HYSTERETIC INFLUENCE ASSOCIATED WITH THERMAL CONDUCTIVITY SENSOR MEASUREMENTS

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ABSTRACT An experimental program was set up to study the properties of the capillary hysteresis of the ceramic block for a newly developed thermal conductivity sensor. The testing program included two groups of tests, one group measured the capillary hysteresis of the ceramic blocks, and the other group measured the hysteresis of the relationships between the output of the sensor and the applied suctions. Test results show an error of up to 40% if the conventional calibration curve is used without considering the effects of capillary hysteresis. Based on the experimental results, a mathematical approach was proposed, which is able to take into account the effects of capillary hysteresis in interpreting the measured suction data.

RÉSUMÉ Un programme expérimental a été mis sur pied pour étudier les propriétés d' hystérésis capillaire d'un bloc en céramique pour un senseur à conductivité thermal nouvellement développé. Le programme inclut deux groupes de tests soit un groupe mesurant l'hystérésis capillaire des blocs en céramique et un autre groupe mesurant l'hystérésis de la relation entre le redement du senseur et les succions appliquées. Les résultats des tests montrent jusqu'à 40% d'erreur si la courbe de calibration conventionnelle est utilisée. Basée sur les résultats de ces tests, une approche mathématique a été proposée, laquelle permet de prendre en considération l'hystérésis capillaire en interprétant des données de succion mesurées.

1. INTRODUCTION

The thermal conductivity sensor proves to be one of the most promising devices for *in situ* measurement of soil suction (Fredlund and Rahardjo, 1988; Fredlund et al., 1992). A thermal conductivity sensor consists of a cylindrical porous block containing a temperature sensing element and a miniature heater (Phene et al., 1971).

The heater at the center of the porous block converts electrical energy to thermal energy. A portion of the thermal energy will be dissipated throughout the porous block. The undissipated thermal energy will result in a temperature rise at the center of the porous block. The temperature sensor measures the temperature rise at a certain elapsed heating time. Since water has a much higher thermal conductivity than air, the rate of dissipation of the thermal energy within the porous block will increase with the water content of the porous block. The higher water content will result in a less temperature rise at the center of the porous block, and, consequently, a lower voltage output of the temperature sensor. Since the water content is directly related with the matric suction, the voltage output of the temperature sensor (i.e., the output of the suction sensor) can be calibrated with the matric suction.

The measurement of matric suction using thermal conductivity sensor is an indirect method. There are three relationships that indirectly relate the output of the sensor to the matric suction:

(1) The output of the suction sensor is a voltage, which is inversely related to the rate of dissipation of the thermal energy within the porous block;

(2) The rate of dissipation of the thermal energy is dependent upon the water content of the porous block; and

(3) The water content of the porous block is a function of the matric suction applied on the porous block by the surrounding soil.

The first two relationships are reversible, whereas, the relationship between the water content of the porous block and the matric suction in the soil exhibits hysteresis for different directions of water movement. The hysteresis between the water content of a porous medium and the matric suction is usually referred to as capillary hysteresis.

As a result of the capillary hysteresis of water movement in the porous block of the thermal conductivity sensor, the same voltage output of the sensor may correspond to different matric suction values, depending on the drying and wetting history of the porous block.

If a single drying or wetting curve is used as the calibration curve of the thermal conductivity sensor, error of measurement will occur when the porous block of sensor undergoes wetting and drying cycles in the field. The influence of the capillary hysteresis on the measurement of matric suction using thermal conductivity sensor has been noticed for many years, but little research has been done on this subject (Wong et al., 1989; Fredlund, 1992; Fredlund et al., 1994). Attempts were made in this study to better understand the properties of the capillary hysteresis of the porous block of a thermal conductivity sensor and its effects on the measurement of matric suction.

2. A DESCRIPTION OF THE BETA-97 SENSORS

A research program was set up at the University of Saskatchewan to develop a new thermal conductivity sensor that would better meet the requirement of field measurement of matric suction. Based on the research conducted on the electronics and the tip porous materials, a new thermal conductivity sensor, Beta-97, was developed and produced for laboratory testing and field monitoring. The Beta-97 sensor consists of an IC (integrated circuit) temperature sensor and a heating resistor embedded at the center of a porous ceramic tip. The ceramic tip has a diameter of 28mm and a height of 38mm. The ceramic is first baked and a hole is then drilled at the center of the ceramic. The electronics are embedded in the ceramic tip using a thermally conductive epoxy (Shuai et al., 1998).

The ceramics of the Beta-97 sensors have a compressive strength of approximately 2100 kPa, a saturated coefficient of permeability of 2.0×10^{-6} m/s, a dry density of 0.81 to 0.84 Mg/m^3, and a porosity of 60 to 61% (Shuai, et al., 1998).

Three ceramics (without any electronics in the ceramics), Ceramic-1 to 3, with the same dimensions as the ceramic tips of the Beta-97 sensors were used in the laboratory tests to investigate the capillary hysteresis of the ceramic. Six finished Beta-97 suction sensors, Sensor-1 to 6, were used to investigate the hysteresis of the relationship between the output of the sensor and the applied matric suction. The basic physical properties of the three ceramic tips are show in table 1.

Table 1 Physical properties of the three ceramics

Ceramic No.	Dry density γ_d (g/cm^3)	Void ratio e	Porosity n (%)	Diameter (mm)	Height (mm)
1	0.814	1.56	60.9	28.5	38.4
2	0.836	1.52	60.3	28.4	38.3
3	0.824	1.53	60.5	28.4	38.5

3. TEST METHOD

The pressure cells designed at the University of Saskatchewan were used to apply matric suction to the ceramic tips of the sensors. The setup of the apparatus is shown in Figure 1. A saturated ceramic disk with an air-entry value of 5-bar was used for the pressure cell. A thin layer of kaolinite paste was placed between the ceramic tip and the high air entry disk to ensure good contact and continuity of water flow between the ceramic tip and the high air entry disk. The kaolinite layer also helps to hold the ceramic tip in place.

The output of the sensors was monitored using a CR-10 data acquisition system. Readings were taken at one-hour intervals. The same pressure cells were used for measuring the capillary hysteresis of the three ceramics. The flow of water was monitored using a burette. At each suction equilibrium condition, the water content of the ceramic was measured by taking the ceramic out and weighing it on a balance.

Suctions below 10kPa were applied by keeping the air pressure in the pressure cell at one atmosphere, while, decreasing the water pressure underneath the air entry disk by lowering the water level in the burette. Suctions higher than 10kPa were applied by increasing the air pressure in the pressure cell while keeping the water pressure underneath the high air entry disk constant.

The three ceramics and the ceramic tips of the six sensors were saturated under vacuum to prevent air bubbles from being entrapped in the pores of the ceramic tips. The dry ceramic tips were first placed in a vacuum chamber and a vacuum of approximately one atmosphere was applied for a few hours in the chamber. Water was then sprayed into the chamber while keeping the vacuum in the chamber constant until the ceramic tips were submerged. The vacuum was released and the ceramics were kept submerged in water for a few hours. The vacuum method significantly shortens the time for saturation while effectively prevents air bubbles from being entrapped in the pores of the ceramic.

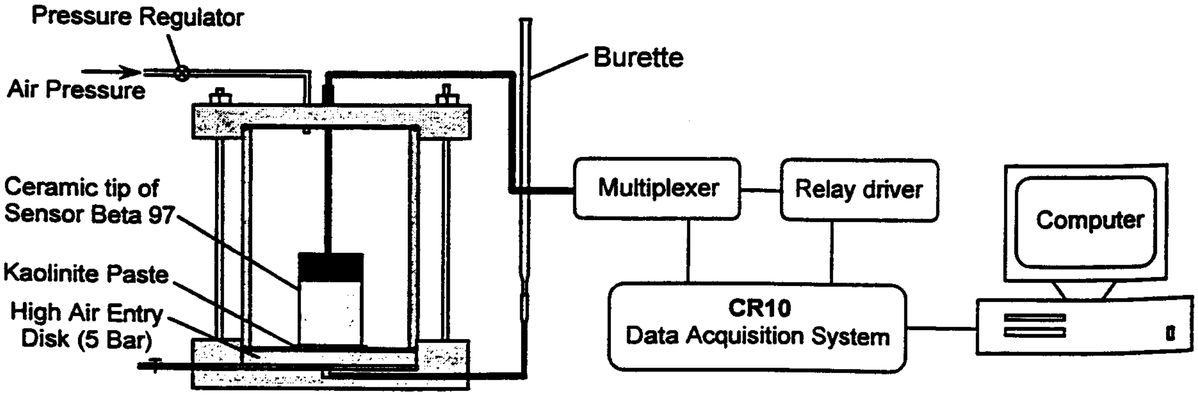


Figure 1 Apparatus for measuring the capillary hysteresis curves of the ceramic tips and the relationships between sensor output and matric suction

The saturated ceramic or the ceramic tip of the sensor was installed in the pressure cell. Desorption and absorption tests were conducted according to the following procedures:

(1) The ceramic tip was dried to a matric suction of 400 kPa, in the following steps of 2, 4, 7, 15, 30, 60, 100, 200, and 400 kPa, giving the initial drying curve;

(2) The suction was then decreased, in the reverse order from 400 kPa to 0.1 kPa giving the main wetting curve. The ceramic was subsequently dried in the same order of suction increments given in (1) to produce the main drying curve. The loop formed by the main wetting curve and the main drying curve is referred to as main hysteresis loop;

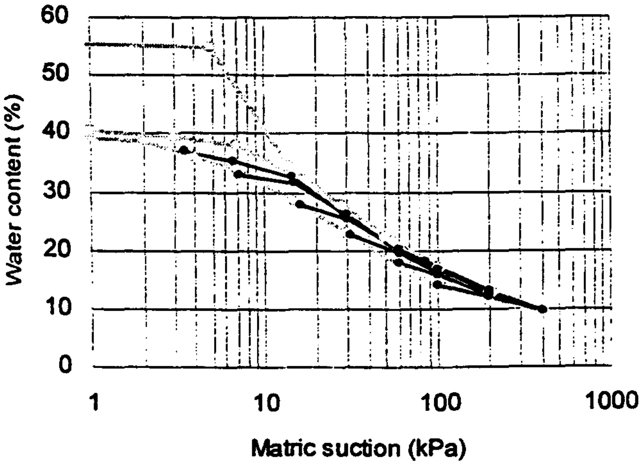
(3) Step (2) was repeated to produce the primary drying scanning curves and the primary wetting scanning curves.

4. TEST RESULTS

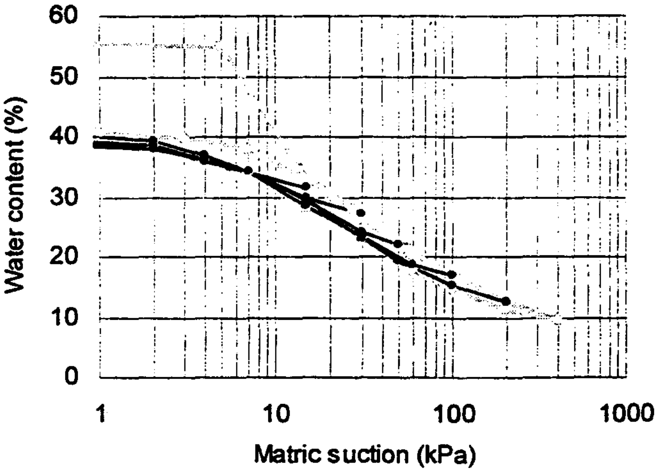
Similar results of hysteresis curves were obtained for the three ceramic tips and for the six sensors. Only the hysteresis curves of Ceramic-1 and Sensor-1 are shown in Figure 2 and Figure 3, respectively.

When the suction was above 400 kPa, the equalization time will be too long, usually more than 20 days. Other factors, such as evaporation and bacteria growth would significantly affect the test results. So the drying processes were stopped at suction of 400 kPa. It can been seen from Figures 2 and 3, the residual saturation had not yet been reached.

As seen in Figures 2 and 3, a relative large gap was found between the initial drying curve and the main hysteresis loop in the low suction range of 0 to 15 kPa. This indicates that a relatively large amount of air was entrapped in the pores of the ceramic when it was first rewetted. The data in Tables 2 and 3 show the relative amount of entrapped air.

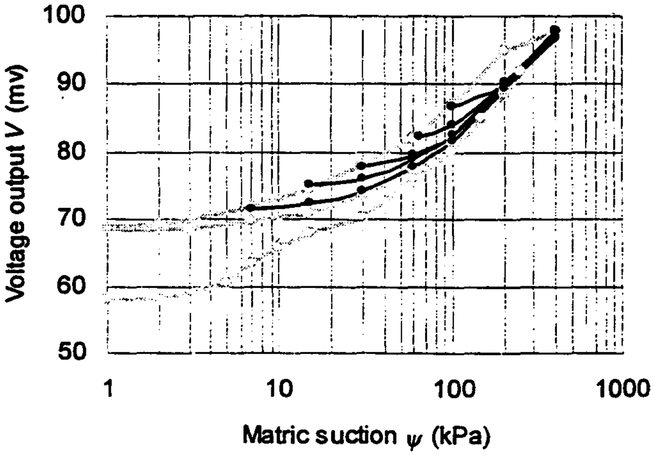


(a) Primary drying scanning curves

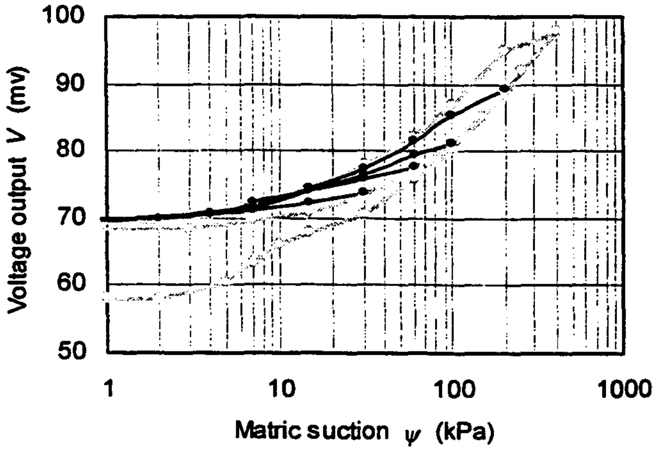


(b) Primary wetting scanning curves

Figure 2 The initial drying curve, main hysteresis loop and primary drying scanning curves for Ceramic-1



(a) Primary drying scanning curves



(b) Primary wetting scanning curves

Figure 3 The initial drying curve, main hysteresis loop and primary scanning curves for Sensor-1

In Table 2, θ_0 and θ' denote respectively the saturated water content of the ceramic and the water content after rewetting to a suction of 0.1 kPa. S_r' represents the degree of saturation when the ceramic is rewetted to a suction of 0.1 kPa. In Table 3, ΔV_0 is the sensor output change when the sensor ceramic tip is fully saturated from initially air-dried condition, and $\Delta V'$ is the sensor output change when the ceramic tip is rewetted in the pressure cell from air-dried condition to a suction of 0.1 kPa. ΔV_0, $\Delta V'$ and $\Delta V'/\Delta V_0$ correspond to θ_0, θ' and S_r', respectively.

Table 2 Water content after initial drying and rewetting to a suction of 0.1 kPa

Specimen	θ_0 (%)	θ' (%)	S_r' (%)
Ceramic-1	41.0	56.0	73.2
Ceramic-1	41.5	59.0	70.3
Ceramic-1	40.0	56.5	70.8

Table 3 Voltage output after initial drying and rewetting to a suction of 0.1 kPa

Specimen	ΔV_0 (mv)	$\Delta V'$ (mv)	$\frac{\Delta V'}{\Delta V_0}$ (%)
Sensor-1	54.0	43.0	79.6
Sensor-2	68.0	53.0	77.9
Sensor-3	65.0	50.0	76.9
Sensor-4	65.3	58.8	90.0
Sensor-5	61.3	53.3	86.9
Sensor-6	56.1	49.1	87.5

A degree of saturation of only 70% to 75% is reached after the ceramic is rewetted to a suction of 0.1 kPa. The sensor output change after the sensor tip is re-wetted from air-dried condition to 0.1 kPa suction is only 70% - 85% of its output change when the sensor tip is fully saturated from air-dried condition. These results indicate that approximately 15 to 30% of the pore space was occupied by the entrapped air when the ceramic tips were rewetted from 400 kPa to 0.1 kPa.

The main hysteresis loop is always located below the initial drying curve for the $\theta - \psi$ relationships of the ceramics, and above the initial drying curve for the V - ψ relationships of the sensors. The difference in water content (for the ceramics) or voltage output (for the sensors) between the initial drying curve and the main drying curve varies with matric suction. This difference is more significant at low matric suctions (i.e., suction lower that 15 kPa) than at high matric suction (i.e., suction higher than 15 kPa).

If the ceramic tip is submerged in water, the entrapped air will gradually escape by diffusing or pore water redistribution. As a result, the ceramic tip will have a water content higher than the water content on the main hysteresis loop at zero suction. The subsequent drying curve will be above the main drying curve. Test results show that as long as the submergence of the ceramic tip in water is approximately less than 10 days, the drying and wetting will take place within the main hysteresis loop. Even the prolonged submergence does occur, a small suction (e.g., 15 kPa) can bring the drying and wetting back to the main hysteresis loop.

The main hysteresis loop and the primary scanning curves were found to be stable and reproducible over a two-year testing period. The thermal conductivity sensor is suitable for long term suction measurement.

5. DISCUSSIONS OF THE TEST RESULTS

Research work has been done on the calibration of the thermal conductivity sensor (Fredlund and Wong, 1989; Fredlund, 1992). Conventionally, the ceramic tip of the sensor is first submerged in a water bath for a few days. The sensor is then mounted in a pressure cell and matric suction is applied by increasing the air pressure in the pressure cell incrementally while maintaining a constant water pressure underneath the high air entry disk. Therefore, the calibration curve using the conventional method is a drying curve.

There are two problems associated with the conventional calibration procedure. The first problem is that the initial degree of saturation of the ceramic tip is not known. Experiments in this study showed that a degree of saturation of only 75.1% was reached for a ceramic after a 2.5-month submergence in water (Feng, 1999) and the degree of saturation was still increasing. Therefore, the calibration curve could be anywhere between the initial drying curve and the main drying curve, or even below the main drying curve, depending on how long the ceramic tip was submerged.

The second problem is that the conventional calibration curve is only a drying curve. It does not take into consideration the effects of capillary hysteresis. When a sensor is installed in the field the ceramic tip of the sensor will experience drying and wetting processes depending on the moisture movement in the surrounding soil. For a certain value of voltage output of the sensor, the corresponding matric suction could be anywhere between the main drying curve and the main wetting curve or even between the initial drying curve and the main wetting curve, depending on the water flow history. Therefore, if only a drying curve is used as the calibration curve, error will occur as a result of the capillary hysteresis.

Assuming that the main drying curve is the calibration curve, the maximum error will occur when the ceramic tip is undergoing a main wetting process at the time the voltage reading is taken. In this case, the actual suction is ψ_w and the measured suction is ψ_d. The maximum possible relative error will be

$$\varepsilon_{max} = \frac{\delta\psi_{max}}{\psi_w} = \frac{\psi_d - \psi_w}{\psi_w} \qquad \text{[Eq. 1]}$$

Where

ψ_d = suction on the main drying curve corresponding to a certain output, i.e., the measured suction if the main drying curve is used as the calibration curve; and

ψ_w = suction on the main wetting curve corresponding to a certain output, i.e., the actual suction if the ceramic tip of the sensor is undergoing a main wetting process.

Figure 4 shows the relationship between the maximum possible relative error, ε_{max}, and the measured matric suction, ψ_d, for the six sensors when the main drying curve is used as the calibration curve.

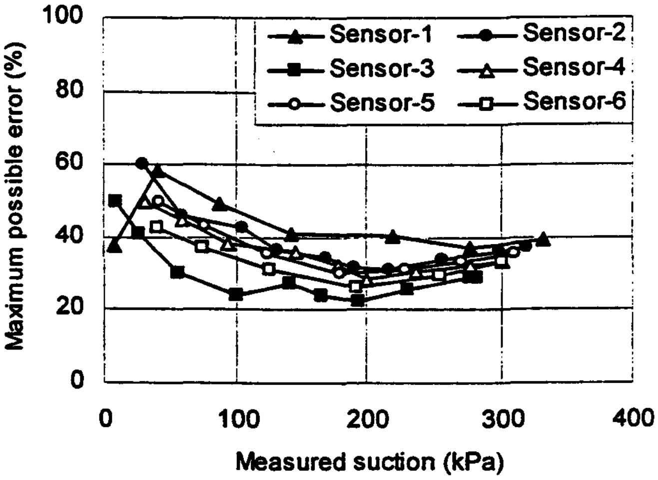


Figure 4 Maximum possible errors caused by the capillary hysteresis for the six sensors

It can be seen that the maximum possible errors decrease when the measured suctions are low. The maximum errors are approximately constant when the measured suctions are above 100 kPa. The average maximum possible error varies from sensor to sensor, from 24% for Sensor-3 up to 40% for Sensor-1. The above discussion indicates that the effects of capillary hysteresis on the measurement of matric suction using thermal conductivity sensors are not negligible.

The calibration curve measured using the conventional method of calibration could be above or even below the main drying curve depending on the length of submergence. The error of measurement using a calibration curve obtained using the conventional calibration procedure could be even higher than indicated in Figure 4.

6. PREDICTION OF THE HYSTERESIS CURVES

The discussions of the previous sections indicate that the hysteresis must be taken into consideration when calibrating the sensor. It is, however, impractical to measure all the hysteresis curves in the calibration. It is desirable to develop an appropriate mathematical approach that can predict the hysteresis curves from a limited amount of calibration data.

Several hypothetical models are found in the literature. An examination of some of the models using the measured hysteresis data of the suction sensors showed that the models either require too much calibration data, or can not make good prediction (Feng, 1999).

The experimental results show that the hysteresis curves are consistent from ceramic to ceramic and from sensor to sensor. The six sensors tested all had similar hysteresis curves, and so did the three ceramics. It would seem logical to find a proper formulation that would fit the measured curves of sensors with known hysteresis curves and then to use the formulation to predict the hysteresis curves of other sensors that have only limited calibration data.

The following equation is used to fit the main drying and main wetting curves (Personal communication with M. Fredlund).

$$\theta(\psi) \text{ or } V(\psi) = \frac{ab + c\psi^d}{b + \psi^d} \qquad \text{[Eq. 2]}$$

This simple equation fits quite well the measured main drying and main wetting curves within the measured suction range (i.e., 0 to 400 kPa).

In Eq. 2, parameter *a* is the ceramic water content or the sensor reading at suction equals to 0 on the main hysteresis loop. Parameter *c* is the ceramic water content or the sensor reading when the ceramic tip is in a dry condition. Parameters *a* and *c* are easy to measure and remain the same for the main wetting and main drying curves. With one branch of the main hysteresis loop measured, only two parameters, *b* and *d*, remain unknown for the other branch. If two points on this branch are measured, this branch can be predicted using Eq. 2.

With one branch of the main hysteresis loop measured and the other branch predicted, the following equations are used to fit the scanning curves.

$$\theta_d(\psi, \psi_1) = \theta_d - (\frac{\psi_1}{\psi})^\alpha (\theta_d - \theta_w) \qquad \text{[Eq. 3]}$$

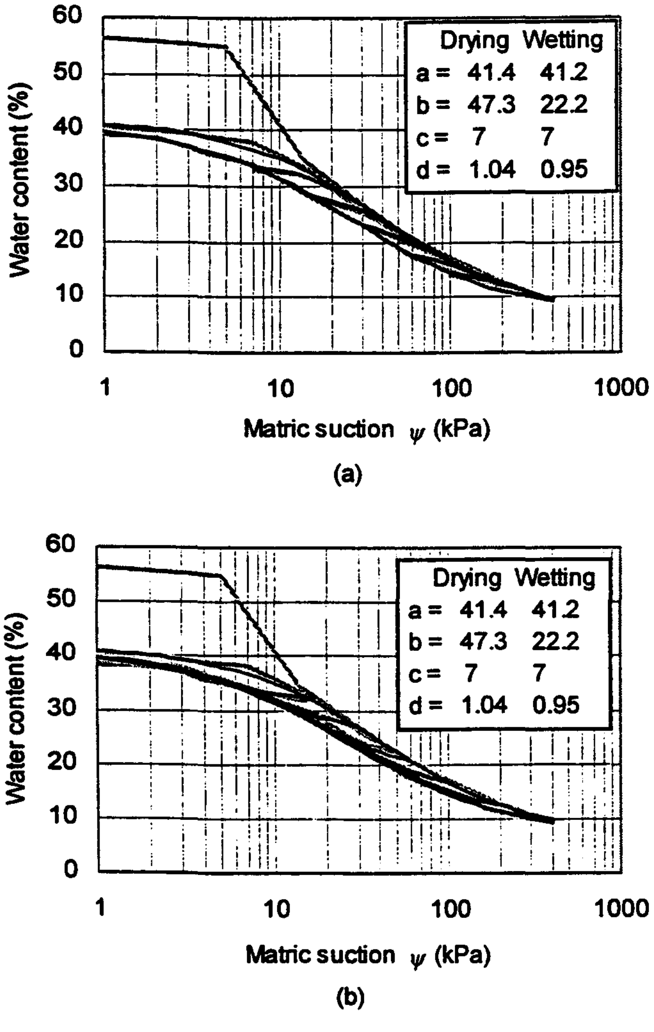
$$\theta_w(\psi, \psi_1) = \theta_w + (\frac{\psi}{\psi_1})^\alpha (\theta_d - \theta_w) \qquad \text{[Eq. 4]}$$

There is one unknown parameter, α, in the above equations. The α parameter is an empirical parameter. It determines the degree of curvature of the scanning curves.

The measured hysteresis curves of the three ceramic tips and the six sensors were fitted using Eqs. 2, 3 and 4. The predicted curves of the main hysteresis loop and primary scanning curves for Ceramic-1 and Sensor-1 are shown in Figure 5 and Figure 6, respectively.

The value of α is assumed to be 1.8 for both the primary drying scanning curves and the primary wetting scanning curves of all three ceramics and six sensors. It can be seen in Figures 5 and 6 that the predicted primary scanning curves are close to the measured ones. The α value of 1.8 is therefore reasonable for predicting the primary scanning curves of other Beta-97 sensors.

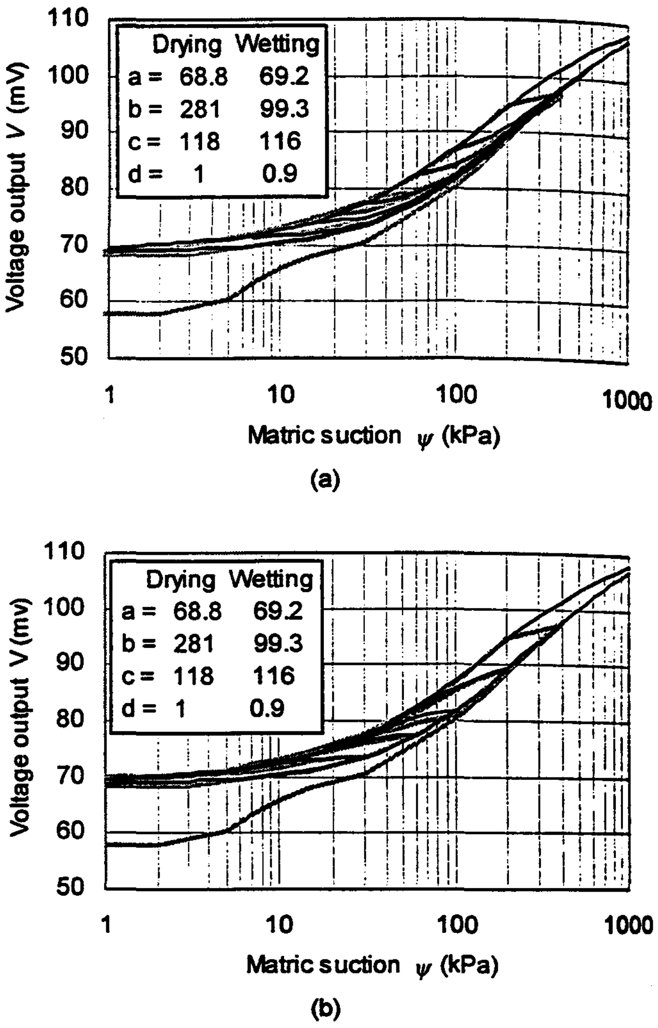
It should be noted, however, the value of 1.8 for the parameter α is valid only for Beta-97 sensors. The α value could be different for other ceramic sensors. It is necessary to investigate the hysteresis properties using some typical sensors to estimate the α value for the type of sensor under consideration.



(a)

(b)

Figure 5 Measured (gray lines) and predicted (black lines) primary drying scanning curves (a) and primary wetting scanning curves (b) for Ceramic-1



(a)

(b)

Figure 6 Measured (gray lines) and predicted (black lines) primary drying scanning curves (a) and primary wetting scanning curves (b) for Sensor-1

7. CONCLUSIONS

A gap was observed between the initial drying curve and the main hysteresis loop in the low suction range. The cause of the gap is believed to be due to air entrapment in the pores of the ceramic.

If a single drying or wetting curve is used as calibration curve, the error caused by the capillary hysteresis of the ceramic will be unacceptable.

The following procedures are recommended in calibrating the Beta 97 sensors.

(1) saturate the sensor ceramic tip by submerging it in water for a few days,
(2) install the sensor ceramic tip in the pressure cell and apply a suction of 50 to 100 kPa,
(3) at equilibrium, reduce the suction to zero,

(4) at equilibrium, increase the suction in increments following the conventional calibration procedure to measure the main drying curve, and
(5) rewetting the sensor ceramic to obtain two points on the main wetting curve.

The main wetting curve is predicted using Eq. 2. The primary scanning curves are predicted using Eqs. 3 and 4 by assuming the α value to be 1.8.

A similar procedure could be used to calibrate other thermal conductivity sensors. However, a study on the hysteretic properties of the sensor should be conducted to best estimate the value of parameter α for Eqs. 3 and 4.

The hysteresis curves of the ceramics and the sensors were stable and reproducible for a two-year experimental period. The thermal conductivity sensor could be a reliable means for long term field measurement of matric suction.

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