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Using limit equilibrium concepts in finite element slope stability analysis

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ABSTRACT: This paper reviews the development of finite element slope stability analyses and proposes that such a method can form a practical procedure for solving slope stability problems. Several slope stability methods have been proposed that make use of the finite element methods; these are summarized in this paper. The proposed finite element method is in a form that can be conveniently used in engineering practice. The procedure lends itself to present day numerical modelling techniques. The method has been updated to take advantage of recent advances in computer technology and algorithms.

The combination of a finite element stress analysis with a limit equilibrium analysis provides greater certainty and flexibility regarding the internal distribution of stresses within the soil mass. The normal force along any selected slip surface can be calculated from the stress distribution that has been calculated using a linear and non-linear stress analysis. The overall factor of safety for a slope, when the finite element method is used, can be defined as the available shear strength of the soil divided by the resisting shear strength. The overall factor of safety is a combination of the local factors of safety within the slope. The resulting overall factor of safety retains the basic assumptions inherent to the limit equilibrium definition of the factor of safety.

The local factors of safety are an expression of the stability of the soil mass at each point along the slip surface. The overall factor of safety computed using the finite element method shows good agreement with the factors of safety computed using any one of several limit equilibrium methods. The finite element method provides additional information regarding the potential performance of a slope; information not available when using traditional limit equilibrium methods. The results indicate that it is important to use the effective shear strength characterization of the soil when performing the slope stability analysis. The computed factor of safety obtained when using a total shear strength characterization of the soil, may not agree with the factor of safety computed when using the finite element stress analysis method.

Key words: slope stability analysis, finite element, enhanced method, direct method, strength method, stress level method, factor of safety, local factor of safety.

1 INTRODUCTION

Limit equilibrium methods of analysis have proven to be a widely used and successful method for the assessment of the stability of a slope. Limit equilibrium methods sum forces and moments related to an assumed slip surface passed through a soil mass (Fredlund and Krahn, 1975; Fredlund et al., 1981). However, these methods do not utilize the stress versus strain characteristics of the soils involved. It is well known, and intuitively understood that the stability of a slope should be influenced by the stress versus strain characteristics of a soil (Kondner 1963). A finite element analysis utilizes a stress versus strain model for the soils involved to calculate the stresses in the soil mass. These stresses can subsequently be used to compute a factor of safety (Fig. 1). The complete stress state from the finite element analysis can be "imported" into a limit equilibrium analysis where the normal stress and the shear stress are computed corresponding to any selected slip surface.

The objective of this paper is to demonstrate a procedure for combining a finite element stress analysis on a slope with the concepts of a limiting equilibrium method of analysis. The final method is called a "finite element method of slope stability analysis" and the results are compared to results obtained when using conventional limit equilibrium method of analysis.

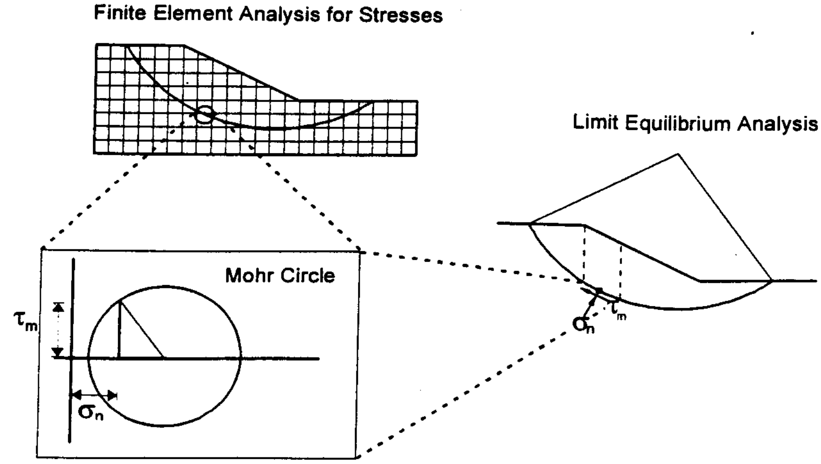


Figure 1. Illustration showing stresses that are "imported" from a finite element analysis into a limit equilibrium analysis.

2 BACKGROUND

Bishop (1952) noted that the stresses from a limit equilibrium method of analysis did not agree with the actual stresses within an earth structure. Other researchers have confirmed this observation both with experimental evidence and with numerical modelling. La Rochelle (1960) estimated the stress conditions in steep slopes using photoelastic tests on gelatine models. The results showed that stresses along a slip surface were over-stressed in the lower portion of the slip circle. Brown and King (1966) produced critical slip surfaces from a finite element stress analysis of slopes using a linear elastic soil model. The critical slip surfaces were produced by using the angle of obliquity, θ, along the slip surface (i.e., θ equal to (45° + ϕ'/α)). Each critical slip surface represented a close approximation to an essentially circular shaped slip surface.

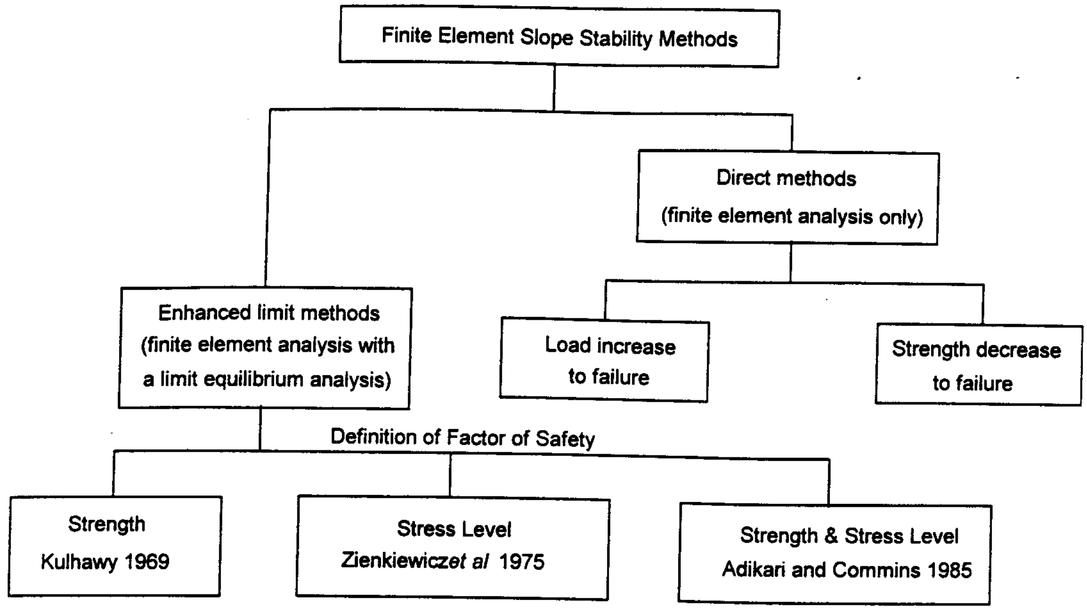
Clough and Woodward (1967) undertook a study to evaluate the effect of incremental loading with single step loading as it related to stresses and deformations. It was concluded that: 1) stresses and deformations in an embankment obtained from a direct application of the gravitational body forces on the complete structure were not completely accurate, and 2) changing Poisson's ratio interferes with the relationship between stresses and displacements, requiring a new analysis for each case. It was concluded that "*meaningful stability analysis can be made only if the stress distribution within the structure can be predicted reliably.*"

Kulhawy (1969) developed a computer program to obtain an independent assessment of the normal and shear stress distribution along an assumed slip surface. The normal and shear stresses from an elastic analysis were used to calculate an overall facto of safety. The formulation of Kulhawy (1969) wa classified as an "Enhanced Limit Strength Method".

A number of finite element slope stability methods have been proposed and the methods can b categorized as "enhanced limit methods" or "direc methods", as shown in Figure 2.

Wright (1969) compared the factors of safety calculated using the "enhanced limit strength" metho with factors of safety calculated using Bishop's Simplified method (1952). A slip surface was selecte for comparative purposes that had a factor of safet of 1.0 when using the Bishop's Simplified method. I was concluded that the factors of safety determine by the "enhanced limit strength" method (Kulhawy 1969) were approximately 3% higher than those determined applying Bishop's Simplified method Wright et al. (1973), using the "enhanced limi strength" method, showed that: 1) along one third o the slip surface, the local factors of safety are les than the overall factor of safety, 2) the factors o safety calculated by the finite element method using linear elastic material properties ranged from 0% t 4.5% higher than those calculated using Bishop's Simplified method, and 3) the factors of safety calculated by the finite element method using nonlinear elastic material properties increased wit Poisson's ratio and are 2% to 8% higher than thos calculated using the Bishop's Simplified method.

Reséndiz (1974) agreed with the concept of using the finite element method to calculate the stability o a slope; however, disagreed with points No. 2 and No. 3 of the results of Wright et al. (1973) because the factor of safety differences were too small. Reséndiz had developed a finite element method o



$$F_K = \frac{\Sigma\,(c' + \sigma' \tan\phi')\,\Delta L}{\Sigma\,\tau\,\Delta L}$$

$$F_Z = \frac{\Sigma\,[\Delta L]}{\Sigma\left\{\frac{(\sigma_1' - \sigma_3')}{(\sigma_1' - \sigma_3')_f}\,\Delta L\right\}}$$

$$F_A = \frac{\Sigma[(c' + \sigma' \tan\phi')\,\Delta L]}{\Sigma\left\{\frac{(\sigma_1' - \sigma_3')}{(\sigma_1' - \sigma_3')_f}\cdot(c' + \sigma' \tan\phi')\Delta L\right\}}$$

Figure 2. Finite element approaches proposed in computing the factor of safety in a slope stability analysis.

slope stability analysis defined as an "enhanced limit stress-level" method" in 1972 (Fig. 2). This method used the maximum principal stress difference of the soil at failure to define the factor of safety. Analyses made using non-linear stress versus strain relationships led to factors of safety which in all cases were higher (i.e., differences as large as 30%) than conventional factors of safety (e.g., Ordinary method or Bishop's Simplified method).

Zienkiewicz et al. (1975) also proposed a finite element method of analysis to compute the factor of safety by using the principal stress difference in the soil at failure to define the factor of safety. The method is an "enhanced limit stress - level method" (Fig. 2). Both the Reséndiz (1972) and Zienkiewicz et al. (1975) formulations are classified as "enhanced limit stress-level" methods.

Naylor (1982) established two types of finite element slope stability methods, a "direct" and an "enhanced limit" method of analysis. The direct method used a finite element nodal formulation to define the slip surface and the factor of safety directly from the analysis. The proposed "direct" slope stability method defined the factor of safety either as the increased load necessary to cause failure, or as the reciprocal of the reduction in the strength properties required in order to achieve failure. These methods have also been studied by Martins et al. (1981) and Tan and Donald (1985).

The "enhanced limit" slope stability methods are based on stresses calculated using a finite element analysis and combined with a limit equilibrium type of analysis along a prescribed slip surface, to define the factor of safety. The prescribed slip surface is the one defined by the lowest factor of safety and is found using a trial and error procedure. The stresses along the slip surface are computed using a finite element analysis and can either be used in a "strength" method or a "stress-level" method. Farias and Naylor (1996) stated that when using the "direct" finite element method it is, "*not easy to obtain a safety factor accurate to within the confidence limits achievable by limit equilibrium methods*". The authors noted that: 1) *a fine mesh is required*, 2) *a code capable of giving reliable results with the Mohr Coulomb elasto-plastic model for loading states close to failure is needed*, and 3) *it is usually necessary to carry out a set of analyses with c′ and tanφ′ progressively reduced by a factor which will become the safety factor when failure is eventually reached*. "Enhanced limit" methods require only one finite element analysis to calculate factors of safety for a slope with various combinations of c' and $\tan\phi'$.

Adikari and Cummins (1985) produced a finite element method that combine the "strength" and the "stress-level" methods as defined by Kulhawy (1969) and Zienkiewicz et al. (1975), respectively (Fig. 2). The Adikari and Cummins (1985) method produced factors of safety that were between the values obtained when applying the Kulhawy (1969) and the Zienkiewicz et al. (1975) methods. It was noted that for near-failure conditions (i.e., as defined by Bishop's Simplified method, 1955), the value of the factor of safety calculated by the Adikari and Cummins (1985) method approached 1.0, while the value of the factor of safety calculated by the Zienkiewicz et al. (1975) method remained high. The factor of safety by the Kulhawy (1969) method also approached unity with the factor of safety being dependent on the percentage of the strength mobilization in the component materials. The main difference in results appears related to using the stresses on the principal plane (Zienkiewicz et al. 1975) rather than on the plane. By definition, failure does not occur on the plane of principal stress and therefore, the Zienkiewicz et al. (1975) method (or any stress-level method) is computing a factor of safety that must be higher than the factors of safety produced by a "strength" method.

Duncan et al. (1996) provided a summary of the limit equilibrium and finite element methods that have been proposed for slope stability analyses.

3 SUGGESTED STUDY FOR COMPARISON BETWEEN THE FINITE ELEMENT AND THE LIMIT EQUILIBRIUM METHODS OF SLOPE STABILITY ANALYSIS

The finite element slope stability method proposed in this paper is of the "enhanced limit strength" type (Scoular, 1997). The finite element method uses the Kulhawy (1969) definition for the factor of safety combined with a finite element stress analysis of the slope. Stress analyses were done using Poisson's ratios equal to 0.33 and 0.48. For each stress analysis, the cohesion and the angle of internal friction of the soil were altered as the stability of the slope was computed. The selected values for cohesion, c', were 10, 20 and 40 kPa, and for the angle of internal friction, ϕ', were 10, 20 and 30 degrees.

The finite element slope stability method produces an overall factor of safety that is an expression of the stability of the slope based on the calculated stresses within the slope. Slope stability problems solved using the finite element method have two important distinctions from limit equilibrium methods. First, the finite element slope stability equation is determinate; therefore, no further assumptions are required to complete the calculations. Second, the factor of safety equation is linear, because the normal stress at the base of a slice is known. On the other hand, limit equilibrium methods, starting with Bishop's Simplified method (1955), have used an estimated factor of safety when computing the normal force at the base of a slice. The final factor of safety is found through an iterative process. The finite element method factor of safety is defined using the normal and shear stresses computed using a finite element analysis.

Finite element numerical stress analyses have been available for many years. The finite element method, however, has not become popular for slope stability studies due to intense computational requirements and difficulties in assessing the stress versus strain characteristics of the soils. In addition, inexpensive and easy to use limit equilibrium methods have provided factors of safety that appear to represent failure conditions in the field in most situations. Microcomputers now have sufficient computational capacity to perform combined stress and limit equilibrium analyses. As a result, it is anticipated that the latter procedure will become more common in engineering practice.

3.1 *Procedure used for the finite element analysis*

The enhanced limit (strength) finite element method proposed by Kulhawy (1969) was selected as the most appropriate method for slope stability analysis. The finite element stress-deformation software, Sigma/W (a proprietary product of Geo-Slope International Ltd., Calgary, Alberta, Canada), was modified to utilizes a search algorithm in order to assign and transfer calculated finite element stresses to a designed point on the slip surface (Bathe, 1982; Krahn et al., 1996). The calculated finite element calculated stresses are used to calculate the normal and shear stresses on the slip surface. The latter stresses are used to calculate local factors of safety at the center of the base of each slice as well as the overall factor of safety for the entire slip surface.

3.2 *Definition of factor of safety*

The overall factor of safety is defined in accordance with the finite element slope stability method described by Kulhawy (1969), and expressed as the ratio of the sum of the incremental resisting shear strengths, S_r, to the sum of the mobilized shear forces, S_m, along the slip surface.

$$F_{FEM} = \frac{\Sigma S_r}{\Sigma S_m} \tag{1}$$

The resisting force for each slice is calculated in terms of the shear strength, τ, at the center of a slice multiplied by the base length of the slice, β. The available resisting shear strength for a saturated/unsaturated soil (Fredlund and Rahardjo, 1993) can be written as:

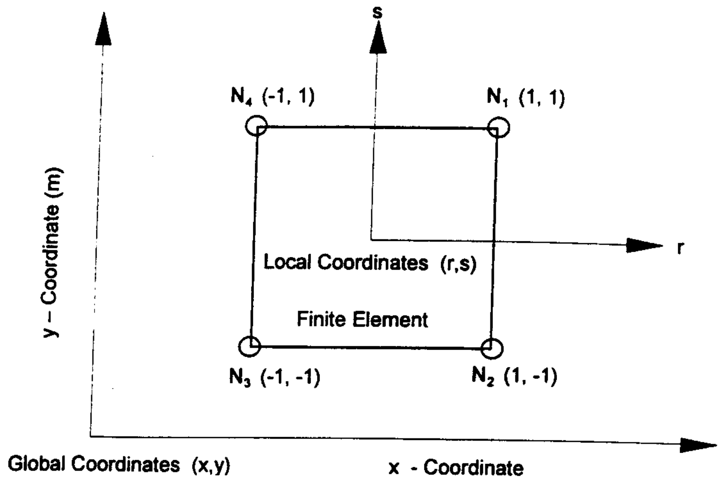


Figure 3. Definition of the global and local coordinates for a rectangular finite element.

$$S_r = \tau\beta = \{c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)\tan\phi^b\}\beta \quad (2)$$

The mobilized shear force, S_m, for each slice is calculated as the mobilized shear stress, τ_m, at the center of a slice multiplied by the base length, β.

$$S_m = \tau_m \beta \quad (3)$$

The local factor of safety is defined as the ratio of the resisting shear force, S_r, at a point along the slip surface divided by the mobilized shear force, S_m, at the same point,

$$F_{Local} = \frac{S_r}{S_m} = \frac{\tau\beta}{\tau_m\beta} \quad (4)$$

The resisting shear force, S_r, and the mobilized shear force, S_m, are both calculated using the stresses computed in the finite element analysis. The normal stress, σ_n, and shear stress, τ_m, can be "imported" as known values to the limit equilibrium analysis and the definition of both the overall and local factor of safety equations are linear.

3.3 *Element identification corresponding to the base of a slice*

Each element must be checked to confirm that the center of the base of the slice is located within the element under consideration. Then the stresses calculated by the finite element analysis can be "imported" into the stability analysis. Once the element embracing the center of a portion along the slip surface is located, stress values from the Gauss points of the element can be transferred to the nodes of the element and consequently to the center of the base. The procedure is in accordance with the method described by Bathe (1982).

A common set of coordinates is used to identify the center of a slice along a slip surface with respect to the surrounding finite element. The global coordinates for the center of the base are calculated in order to determine the location of the base center within the slope, and to determine which element is associated with the center of the base. The local coordinates of the center of the base are then calculated within the element that encompasses the center of the base (Fig. 3).

The global coordinates for the center of the base of a slice are related to the global coordinates of the finite element nodal points through use of the shape functions.

$$x = \langle N \rangle \{X\} \quad (5)$$

$$y = \langle N \rangle \{Y\} \quad (6)$$

Where x = global x coordinates for the center of the base of a slice; y = global y coordinates for the center of the base of a slice; $\{X\}$ = global x coordinates for the element nodal points; $\{Y\}$ = global y coordinates for the element nodal points; and $\langle N \rangle$ = matrix of shape functions.

The shape functions $\langle N \rangle$ are defined in terms of the local coordinates (r, s). Since the global coordinates for the center of the base of a slice and the nodes are known, the local coordinates can be obtained by solving Equations (5) and (6), simultaneously. The shape functions for a rectangular finite element with four nodes are as follows (Bathe 1982):

$$N_1 = \frac{1}{4}(1+r)(1+s) \quad (7)$$

$$N_2 = \frac{1}{4}(1-r)(1+s) \quad (8)$$

$$N_3 = \frac{1}{4}(1 - r)(1 - s) \qquad (9)$$

$$N_4 = \frac{1}{4}(1 + r)(1 - s) \qquad (10)$$

where r and s = local coordinates within the element.

The local coordinates vary between -1 and +1 (Fig. 3). A knowledge of the local coordinates is crucial to identifying the element overlapping the center of the base of a slice. By definition, an element surrounds the center of the base of a slice if the following conditions are met:

For a triangular element,

$$(0 \le r \ge 1) \text{ and } (0 \le s \ge 1) \qquad (11)$$

For a rectangular element,

$$(-1 \le r \ge 1) \text{ and } (-1 \le s \ge 1) \qquad (12)$$

The center of the base is outside an element if the local coordinates are not within the above specified ranges. The search continues until an element is found that satisfies these conditions.

3.4 *Transfer of element stresses to the center of the base of a slice*

Calculated stresses are stored within the computer software relative to the Gauss points of an element. Stresses must be transferred from the Gauss points of an element to the nodes of the element and then to the center of the base of a slice.

The local coordinates of a point within a finite element are defined in relationship to the global coordinates at the nodes of the element by using the shape functions, as per Equations (5) and (6):

$$x = \langle N_1\, N_2\, N_3\, N_4 \rangle \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{Bmatrix} \qquad (13)$$

$$y = \langle N_1\, N_2\, N_3\, N_4 \rangle \begin{Bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \end{Bmatrix} \qquad (14)$$

where x and y = global coordinate positions within the element that are known as the center of base of a slice (Fig. 4); X and Y = global coordinate at the element nodes; and N_1, N_2, N_3 and N_4 = the shape functions defined in Equations 7 to 10.

The stresses from a finite element analysis are stored at the Gauss points. The shape functions can be used to describe the change of a variable within an element in terms of nodal values. The finite element slope stability calculations require that stresses at the center of the base for each slice be within an element. This is achieved using the following procedure:

$$\{\sigma\}_n = \langle N \rangle \{F\} \qquad (15)$$

where $\{\sigma\}_n$ = stresses at the element node; $\langle N \rangle$ = matrix of the shape functions; and $\{F\}$ = stress values at the Gauss points.

The local Gauss point integration coordinates are (0.577, 0.577), however, when the local Gauss point integration coordinates are projected outward to the element nodes, the local coordinates become (1.7320, 1.7320) (Fig. 5). This projection is carried out for each element and the values for the stresses from each contributing element are averaged at each node. Accordingly, the values of σ_x, σ_y, and τ_{xy} can be computed at each node of the finite element mesh. The nodal stresses, σ_x, σ_y, and τ_{xy}, of an element are transferred to the center of the base of a slice along the slip surface.

$$\{\sigma\} = \langle N \rangle \{\sigma\}_n \qquad (16)$$

where $\{\sigma\}$ = stresses at the center of the base of a slice.

The stresses, σ_x, σ_y, and τ_{xy}, can now be computed at the center of the base for each slice.

3.5 *The normal and shear stresses at the center of a slice*

Once the stresses, σ_x, σ_y, and τ_{xy}, are known at the center of the base for each slice, the normal stress, σ_n, and the mobilized shear stress, τ_m, can be calculated using Equations (17) and (18), respectively (Higdon et al. 1976):

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y \cos 2\theta}{2} + \tau_{xy}\sin 2\theta \qquad (17)$$

$$\tau_m = \tau_{xy}\cos 2\theta - \frac{\sigma_x - \sigma_y \sin 2\theta}{2} \qquad (18)$$

where σ_x = total stress in the x-direction at the center of the base; σ_y = total stress in the y-direction at the center of the base; τ_{xy} = shear stress in the x- and y-direction at the center of the base; and θ = angle measured from the positive x-axis to the line of application of the normal stress.

The above steps provide the necessary information required to calculate the stability of a slope using the finite element stresses. The calculated values for the normal stress, σ_n, and the mobilized shear stress, τ_m, at the center of the base of a slice are entered into Equations (2) and (3) to give the resisting shear force

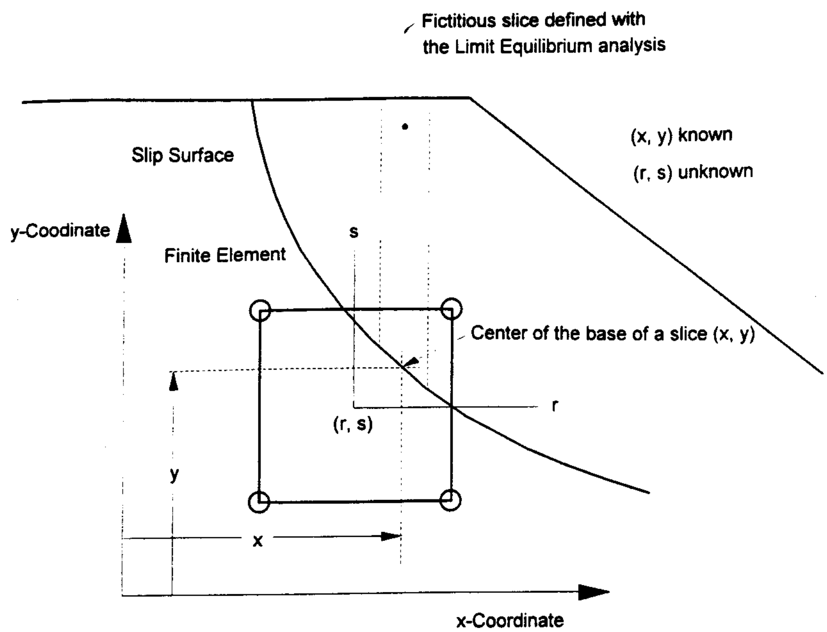


Figure 4. Location of the center of the base along the slip surface within a particular finite element.

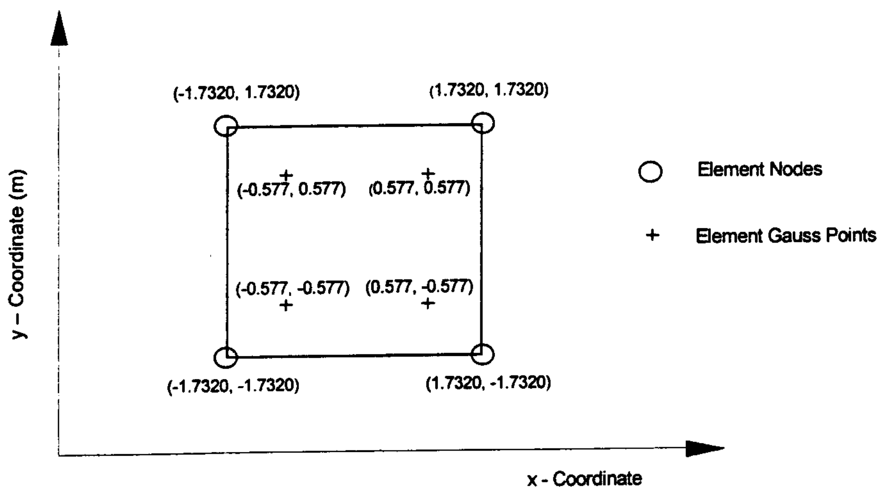


Figure 5. Gauss point projections to the nodes of a finite element.

(strength) and the mobilized shear force (actuating shear), respectively.

The local factor of safety is computed as the ratio of the resisting shear force to the mobilized shear force. The overall factor of safety is the sum of the shear force resistance values divided by the sum of the actuating shear forces along the slip surface.

4 PARAMETRIC STUDIES ON A SIMPLE 2:1 SLOPE

A slope at 2 horizontal to 1 vertical is analyzed for 4 conditions (Scoular, 1997). The first case is a free-standing slope with zero pore-water pressures and the slope is referred to as a dry slope (Fig. 6). The second case is a free-standing slope with a piezometric line at three quarters of the slope height, and the slope is referred to as a wet slope (Fig. 6).

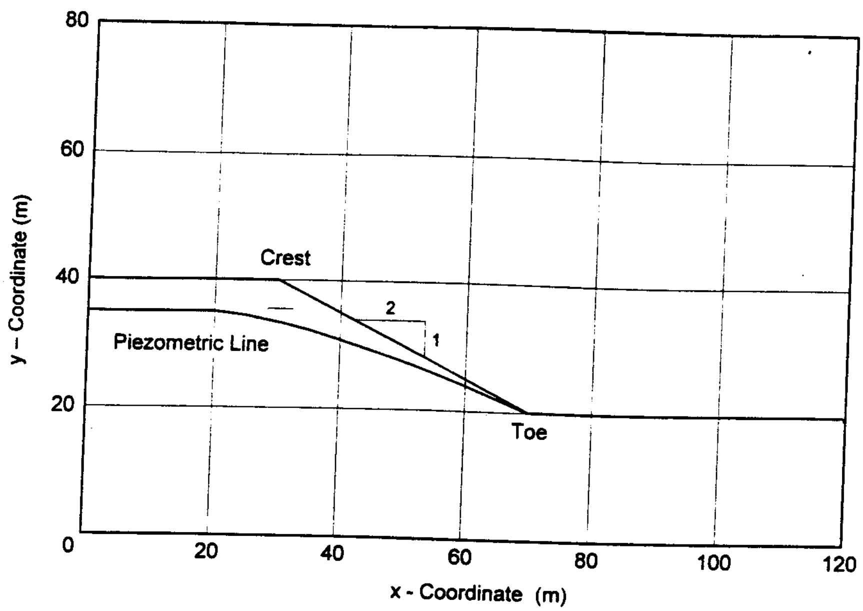


Figure 6. Selected 2:1 free-standing slope with a piezometric line exiting at the toe of the slope.

The third case is a slope partially submerged in water with zero pore-water pressures in the slope (referred to as dry) (Fig. 7). The fourth case is a partially submerged slope with a piezometric line at one half of the slope height (referred to as wet) (Fig.7). The partially submerged slope is covered with water to one half of the slope height, providing support for the slope and increasing the factors of safety. The cohesion of the soil was varied from 10 to 40 kPa and the angle of internal friction was varied from 10 to 30 degrees for each slope type.

4.1 *Limit equilibrium analysis*

The limit equilibrium analyses are performed using the General Limit Equilibrium method (GLE), (Fredlund & Krahn 1977) which provides a combined moment and force equilibrium solution. An empirical finite element interslice force function, based on an independent stress analysis (Fan et al. 1986) was used. The General Limit Equilibrium method along with a finite element interslice force function provides a method of comparison between the finite element based analysis and the limit equilibrium analysis.

4.2 *Finite element stress analysis*

The finite element stress analysis was performed by "switching-on" gravity for the free-standing slope and for the partially submerged slope. The load of the water and the lateral support it provides to the slope is simulated by point loads equal to the weight of water on the slope. The analyses are performed using Poisson's ratios of 0.33 and 0.48, and a Young's modulus equal to 20,000 and 200,000 kPa. The results showed that the stresses change with a changing poisson's ratio, but are constant for changes in the Young's modulus. This observation is consistent with the observations of Matos (1982).

5 RESULTS OF THE FINITE ELEMENT SLOPE STABILITY METHOD

The local factors of safety differs along the overall slip surface (Fig. 8). Local factors of safety were computed for a 2:1 (dry) slope with a cohesion equal to 40 kPa and an angle of internal friction equal to 30 degrees. While the local factors of safety differ along the slip surface, the overall finite element factors of safety fall within the range of the limit equilibrium factors of safety. The difference between the local factors of safety for Poisson's ratios of 0.33 and 0.48, calculated using the finite element method, is reflected in Figure 8. The factor of safety computed by the limit equilibrium method and the finite element method appear to be very similar. The results appear to be within the limits of uncertainty associated with slope stability calculations. The finite element method incorporates the stress-strain characteristics of the soil when computing the shear strength and actuating shear force of the soil in the calculation of the factor of safety (Fig. 9).

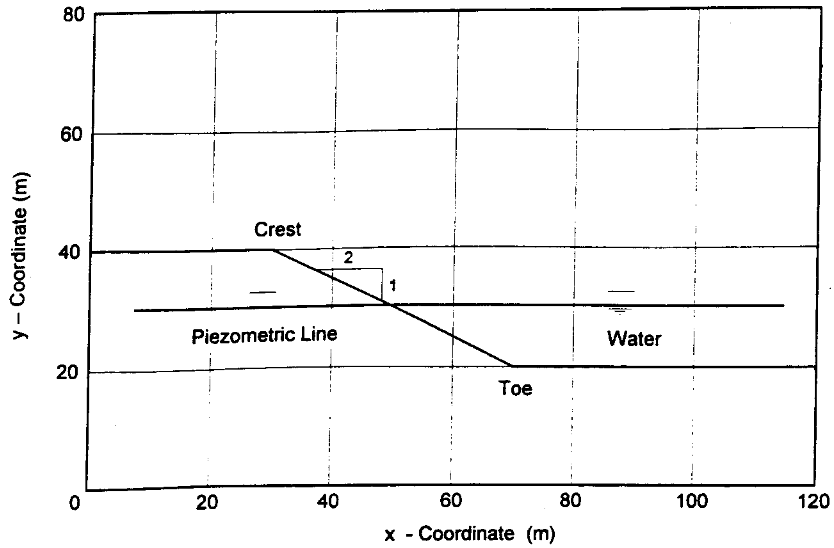


Figure 7. Selected 2:1 partially submerged slope with a horizontal piezometric line at mid-slope.

The factor of safety results computed using the finite element method (i.e., F3 corresponding to a Poisson's ratio of 0.33, F4 corresponding to a Poisson's ratio of 0.48) are compared to the factors of safety computed using the limit equilibrium method (GLE) and are shown in Tables 1 and 2. To assess the variations in the factor of safety by each method of analysis, the results are grouped according to cohesion and angle of internal friction. The factors of safety grouped according to cohesion, c', are plotted versus the stability number, $[(\gamma H \tan\phi')/c']$, (Janbu, 1954). The factors of safety grouped according to the angle of internal friction, ϕ', are plotted versus the stability coefficient, $(c'/\gamma H)$ (Taylor, 1937), where γ is the unit weight of the soil, H is the height of the slope, ϕ' is the angle of internal friction, and c' is the cohesion.

The factors of safety are grouped according to the soil parameters and plotted versus the stability number and the stability coefficient. The greatest difference in factors of safety is noticed at high angles of internal friction, at low values of cohesion and at the maximum values of Poisson's ratio.

The factors of safety for the (dry) free-standing slope, when grouped according to cohesion and plotted versus the stability number (Fig. 10) show a

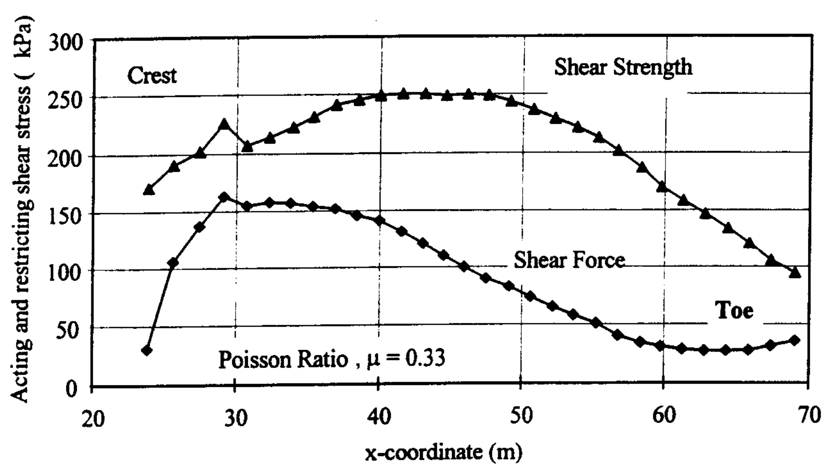


Figure 8. Presentation of the local and global factors of safety for a 2:1 dry slope.

Table 1. 2:1 free-standing slope

Soil c' kPa	Parameters ϕ' degree	Dry GLE Finite element function	Dry F3 $\mu = 0.33$	Dry F4 $\mu = 0.48$	Wet GLE Finite element function	Wet F3 $\mu = 0.33$	Wet F4 $\mu = 0.48$
10	10	0.669	0.662	0.672	0.488	0.456	0.467
20	10	0.882	0.867	0.874	0.677	0.634	0.647
10	20	1.131	1.125	1.151	0.782	0.745	0.755
40	10	1.260	1.230	1.239	0.995	0.930	0.953
20	20	1.370	1.352	1.368	1.021	0.969	0.988
10	30	1.615	1.639	1.696	1.102	1.077	1.101
40	20	1.794	1.765	1.775	1.335	1.260	1.293
20	30	1.892	1.884	1.918	1.374	1.287	1.312
40	30	2.356	2.324	2.339	1.741	1.627	1.661

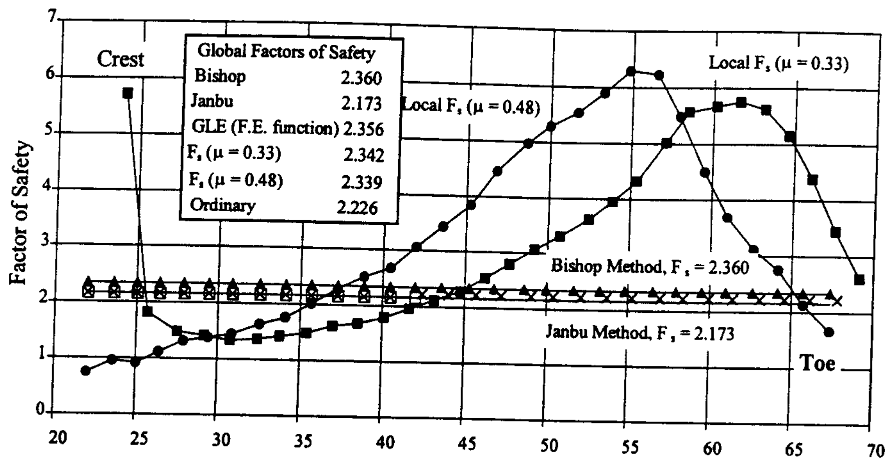


Figure 9. Shear strength and shear force for a 2:1 dry slope calculated using the finite element method.

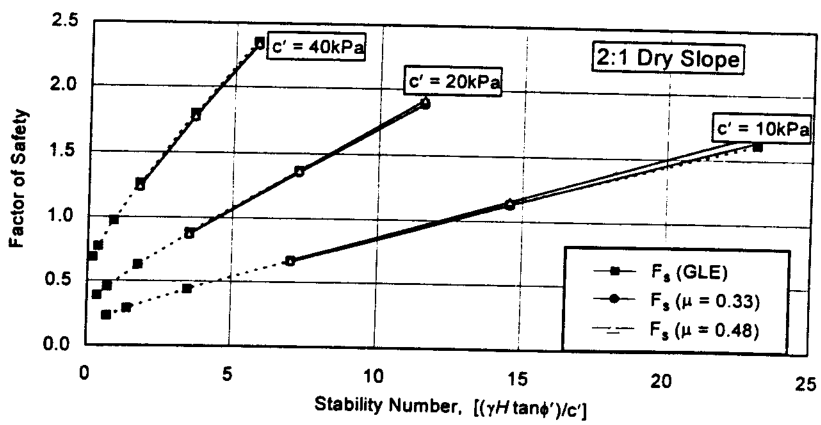


Figure 10. Factors of safety versus stability number for a 2:1 dry slope as a function of cohesion.

Table 2. 2:1 partially submerged slope

Soil	Parameters	Dry			Wet		
c'	ϕ'	GLE	F3	F4	GLE	F3	F4
kPa	degree	Finite element function	$\mu = 0.33$	$\mu = 0.48$	Finite element function	$\mu = 0.33$	$\mu = 0.48$
10	10	0.845	0.843	0.827	0.649	0.635	0.641
20	10	1.149	1.115	1.085	0.886	0.874	0.880
10	20	1.344	1.425	1.422	1.050	1.046	1.068
20	20	1.618	1.586	1.575	1.318	1.314	1.343
40	10	1.721	1.722	1.691	1.322	1.296	1.316
10	30	1.865	2.081	n.s.a.*	1.482	1.505	1.530
40	20	2.297	2.385	2.368	1.800	1.774	1.795
20	30	2.337	2.268	2.204	1.783	1.763	1.786
40	30	3.006	2.970	2.899	2.303	2.260	2.274

*n.s.a.: no solution achieved

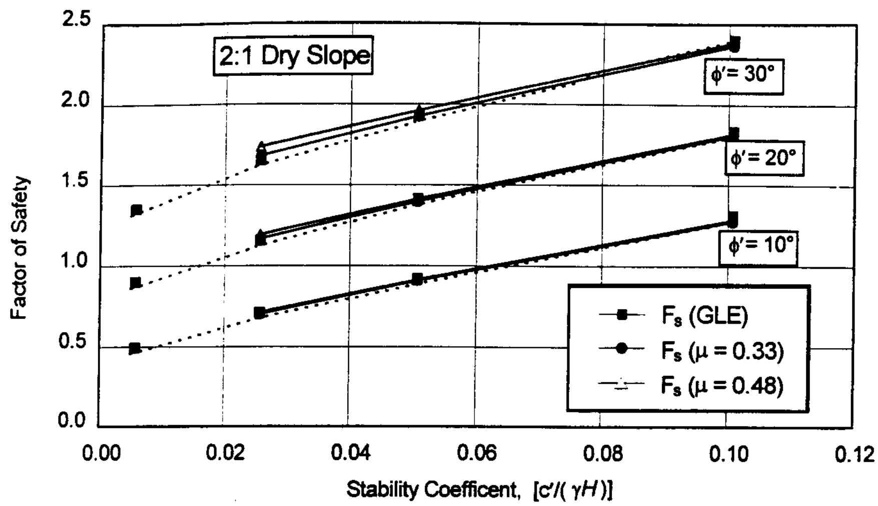


Figure 11. Factor of safety versus stability coefficient for a 2:1 dry slope as function of angle of internal friction.

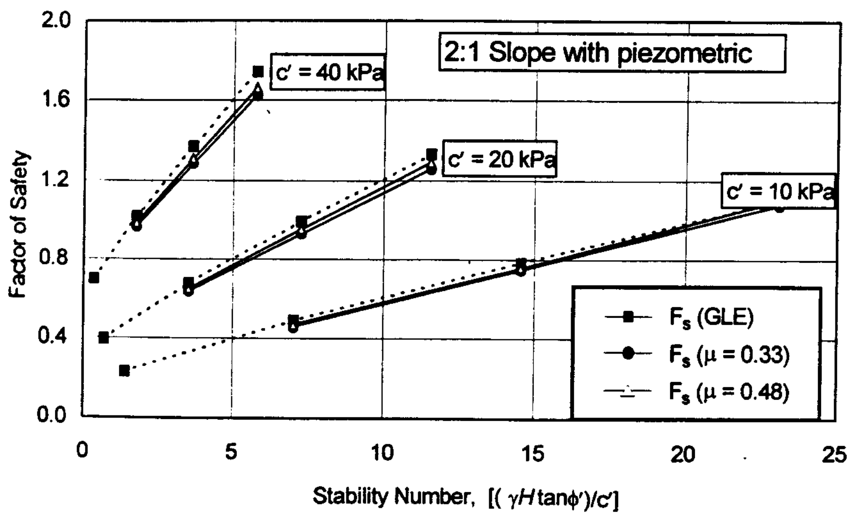


Figure 12. Factor of safety versus stability number as a function of cohesion for a 2:1 slope with the piezometric line at ¾ of the slope height.

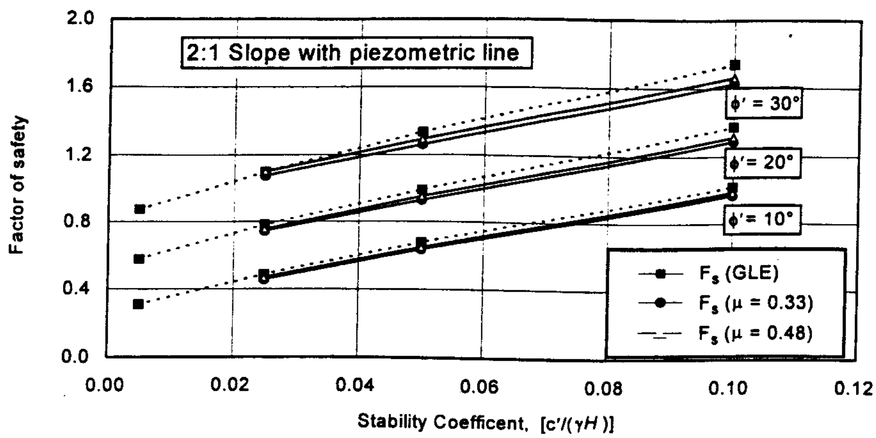


Figure 13. Factor of safety versus stability coefficient as a function of the angle of internal friction for a 2:1 slope with the piezometric line at ¾ of the slope height.

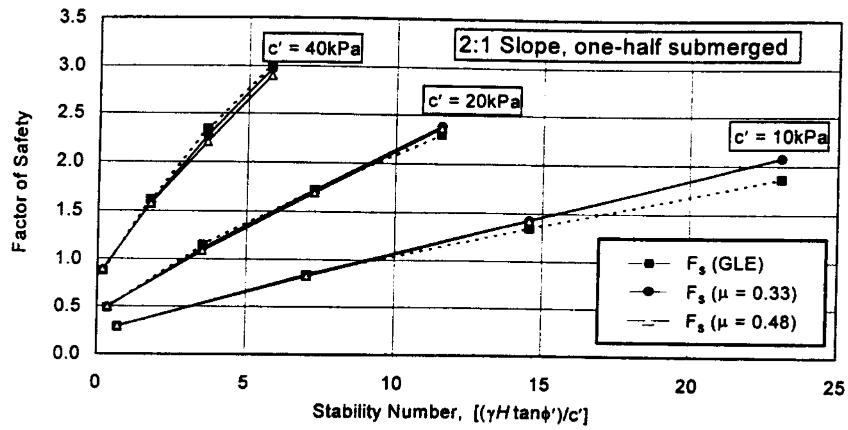


Figure 14. Factor of safety versus stability number as a function of cohesion for a 2:1 dry slope ½ submerged with water.

slight divergence in the factors of safety when the cohesion approaches 10 kPa and the angle of internal friction approach 30 degrees.

The factors of safety by the finite element method, with a high Poisson's ratio, is greater than the General Limit Equilibrium solution. The slight divergence is evident when the factors of safety are grouped according to the angle of internal friction and plotted versus the stability coefficient (Fig. 11). It is also evident that at high values of cohesion, (i.e., c' equal to 40 kPa), The factors of safety computed when using the General Limit Equilibrium method are greater than those from the finite element methods with either Poisson's ratio value.

The factors of safety for the (wet) free-standing slope with a piezometric line at three quarters of the slope height, are grouped according to the cohesion and plotted versus the stability number (Fig. 12). The results show a slight divergence between the finite element factors of safety and the General Limit Equilibrium factors of safety when the cohesion is 40 and 20 kPa. The difference between the factors of safety by both methods is constant at all values of cohesion until the angle of internal friction becomes equal to 30 degrees and cohesion becomes equal to 10 kPa (Fig. 13).

The grouping of the factors of safety according to the angle of internal friction, plotted versus the stability coefficient (Fig. 15), shows the same pattern as for the (dry) free-standing slope (Fig.10). The differences in the results are more pronounced as the cohesion become less than 10 kPa.

The factors of safety for the partially submerged slope with a piezometric line at one half of the slope height were grouped by cohesion and plotted versus the stability number (Fig. 16). The results show close agreement between the General Limit Equilibrium method and the finite element method. The

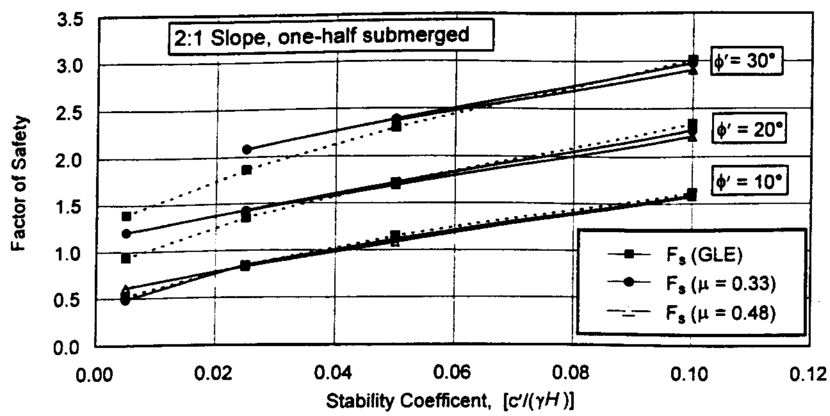


Figure 15. Factor of safety versus stability coefficient as a function of internal friction for a 2:1 dry slope ½ submerged in water.

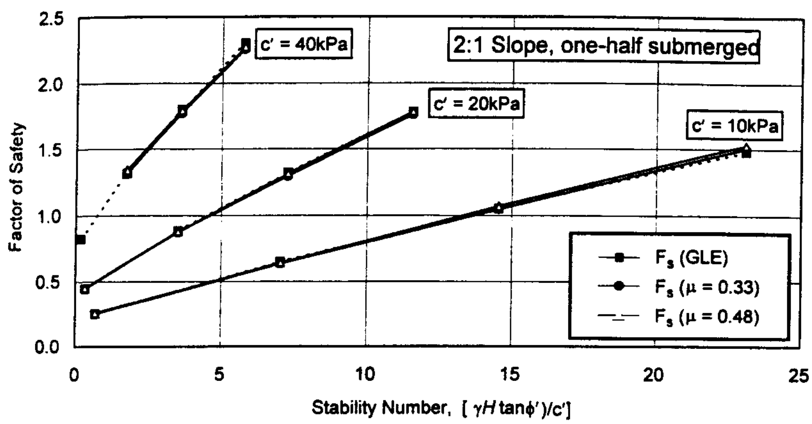


Figure 16. Factor on safety versus stability number as a function of cohesion for a 2:1 slope half submerged with a horizontal piezometric line.

same pattern of divergence is evident as was shown for the dry soil slope which is partially submerged (Fig. 14). However, the divergence is not quite as extensive. The same comments apply to the factor of safety versus the stability coefficient as shown in Figure 17.

Plotting the factors of safety for the various slope conditions, (i.e., dry free-standing, wet free-standing and dry partially submerged), versus stability number on Figure 18, shows the ranking of slopes by factors of safety. The factor of safety can be estimated for a slope that is similar to one of these cases by calculating the stability number and selecting the appropriate value of cohesion and angle of internal friction.

Both the General Limit Equilibrium method and the finite element method of slope stability produce factors of safety that are in close agreement. The advantage of the finite element method is that the stress-stain characteristics of the soil are used to determine the stress state in the slope. If the limit equilibrium and finite element factors of safety are similar for a simple slope than results from the two methods can be interpreted in similar manners. This study then sets the stage for using the finite element method for situations where the limit equilibrium methods is known to not yield satisfactory results. The finite element method also produces graphs of the local factors of safety that can be combined with the shear strength-actuating shear force plots to help explain the best support mechanism for the slope.

The close agreement between the factors of safety when using the limit equilibrium method or the finite element method, has historically favored the use of limit equilibrium methods. Examination of the critical slip surfaces reveals that while the factors of safety values are close, the location of the critical slip surfaces may be different.

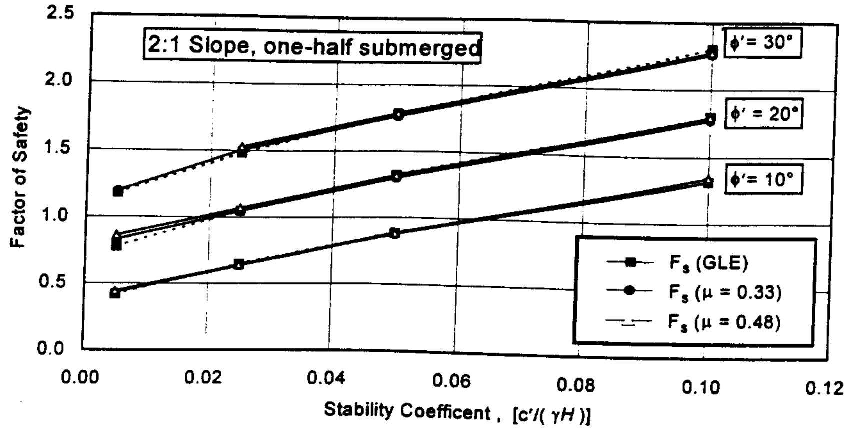


Figure 17. Factor of safety versus stability coefficient as a function of the angle of internal friction for a 2:1 slope half submerge with a horizontal piezometric line.

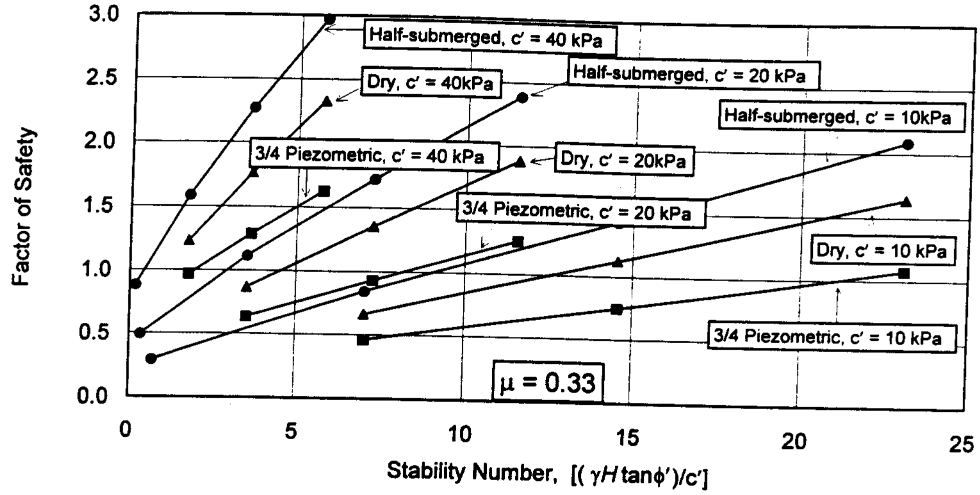


Figure 18. Factor of safety versus stability number as a function of cohesion for a 2:1 slope, evaluated for dry, piezometric and submerged conditions.

6 ANALYSIS FOR THE LOCATION OF THE CRITICAL SLIP SURFACE

The location of the critical circle changes depending on the situation being analyzed. The biggest change in location of critical slip surface was experienced for the (wet) free-standing slope (Figs. 19 and 20) and the (wet) supported slope (Figs. 21 and 22).

In general, the finite element method slip surfaces go deeper than the limit equilibrium slip surfaces for the (wet) free-standing slope. The partially submerged slopes show that the limit equilibrium slip surfaces go deeper than the finite element method slip surfaces. For the free- standing slope, the finite element method with a Poisson's ratio equal to 0.48 showed the deepest slip surface. For the partially submerged slope, the finite element method with a Poisson's ratio equal to 0.48, showed a considerably shallower slip surface.

7 CONCLUSION

The finite element method of slope stability is a viable method of analysis that is now available for engineering practice. The use of the finite element method yields more detailed information on the stress state in the soil than is available from conventional limit equilibrium methods. This information can assists engineers in the design of slopes and slope retaining structures.