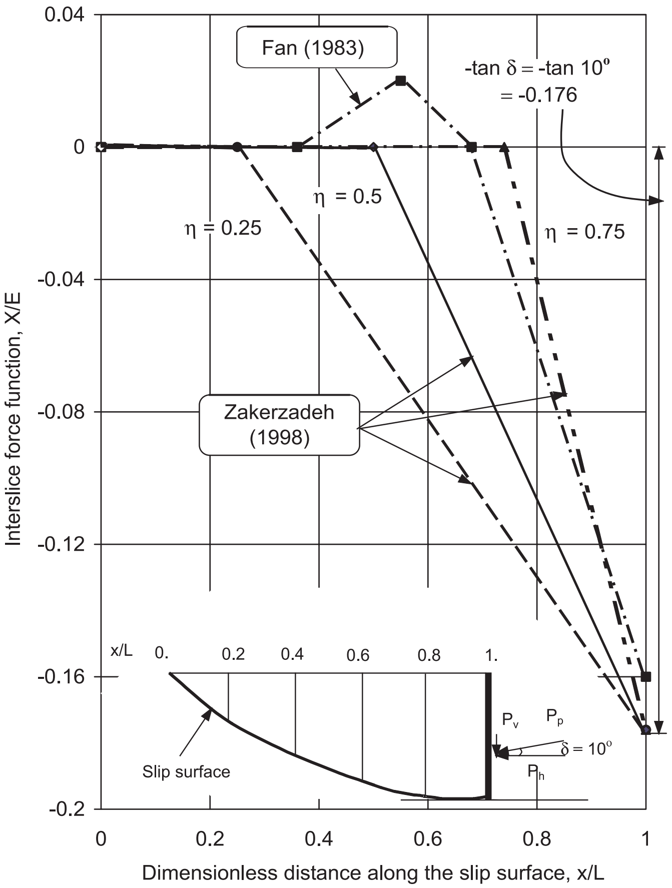

Fig. 6. Comparison between interslice force functions suggested by Zakerzadeh (1998) and those used by Fan (1983) for the passive case when δ = 10°.

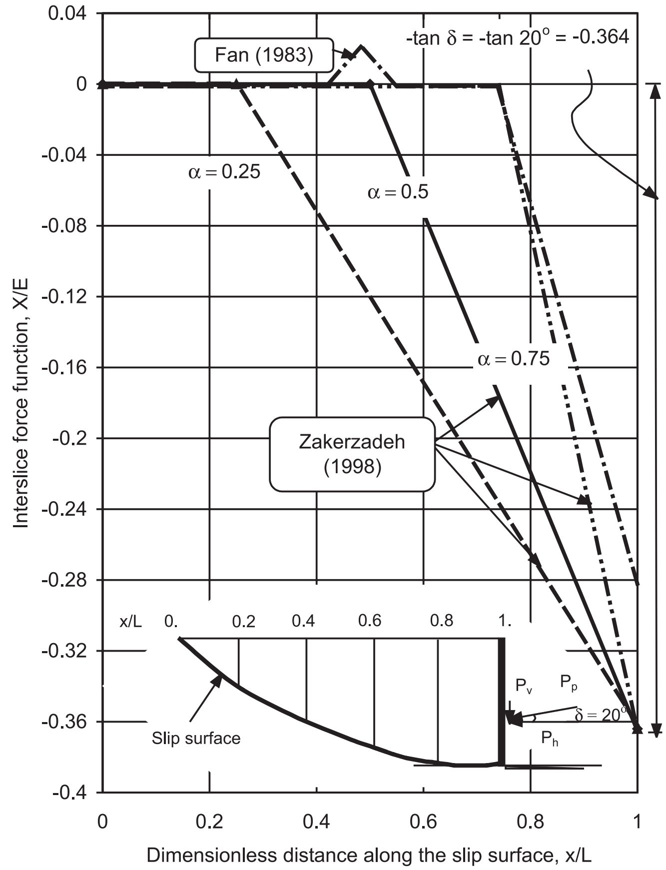

Fig. 7. Comparison between interslice force functions suggested by Zakerzadeh (1998) and those used by Fan (1983) for the passive case when δ = 20°.

where x_{ion} and y_{ion} are the x and y coordinates respectively, of the heel of the retaining wall and x_c and y_c are the coordinates of the centre of rotation.

Once again, it would be desirable to directly solve for the distance, *d*; however, the Slope/W computer software solves for the factor of safety, F_{sm} (i.e., eq. [12]), and it is necessary to use a trial and error procedure to solve for *d*.

The point of application of the lateral earth force, *d*, is dependent upon the interslice shear force, *X* (i.e., the point of application, *d*, is the function of normal force, *N*, which is a function of the interslice shear force, *X* (eq. [4])). Therefore, the point of application of the lateral earth force is dependent on the interslice force function used in the analysis.

Appropriate interslice force functions

As previously mentioned, the interslice force function must be fixed to balance the number of known and unknown variables in the lateral earth force problem. Interslice force functions are used that are consistent with the results of stress analyses simulating active and passive earth pressure conditions.

Fan (1983) computed the interslice force function, *X*/*E*, for the active and passive cases based on a linear elastic, stress–strain analysis. The geometry of the problem studied consisted of a vertical wall with a horizontal backslope. Figure 3 shows the finite element mesh and geometry of the problem used by Fan. Horizontal forces were applied at the nodes along the vertical wall, based on the Rankine (1857) theory. Loads associated with the unit weight of the soil were simulated by "switching on" gravity. The normal and shear stresses in elements of the soil mass were calculated. The slip surface was divided into vertical sections and the interslice side force ratio for each section (i.e., a vertical slice above a slip surface) was obtained by taking the ratio of the interslice shear and normal forces (*X*/*E*). The interslice force function was determined by plotting the interslice force ratios for all the vertical sections across the slip surface. The angle of shearing resistance of the soil, ϕ′, was varied to examine the effects of soil strength on the distribution of the interslice force function. The resulting interslice force functions computed by Fan are shown in Figs. 4–7. The interslice force functions used in this study are also shown in Figs. 4–7.

Interslice force function for the active earth force case

Figure 4 shows the interslice force functions computed by Fan (1983) for the active case when the wall friction angle is 10°. The interslice force functions for different angles of shearing resistance of the soil, ϕ′, vary linearly from zero at the starting point of the slip surface (i.e., $x/L = 0$, where L is the horizontal length along the slip surface) to a value about two times that of tan δ (i.e., 0.176) close to the wall.

In the present study, the interslice force function is set to zero at the starting point of the slip surface (i.e., $x/L = 0$).

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Fig. 8. Typical interslice force function, *X*/*E*, for (*a*) the active case, and (*b*) the passive case.

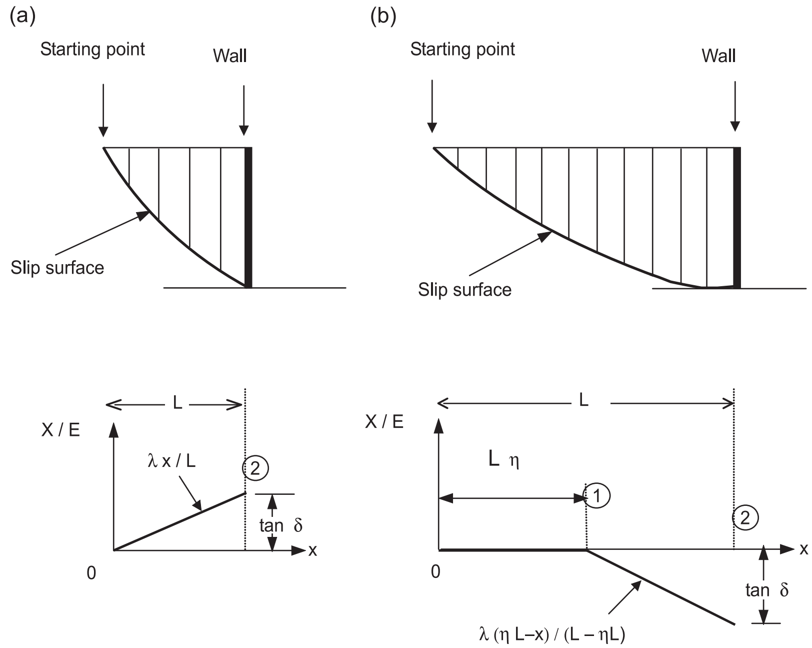
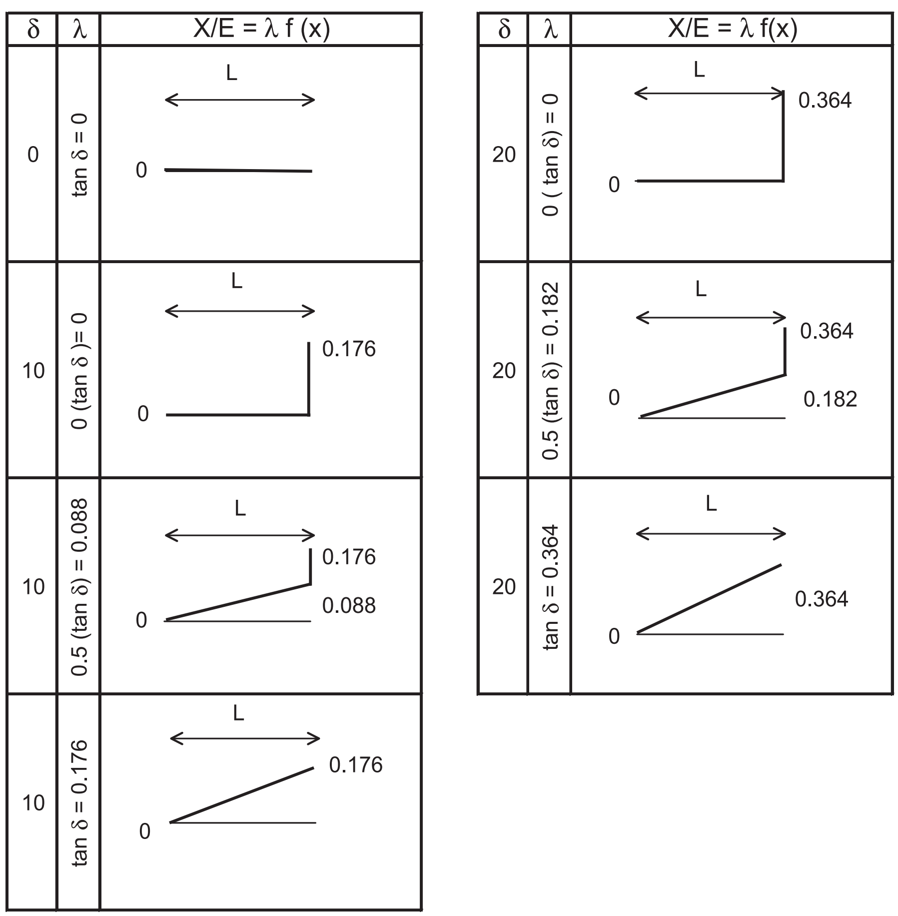


Fig. 9. Selected interslice force functions by Zakerzadeh (1998) for the active case.



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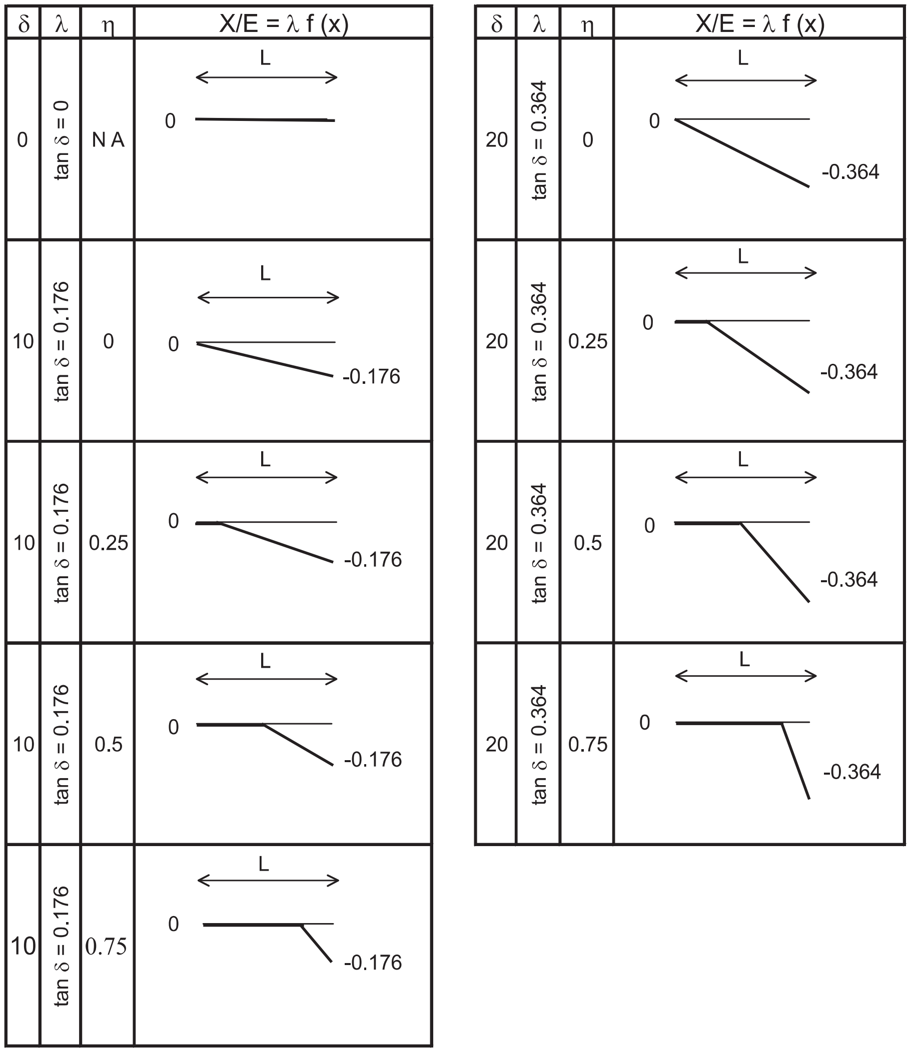

Fig. 10. Selected interslice force functions by Zakerzadeh (1998) for the passive case. NA, not applicable.

Close to the wall (i.e., x/L = 1.0) the interslice force function is set to tan δ (i.e., 0.176 for δ = 10°). Between those two points, the interslice force function is assumed to vary linearly.

Figure 5 compares the interslice force functions computed by Fan (1983) for the active case when the wall friction angle is 20° with those functions used in this study. The interslice force functions computed by Fan vary nonlinearly from zero at the starting point of slip surface (i.e., x/L = 0) to a value higher than tan δ (i.e., 0.364) close to the wall (i.e., x/L = 1.0); however, the interslice force function is approximated with a linear function in the present study (Zakerzadeh 1998).

Typical interslice force functions for the active case are shown in Fig. 8*a*. The magnitude of the force function is set to zero at the starting point of the slip surface (i.e., the extreme left point in Fig. 2). The magnitude of the force function at the wall is set equal to the tangent of the wall friction angle (tan δ). Between these two boundaries, the interslice force function varies linearly.

The shear force mobilized on the base of the slice, S_m, acts upward along the assumed slip surface for the active case, since the soil mass slides downward under the influence of gravity. The displacement of the soil mass mobilizes positive shear stresses for all elements within the sliding mass. This behavior accounts for the positive interslice force ratios across the entire sliding mass for the active case. Using eq. [7] to define the interslice force ratio, λ can be set equal to tan δ, and the interslice force function, $f(x)$, can be set to reach a maximum value of 1.0 at the wall. To evaluate the effect of the interslice force function, λ was assumed equal to 0, 0.5 tan δ, and tan δ. However, it should be emphasized that tan δ is the most meaningful value for λ. The interslice force functions used in the analyses for different values of λ for the active case are shown in Fig. 9. When using λ equal to tan δ, the relationship for the interslice force function can be expressed as follows:

$$[15] \qquad \frac{X}{E} = (\tan\delta)\frac{x}{L} \qquad \text{for} \qquad 0 \le x \le L$$

where L is the horizontal length along the slip surface.

Interslice force function for the passive earth force case

Figure 6 shows the interslice force function computed by Fan (1983) for the passive case when the wall friction angle,

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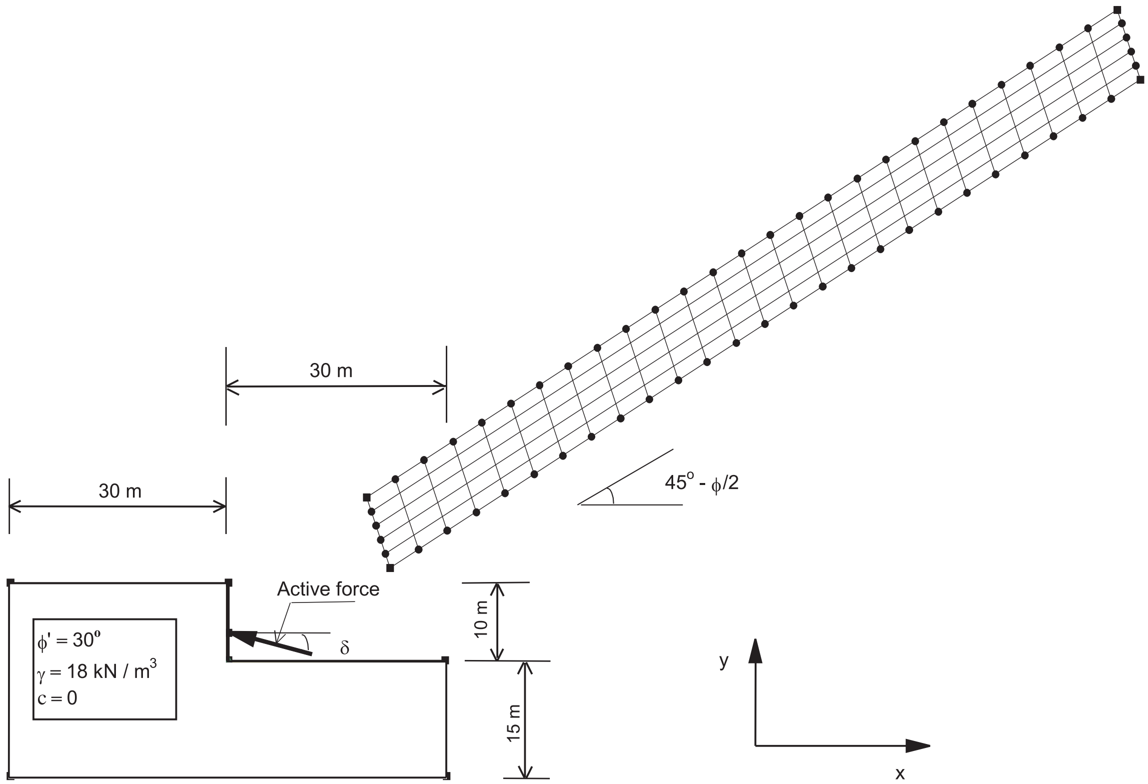
Fig. 11. Geometry of the example problem for the active case.



δ, is equal to 10°. The interslice force function is zero at the starting point of the slip surface (i.e., $x/L = 0$). The interslice force function is slightly higher than –tan δ (i.e., by approximately 9%) at the end point of the slip surface (i.e., $x/L = 1.0$). In the region between those two points, the interslice force function can be slightly positive or remain at zero and then vary linearly to reach the value of the interslice force function computed at the wall (i.e., –0.16). The positive value for the interslice force function is very low, with a maximum of about 0.02.

In the present study, the interslice force function is set to zero at the starting point of the slip surface (i.e., $x/L = 0$). Close to the wall (i.e., $x/L = 1.0$) the interslice force function is set to –tan δ (i.e., –0.176 for δ = 10°). The interslice force function between those two points is set to zero and then assumed to vary linearly. The linear part of the function starts at a value specified by the η parameter. Interslice force functions using different η values (e.g., 0.25, 0.5, and 0.75) are shown in Fig. 6. A value of η equal to zero is also assumed in the analyses for completeness.

Figure 7 shows the interslice force function computed by Fan (1983) for the passive case when wall friction angle, δ, is 20°. The interslice force function is zero at the starting point of the slip surface (i.e., $x/L = 0$). The interslice force function is higher than the –tan δ value by about 23% at the end point of the slip surface (i.e., $x/L = 1.0$). In the region between those two points, the interslice force function can be positive or remain at zero. The function is then assumed to vary linearly to reach the value of the interslice force function computed at the wall (i.e., –0.28). The positive value of the interslice force function is again very low, with a maximum of about 0.02.

In the present study, the procedure for the case of δ = 20° is the same as that used for a wall friction angle of 10°, except at the wall where the interslice force function is set at –tan 20° (i.e., –0.364).

A typical interslice force function used in this paper is shown in Fig. 8*b*. The X/E ratio at the starting point of the slip surface is set equal to zero. At the end point of the slip surface (i.e., close to the wall), the X/E ratio is set equal to –tan δ. For the passive case, the shear force mobilized on the base of the slice, S_m, acts downward, since the soil mass is being moved upward. The direction of the wall friction is negative and as a result the interslice force function is negative. The magnitude of the side force ratio reduces from a maximum value at the wall to zero at some distance from the edge of the wall (i.e., point 1 in Fig. 8*b*). To the left of point 1, where the shear force vanishes, the interslice side force ratio remains essentially at zero. The interslice force function can be assumed to vary linearly between point 1 and point 2. Using eq. [7] to define the interslice force ratio, λ can be set equal to tan δ, and the interslice force function, $f(x)$, will reach a minimum of –1.0 at the wall. The relationship for the interslice force function can be expressed as

$$[16] \quad \frac{X}{E} = 0 \qquad \text{for} \qquad 0 \le x \le \eta$$

$$[17] \quad \frac{X}{E} = \tan\delta \frac{(\eta L - x)}{L - \eta L} \qquad \text{for} \qquad \eta \le x \le L$$

where η is the length from the starting point of the slip surface to any point on the slip surface (i.e., point 1) as a ratio of the total distance across the slip surface, L.

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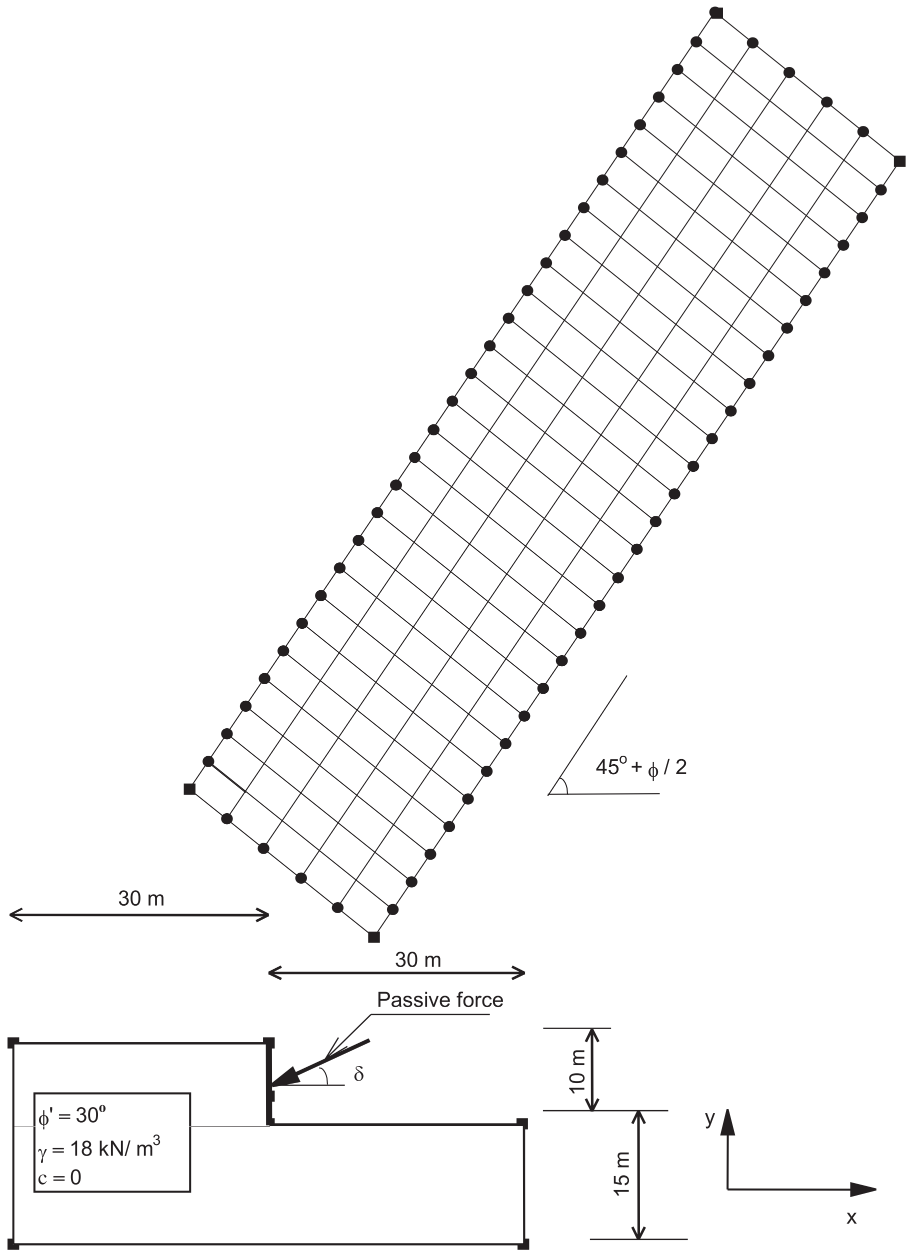
Fig. 12. Geometry of the example problem for the passive case.



The location of point 1 is first considered to be a variable, and therefore values of η were varied (i.e., 0, 0.25, 0.5, and 0.75) for the analyses. The interslice force functions used for the passive earth force case with different values of η are indicated in Fig. 10. The computed values for the lateral earth force and the point of application were compared with those from the classical solutions to evaluate the effect of η.

An example problem to illustrate lateral earth force calculations

Figures 11 and 12 show the geometry of the problem used in this study for the active and passive cases. A rigid wall retaining a granular backfill was selected for the study. The angle of shearing resistance (ϕ') of the soil is assumed to be 30° and the unit weight is 18 kN/m^3. The slip surfaces are assumed to be circular and to go through the base of the wall. The grid of centres of rotation is shown in Figs. 11 and 12, respectively. Each point on the grid is related to one slip surface through the soil. The inclination angle of the grid of centres of rotation is 45 – $\phi/2$ degrees to the horizontal for the active case (Fig. 11) and 45 + $\phi/2$ degrees for the passive case (Fig. 12). The angle of wall friction ranges from 0° to a maximum of 20° (i.e., two-thirds the angle of shearing resistance of the soil). The Slope/W computer program was used, along with appropriate interslice force functions, to compute the lateral earth forces.

Procedure to compute the magnitude of the lateral force

Figure 13 shows the flow chart outlining the procedure used to solve for the lateral earth force using the GLE method. The procedure outlined is somewhat involved

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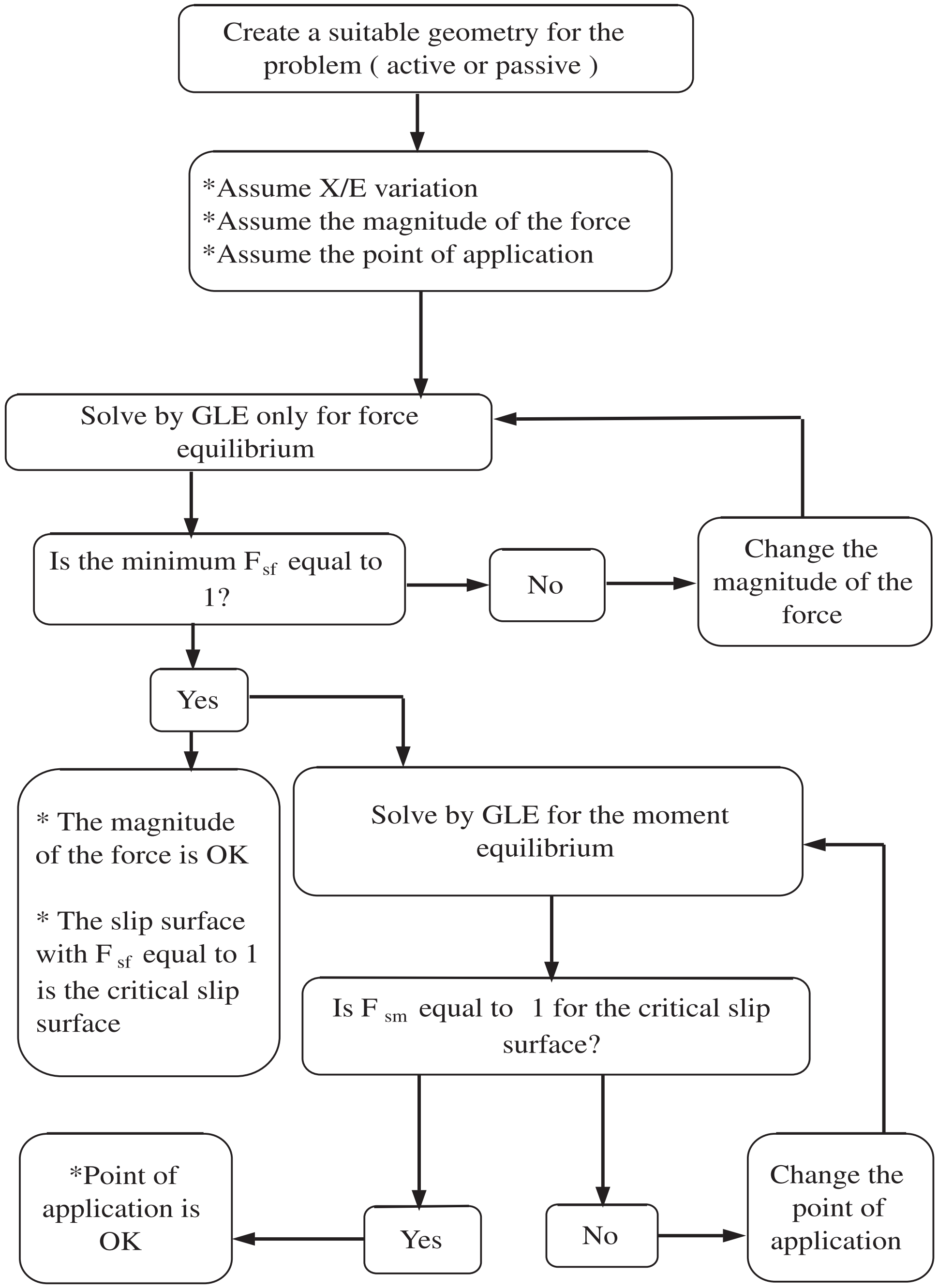
Fig. 13. Flow chart used to solve the lateral earth pressure problem by limit equilibrium analysis (Zakerzadeh 1998).



because of the inflexibility associated with using Slope/W for solving lateral earth force problems.

The magnitudes of the active and passive earth forces are computed using the force equilibrium solution. The external lateral earth force, P_a or P_p, that brings the soil mass into the state of limiting equilibrium is first computed. In other words, for trial values of lateral earth force, the force factors of safety, F_{sf}, are computed. If the minimum factor of safety is not equal to 1.0, another lateral earth force is applied and the computations are repeated. Once force equilibrium has been established, the magnitude of the lateral earth force and the location of the critical slip surface have been identified. The shear strength is assumed to be fully mobilized for both the active and passive cases when computing the factor of safety.

When computing the force equilibrium factor of safety, F_{sf}, for one assumed force and one assumed slip surface, the calculations begin at the extreme left-end slice where the slip surface intersects the ground surface. For the first iteration, the initial value of the force, P_a or P_p, is set to an assumed value (e.g., Coulomb's value). The interslice shear force, X_L, for the first slice and the values for the change in shear, $X_L - X_R$, for the first iteration are set to zero. The normal force, N, is computed using eq. [4], and the shear force mobilized on the base of the slice, S_m, is computed using eq. [3], assuming an initial factor of safety, F_{sf}, of 1.0.

Using eq. [6] and assuming a normal interslice force, E_L, equal to zero for the first slice, the value of the right-side normal interslice force, E_R, is computed. Using the ratio of

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Fig. 14. Active force versus wall friction angle for different interslice force functions specified by λ.

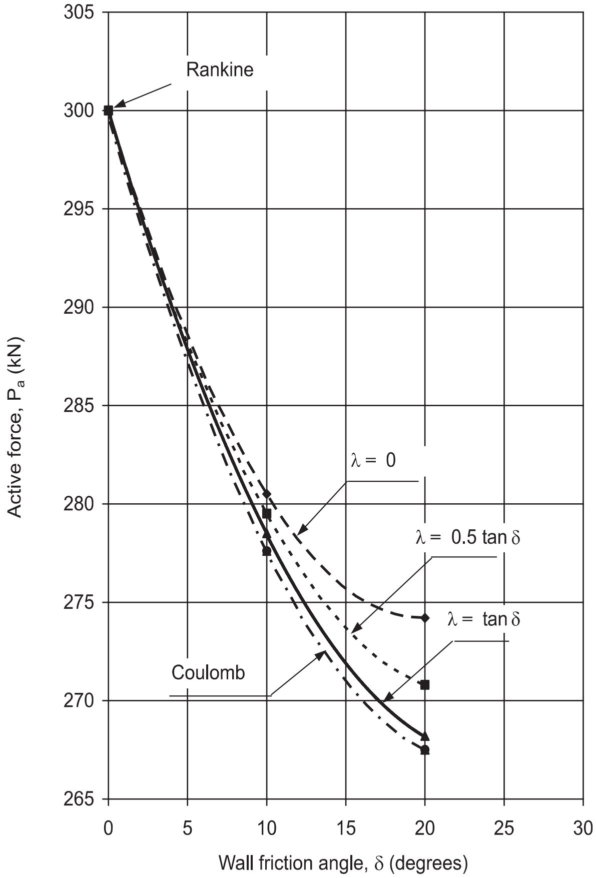
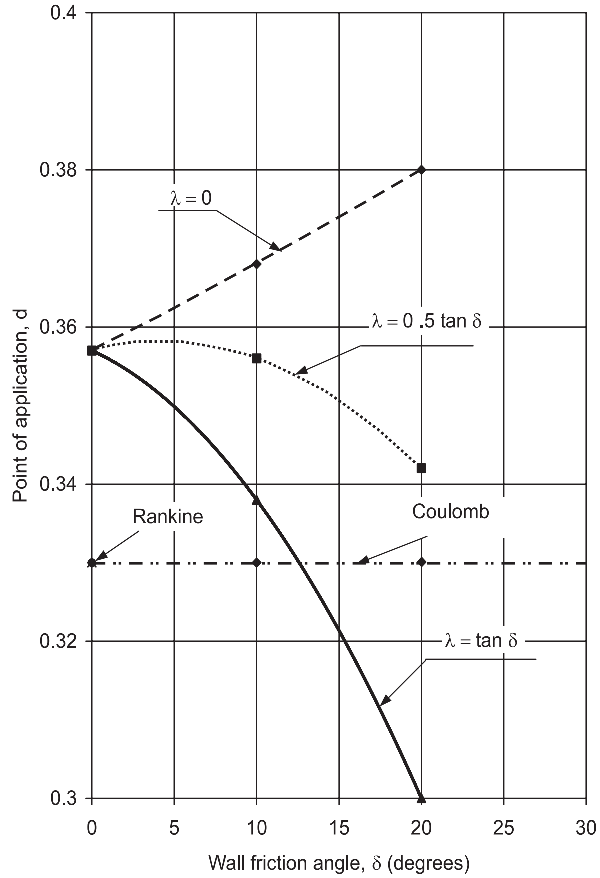


Fig. 15. Point of application of the force versus wall friction angle for different interslice force function specified by λ for the active case.



shear force to normal force (i.e., eq. [7]), the shear force at the right side of the slice, X_R, is calculated. Computations for other slices are repeated in a similar manner. At the end of this procedure, the new value for F_{sf} is computed using eq. [9]. The computed value for F_{sf} is used to repeat the procedure and to compute another value of F_{sf}. Computations are repeated until the difference between two successive values for F_{sf} is less than 0.001.

Procedure to compute the location of the lateral earth force

The moment equilibrium solution is used to compute the point of application, *d*, for the active or passive earth force cases. The point of application of the lateral earth force is determined by varying its location until the moment factor of safety for the critical slip surface is equal to 1.0. Equation [12] is used to calculate the value of the moment equilibrium factor of safety, F_{sm}.

Several trials are necessary to compute the lateral earth force and the point of application as described above. Specially designed software could greatly reduce the time needed for each trial.

Results for the active force case

The calculated forces associated with the active case are compared with results of the Coulomb and Rankine methods. Figure 14 shows that for a wall without friction, the force computed using the Coulomb and Rankine methods compares well with those obtained using the limit equilibrium method and selected interslice force functions.

The values for the active force are essentially the same, and therefore it can be concluded that for a smooth wall there is no need to use an interslice force function. For any given value of wall friction, the active force decreases with an increasing value of λ. The effect is slightly more pronounced with increasing values of wall friction. All values of the active force computed using the GLE method are slightly higher than those computed using the Coulomb method. The difference appears to be related to the curvature in the assumed slip surface. The results agree most closely with Coulomb's analysis when using an interslice force function with λ = tan δ. The selected function when using λ = tan δ appears to be acceptable.

Figure 15 shows that the variation in the point of application for the active force with increasing angles of wall friction depends upon the values of λ. The point of application for the active force when using the GLE analysis for λ = 0 is higher than the third point because of the circular nature of the slip surface. The point of application increases with an increase in the wall friction angle when λ = 0. However, for λ = 0.5 tan δ or λ = tan δ, the point of application decreases with increasing wall friction angles, δ.

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Fig. 16. Passive force versus wall friction angle for different interslice force functions specified by η (λ = tan δ).

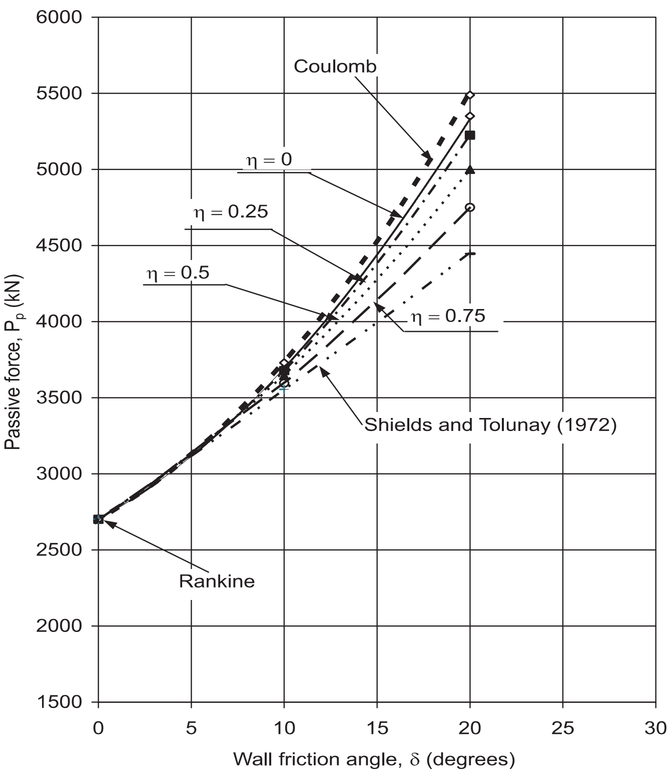
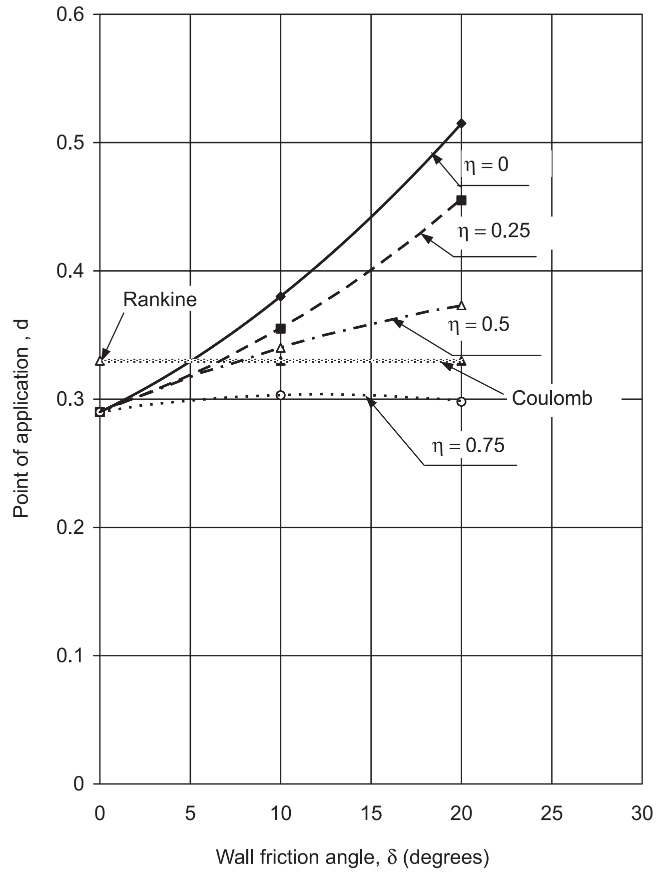


Fig. 17. Point of application of the force versus wall friction angle for different interslice force functions specified by η for the passive case (λ = tan δ).



Results for the passive earth force case

Figure 16 shows passive force versus wall friction angle for different interslice force functions. Various force functions are used for the passive case and comparisons are made with the methods of Rankine (1857), Coulomb (1776), and Shields and Tolunay (1972). The values from Shields and Tolunay for the passive force are based on the logarithmic spiral method.

The Coulomb (1776) analysis provides the highest values for the passive force, while the values from Shields and Tolunay (1972) fall on the lower side of the GLE analysis. Increasing the η value decreases the calculated passive force. The effect is more pronounced with increasing values of wall friction. The value of λ has been set equal to tan δ for all interslice force functions used in the analysis of the passive case. When δ = 0, the passive force computed by the GLE method is essentially the same as that for the Rankine, Coulomb, and Shields and Tolunay solutions. Therefore, as might be assumed, for a smooth wall there is no need to use an interslice force function. Values of η equal to 0.5 or 0.75 produce results that appear to be most acceptable.

Figure 17 shows the point of application of the passive force versus wall friction angle. For a wall friction angle of zero, different interslice force functions have no effect on the computed point of application. Again, for a smooth wall there is no need to use an interslice force function.

For a wall friction angle of zero, the point of application for the passive case appears to be slightly lower than the third point because of the circular nature of the slip surface. The point of application rises with an increase in the wall friction angle. The effect is more pronounced for lower values of η. James and Bransby (1970) show that the point of application of the passive force is generally higher than the third point. Therefore, the interslice force function defined by using η = 0.5 provides reasonable values for the point of application.

Conclusions and summary of findings on earth force calculations

The results indicate that the GLE method of slices, along with the proposed interslice force function, can provide a versatile and reliable means of calculating earth forces for both the active and passive cases. For each case, the interslice force function that should be used in the analysis is described by eq. [7] along with eqs. [18] and [19] for the active case and eqs. [20]–[23] for the passive case.

For the active case, the interslice force function can be written as follows:

[18] $f(x) = x/L$

[19] $\lambda = \tan \delta$

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For the passive case, the interslice force function can be written as follows:

[20] $f(x) = 0$

and

[21] $\lambda = \tan \delta \quad \text{for} \quad 0 \le x \le 0.5L$

and

[22] $f(x) = (L - 2x)/L$

and

[23] $\lambda = \tan \delta \quad \text{for} \quad 0.5L \le x \le L.$

The calculated results for the lateral earth force and the point of application show that selecting a value of λ equal to the wall friction value (i.e., tan δ) and a reasonable function $f(x)$ (eq. [18]) gives satisfactory results for the active case. Selecting λ equal to the wall friction value (i.e., tan δ), setting η equal to 0.5, and using a reasonable function $f(x)$ (eqs. [20] and [22]) gives satisfactory results for the passive case.

A procedure to calculate the lateral earth force and the point of application of the force has been described. In the general limit equilibrium (GLE) method, force equilibrium gives the magnitude of lateral earth force, P_a or P_p. Moment equilibrium gives the point of application of the lateral earth force, d.

It should be noted that this analysis has been done for a simple geometry and a homogenous soil. Application of the above proposed functions for other cases of more complex geometries and soil conditions should be studied further.

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List of symbols

a: horizontal distance from the centreline of each slice to the centre of rotation or to the centre of moments
b: width of the slice
c: cohesion of the soil
c': effective cohesion at failure
d: vertical distance from the heel of the retaining wall to the point of application of the lateral earth force
E: normal force on the slides of each vertical slice of the sliding mass
E_L: horizontal normal interslice force on the left side of the slice
E_R: horizontal normal interslice force on the right side of the slice
$f(x)$: interslice force function
F_s: factor of safety (i.e., the factor by which the shear strength of the soil must be reduced to bring the soil mass into a state of limiting equilibrium along the assumed slip surface)
F_{sf}: factor of safety associated with force equilibrium
F_{sm}: factor of safety associated with moment equilibrium
h: height of the slice at the centreline of that slice
N: total normal force on the base of the slice
P_a: lateral earth force (active case)
P_p: lateral earth force (passive case)
P_h: horizontal component of the lateral earth force
P_v: vertical component of the lateral earth force
r: moment arm associated with the lateral earth force
R: radius for a circular slip surface or the moment arm associated with the mobilized shear force, S_m, for any shape of slip surface
S_m: shear force mobilized on the base of the slice
W: weight of the slice
x: length from the starting point of the slip surface to a point on the slip surface
X: shear force on the slides of each vertical slice of the sliding mass
X_L: vertical shear interslice force on the left side of the slice

X_R: vertical shear interslice force on the right side of the slice
x_c, y_c: x and y coordinates of the centre of rotation
x_{ion}, y_{ion}: x and y coordinates of the heel of the retaining wall
α: angle between the tangent to the centre of the base of the slice and the horizontal
β: length of the slice at the base
δ: all friction angle
ϕ: angle of shearing resistance of the soil
ϕ': effective angle of shearing resistance of the soil
λ: coefficient representing the fraction of the function used to calculate lateral earth force
σ_n: normal stress at failure
τ: shear strength at failure
ω: inclination angle of the lateral earth force with the horizontal (wall friction angle

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