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Suitable Interslice Force Functions to Solve Lateral Earth Force Problems

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ABSTRACT: This paper presents interslice force functions (i.e., the ratio of the shear force to the normal force of vertical slices along the slip surface) that can be used to compute the active and the passive earth force by the limit equilibrium method. The procedure to compute the lateral earth force and the point of application of the force is recommended. An example problem involving a vertical wall with a horizontal backslope is analyzed using the General Limit Equilibrium (GLE) method.

The computed lateral earth forces, and the point of application, are compared with the classical solutions. For the active case, reasonable results were obtained when using an interslice force function that varies linearly from the starting point of the slip surface to the end point of the slip surface (adjacent to the wall). For the passive case, reasonable results were obtained when using an interslice force function that remains at zero from the starting point of the slip surface to the mid point of the slip surface and then varies linearly from the mid point of the slip surface to the end point of the slip surface at the wall.

KEYWORDS: Active force, Passive force, General limit equilibrium method, Interslice force function

1 INTRODUCTION

Some recent developments to calculate lateral earth forces on retaining walls and anchors involve the method of slices and limit equilibrium concepts.

The General Limit Equilibrium (GLE) method satisfies both moment and force equilibrium. The force equilibrium solution is used to compute the active or the passive earth force, while the moment equilibrium solution is used to compute the point of application of these forces. An important issue in formulating the solution is the selection of an appropriate interslice force function to relate the normal and the shear stresses between the slices of the sliding mass.

In this paper, appropriate interslice force functions are proposed for calculating lateral earth forces. The paper describes the procedure that is used to calculate the active and the passive forces and the point of application of the lateral earth forces. Mathematical forms are recommended for the interslice force function for both the active and the passive cases. An example problem is presented to illustrate the method for solving for the lateral earth forces

2 BACKGROUND

Rankine (1857) presented a method of calculating active and passive forces for a smooth wall used a planar slip surface. Coulomb's method (1776) satisfies force equilibrium for a sliding wedge. This method is adequate for a problem that involves a modest amount of wall friction in the active and the passive cases. However, the Coulomb (1776) solution is not suitable when computing the passive resistance of a wall with large values of wall friction due to the assumption of a planar slip surface (Terzaghi and Peck, 1967).

Janbu (1957) suggested the use of the generalized method of slices to solve lateral earth force problems. Shields and Tolunay (1972) computed values for the passive earth pressure coefficient, K_p, based on the logarithmic spiral method.

Fredlund, et al. (1981) proposed the use of the GLE method of slices for slope stability problems. Rahardjo (1982) extended the GLE formulation to lateral earth pressure problems. Fan (1983) used the finite element method and a linear elastic soil model to compute an approximate directional function for the resultant interslice forces associated with the active and the passive cases.

In this paper, based on Zakerzadeh (1998), interslice force functions are used along with the GLE method to calculate the lateral earth force for an example problem. A vertical wall with a horizontal backfill composed of a granular material (without pore-water pressure) is used in the example. The Coulomb, Rankine, and Shields and Tolunay (1972) will be used as a basis for comparison.

3 THEORY OF THE GLE METHOD TO COMPUTE LATERAL EARTH FORCES

An arbitrary slip surface is assumed, and the soil mass above the slip surface is divided into several slices. Force and moment equilibrium equations are applied to the soil mass that is at the point of incipient failure. The forces acting on each slice are identified and the magnitude of the shear force mobilized on the base of each slice is computed in accordance with the Mohr-Coulomb failure criterion.

Figure 1 shows the forces acting on one slice of an assumed slip surface for the active case. The normal force, N, and the weight of each slice, W, are assumed to act through the center of the base of each slice. The lateral earth force, P, acts on the last slice at an inclination angle, ω to the horizontal. The normal force on the base of the slice, N, is computed using vertical force equilibrium. When the wall is vertical, the angle, ω, is equal to the wall friction angle, δ.

Assuming a factor of safety, F_{sf}, equal to 1.0 (or a value near to 1.0), the magnitude of the lateral earth force, P, can be calculated from the overall equilibrium of forces in the horizontal direction.

It is desirable to directly solve for the lateral force, P; however, the Slope/W computer software (i.e., proprietary slope stability computer program developed by Geo-slope International Ltd., Canada) used in this study solves for the factor of safety, F_{sf}. Therefore, it is necessary to use a trial and error procedure to compute the lateral earth force corresponding to a designated factor of safety (e.g., F_{sf} = 1.0). Figure 2 is a flowchart showing the conceptual procedure for solving for the active or passive earth pressure (Zakerzadeh, 1998).

The point of application of the lateral earth force, d, can be calculated from moment equilibrium about the center of rotation (x_c, y_c in Figure 1). Assuming a factor of safety, F_{sm}, equal to 1.0 (or a value near to 1.0), the point of application, d, is calculated.

Once again, it would be desirable to directly solve for the distance, d; however, the Slope/W computer software solves for the factor of safety, F_{sm}, and it is necessary to use a trial and error procedure to solve for d (Figure 2).

The point of application of the lateral earth force, d, is a function of the interslice shear force, X, and therefore the point of application of the lateral earth force is dependent on the interslice force function used in the analysis.

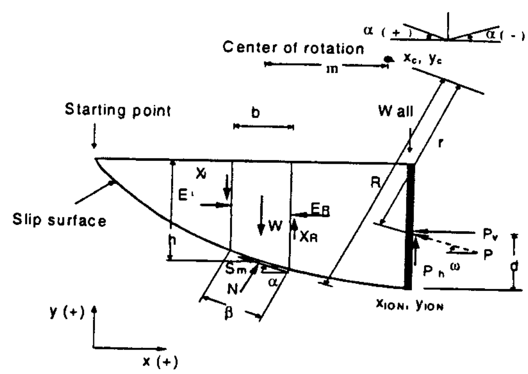

Figure 1. Forces acting on a slice of a slip surface and a vertical wall in the lateral earth pressure problem (active case).

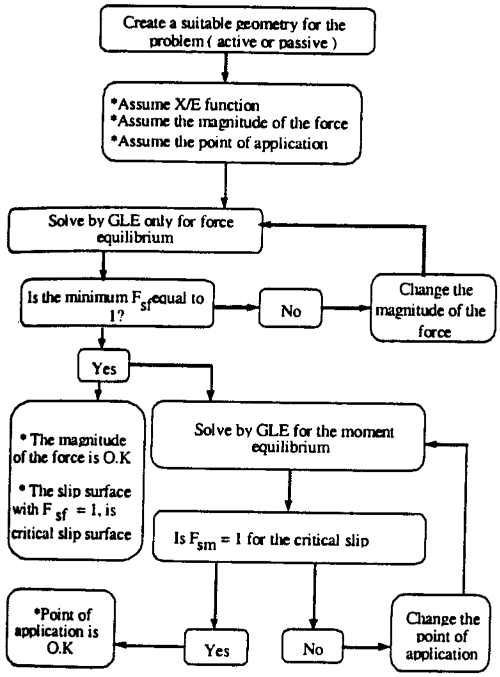

Figure 2. Flowchart to solve the lateral earth pressure problem by limit equilibrium analysis.

4 AN EXAMPLE PROBLEM TO ILLUSTRATE LATERAL EARTH FORCE CALCULATIONS

Figures 3 and 4 show the geometry of the problem used in this study for the active and passive cases, respectively (Zakerzadeh, 1998). The slip surfaces are assumed to be circular and to go through the base of the wall. The grid of centers of rotation is shown in Figure 3 and Figure 4 respectively. Each point on the grid is related to one slip surface through the soil. The inclination angle of the grid of centers of rotation for the active case is [45 - ϕ/2] to the horizontal (Figure 3). For the passive case the inclination angle of the grid of centers of rotations is [45 + ϕ/2] to the horizontal (Figure 4). The angle of wall friction ranges from zero to a maximum of 20 degrees (i.e., 2/3 the angle of shearing resistance of the soil).

5 THE INTERSLICE FORCE FUNCTIONS

The interslice force direction can be described as follows using an arbitrary interslice force function, $f(x)$, (Morgenstern and Price, 1965).

$$\frac{X}{E} = \lambda\, f(x) \qquad (1)$$

where: $f(x)$ = a mathematical function that describes the relationship between the shear force, X, and normal force, E, across the sliding mass, x = the length from the starting point of slip surface to a desired point on the slip surface (Figure 5a), and λ = a coefficient representing the fraction of the function used in the calculation of the lateral earth force.

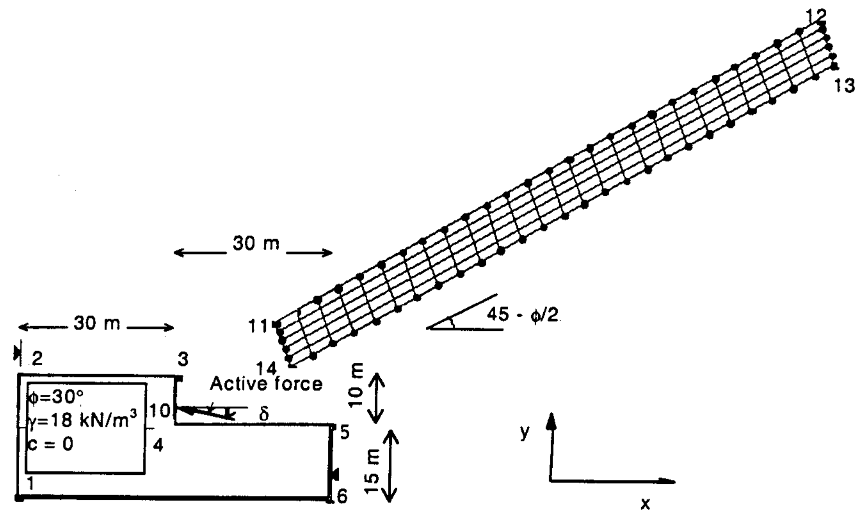

Figure 3. Geometry and grid of center of rotation of the example problem for the active case.

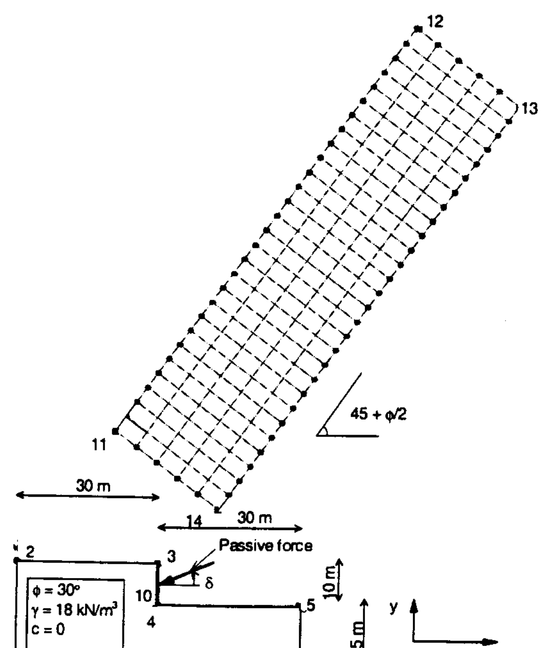

Figure 4. Geometry and grid of center of rotation of the example problem for the passive case.

5.1 Appropriate Interslice Force Function for the Active Earth Force Case

The magnitude of the force function is set to zero at the starting point of the slip surface (i.e., the extreme left point in Figure 5a). The magnitude of the force function at the wall is set equal to the tangent of the wall friction angle. Between these two boundaries, the interslice force function is assumed to vary linearly. The displacement of the soil mass mobilizes positive shear stresses for all elements within the sliding mass. This behavior accounts for the positive interslice force ratios across the entire sliding mass for the active case. Typical interslice force functions that were applied for the active case are indicated in Figure 5a.

Using (1) to define the interslice force ratio, the λ variable can be set equal to ($\tan\delta$) and the interslice force function, $f(x)$, will reach a maximum value of 1.0 at the wall. To evaluate the effect of the interslice force function, the λ variable was varied from 0 to (0.5 $\tan\delta$) to ($\tan\delta$). When using λ equal to ($\tan\delta$), the relationship for the interslice force function can be expressed as follows:

$$\frac{X}{E} = (\tan\delta)\frac{x}{L} \quad \text{for} \quad 0 \le x \le L \qquad (2)$$

where: L = horizontal length along the slip surface.

5.2 Appropriate Interslice Force Function for the Passive Earth Force Case

For the passive case the X/E ratio at the starting point of the slip surface is set equal to zero. At the end point of the slip surface (i.e., close to the wall), the X/E ratio is set equal to (-tan δ). For the passive case, the shear force mobilized, S_m, acts downward, since the soil mass is being moved upward. The direction of the wall friction is negative, and as a result the interslice force function is negative. The magnitude of the side force ratio reduces from a maximum value at the wall to zero at some distance from the edge of the wall (i.e., point 1 in Figure 5b). To the left of point 1, where the shear force vanishes, the interslice side force ratio remains essentially at zero. The interslice force function can be assumed to vary linearly between points 1 and 2. Typical interslice force functions that were applied for the passive case are indicated in Figure 5b.

Using (1) to define the interslice force ratio, the λ variable can be set equal to (tan δ), and the interslice force function, $f(x)$, will reach a minimum of -1.0 at the wall. The relationship for the interslice force function can be expressed as:

$$\frac{X}{E}=0 \quad \text{for} \quad 0\leq x\leq \eta \qquad (3)$$

$$\frac{X}{E}=\tan\delta\,\frac{(\eta L-x)}{L-\eta L} \quad \text{for } \eta\leq x\leq L \qquad (4)$$

where: η = the ratio of the length from the starting point of the slip surface (0 to point 1 on Figure 5b) to the total distance across the slip surface, L.

The location of point 1 can also be considered to be a variable, and therefore values of η were varied (i.e., η = 0, 0.25, 0.5, and 0.75) for the analyses.

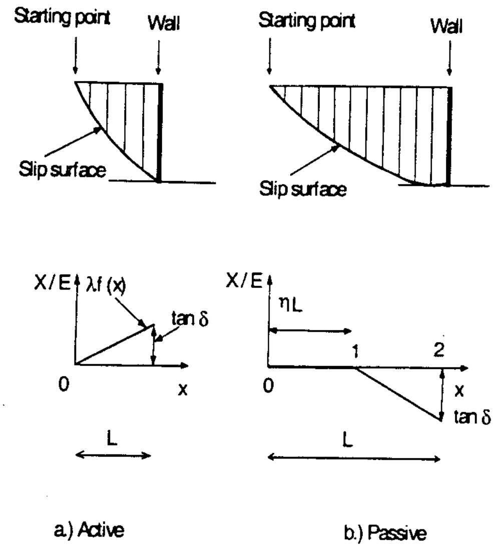

Figure 5. Typical interslice force function, X/E, for a.) active case and the b.) passive case

6 PROCEDURE TO COMPUTE THE MAGNITUDE AND THE POINT OF APPLICATION OF THE LATERAL EARTH FORCE

The magnitude of the active and the passive earth forces are computed using the force equilibrium solution. The external load, P, that brings the soil mass into the state of limiting equilibrium is first computed. In other words, for a trial value of lateral earth force, the force factors of safety, F_{sf}, for different slip surfaces are computed, and the minimum factor of safety amongst different slip surfaces is determined. If the minimum factor of safety is not equal to 1.0, another lateral earth force is applied and the new value for the factor of safety, F_{sf}, is computed. The computed value for the factor of safety, F_{sf}, is used to repeat the procedure and to compute another value of F_{sf}. Computations are repeated until the difference between two successive values for the factor of safety, F_{sf}, is less than 0.001.

Once force equilibrium has been satisfied, the magnitude of the lateral earth force and the location of the critical slip surface, have been identified (Figure 2).

The moment equilibrium solution is used to compute the point of application, d, for the active or passive earth force cases. The point of application of the lateral earth force is determined by varying its location until the moment factor of safety for the critical slip surface is equal to 1.0. (Figure 2)

7 RESULTS FOR THE ACTIVE FORCE CASE

The forces calculated for the active case are compared with results of the Coulomb and Rankine methods (Zakerzadeh, 1998). Figure 6 shows that for a wall without friction, the force computed using the Coulomb and Rankine methods compare well with the solution using the limit equilibrium method and selected interslice force functions.

The values for the active force are essentially the same, and therefore it can be concluded that for a smooth wall there is no need to use any interslice force function. For any given value of wall friction, the active force decreases with an increasing value of λ. The effect is slightly more pronounced with increasing values of wall friction. All values of the active force computed using the GLE method are slightly higher than the values computed using the Coulomb method. The selected function using λ equal to ($\tan \delta$), appears to be acceptable when simulating the Coulomb case.

Figure 7 shows that the point of application of the active force may increase or decrease with increasing angles of wall friction, depending upon the λ values. The point of application for the active force when using the GLE analysis for δ equal zero is higher than the third point because of the circular nature of the slip surface. The point of application increases with an increase in the wall friction angle when λ is equal to zero. However, for λ equal to ($0.5 \tan \delta$) or λ equal to ($\tan \delta$), the point of application decreases with increasing wall friction angles, δ.

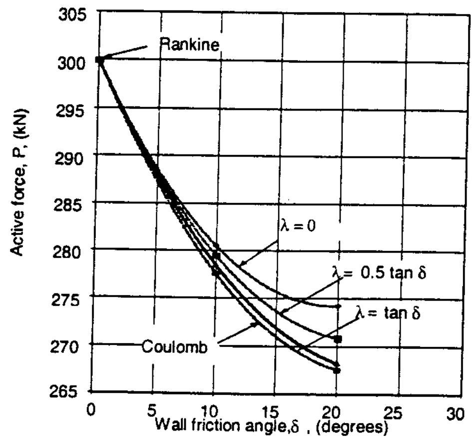

Figure 6. Active force versus wall friction angle for different interslice force functions specified by λ.

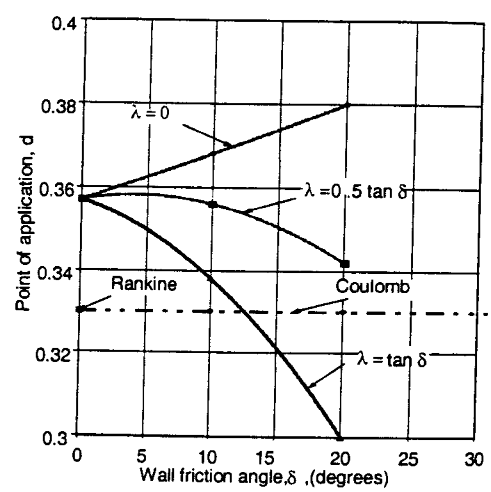

Figure 7. Active case: point of application of the force versus wall friction angle for different interslice force functions. specified by λ

8 RESULTS FOR THE PASSIVE EARTH FORCE CASE

Figure 8 shows the passive force versus wall friction angle for different interslice force functions. Various force functions are used for the passive case and comparisons are made with the Rankine, Coulomb, and Shields and Tolunay (1972) methods.

The value of λ has been set equal to ($\tan\delta$) for all interslice force functions used in the analysis of the passive case. When the wall friction angle, δ, is equal to zero, the passive force computed by the GLE method is essentially the same as that for the Rankine, Coulomb, and Shields and Tolunay (1972) solutions. Therefore, it can be assumed, for a smooth wall there is no need to use any interslice force function. Increasing η values decrease the calculated passive force. The effect is more pronounced with increasing values of wall friction.

The Coulomb analysis provides the highest values for the passive force which are on the unsafe side, while Shields and Tolunay (1972) values fall on the lower side of the GLE analysis. Values of η equal to 0.5 or 0.75 produce results that appear to be acceptable.

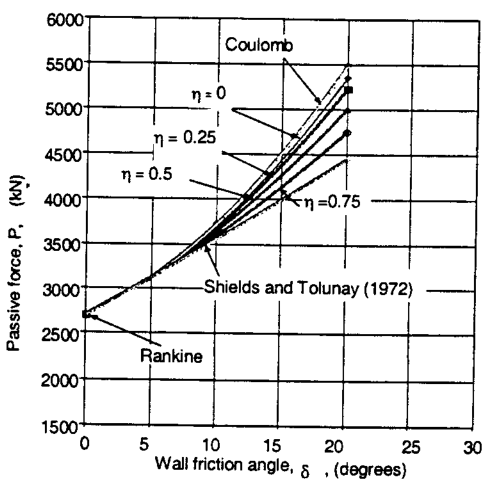

Figure 8. Passive force, versus wall friction angle, for different interslice force functions specified by η; (λ = tan δ)

Figure 9 shows the point of application of the passive force versus wall friction angle. For a wall friction angle equal to zero, different interslice force functions have no effect on the computed point of application. For a wall friction angle equal to zero, the point of application for the passive case appears to be slightly lower than the third point because of the circular nature of slip surface. The point of application increases with an increasing wall friction angle. The effect is more pronounced for lower values of η. James and Brandsby (1970) show that the point of application of the passive force is generally higher than the third point. Therefore, the interslice force function defined by using η equal to 0.5 provides reasonable values for the point of application.

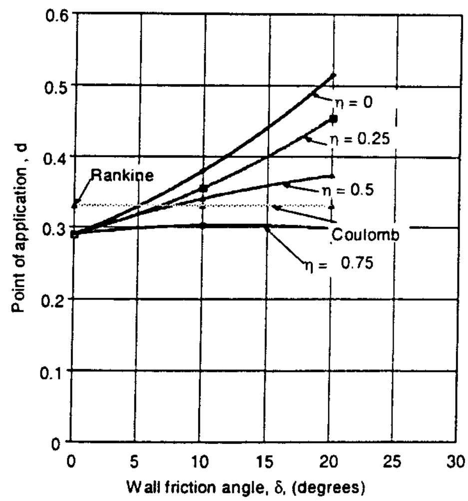

Figure 9. Passive case: point of application of the force versus wall friction angle for different interslice force functions. specified by η; (λ = tanδ)

9 SUMMARY OF FINDINGS ON EARTH FORCE CALCULATIONS

The results indicate that the GLE method can provide a versatile and reliable means of calculating earth forces for both the active and passive cases. For each case, the interslice force function that should be used in the analysis is described by (1).

A reasonable interslice force function, $f(x)$, and a λ value equal to the wall friction value (tan δ) appears to give satisfactory results for the active and the passive cases.

For the active case:

$$f(x) = x / L \quad (5)$$

For the passive case:

$$f(x) = 0 \text{ for } 0 \leq x \leq 0.5L \quad (6)$$

$$f(x) = \frac{(L-2x)}{L} \tan\delta \text{ for } 0.5L \leq x \leq L \quad (7)$$

In the GLE method, force equilibrium, along with (5) to (7) for the selected interslice force function, gives the magnitude of lateral earth force, P. Moment equilibrium, along with (5) to (7) for the selected interslice force function gives the point of application of the lateral earth force, d.

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