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INTERPRETATION OF THE SHEAR STRENGTH OF UNSATURATED SOILS IN UNDRAINED LOADING CONDITIONS

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ABSTRACT

Theoretical procedures are presented in this paper to interpret the undrained shear strength of unsaturated soils in terms of two stress state variables, net normal stress, $(\sigma - u_a)$ and matric suction, $(u_a - u_w)$. The proposed theory is tested on a glacial till representing optimum water content conditions both in confined and unconfined loading conditions. The studies presented in this paper form a rational basis to understand the contribution of matric suction towards the undrained shear strength of unsaturated soils.

RÉSUMÉ

Des procédures théoriques pour interpréter la force de cisaillement des sols insaturés non-drainés en termes de deux variables d'état de stresse, le stresse normal net $(\sigma - u_a)$ et la matrice de succion $(u_a - u_w)$, sont présentées dans cet article. La théorie proposée est testée sur un glacier représentant des conditions de contenu en eau optimum dans des conditions de charge confinée et non-confinée. Les études présentées dans cet article forment une base rationnelle dans la compréhension de la contribution de la matrice de succion vers la force de cisaillement des sols insaturés non-drainés.

1. INTRODUCTION

Several semi-empirical procedures have been developed in the last five years to predict the shear strength behavior of unsaturated soils using the soil-water characteristic curve and the saturated shear strength parameters (Vanapalli et al. 1996a, Fredlund et al. 1996, Oberg and Sallfours 1997, Bao et al. 1998, Khalili and Khabbaz 1998). Fredlund et al. (1978) shear strength theory was extended for the analysis by all the above investigators. While these procedures are useful to understand the shear strength behavior in drained loading conditions in terms of stress state variables, net normal stress, $(\sigma - u_a)$ and matric suction, $(u_a - u_w)$, there are few studies related to the interpretation of undrained shear strength of unsaturated soils.

The ϕ equals zero concept is frequently used for undrained total stress analyses in assessing the stability of embankments, slopes and foundations located on saturated fine-grained soils. The design of pavements and the assessment of the ultimate bearing capacity of clays are two other examples that often utilize the undrained shear strength. The undrained shear strength, c_u, is commonly assumed to be half the unconfined compressive strength of the field specimens obtained at different depths assuming that the soil is in a state of saturated condition. However, this assumption is not adequate for unsaturated soils.

Most of the research related to the undrained shear strength of unsaturated soil comes from studies on compacted soils. Several properties such as initial water content, degree of saturation, soil structure, stress history etc., affect the undrained shear strength of unsaturated soils. The concept of particle structure introduced by Lambe (1960) has been used as a tool to explain undrained strength behavior. While these researchers described the nature of their results in terms of structure and molding water content, predictive capabilities using more fundamental measurable properties were not developed. In addition, there does not appear to be any relationships derived to express the undrained shear strength in terms of stress state variables.

Geotechnical engineers are aware that it is the matric suction that holds the soil together in unconfined compression. The independent contribution of matric suction towards the undrained shear strength is not consciously considered in most analyses. The matric suction in the specimen tested in the laboratory is a function of the insitu pore-water pressure and the change in pore-water pressure resulting from unloading the soil during sampling. Hence, the measured undrained shear strength must be interpreted, taking into account the influence of matric suction, $(u_a - u_w)$.

In this paper, relationships are developed using the Fredlund et al. (1978) equation to interpret the shear strength of unsaturated soils under undrained loading conditions. The proposed theory is tested on glacial till specimens compacted at optimum initial water content conditions.

2. UNDRAINED SHEAR STRENGTH

Volume change in unsaturated soils under undrained loading is due mainly to the compression of air. Compression of soil solids and water can be neglected for the stress ranges generally encountered in engineering practice. Undrained pore pressures are assumed to be generated immediately after loading. The applied total stress is carried by the soil structure, pore-air and pore-water depending on their relative compressibilities. Matric suction changes in an unsaturated soil under undrained loading conditions is analogous to the changes in pore-water pressures in saturated soils under similar conditions of loading.

The concept of pore pressure parameters was introduced by Skempton (1954) and Bishop (1954). The pore pressure parameters may vary depending on the stress path and are different for the air and water phases depending on the degree of saturation of the soil (Bishop 1961).

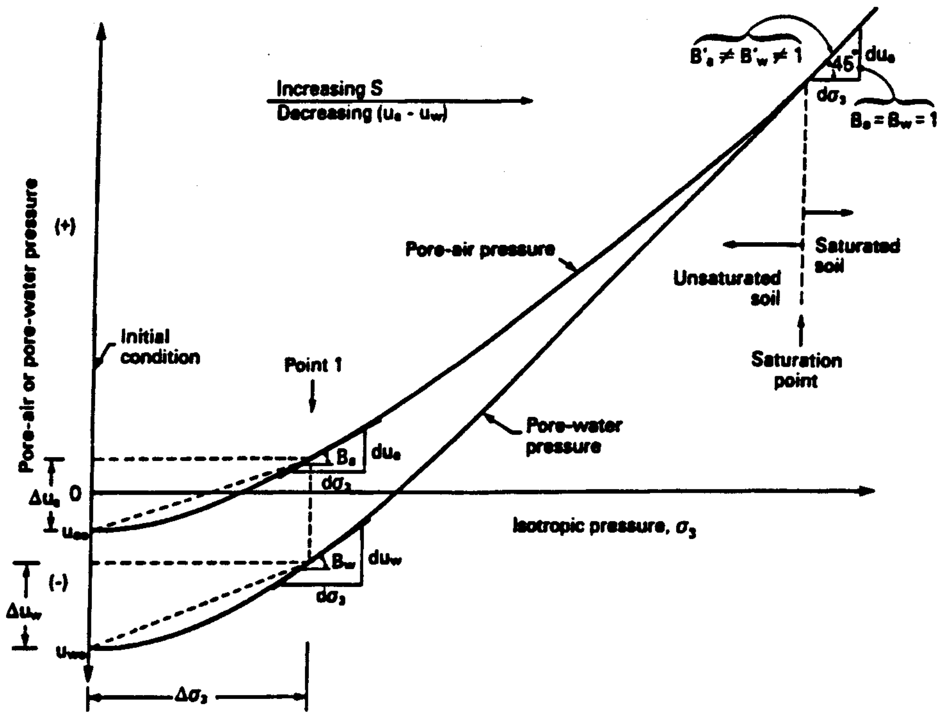

Figure. 1. Development of pore-pressures due to undrained loading conditions

Figure 1 shows the development of pore-air and pore-water pressures during undrained compression under an applied isotropic stress. Tangent pore pressure parameters for air and water phases for undrained loading conditions are:

$$B_a = \frac{du_a}{d\sigma_3} \qquad [1]$$

$$B_w = \frac{du_w}{d\sigma_3} \qquad [2]$$

where:

B_a = tangent pore-air pressure parameter

du_a = increase in pore-air pressure due to an infinitesimal increase in isotropic pressure, $d\sigma_3$

$d\sigma_3$ = infinitesimal increase in isotropic pressure

B_w = tangent pore-water pressure parameter

du_w = increase in pore-water pressure due to an infinitesimal increase in isotropic pressure, $d\sigma_3$

Pore pressure parameters B_a and B_w take into account the changes in matric suction occurring under increasing total stress. The pore pressure parameters are mainly a function of degree of saturation, compressibility and loading and change at varying rates in response to the applied total stress. B_a and B_w are less than one at degrees of saturation less than 100% and at complete saturation B_a equals B_w and approach one. The matric suction, $(u_a - u_w)$, in soil approaches zero once the soil approaches saturation under the applied loading condition.

For a compacted soil, the initial pore-air pressure can be assumed to be zero. The negative pressure in the soil with reference to atmospheric pressure is the matric suction. Under the application of total stress, the degree of saturation of the soil increases due to a decrease in total volume. Changes in pore-air, du_a, and pore-water pressure, du_w, due to a finite change in total isotropic pressure can be computed knowing the initial conditions of the soil (Hasan and Fredlund, 1980 and Fredlund and Rahardjo, 1993). A marching forward technique can be applied with finite increments of total stress to estimate the changes in pore pressures commencing from a known initial unsaturated condition of the soil.

Matric suction changes can also occur in an unsaturated soil during the shearing stage of triaxial loading conditions.

These changes are expressed in terms of D pore pressure parameters. Tangent pore pressure parameters under triaxial undrained loading conditions. The pore-air pressure parameter, D_a, can be defined as:

$$D_a = \frac{du_a}{d(\sigma_1 - \sigma_3)} \quad [3]$$

where:

D_a = tangent pore-air pressure for uniaxial, undrained loading
$d\sigma_1$ = finite increment in major principal stress
$d(\sigma_1 - \sigma_3)$ = finite increment in deviator stress

The pore-water pressure parameter, D_w, is defined as:

$$D_w = \frac{du_w}{d(\sigma_1 - \sigma_3)} \quad [4]$$

In a triaxial test the total stress increment $d\sigma_2$ equals $d\sigma_3$. The major principal stress increment of total stress, $d\sigma_1$, is applied axially. The development of pore pressures in the undrained triaxial test are influenced both by the total stress increment, $d\sigma_3$ and from the change in the deviator stress, $d(\sigma_1 - \sigma_3)$.

The changes in pore pressures are given by:

$$du_a = B_a d\sigma_3 + D_a d(\sigma_1 - \sigma_3) \quad [5]$$

$$du_w = B_w d\sigma_3 + D_w d(\sigma_1 - \sigma_3) \quad [6]$$

Equations 5 and 6 can be expressed in a different form as:

$$du_a = B_a\{d\sigma_3 + A_a d(\sigma_1 - \sigma_3)\} \quad [7]$$

$$du_w = B_w\{d\sigma_3 + A_w d(\sigma_1 - \sigma_3)\} \quad [8]$$

where:

$$B_a A_a = D_a \quad [9]$$

$$B_w A_w = D_w \quad [10]$$

Shear strength can be related to the total stresses when the pore pressures at failure are unknown. In undrained triaxial tests, the specimens when subjected to increasing loading, undergo changes in pore-air and pore-water pressures which results in changes of matric suction of the soil. Such a situation in laboratory testing is demonstrated using Figure 2. Four "identical" soil specimens subjected to different confining pressures during an undrained test are shown. The pore pressures in these soil specimens increase with the increase in confining pressure. The shear strength also increases with the increasing confining pressure. The matric suction in the soil decreases with the increase in degree of saturation accompanied by a reduction in the volume. The four identical soil specimens are brought to different initial states of stress, due to the changes in pore pressures under undrained loading conditions.

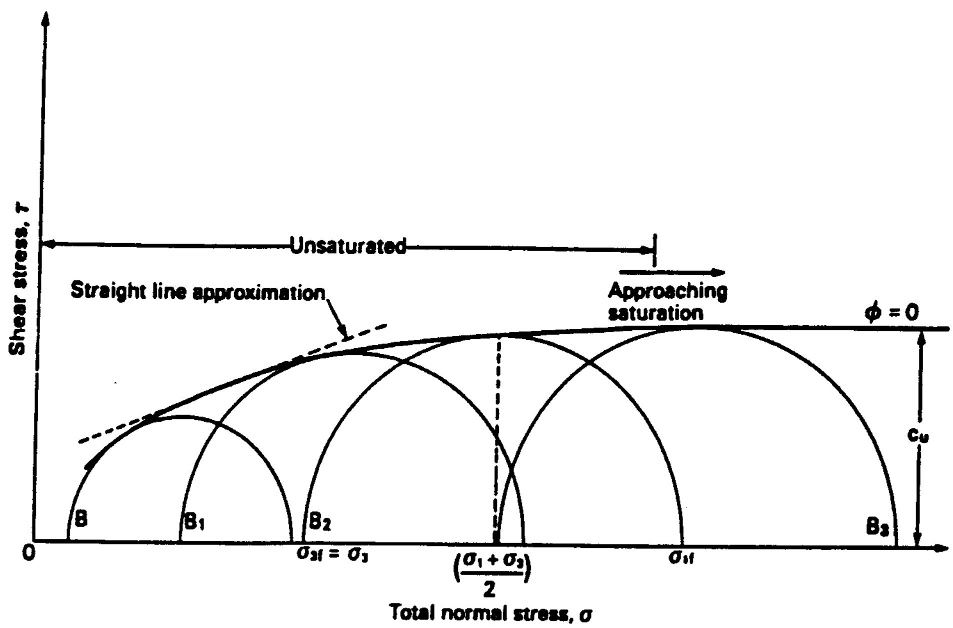

Figure 2. Shear stress versus total normal stress relationship for the undrained tests

In undrained loading conditions for unsaturated soils the increase in shear strength caused by an increase in confining pressure is greater than the reduction in shear strength caused due to a decrease in matric suction. In Fig. 2 the diameter of Mohr's circles increase with the increase in confining pressure. The envelope defines a curved relationship between the shear strength and the total normal stress for unsaturated soils tested under undrained conditions. Once the soil becomes saturated under the application of confining pressure (i.e., B_a equals B_w equals 1) a horizontal envelope develops on the shear strength axis. In the saturated condition, where the single stress state variable controls the strength, an increase in the confining pressure will be equally balanced by a pore-water pressure increase. The effective stress, (i.e., $(\sigma_3 - u_w)$), remains constant regardless of the applied confining pressure, σ_3. Once the soil is saturated, the shear strength behavior is in accordance with $\phi = 0$ concept (Skempton, 1948).

During the shearing stage, matric suction can in increase, decrease or remain constant depending upon the *A* pore pressure parameter of the soil. The *A* parameter depends on several factors such as soil type, stress history, magnitude of strain, and time for saturated soils (Lambe and Whitman, 1979). The factors affecting saturated soils can also be expected to affect an unsaturated soil. All analyses in this paper are addressed in terms of *B* pore pressure parameters. The influence of the *A* parameter is not considered, assuming the changes in matric suction caused during axial loading (i.e., $d(\sigma_1 - \sigma_3)$) as negligible.

3. THEORY

A linear shear strength equation was proposed by Fredlund et al. (1978) for unsaturated soil in terms of the stress state variables as:

$$\tau_f = c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)\tan\phi^b \quad [11]$$

where:

τ_f	= shear strength of an unsaturated soil
c'	= effective cohesion of saturated soil
ϕ'	= effective angle of shearing resistance for a saturated soil
ϕ^b	= angle of shearing resistance with respect to matric suction
$(\sigma_n - u_a)$	= net normal stress on the plane of failure, at failure
$(u_a - u_w)$	= matric suction of the soil at the time of failure

The experimental behavior of the shear strength under drained conditions were found to be non-linear when the soil is tested under large ranges of suction (Gan et al. 1988 Vanapalli, 1994). Equation [11] is also valid to describe the non-linear variation of shear strength of unsaturated treating the frictional angle of shear resistance with respect to suction, $\tan\phi^b$, as a variable (Vanapalli et al. 1996a).

Undrained shear strength analysis is presented in this paper extending Fredlund et al. (1978) shear strength equation (i.e., Eqn. 11). Effective strength parameters of the soil along with the initial matric suction and the results from unconfined and confined compression tests are required for the analysis. Changes in matric suction due to applied total isotropic pressure can be computed knowing the initial conditions of the soil using a marching forward technique. This procedure is detailed in Fredlund and Rahardjo (1993) and is not repeated here. This technique is not necessary if the matric suction at failure are measured.

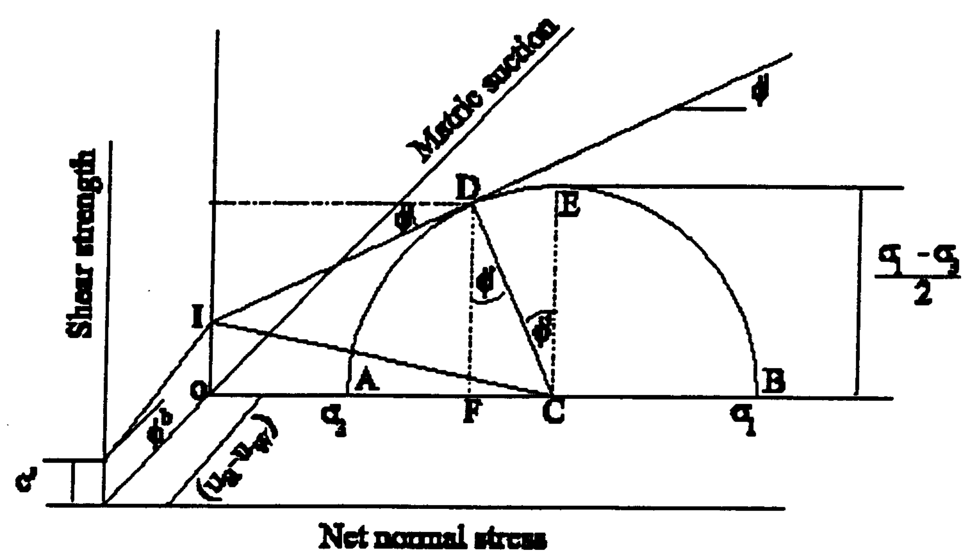

Figure 3. Three-dimensional representation of confined compression test expressed in terms of stress state variables.

Two methods of analysis for shear strength are proposed and presented in this paper. First procedure described assumes that ϕ^b is a constant value using a planar failure envelope for shear strength. The second procedure details the methodology for predicting the non-linear variation of ϕ^b as a function of matric suction, $(u_a - u_w)$.

3.1 Interpretation of Confined Compression Tests (Assuming a Planar Failure Envelope)

The failure conditions for a confined compression test are shown in Fig. 3. When the saturated shear strength parameters are known, a tangent with a slope of angle of shearing, ϕ', can be drawn to the Mohr's envelope. The intercept, OI, obtained from such a construction is equal to the summation of cohesion, c', and the product of matric suction, $(u_a - u_w)$, and $\tan\phi^b$. Thus from Figure 3:

$$OI = \left\{c' + (u_a - u_w)\tan\phi^b\right\} \quad [12]$$

The shear strength, τ_f, for this loading condition is equivalent to DF from Fig. 3. From the geometry of the diagram,

$$\tau_f = DF = DC\cos\phi' = \left(\frac{\sigma_1 - \sigma_3}{2}\right)\cos\phi' \quad [13]$$

For shear strength analysis of triaxial tests [11] can be expressed as:

$$\tau_f = \left\{c' + (u_a - u_w)\tan\phi^b\right\} + (\sigma_n - u_a)\tan\phi' \quad [14]$$

where:

$$\sigma_n = \left(\frac{\sigma_1 + \sigma_3}{2}\right) - \left(\frac{\sigma_1 - \sigma_3}{2}\right)\sin\phi'$$

A relationship for frictional angle, ϕ^b, is obtained by equating [13] and [14].

$$\tan\phi^b = \frac{\left\{c_u(\cos\phi' + \sin\phi'\tan\phi') - (c_u + \sigma_3 - u_a)\tan\phi' - c'\right\}}{(u_a - u_w)} \quad [15]$$

where:

$c_u = \left(\frac{\sigma_1 - \sigma_3}{2}\right)$, failure deviator stress from the undrained triaxial test

$(u_a - u_w)$ = is the matric suction in the specimen at failure condition

If we assume that the pore-air dissolves in the water of the specimen and the pore-air pressure, u_a, equals zero, [15] takes the form:

$$\tan\phi^b = \frac{\left\{c_u(\cos\phi' + \sin\phi'\tan\phi') - (c_u + \sigma_3)\tan\phi' - c'\right\}}{(u_a - u_w)} \quad [16]$$

3.2 Interpretation of Unconfined Compression Tests (Assuming a Planar Failure Envelope)

Equation [16] can be applied for unconfined compression tests also by setting σ_3 equal zero.

$$\tan\phi^b = \frac{\left[\frac{\sigma_1}{2}(\cos\phi' + \sin\phi'\tan\phi') - \frac{\sigma_1}{2}\tan\phi' - c'\right]}{(u_a - u_w)} \quad [17]$$

For unconfined compression tests, the pore-air pressure can be assumed to be atmospheric and the results can be interpreted assuming constant matric suction.

The theory presented can be used for practical applications to determine the contribution of matric suction towards undrained shear strength, ϕ^b, knowing the unconfined and confined compressive strengths and matric suction values of the soil in the field along with the saturated shear strength parameters, c' and ϕ'.

3.3 Interpretation of Unconfined and Confined Compression Tests (Assuming Non-linear Failure Surface with respect to Matric Suction)

The non-linear variation of the shear strength under drained loading conditions with matric suction can be predicted using the soil-water characteristic curve (drying path) and the saturated shear strength parameters (Vanapalli et al. 1996a, Fredlund et al. 1996). The theory can be extended to predict non-linear variation of ϕ^b using the unconfined and confined undrained shear strength test results that follow a trend of wetting.

The degree of saturation versus matric suction variation (i.e., the soil-water characteristic curve relationship) can be obtained from the triaxial test results at failure conditions. The degree of saturation changes in these specimens without any change in water content. The rate at which suction contributes towards shear strength can be related to the area of water available within the voids (Vanapalli 1996a). With the compression of air voids the available constant water content communicates suction as a stress state variable over a greater area.

Thus, the soil-water characteristic curve generated from the undrained tests may be assumed to be similar to the wetting curve (i.e., obtained from confined compression shear test results) can be used for predicting the shear strength of unsaturated soils in undrained loading conditions. However, results from desiccator tests (for higher suction range) is necessary to have the entire range suction range of soil-water characteristic curve data (i.e., from 0 to 1,000,000 kPa). The variation of soil-water characteristic curve behavior at suctions greater than 3000 kPa suction can be assumed to be similar both in drying and wetting conditions.

The shear strength contribution due to matric suction and cohesion is given as:

$$\tau_f = c' + (u_a - u_w)[(\Theta^{\kappa})\tan\phi'] \qquad [18]$$

where:

Θ = normalized volumetric water content, defined as the ratio of volumetric water content, θ and volumetric water content, θ_s, at a saturation of 100%.

κ = fitting parameter used for obtaining a good correlation between the experimental and predicted shear strength values.

$$\Theta = [C(\psi)]\left[\frac{1}{\ln\left(e + \left(\frac{\psi}{a}\right)^n\right)}\right]^m \qquad [19]$$

where:

θ = volumetric water content
θ_s = saturated volumetric water content
a = a suction related to the air-entry value of the soil
n = soil parameter related to the slope at the inflection point on the soil-water characteristic curve
ψ = soil suction
m = soil parameter related to the residual water content
θ_r = volumetric water content at residual conditions
e = natural number, 2.71828...
$C(\psi)$ = a correction function which forces the soil-water characteristic curve through a suction of 1,000,000 kPa and zero water content

The correction factor is defined as:

$$C(\psi) = \left[1 - \frac{\ln\left(1 + \frac{\psi}{C_r}\right)}{\ln\left(1 + \frac{1{,}000{,}000}{C_r}\right)}\right] \qquad [20]$$

where:

C_r = the suction value corresponding to residual water content, θ_r

However, the degree of saturation, S, is also equal to the normalized volumetric water content, Θ. Fredlund and Xing (1994) have shown that the soil-water characteristic curve data can be best-fit for the entire range of suction (i.e., 0 to 1,000,000 kPa) using the equation given below.

The value of $\tan\phi^b$ at any value of matric suction is:

$$\tan\phi^b = \frac{d\tau}{d(u_a - u_w)} = \left[\left(\Theta^{\kappa}\right) + (u_a - u_w)\frac{d\left(\Theta^{\kappa}\right)}{d(u_a - u_w)}\right]\tan\phi' \qquad [21]$$

Vanapalli et al. 1996b have shown that for drained direct shear tests in unsaturated conditions to soil suction has no influence on the angle of shearing resistance, ϕ'. Thus, the difference between the shear strength at failure conditions and the strength contribution due to net normal stress from the undrained triaxial tests can be assumed to be equivalent to the shear strength contribution due to matric suction and effective cohesion c'. This can be expressed mathematically as below:

$$\tau_f - \{(\sigma_n - u_a)\tan\phi'\} = c' + (u_a - u_w)\tan\phi^b \qquad [22]$$

where:

$$\tau_f = \left(\frac{\sigma_1 - \sigma_3}{2}\right)\cos\phi'$$

$$\sigma_n = \left(\frac{\sigma_1 + \sigma_3}{2}\right) - \left(\frac{\sigma_1 - \sigma_3}{2}\right)\sin\phi'$$

The validity of the assumptions made for this analysis can be checked by comparing the results obtained from [18] and [22].

Undrained tests follow the trends of wetting and under this condition gravimetric water content is constant in the specimens. However, the degree of saturation is increasing at a constant water content under the applied confining pressures.

4. PRESENATATION AND DISCUSSION OF RESULTS

A glacial till obtained from Indian Head, Saskatchewan was used for the study. The soil was air-dried for several days, pulverized using a rubber mallet, and passed through a 2 mm sieve. The liquid limit, ω_L, and the plastic limit, ω_P, are 35.5% and 16.8% respectively. The percentages of sand, silt, and clay are 28, 42, and 30 respectively. The AASHTO standard compacted density was 1.80 Mg/m^3 with an optimum content of 16.3%. The specific gravity of soil was 2.73. The soil is classified as a CL. The effective cohesion, c' was found to be equal to 15 kPa and angle of internal friction, ϕ' was equal to 23 degrees. Undrained tests both in confined and unconfined compressions laoding conditions were undertaken on the specimens that were compacted at an initial water content of 16.3%. More details of soil and specimen preparation details are available in Vanapalli (1994).

4.1 Unconfined Compression Tests

Figure 3 shows the results of three unconfined compression tests conducted. The initial matric suction in these specimens was 152 kPa. The specimens failed after reaching a peak strength at about 3 to 5% axial strain.

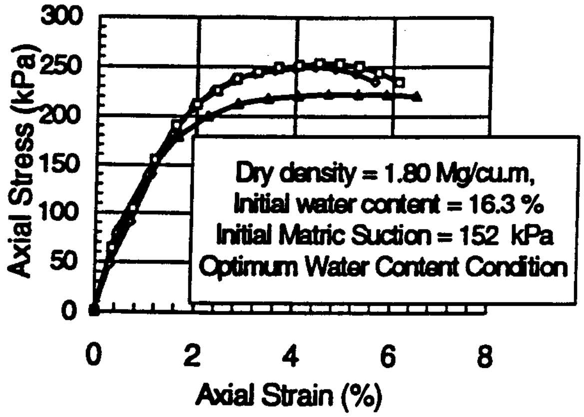

Figure 4. Stress versus strain relationships from unconfined compression tests

The shear strength contribution due to matric suction, (i.e., ϕ^b) from the unconfined compression test results is 23° using Eq. [17]. The average value of ϕ^b is same as the effective angle of friction, ϕ' due to high degree of saturation (i.e., 95.4%). The value of ϕ^b for specimens tested under drained conditions at the same value of suction is approximately the same (Vanapalli et al. 1996b).

4.2 Confined Compression Tests Assuming a Planar Failure Envelope

Table 1 shows the shear strength contribution due to matric suction in terms of ϕ^b using Eq. [16]. The results indicate that ϕ^b is increasing with an increasing confining pressure up to 200 kPa.

Table 1. ϕ^b from Confined Compression Tests Assuming a Planar Failure Envelope for Optimum Water Content Specimens Using Equation [16].

Confining pressure (kPa)	Predicted pore-water pressure (kPa)	Confined compressive strength (kPa)	ϕ^b (degrees)
0	-152.0	120.8	23.1
100	-74.4	159.6	32.9
200	-21.4	178.1	40.1
400	90.0	227.9	20.7
600	268.9	237.9	22.7
750	382.1	236.8	24.8

With the increasing confining pressure, the degree of saturation increases which results in a decrease in matric suction, and hence ϕ^b should be increasing. At higher degrees of saturation, ϕ^b should approach and equal ϕ'. However, ϕ^b values higher than ϕ' were calculated for specimens tested at confining pressure values of 100 and 200 kPa. These calculations were based on predicted matric suction values rather than measured values.

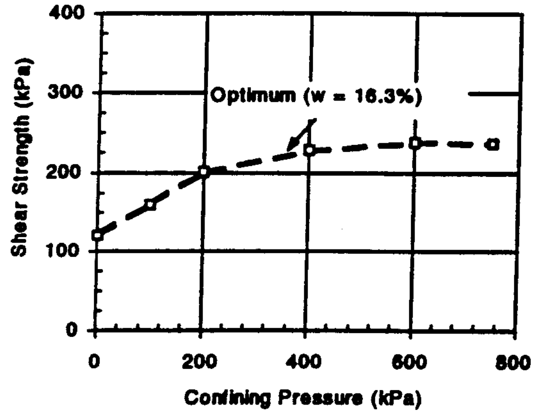

Figure 5. Variation of shear strength with the applied confining pressure

Positive pore-water pressures were predicted for specimens subjected to confining pressures greater than 400 kPa. This is likely to be true due to the fact that the measured degrees of saturations for specimens failed at these confining pressures were 100% and the variation of confined compressive strength with the applied confining pressure, is horizontal (Figure 3). The angle ϕ^b values predicted for higher confining pressures are close to ϕ'.

4.3 Non-linear Failure Envelope (Using the Soil-Water Characteristic Curve - Wetting)

The degree of saturation versus matric suction relationship obtained from the testing program along with the osmotic dessicator data were used as the wetting soil-water characteristic curve.

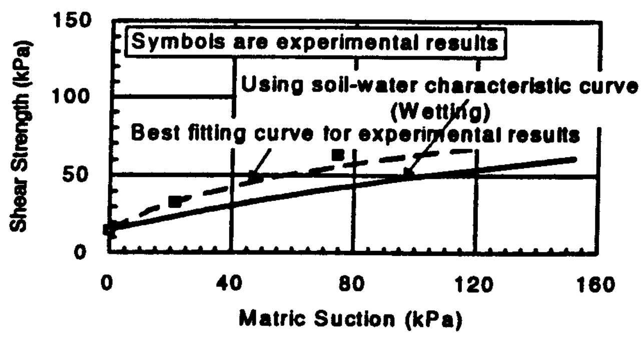

Figure 6. Relationship between the shear strength and matric suction

Using this soil-water characteristic curve data and the saturated shear strength parameters c' equal 15 kPa and ϕ' equal 23 degrees, the variation of shear strength with matric suction is predicted using Eq. [18]. A value of κ equal to 2.2 has been used for predicting the shear strength for the same soil under drained loading conditions with different net normal stresses and initial water content conditions (Vanapalli et al. 1996a). There is a reasonably good comparison between the predicted and the measured shear strength contribution due to suction.

5. SUMMARY AND CONCLUSIONS

A framework for the interpretation of undrained shear strength of unsaturated soils using stress state variable theory proposed by Fredlund et al. (1978) is presented and applied to the results on glacial till specimens compacted at optimum water content conditions. The results appear to be satisfactory. The presented theory can be used for practical applications to determine the contribution of matric suction towards the undrained shear strength, ϕ^b. Further studies are however required to examine the proposed framework in this research paper on different soils.

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