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# USING A FINITE ELEMENT STRESS ANALYSIS TO COMPUTE THE FACTOR OF SAFETY

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ABSTRACT: This paper reviews the development of finite element slope stability analyses and proposes that such a method can form a practical procedure for solving slope stability problems. The combination of a finite element stress analysis with a limit equilibrium analysis provides greater certainty and flexibility regarding the internal distribution of stresses within the soil mass. The normal force along any selected slip surface can be calculated from the stress distribution that has been calculated using a linear and non-linear stress analysis. The overall factor of safety for a slope can be defined as the available shear strength of the soil divided by the resisting shear strength. The resulting overall factor of safety retains the basic assumptions inherent to the limit equilibrium definition of the factor of safety.

The overall factor of safety computed using the finite element method shows good agreement with the factors of safety computed using any one of several limit equilibrium methods.

RÉSUMÉ: Cet atricle revoit le développement des analyses de stabilité de pente d'élément fini et propose qu'une telle méthode puisse former une procédure pratique pour résoudre les problémes de stabilité de pente. La combinaison d'analyse de stresse d'élément fini et d'analyse d'équilibre limite fournit une plus grande certitude et flexibilité concernant la distribution interne des stresses dans la masse de sol. La force normale le long de toute "slip" surface sélectionnée peut être calculée à partir de la distribution du stresse qui a été calculée en utilisant une analyse de stresse linéaire et non-linéaire. Le facteur d'ensemble de sécurité pour une pente peut être défini comme étant la force de cisaillement disponible du sol divisée par la force de cisaillement résistante. Le facteur d'ensemble de sécurité résultant retient les assomptions de base inhérentes à la définition d'équilibre limite du facteur de sécurité.

Le facteur d'ensemble de sécurité automatisé utilisant la méthode d'élément fini se montre en accord avec les facteurs de sécurité automatisés utilisant une des nombreuses méthodes d'équilibre limite.

## 1. INTRODUCTION

Limit equilibrium methods of analysis have proven to be a widely used and successful method for the assessment of the stability of a slope. Limit equilibrium methods sum forces and moments related to an assumed slip surface passed through a soil mass (Fredlund and Krahn, 1975; Fredlund et al., 1981). However, these methods do not utilize the stress versus strain characteristics of the soils involved. It is well known, and intuitively understood that the stability of a slope should be influenced by the stress versus strain characteristics of a soil (Kondner 1963). A finite element analysis utilizes a stress versus strain model for the soils involved to calculate the stresses in the soil mass. These stresses can subsequently be used to compute a factor of safety (Figure 1). The complete stress state from the finite element analysis can be "imported" into a limit equilibrium analysis where the normal stress and the shear stress are computed corresponding to any selected slip surface.

The objective of this paper is to demonstrate a procedure for combining a finite element stress analysis on a slope with the concepts of a limiting equilibrium method of analysis. The final method is called a "finite element method of slope stability analysis" and the results are compared to results obtained when using conventional limit equilibrium methods of analysis.

## 2. BACKGROUND

Bishop (1952) noted that the stresses from a limit equilibrium method of analysis did not agree with the actual stresses within an earth structure. Other researchers have confirmed this observation both with experimental evidence and with numerical modelling. La Rochelle (1960) estimated the stress conditions in steep slopes using photoelastic tests on gelatine models. The results showed that stresses along a slip surface were over-stressed in the lower portion of the slip circle. Brown and King (1966) produced critical slip surfaces from a finite element stress analysis of slopes using a linear elastic soil model. The critical slip surfaces were produced by using the angle of obliquity along the slip surface (i.e., $\theta$ equal to ($45° + \phi'/\alpha$)). Each critical slip surface represented a close approximation to an essentially circular shaped slip surface.

Kulhawy (1969) developed a computer program to obtain an independent assessment of the normal and shear stress distribution along an assumed slip surface. The normal and shear stresses from an elastic analysis were used to calculate an overall factor of safety. The formulation of Kulhawy (1969) was classified as an "Enhanced Limit Strength Method".

A number of finite element slope stability methods have been proposed and the methods can be categorized as "enhanced limit methods" or "direct methods", as shown in Figure 2.

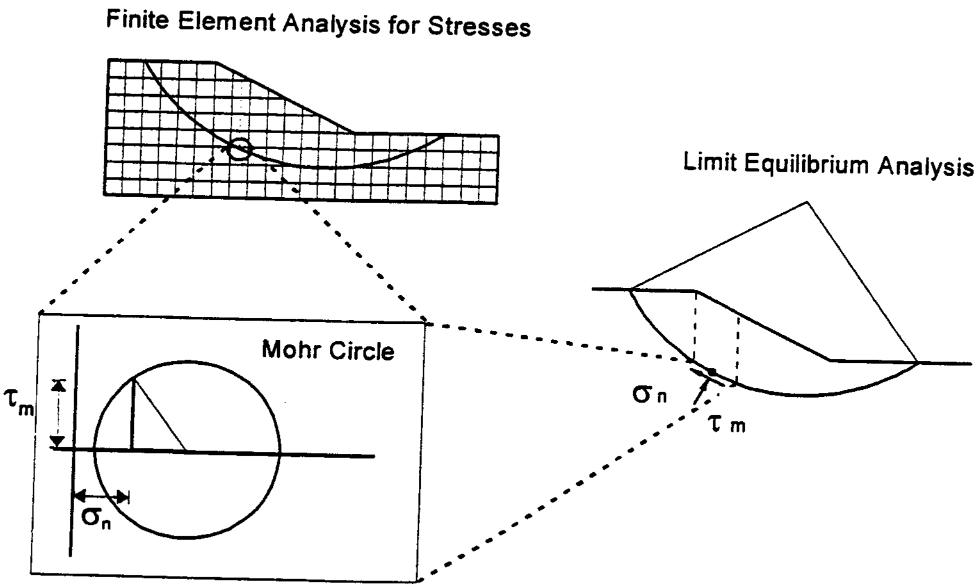
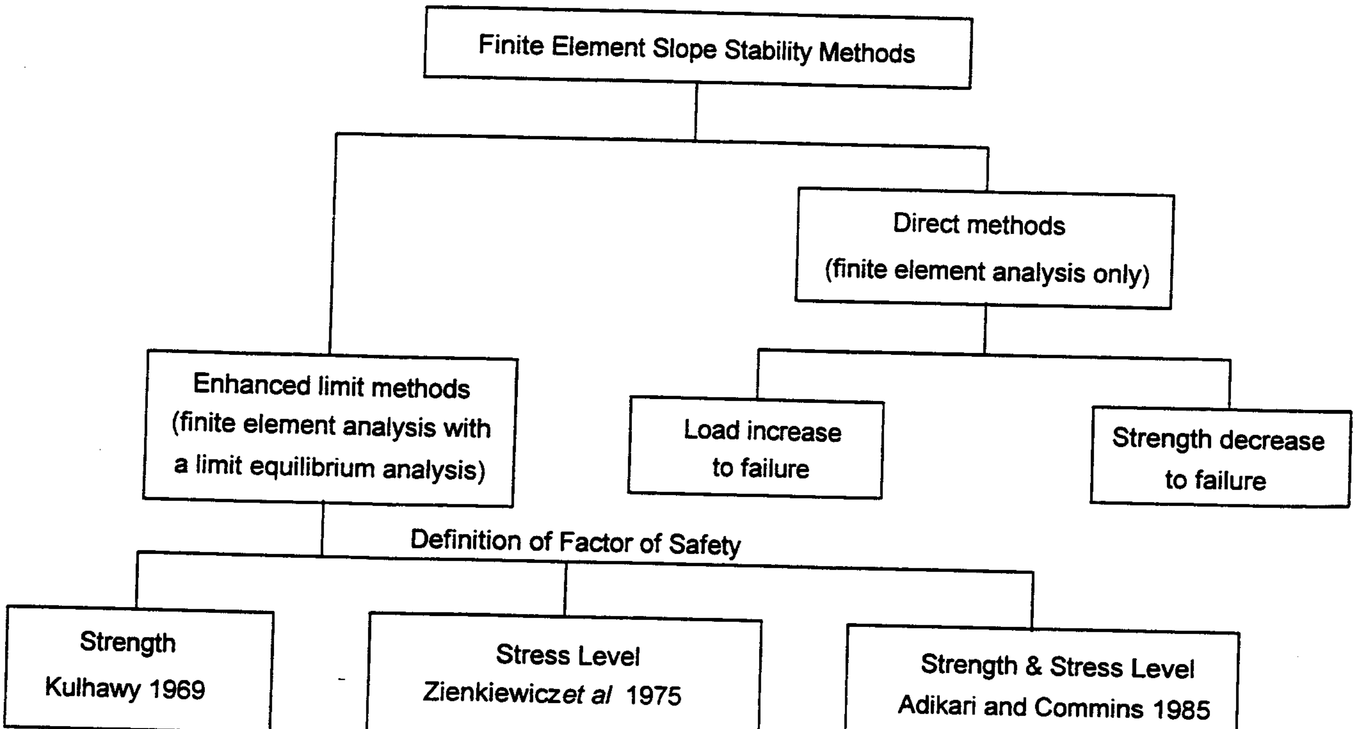



Figure 1. Illustration showing stresses that are "imported" from a finite element analysis into a limit equilibrium analysis.




$$F_K = \frac{\sum (c' + \sigma' \tan\phi')\,\Delta L}{\sum \tau \Delta L}$$

$$F_Z = \frac{\sum [\Delta L]}{\sum \left\{ \frac{(\sigma_1' - \sigma_3')}{(\sigma_1' - \sigma_3')_f} \Delta L \right\}}$$

$$F_A = \frac{\sum \{(c' + \sigma' \tan\phi')\,\Delta L\}}{\sum \left\{ \frac{(\sigma_1' - \sigma_3')}{(\sigma_1' - \sigma_3')_f} \cdot (c' + \sigma' \tan\phi')\,\Delta L \right\}}$$

Figure 2. Finite element approaches proposed in computing the factor of safety in a slope stability analysis.

Wright (1969) compared the factors of safety calculated using the "enhanced limit strength" method with factors of safety calculated using Bishop's Simplified method (1955). It was concluded that the factors of safety determined by the "enhanced limit strength" method (Kulhawy, 1969) were approximately 3% higher than those determined applying Bishop's Simplified method.

Zienkiewicz et al. (1975) also proposed a finite element method of analysis to compute the factor of safety by using the principal stress difference in the soil at failure to define the factor of safety. The method is an "enhanced limit stress level method" (Figure 2).

Naylor (1982) established two types of finite element slope stability methods, a "direct" and an "enhanced limit" method of analysis. The direct method used a finite element nodal formulation to define the slip surface and the factor of safety directly from the analysis. The proposed "direct" slope stability method defined the factor of safety either as the increased load necessary to cause failure, or as the reciprocal of the reduction in the strength properties required in order to achieve failure.

The "enhanced limit" slope stability methods are based on stresses calculated using a finite element analysis and combined with a limit equilibrium type of analysis along a prescribed slip surface, to define the factor of safety. The prescribed slip surface is the one defined by the lowest factor of safety and is found using a trial and error procedure. The stresses along the slip surface are computed using a finite element analysis and can either be used in a "strength" method or a "stress-level" method". Enhanced limit" methods require only one finite element analysis to calculate factors of safety for a slope with various combinations of $c'$ and tan $\phi'$.

Adikari and Cummins (1985) produced a finite element method that combine the "strength" and the "stress level" methods as defined by Kulhawy (1969) and Zienkiewicz et al. (1975), respectively (Figure 2). By definition, failure does not occur on the plane of principal stress and therefore, the Zienkiewicz et al. (1975) method (or any stress-level method) is computing a factor of safety that must be higher than the factors of safety produced by a "strength" method. Duncan et al. (1996) provided a summary of the limit equilibrium and finite element methods that have been proposed for slope stability analyses.

## 3. COMPARISON BETWEEN THE FINITE ELEMENT AND THE LIMIT EQUILIBRIUM METHODS OF SLOPE STABILITY ANALYSIS

The finite element slope stability method proposed in this paper is of the "enhanced limit strength" type (Scoular, 1997). The finite element method uses the Kulhawy (1969) definition for the factor of safety. The finite element slope stability method produces an overall factor of safety that is an expression of the stability of the slope based on the calculated stresses within the slope. Slope stability problems solved using the finite element method have two important distinctions from limit equilibrium methods. First, the finite element slope stability equation is determinate; therefore, no further assumptions are required to complete the calculations. Second, the factor of safety equation is linear, because the normal stress at the base of a slice is known. On the other hand, limit equilibrium methods, starting with Bishop's Simplified method (1955), have used an estimated factor of safety when computing the normal force at the base of a slice. The final factor of safety is found through an iterative process. The finite element method factor of safety is defined using the normal and shear stresses computed, using a finite element analysis.

Finite element numerical stress analyses have been available for many years. The finite element method, however, has not become popular for slope stability studies due to intense computational requirements and difficulties in assessing the stress versus strain characteristics of the soils. In addition, inexpensive and easy to use limit equilibrium methods have provided factors of safety that appear to represent failure conditions in the field in most situations. Microcomputers now have sufficient computational capacity to perform combined stress and limit equilibrium analyses. As a result, it is anticipated that the latter procedure will become more common in engineering practice.

### 3.1 Procedure Used for the Finite Element Analysis

The enhanced limit (strength) finite element method proposed by Kulhawy (1969) was selected as the most appropriate method for slope stability analysis. The finite element stress-deformation software, Sigma/W, was modified to utilizes a search algorithm in order to assign and transfer calculated finite element stresses to a designed point on the slip surface (Bathe, 1982; Krahn et al., 1996). The calculated finite element stresses are used to compute the normal and shear stresses on the slip surface. The latter stresses are used to calculate local factors of safety at the center of the base of each slice as well as the overall factor of safety for the entire slip surface. The overall factor of safety is defined in accordance with the finite element slope stability method described by Kulhawy (1969), and expressed as the ratio of the sum of the incremental resisting shear strengths, $S_r$, to the sum of the mobilized shear forces, $S_m$, along the slip surface.

$$[1]\quad F_{FEM} = \frac{\Sigma S_r}{\Sigma S_m}$$

The resisting force for each slice is calculated in terms of the shear strength, $\tau$, at the center of a slice multiplied by the base length of the slice, $\beta$. The mobilized shear force, $S_m$, for each slice is calculated as the mobilized shear stress, $\tau_m$, at the center of a slice multiplied by the base length, $\beta$.

$$[2]\quad S_m = \tau_m \beta$$

The local factor of safety is defined as the ratio of the resisting shear force, $S_r$, at a point along the slip surface divided by the mobilized shear force, $S_m$, at the same point,

$$[3]\quad F_{Local} = \frac{S_r}{S_m} = \frac{\tau \beta}{\tau_m \beta}$$

The resisting shear force, $S_r$, and the mobilized shear force, $S_m$, are both calculated using the stresses computed in the finite element analysis. The normal stress, $\sigma_n$, and shear stress, $\tau_m$, can be "imported" as known values to the limit equilibrium analysis and the definition of both the overall and local factor of safety equations are linear.

## 4. PARAMETRIC STUDIES ON A SIMPLE 2:1 SLOPE

A slope at 2 horizontal to 1 vertical is analyzed for a free standing slope with zero pore-water pressures, dry slope, and a free-standing slope with a pizometric line at three quarters of the slope height, wet slope, (Figure 3) (Scoular, 1997). The effective cohesion of the soil, $c'$, was varied from 10 to 40 kPa and the effective angle of internal friction, $\phi'$, was varied from 10 to 30 degrees for each slope type.

### 4.1 Limit Equilibrium Analysis

The limit equilibrium analyses are performed using the General Limit Equilibrium method (GLE), (Fredlund and Krahn 1977) which provides a combined moment and force equilibrium solution. An empirical finite element interslice force function, based on an independent stress analysis (Fan et al. 1986) was used. The General Limit Equilibrium method along with a finite element interslice force function provides a method of comparison between the finite element based stress analysis and the limit equilibrium analysis.

### 4.2 Finite Element Stress Analysis

The finite element stress analysis was performed by "switching-on" gravity for the free-standing slope and for the partially submerged slope. The load of the water and the lateral support it provides to the slope is simulated by point loads equal to the weight of water on the slope. The analyses are performed using Poisson's ratios of 0.33 and 0.48, and a Young's modulus equal to 20,000 and 200,000 kPa. The results showed that the stresses change with a changing Poisson's ratio, but are constant for changes in the Young's modulus. This observation is consistent with the observations of Matos (1982).

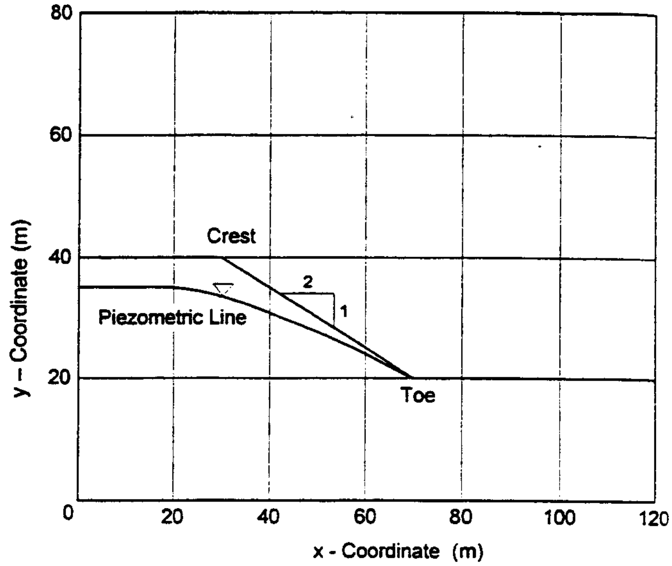



Figure 3. Selected 2:1 free-standing slope with a piezometric line exiting at the toe of the slope.

## 5. RESULTS OF THE FINITE ELEMENT SLOPE STABILITY METHOD

While the local factors of safety differ along the slip surface, the overall finite element factors of safety fall within the range of the limit equilibrium factors of safety. The factor of safety computed by the limit equilibrium method and the finite element method appear to be very similar. The results appear to be within the limits of uncertainty associated with slope stability calculations. The finite element method incorporates the stress-strain characteristics of the soil when computing the shear strength and actuating shear force of the soil in the calculation of the factor of safety (Figure 4). The greatest difference in factors of safety is noticed at high angles of internal friction, at low values of cohesion and at the maximum values of Poisson's ratio.

The factors of safety grouped according to cohesion, $c'$, and plotted versus the stability number, $[(\gamma H \tan\phi') / c']$, (Janbu, 1954), where $\gamma$ is the unit weight of the soil, $H$ is the height of the slope, $\phi'$ is the angle of internal friction, and $c'$ is the cohesion.

The factors of safety for the (dry) free-standing slope (Figure 5) show a slight divergence in the factors of safety when the cohesion approaches 10 kPa and the angle of internal friction approach 30 degrees. The factors of safety by the finite element method, with a high Poisson's ratio, is greater than the General Limit Equilibrium solution. It is also evident that at high values of cohesion, (i.e., $c'$ equal to 40 kPa), the factors of safety computed when using the General Limit Equilibrium method are greater than those from the finite element methods with either Poisson's ratio value.

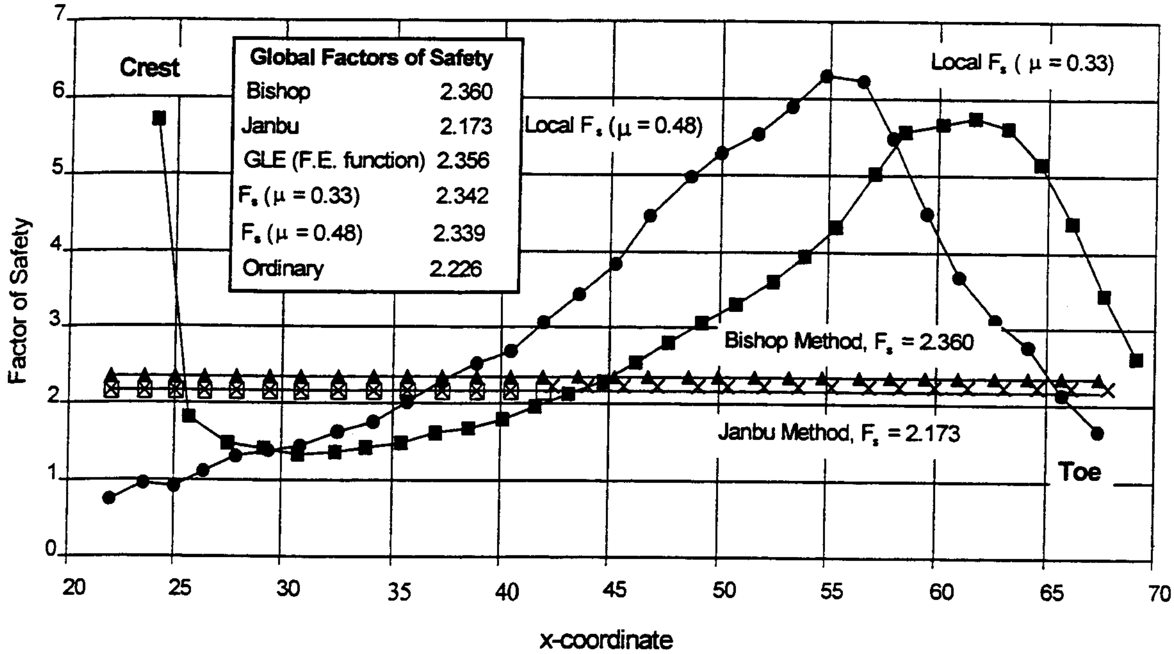



Figure 4. Shear strength and shear force for a 2:1 dry slope calculated using the finite element method.

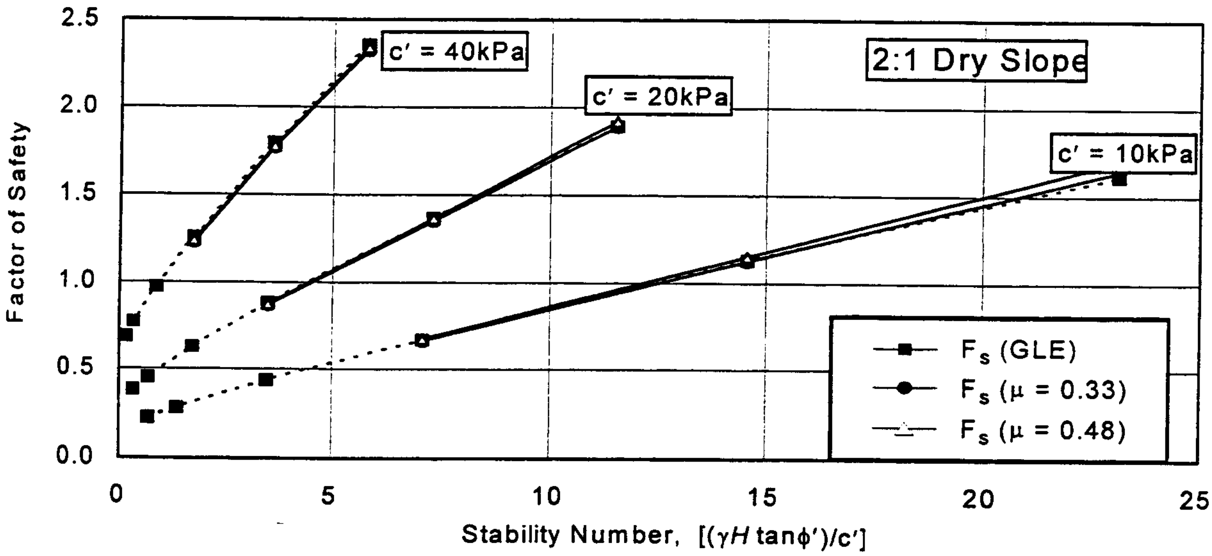



Figure 5. Factors of safety versus stability number for a 2:1 dry slope as a function of cohesion.

The factors of safety for the (wet) free-standing slope with a piezometric line at three quarters of the slope height (Figure 6) show a slight divergence between the finite element factors of safety and the General Limit Equilibrium factors of safety when the cohesion is 40 and 20 kPa. The difference between the factors of safety by both methods is constant at all values of cohesion until the angle of internal friction becomes equal to 30 degrees and cohesion becomes equal to 10 kPa.

Both the General Limit Equilibrium method and the finite element method of slope stability produce factors of safety that are in close agreement. The advantage of the finite element method is that the stress-stain characteristics of the soil are used to determine the stress state in the slope. If the limit equilibrium and finite element factors of safety are similar for a simple slope, the results from the two methods can be interpreted in similar manners. This study then sets the stage for using the finite element method for situations where the limit equilibrium method is known to not yield satisfactory results. The finite element method also produces graphs of the local factors of safety that can be combined with the shear strength-actuating shear force plots to help explain the best support mechanism for the slope.

The close agreement between the factors of safety when using the limit equilibrium method or the finite element method, has historically favored the use of limit equilibrium methods. Examination of the critical slip surfaces reveals that while the factors of safety values are close, the location of the critical slip surfaces may be different.

## 6. ANALYSIS FOR THE LOCATION OF THE CRITICAL SLIP SURFACE

The location of the critical circle changes depending on the situation being analyzed. The biggest change in location of critical slip surface was experienced for the (wet) free-standing slope (Figure 7).

In general, the finite element method slip surfaces go deeper than the limit equilibrium slip surfaces for the (wet) free-standing slope. For the free- standing slope, the finite element method with a Poisson's ratio equal to 0.48 showed the deepest slip surface

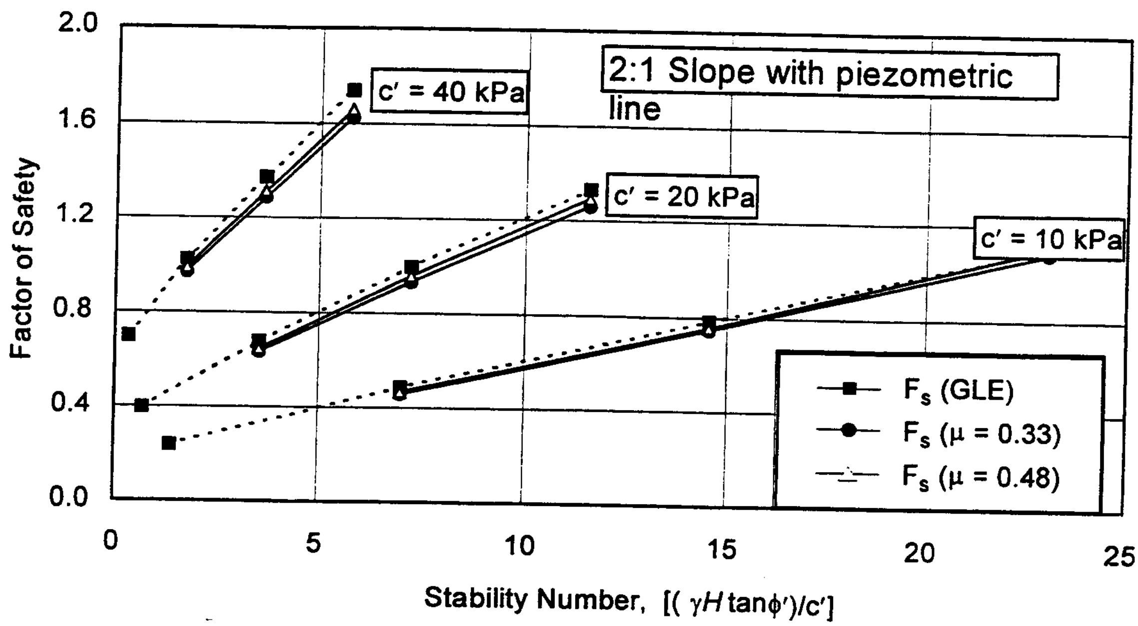



Figure 6. Factor of safety versus stability number as a function of cohesion for a 2:1 slope with the piezometric line at ¾ of the slope height.

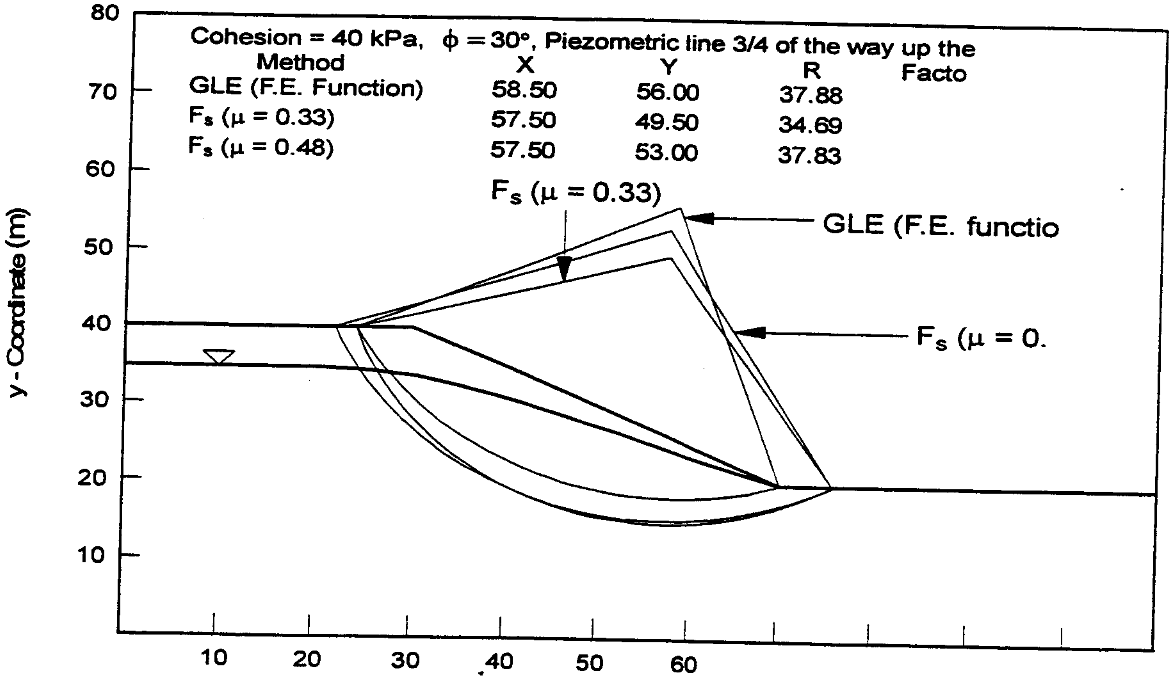



Figure 7. Location of the critical slip surface for a slope with a piezometric line where the soil properties are c' = 40 kPa and $\phi' = 30°$.

## 7. CONCLUSION

The finite element method of slope stability is a viable method of analysis that is now available for engineering practice. The use of the finite element method yields more detailed information on the stress state in the soil than is available from conventional limit equilibrium methods. This information can assists engineers in the design of slopes and slope retaining structures.

The value of Poisson's ratio can affect the calculation of the factor of safety as well as the location of the slip surface. With an increasing application of the finite element method to slope stability problems, a better understanding is required regarding the effect of Poisson's ratio and the overall deformation model on the stability of slopes.

The finite element stress analysis provides input information for the calculation of the stability of a slope. Further research must be undertaken on the stress analysis in order to ensure that the proper boundary conditions are being used and that a reasonable stress-deformation model is being used. With this assurance, soil structures can be better designed to account for a variety of stress conditions.

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