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Permeability Functions For Unsaturated Residual Soils

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ABSTRACT

A summary of the difficulties of both direct and indirect measurements for permeability of unsaturated soils is presented. The coefficient of permeability for an unsaturated soil is primarily determined by the pore-size distribution of the soil and can be predicted from the soil-water characteristic curve. The prediction of the coefficient of permeability functions for some Hong Kong soils is presented in this paper. The laboratory soil-water characteristic curve data was fitted using Fredlund and Xing (1994) equation. A brief description of the Modified Tempe cell used for obtaining the laboratory soil-water characteristic curve data is presented. The prediction of each coefficient of permeability for the soils was computed from the fitted soil-water characteristic curve, using the saturated coefficient of permeability, k_s, as the initial value. The results show that most of the residual soils in Hong Kong have a values (a is an approximate value of an air-entry value of a soil) of about 1-30 kPa. The desaturation rate parameter, n, values range between 0.8 to 5. The paper also shows the influence of matric suction, $(u_a - u_w)$ on the coefficient of permeability function of a soil.

Keywords
Permeability, soil-water characteristic curve, air-entry value, matric suction, unsaturated soils.

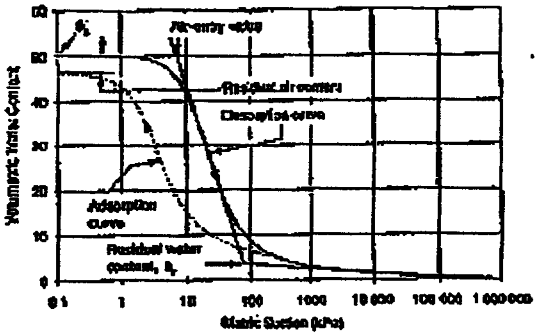
INTRODUCTION

There is no engineering soil property that can vary more widely than that of the coefficient of permeability. For saturated soils, the coefficient of permeability can vary more than 10 orders of magnitude when considering soils that range from gravel to clay (Fredlund et. al, 1994). This wide range in coefficient of permeability has proven to be a major obstacle in analyzing seepage problems.

Soils that become unsaturated are even more difficult to analyze. In this case it is possible for a single soil to have a coefficient of permeability that ranges over 10 orders of magnitude. Initial consideration of problems involving unsaturated soils might lead an engineer to conclude that no useful analyses are possible when the soil becomes unsaturated. However, experience has now shown that many important questions can be addressed using seepage analyses on unsaturated soils.

The permeability function of a soil affects the infiltration characteristics and will determine the shear strength and volume change behaviour of the soil as the water content changes. The coefficient of permeability of a soil is a maximum at saturation. As the soil desaturates, the coefficient of permeability of the soil decreases. The measurement of the coefficient is difficult. However, the relationship between the coefficient of permeability and the degree of saturation can be related to the soil-water characteristic curve (i.e., a plot of volumetric water content versus matric suction) of the soil. The coefficient of permeability function can be computed from the soil-water characteristic curve (Fig. 1), using the saturated coefficient of permeability, k_s, as the initial value.

Figure 1 Typical soil-water characteristic curve for



a silty soil.

PERMEABILITY OF UNSATURATED SOILS

Permeability of saturated soil, k_s, is a function of void ratio, e, only and remains essentially constant throughout the process of seepage flow. For unsaturated soils, the coefficient of permeability with respect to water, k_w, is a function of both void ratio, e, and water content, w. Since void ratio, water content and degree of saturation, S, are interelated, k_w can be expressed as a function of any two of three parameters (Leong and Rahardjo, 1997), i.e.,

$$k_w = f(e, w); \quad k_w = f(S, e); \quad k_w = f(S, w) \qquad [1a\text{-}c]$$

If the soil structure is assumed to be incompressible, then it is possible to decouple the two parameters in Eq. 1 where the saturated coefficient of permeability, k_s, will quantify the effect of void ratio and another function will account for the effect of water content in soil.

For unsaturated soils the coefficient of permeability, k_w, is also a function of matric suction, $(u_a - u_w)$. The coefficient of permeability for unsaturated soil can be expressed as a function of a function of matric suction, $(u_a - u_w)$ and the saturated coefficient of permeability, k_s (Kasim et. al, 1998).

$$k_w = f\{(u_a - u_w),\ k_s\} \qquad [2]$$

When the pore-air, u_a, remains constant at atmospheric condition (i.e., $u_a = 0$), the coefficient of permeability, k_w, for unsaturated soil can be defined as a function of negative pore-water pressure and the saturated coefficient of permeability, i.e., $k_w = f\{(-u_w), k_s\}$.

Difficulties of Measuring Permeability of Unsaturated Soils

Measurement of permeability of unsaturated soils is a very time-consuming process. The duration of the test increases with decreasing water content of the soil. The permeability values can differ by several orders in magnitude causing direct measurement to be very difficult, as there is no apparatus that can measure such a wide range of permeability values efficiently. In this paper, only laboratory measurements are discussed.

There are direct and indirect measurements of permeability. In direct measurement, there are steady-state and unsteady-state methods (Leong and Rahardjo, 1997).

a) In the steady-state method, a matric suction is first imposed on a soil specimen using the axis-translation technique (Fredlund and Rahardjo, 1993). At equilibrium, denoted by constant water content, a hydraulic gradient is then imposed across the soil specimen. The flow rate is then measured and the permeability is obtained via Darcy's law (1856).
b) Using the unsteady-state method or instantaneous profile method, a cylindrical specimen is subjected to a continuous flow of water from one end. The hydraulic gradient and the flow rate at various points along the specimen are computed by monitoring water content and/or pore-water pressure at these points.

The associated problems of direct measurement (Leong and Rahardjo, 1997) are:

1. A long time is required to complete a series of permeability measurements as the coefficient of unsaturated soil is very low, especially at high matric suction.
2. Because of the low flow rate, the measurement of volume change of water must be very accurate. Water loss from or within the apparatus and air-diffusion through the water can introduce serious errors in the measurement of volume.
3. In some cases, an osmotic suction gradient may develop between the pore-water within the soil and the pure-water that is used as the permeating fluid. This gradient will induce an additional osmotic flow across the specimen. The osmotic flow becomes more significant as the water content of the specimen decreases.
4. As matric suction increases, the specimen may shrink from the wall of the cell and also from the high air-entry disk. The air gap will disrupt the continuity of water flow, as air is nonconductive to water flow. For the instantaneous profile method, the soil may shrink away from the instruments that are used to measure pore-water pressure changes.

In indirect measurement, the water content of the soil specimen at various matric suction values is determined. The permeability is then inferred from the soil-water characteristic curve, using a statistical method. Compared with direct measurement using the steady-state method, the duration of test is greatly reduced as the test only lasts up to the time the water content in the soil specimen equilibrates with imposed matric suction. The problems associated with the indirect measurement:

1. Determination of the end point where the water content becomes constant at imposed matric suction can be difficult.
2. Similar to direct measurement, measurement of water volume changes must be accurate. This is especially so at a very high matric suction.
3. Air diffusion through the ceramic disk reduces the accuracy of water volume determination.
4. As most indirect measurement devices use an 'open system' with the water pressure at atmospheric, losses of water through evaporation and in some cases drying of the ceramic disk will cause a large error in the determination of water content of the soil specimen.
5. The soil specimen will shrink at high matric suction and the volumetric water content requires the determination of the volume of the soil specimen. The volume of the soil specimen is difficult to determine, causing some errors in the soil-water characteristic curve determination.

DETERMINATION OF LABORATORY SOIL-WATER CHARACTERISTIC CURVE

The soil samples used in this study were taken from 12 selected sites in Hong Kong. The Geotechnical Engineering Office (GEO) of Hong Kong provided the samples. The soil-water characteristic curve data for each specimen were obtained from the Modified Tempe cell tests in 1996. The Modified Tempe cell were designed and fabricated at the Department of Civil Engineering, University of Saskatchewan in the early 1990's to withstand air-pressures of 0-5 Bar (i.e., 0 to 500 kPa). The cross section of the Modified Tempe cell is shown in Fig. 2. The cell was modified from the

Tempe cell of the Soil Moisture Corporation. The Tempe cell of Soil Moisture Corporation can be used for the matric suction up to 100 kPa.

Three specimens from each soil sample were tested using high air-entry ceramic disks of 1-Bar, 3-Bar and 5-Bar, respectively. Each specimen was subjected to 'drying' test. In the drying test the soil starts at a saturation condition and the matric suction is gradually increased leading to a reduction in the water content in the soil specimen.

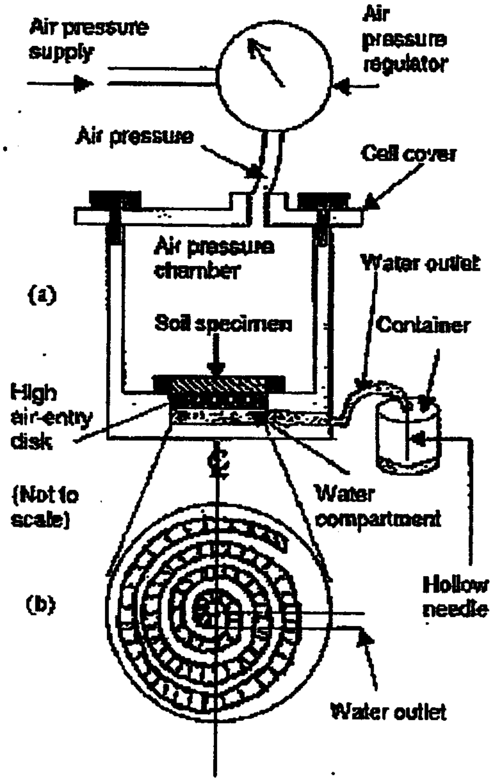


Figure 2 (a) Modified Tempe cell.
(b) Plan view of the base of the grooved water compartment underneath the high air-entry ceramic disk.

A soil specimen is placed on the high air-entry ceramic disk inside the retaining cylinder. A test is started by saturating the high air-entry ceramic disk and the soil specimen. After saturating the ceramic disk and the soil specimen, excess water is removed from the cell. The cell cover is then mounted and tightened into place, and ir-pressure is applied to the soil specimen. The air pressure is applied to the soil specimen. The air pressure is set equal to the desired matric suction value (i.e., 0 to 500 kPa).

Once the air pressure is applied, water starts draining from the specimen through the high air-entry ceramic disk until equilibrium is reached. The drained water is collected in the water container. The matric suction in the soil specimen is then equal to the applied air-pressure. The time required to reach equilibrium depends upon thickness and permeability of the specimen and the permeability of the high air-entry ceramic disk. The change in water content is measured by weighing the collected water and the water container after equilibrium is reached (Vanapalli, 1994).

The procedure is then repeated at higher applied air pressures (i.e., higher matric suctions). Once the highest matric suction has been applied, the air pressure in the cell is released and the soil specimen is removed. The water content corresponding to the highest matric suction is determined by oven-drying the soil specimen. This water content, together with the previous changes in weight, is used to back-calculate the water contents corresponding to the other matric suction values. The matric suctions are then plotted against their corresponding water contents to dive the soil-water characteristic curve data.

DETERMINATION OF COEFFICIENT OF PERMEABILITY FUNCTION

The parameters required for obtaining the coefficient of permeability function from the soil-water characteristic curve are shown in Fig. 3. Equation 3, proposed by Fredlund and Xing (1994) was used to fit the data obtained from the Modified Tempe cell tests to compute the best-fit soil-water characteristic curve.

$$\theta = \left\{1 - \frac{\ln\left(1+\dfrac{h}{h_r}\right)}{\ln\left(1+\dfrac{1000000}{h_r}\right)}\right\}\left\{\frac{1}{\ln\left(e+\left(\dfrac{h}{a}\right)^n\right)}\right\}^m \qquad [3]$$

where:

θ = volumetric water content
h = matric suction head
e = natural number, 2.71828 ......
h_r = matric suction corresponding to the residual water content
a = matric suction value at the inflection point of the soil-water characteristic curve. This matric suction value is closely related to the air-entry

$$c = \frac{\theta(e^y) - \theta(\psi)}{e^y}\theta'(e^y)dy$$

value of the soil. The air-entry value is a matric suction beyond which the soil starts to desaturate (Fig. 3)
n = soil desaturation rate parameter, a parameter

$$\left(d = \int_{\log(\psi\_\_)}^{b} \frac{\theta(e^y) - \theta_s}{e^y}\theta'(e^y)dy \right)$$

designating the slope at the inflection point of the soil-water characteristic curve (Fig. 3)

m = a parameter which is associated with the residual water content (Fig. 3)

The corresponding coefficient of permeability function is computed from the best-fit soil-water characteristic curve, starting with the value of the saturated coefficient of permeability by using the computer program, CFVIEW. The computation of the permeability function was done using the following modified Mualem equation:

$$k_r(\psi) = \int_{\log(\psi)}^{b} cd \qquad [4]$$

where:

ψ = matric suction
k_r = relative permeability as a function of matric suction, ψ
b = log (1000000)
y = dummy variable of integration representing the logarithm of suction
aev = air-entry value

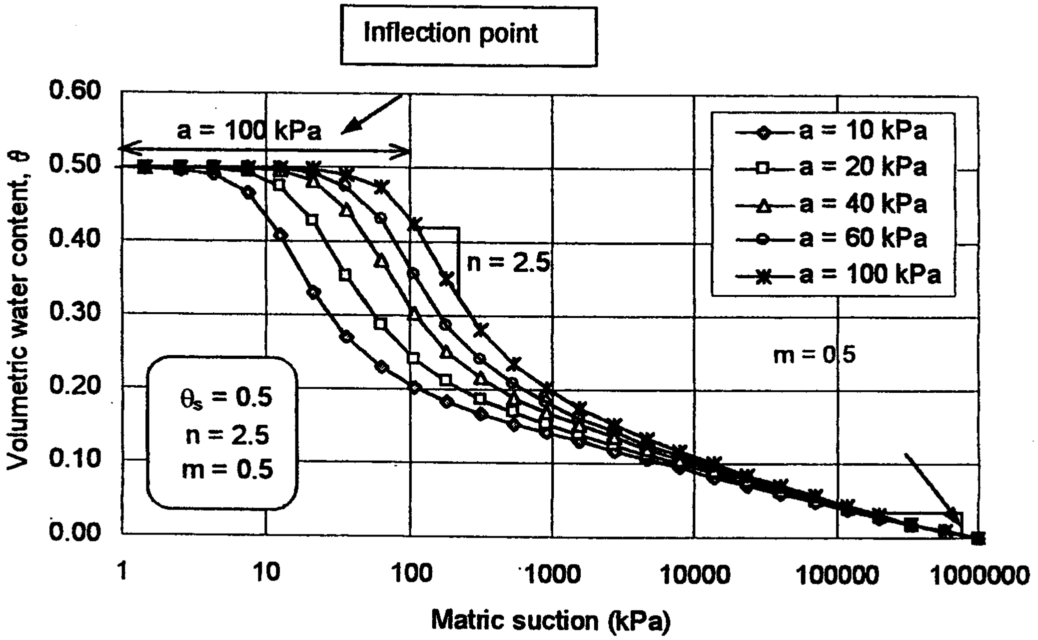


Figure 3 Parameters (*a*, *n* and *m*) used in the determination of the best-fit soil-water characteristic curve and the coefficient of permeability function.

PRESENTATION AND DISCUSSIONS OF RESULTS

Figures 4a to 4c present the experimental soil-water characteristic curve data and the corresponding best-fit soil-water characteristic curves for the Kwun Lung Lau, Chai Wan Fei Tsui and Junk Bay soil specimens, respectively, tested in the 3-Bar Modified Tempe cell. The results of the best-fit parameters (i.e., *a*, *n* and *m*) and the average saturated coefficient of permeability, k_s, for various Hong Kong soil samples are shown in Table 1. Three specimens from each soil sample (except the Junk Bay, the Kwun Lung Lau and the Nam Long Shan soil samples) were tested in the 1-Bar, 3-Bar and 5-Bar Modified Tempe cell.

Figure 5 presents the best-fit soil-water characteristic curve and the corresponding predicted coefficient of permeability function for the Butterfly Valley soil specimen tested in the 3-Bar Modified Tempe cell.

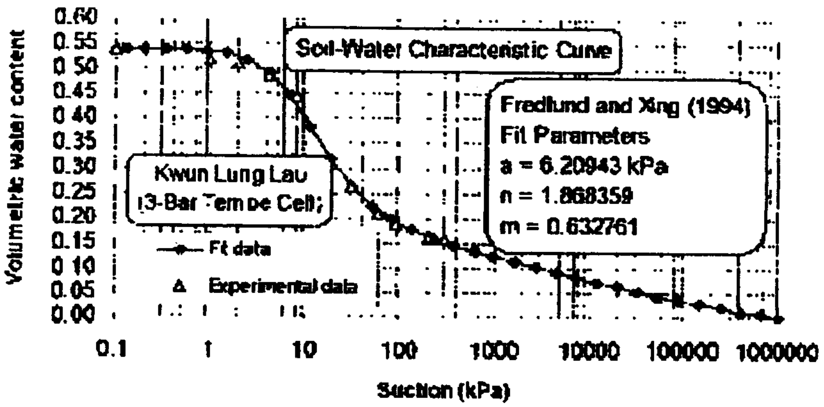
Figures 4a-c and Table 1 shows that most of the Hong Kong soils have *a* values of 1 kPa to 30 kPa, except for the Chai Wan Fei Tsui and the Junk bay soils. The results illustrate that most of Hong Kong soils are granular and porous and contain high percentages of sand. For examples, *a* values for the soil samples from the Shousun Hill are were found to be about 1 kPa to 3 kPa. The Shousun Hill soil starts to desaturate at suction values of about 1 kPa to 3 kPa and it contains sand and some gravels formed from decomposed granite. The Chai Wan Fei Tsui and the Junk Bay soil samples were taken from the coastal areas. Therefore, the *a* values for the soil samples were about 5 kPa to 365 kPa (higher than the *a* values for the Shousun Hill soil samples). The *a* values (close to an air-entry value of a soil) increases with increasing percentages of fine particles such as silt and clay. The coefficient of permeability of any soils is constant at the value of the saturated coefficient of permeability, k_s, until the suction value reaches the air-entry value of the soil. In other words, a fine-grained soil remains at saturation condition at a larger range of suction values than does a course-grained soil.

The desaturation rate parameter, n values for the Chai Wan Fei Tsui and the Junk Bay soils were less than 2.0. The desaturation rate parameter, n values for the Shousun Hill soil and other course-grained soils were about 0.6 to 4.73. The course-grained soils tend to have higher n values than does the fine-grained soils. Within a same range of suction value in the desaturation phase, the coefficients of permeability for a course-grained soil vary over larger orders of magnitude compares to the variation in the coefficients of permeability for a fine-grained soil. For example, the coefficients of permeability for the Shousun Hill and the Chai Wan Fei Tsui soils decrease by about 5 and 2 orders of magnitude, respectively, when the suction value increases from 0 kPa (i.e., saturated condition) to 100 kPa (Fig. 6).

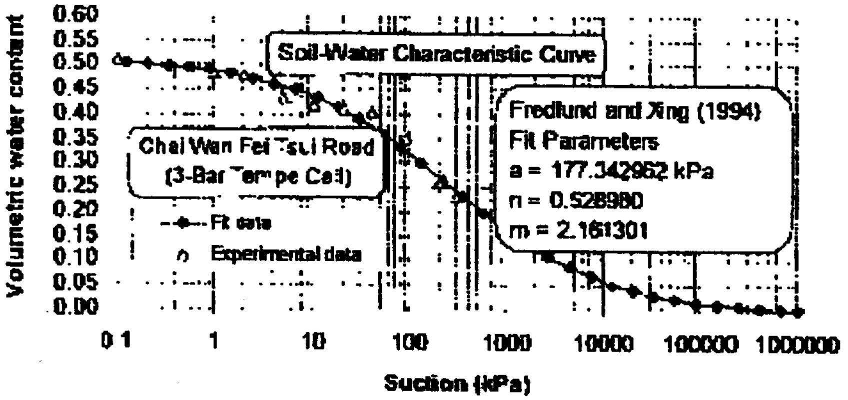
The m values for the Hong Kong residual soils range from 0.1 to 2.95. The m value (related to a residual water content of a soil) does not influence the variation of the coefficient of permeability of unsaturated soils significantly because the field suction available usually is less than 1000 kPa whereas the residual water content is assumed occurs at suction value of more than 3000 kPa. If the m value is unknown, an m value of 0.5 is recommended.

Table 1 The results of the best-fit parameters (a, n and m) and the saturated coefficients of permeability for various Hong Kong soils.

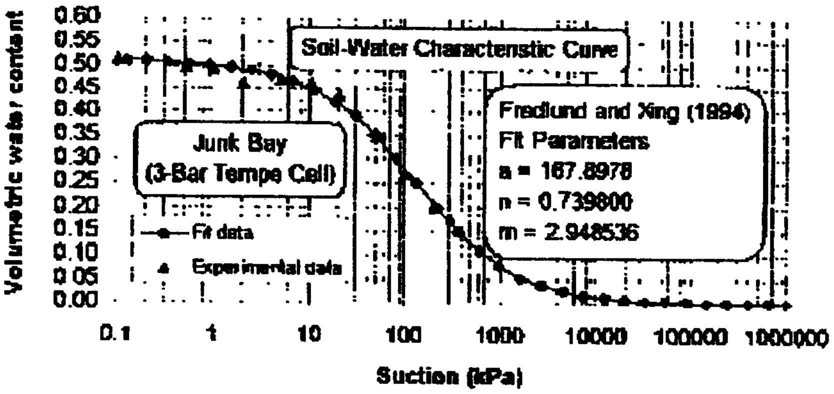
Hong Kong soils	**Modified Tempe cell tests**	***a* (kPa)**	***n***	***m***	**Average k_s (m/s)**
Butterfly Valley	1-Bar	1.58	1.97	0.32	3.30E-05
	3-Bar	1.95	1.42	0.39	
Chai Wan Fei Tsui	1-Bar	133.27	1.57	1.39	7.03E-06
	3-Bar	177.34	0.53	2.16	
	5-Bar	364.53	0.67	2.28	
Junk Bay	1-Bar	4.75	0.79	0.56	3.14E-06
	3-Bar	167.90	0.74	2.95	
	5-Bar	58.12	0.55	2.09	
Kwun Lung Lau	1-Bar (a)	10.45	1.21	0.26	9.47E-06
	1-Bar (b)	6.85	1.73	0.65	
	3-Bar	6.21	1.87	0.63	
Nam Long Shan	1-Bar	2.05	3.36	0.12	2.93E-05
	3-Bar	1.85	1.84	0.16	
Shousun Hill	1-Bar	0.80	3.68	0.29	3.80E-04
	3-Bar	0.82	4.73	0.29	
	5-Bar	2.95	1.06	0.56	
Shaukel Wan	1-Bar	1.04	2.74	0.19	2.27E-05
	3-Bar	11.23	0.58	0.82	
	5-Bar	7.65	0.82	0.77	
Shum Wan	1-Bar	1.16	2.04	0.19	1.71E-05
	3-Bar	26.78	0.7	0.88	
	5-Bar	5.01	0.79	0.51	



(a) Kwun Lung Lau specimen tested in the 3-Bar Modified Tempe cell.



(b) Chai Wan Fei Tsui Road specimen tested in the 3-Bar Modified Tempe cell.



(c) Junk Bay specimen tested in the 3-Bar Modified Tempe cell

Figure 4 The experimental and fitted Soil-Water Characteristic Curves (SWCC) for various Hong Kong soil samples.

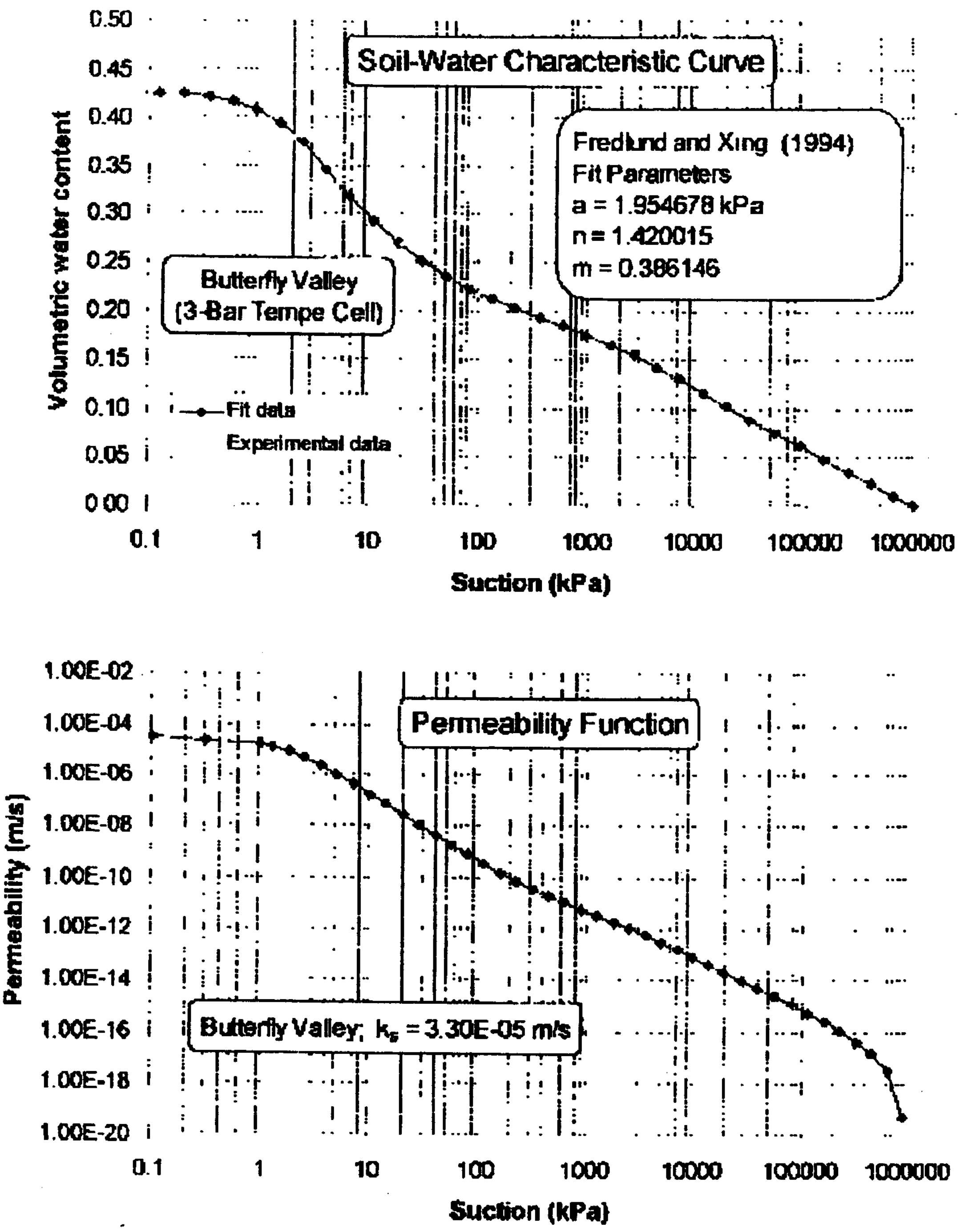


Figure 5 The soil-water characteristic curve and the corresponding coefficient of permeability function for the Butterfly Valley soil specimen.

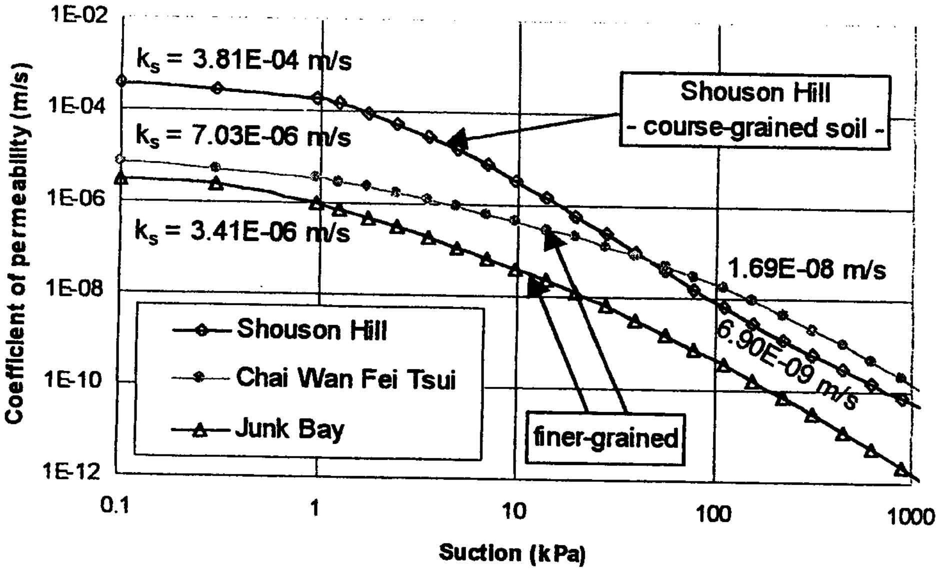


Figure 6 Decreases in the coefficient of permeability with increases in the suction values for various Hong Kong soils.

CONCLUSIONS

Several conclusions can be made as follows:

1. Coefficient of permeability is required for many geotechnical applications. However, such measurements for unsaturated soils are time-consuming and tedious. Measurement of coefficient of permeability for unsaturated soil at low water content is not easy to perform and demand a highly accurate means of determination of water volume change.
2. Since establishing the soil-water characteristic curve is generally not as difficult as measuring the coefficient as measuring the coefficient of permeability at various suction levels, the task of defining the coefficient of permeability function can be simplified by calculating the coefficient of permeability from the soil-water characteristic curve.
3. Most of residual soils have low air-entry value and therefore the soils at low suction values. As the suction value increases, the coefficient of permeability decreases. It is possible for a single soil to has a coefficient of permeability that ranges over 5 orders of magnitude when the suction value increases from 0 kPa (i.e., saturated condition) to 100 kPa.

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