Development and verification of a coefficient of permeability function for a deformable unsaturated soil

Shangyan Huang, S.L. Barbour, and D.G. Fredlund

Abstract: The modelling of flow through saturated/unsaturated soils has become routine in geotechnical and geo-environmental engineering. The analysis requires that the coefficient of permeability for an unsaturated soil be defined. The coefficient of permeability can be estimated based on currently available procedures. However, each procedure has limitations and consequently cares should be taken in the selection of a proper procedure. The coefficient of permeability of a *saturated* soil is a function of void ratio. The coefficient of permeability of an *unsaturated* soil of constant volume, is a function of the degree of saturation. However, soil is deformable and both the degree of saturation and the void-ratio influence the coefficient of permeability of a compressible, unsaturated soil. In this paper, the literature pertaining to the coefficient of permeability function for an *unsaturated* soil of constant volume and the coefficient of permeability for a deformable *saturated* soil are reviewed. A new coefficient of permeability function for a deformable *unsaturated* porous medium is then developed. A series of triaxial permeameter tests on unsaturated silty sand are described and the results from the experimental program are analyzed using the general form of the newly developed permeability function. The results show good agreement between the experimental data and the proposed model for a deformable unsaturated porous medium.

Key words: unsaturated soil, coefficient of permeability, permeability function, soil–water characteristic curve, triaxial permeameter, deformable porous medium.

Résumé : Modéliser un écoulement à travers des sols non saturés ou saturés est devenu courant en géotechnique ou en génie géo-environnemental. L'analyse exige que le coefficient de perméabilité d'un sol non saturé soit défini. Le coefficient de perméabilité peut être estimé à partir de procédures disponibles actuellement. Cependant chaque procédure a ses limites et il faut donc faire attention quand on choisit une méthode. Le coefficient de perméabilité d'un sol saturé est fonction de son indice des vides. Pour un sol non saturé, c'est une fonction de son degré de saturation. Cependant le sol est déformable et le coefficient de perméabilité d'un sol compressible et non saturé dépend à la fois du degré de saturation et de l'indice des vides. Dans cet article, on passe en revue la littérature existant sur la fonction coefficient de perméabilité pour un sol non saturé à volume constant et sur le coefficient de perméabilité pour un sol déformable saturé. Puis on définit une nouvelle fonction coefficient de perméabilité pour un milieu déformable non saturé. Une série d'essais au perméamètre triaxial sur un sable silteux non saturé est décrite et les résultats du programme expérimental sont analysés en utilisant la forme générale de la nouvelle fonction présentée. Les résultats montrent un bon accord entre les données expérimentales et le modèle proposé pour un milieu poreux déformable non saturé.

Mots clés : sols non saturés, coefficient de perméabilité, fonction perméabilité, courbe caractéristique sol–eau, perméamètre triaxial, milieu poreux déformable.

[Traduit par la rédaction]

Introduction

The prediction of seepage in an unsaturated soil is of increasing interest in geotechnical and geo-environmental engineering. The coefficient of permeability function (referred to as the *permeability function* in the following text) for an unsaturated soil is a key parameter in performing seepage analyses in saturated/unsaturated soil. Examples of engineering applications, in which the permeability function is required, include the consolidation of compacted soils, the modelling of flow and volume change in collapsing soils, the prediction of heave in expansive clays, and the modelling of the migration of contaminants within the vadose zone.

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The coefficient of permeability of a saturated soil is a function of void ratio. The coefficient of permeability of an unsaturated incompressible soil is a function of the degree of saturation. However, soils are deformable and the coefficient of permeability for an unsaturated deformable soil is a function of both the degree of saturation and the void ratio.

The variation of the coefficient of permeability with void ratio for a saturated soil has been widely investigated in geotechnical engineering. Little effort has been made by

Table 1. Empirical relationships for the coefficient of permeability as a function of soil suction or volumetric water content.

Form of empirical relationship	Reference
Coefficient of permeability as a function of suction	
Linear function	Richards (1931).
Single power function	Wind (1955), Weeks and Richards (1967).
Double power function	Ahuja et al. (1980, 1988).
Power function and a constant	Gardner and Fireman (1958), Barden (1965), Arbhabhirama and Kridakorn (1968).
Exponential function	Christensen (1943), Gardner (1958), Philip (1986).
Coefficient of permeability as a function of volumetric water content	
Power function	Gardner (1956), Campbell (1974), Ahuja (1973;1974), Gillham et al. (1976), Zachumann et al. (1981), Hillel (1982).
Exponential	Davidson et al. (1969) and Dane and Klute (1977).

geotechnical engineers to extend this understanding to the permeability function for an unsaturated soil. The permeability function for an incompressible, unsaturated soil has been extensively investigated in soil science. Limited research has been undertaken to develop a permeability function for a deformable unsaturated porous medium.

In this paper, the development of the permeability function is reviewed from a historical standpoint. Methods of estimating the permeability function for an unsaturated soil of constant volume are first reviewed. The influence of the void ratio on the coefficient of permeability of a saturated soil is then outlined, followed by the development of a permeability function for a deformable unsaturated porous medium. An experimental program to define the permeability function of a silty sand is presented. The experimental data are used to evaluate the applicability of the proposed permeability function for a deformable unsaturated porous soil.

Permeability function for a constant volume porous medium

The coefficient of permeability of an unsaturated soil with an incompressible structure is a function of the degree of saturation (or the volumetric water content). The degree of saturation expressed as a function of soil suction is called a soil–water characteristic curve. A number of empirical relationships have been proposed for the coefficient of permeability as a function of degree of saturation, volumetric water content, or soil suction.

The soil–water characteristic curve has also been used to define various factors related to the hydraulics of flow within liquid-filled pores. Models developed on this basis are referred to as macroscopic models.

The soil–water characteristic curve can also be used to define the distribution of fluid-filled pores at any suction (Childs and Collis-George 1950). Statistical models have been developed by considering the probability of liquid phase continuity between pores of various sizes in formulating the permeability function. Each of these types of models for the permeability function is described briefly in the following sections.

Empirical relationships

A variety of general functions have been used to describe empirical measurements of the permeability function. Theoretical developments using macroscopic or statistical approaches have provided some support for many of these empirical relationships. Examples from the literature of the various empirical forms for the permeability function are given in Table 1.

Macroscopic/mechanistic model

Macroscopic models of permeability function are based on a mechanistic view that the fluid-filled pores could be represented as bundles of capillary tubes of various sizes. The distribution of the fluid-filled pores is defined by applying the capillary model to the soil–water characteristic curve. A well-known example of this type of development is that of Brooks and Corey (1964), which was built on the work of Burdine et al. (1950), and Burdine (1953), which studied the relative permeabilities of hydrocarbons and water in petroleum reservoirs.

In the Brooks and Corey (1964) development, the Poisseuille equation was used to describe the flow through fluid-filled tubes (or pores). The hydraulic radius of these pores was obtained by integration across all the pore-water volume represented by the soil–water characteristic curve. A tortuosity factor was included in the derivation to account for the difference between the actual and mean pore velocities and the actual and mean pressure gradients. Based on the experimental data of Burdine et al. (1950) and the analytical value obtained by Wyllie and Gardner (1958) the following relationship for tortuosity was proposed.

$$[1] \qquad \frac{T_{S=1}}{T_S} = \left[\frac{S - S_r}{1 - S_r}\right]^2 = S_e^2$$

where T_S is the tortuosity as a function of the degree of saturation, and ($T_{S=1}$) is the tortuosity at saturation. The term, $\frac{S - S_r}{1 - S_r}$ is the effective degree of saturation, S_e, where S is the degree of saturation and S_r is the residual degree of saturation.

Brooks and Corey (1964) developed the following expression for the relative permeability, k_r:

$$[2a] \qquad k_r = \frac{k(\psi)}{k_s}$$

$$[2b] \qquad k_r = \left[\frac{S - S_r}{1 - S_r}\right]^2 \frac{\int_0^s dS/\psi^2}{\int_0^1 dS/\psi^2}$$

where k_r is the relative permeability, *k(ψ) is the permeability function*, k_s is the coefficient of permeability at saturation, and ψ is soil suction. It was found that the soil–water characteristic curve could be represented as a function of S_e:

$$[3a] \qquad S_e = 1 \qquad \text{for} \qquad \psi \le \psi_{aev}$$

where ψ_{aev} is the suction corresponding to the air-entry value and

$$[3b] \qquad S_e = \left[\frac{\psi_{aev}}{\psi}\right]^\lambda \qquad \text{for} \qquad \psi > \psi_{aev}$$

where λ is the slope [$\Delta\log S_e/\Delta\log\psi$] of the soil–water characteristic curve on a log–log plot. The parameter, λ, is called the pore-size distribution index. It is important to note that to get a bilinear curve on a log–log plot, S_r must be adjusted as a curve-fitting parameter and consequently should not be considered a true or singular indicator of the residual degree of saturation. The effective degree of saturation, rather than the degree of saturation, is commonly used in macroscopic models to account for the "mobile" water phase in the soil (Mualem, 1986).

Substituting the effective degree of saturation, S_e, into the equation for relative permeability gives the following relationship:

$$[4] \qquad k_r = S_e^2 \frac{\int_0^s dS_e/\psi^2}{\int_0^1 dS_e/\psi^2}$$

Using the relationship for the effective degree of saturation given in eq. [3], the relative permeability can be written in a simplified form,

$$[5] \qquad k_r = S_e^{(2+3\lambda)/\lambda} = S_e^\delta$$

where δ is an empirical index.

An equivalent expression for relative permeability in terms of suction can be given as:

$$[6a] \qquad k_r = 1 \qquad \text{for} \qquad \psi < \psi_{aev}$$

$$[6b] \qquad k_r = \left[\frac{\psi_{aev}}{\psi}\right]^\eta \qquad \text{for} \qquad \psi \ge \psi_{aev}$$

where η is equal to (2+3λ) and is called the pore-size distribution coefficient.

The permeability function in terms of the degree of saturation (eq. 5) is referred to in this work as a "macroscopic model". Equation [6] has been widely used because of its strong theoretical basis and experimental background.

Various values have been proposed for the index, δ. Averjanov (1950) considered the flow through a partly filled pipe and derived a macroscopic model using a δ value of 3.5. Yuster (1951) developed a similar model with a δ value of 2. Irmay (1954) replaced the porosity in Kozeny's (1927) equation and matched the predicted coefficient of permeability to the coefficient of permeability at saturation using a δ value of 3. Mitchell et al. (1965) derived a permeability function based on Kozeny's (1927) equation. When the effective degree of saturation is introduced into the model, a δ value of 3 was obtained. Corey (1954) and Singh (1965) suggested essentially the same model based on their permeability measurements. Mualem (1978) investigated the hydraulic radius in the Hagen–Poiseuille equation and the solid surface area in Kozeny's (1927) equation. A macroscopic model was proposed in which the index, δ, was statistically obtained as being equal to $\left(3.0 + 0.015\int_{\theta_{15atm}}^{\theta_S} \psi \, d\theta\right)$, where θ is the volumetric water content.

Experimental evidence has indicated that the index δ is dependent on the pore-size distribution of the porous medium (Laliberte et al. 1966; Mualem 1978; Huang 1994). Consequently, the macroscopic models proposed by Brooks and Corey (1964) and Mualem (1978), which have a varying δ index, appear to be of wider applicability than other models.

Statistical models for a constant volume porous medium

Mualem (1986) provided an extensive summary of the various statistical models proposed for permeability function. The statistical models are based on the probable contribution of various pore sizes to the coefficient of permeability. The models are based on the following common assumptions (Mualem 1986):

(*i*) The flow system in a porous medium is simulated as a set of interconnected, randomly distributed pores with a frequency distribution, $f(r)$. The areal distribution is also equal to $f(r)$ and is the same for any cross section in the medium.

(*ii*) The Hagen–Poiseuille equation is valid at the level of a singular pore.

(*iii*) The soil–water characteristic curve is a representation of the pore-size distribution function based on the capillary model.

From a rigorous standpoint, the macroscopic model proposed by Burdine (1953) can also be seen as a special case of a statistical model. The model considers the pores in a porous medium as a bundle of parallel capillaries. The different sized pores are not considered as being randomly connected. Rather, large pores are assumed to connect to large pores and small pores are assumed to connect to small pores. That is, the probability for a pore being connected to a pore of the same size is one while the probability that the pore being connected to a pore of different size is zero.

Childs and Collis-George (1950) investigated the influence of the random distribution of pores on the coefficient of permeability. The pores on two imaginary faces are assumed to be randomly connected by a series of capillaries. The pore-size distribution on each face is considered to be identical. The probability that pores with a radius, r, on one face are connected to pores with a radius, ρ, on the other face is set to be $f(r)f(\rho)$, where $f(r)$ represents the probability of pores with a radius, r, on one face and $f(\rho)$ represents the probability of pores with a radius, ρ, on the other face. Two assumptions were made (*i*) the effective resistance to flow in the pore sequence is confined to the smaller of the pores, and (*ii*) the only contribution to permeability is by a direct pore sequence so that the possibility of by-passing sequences of pores are ignored. These assumptions lead to the derivation of the following equation:

$$[7] \qquad k(\theta) = M\left[\int_{\rho=R_{\min}}^{\rho=R(\theta)} \int_{r=\rho}^{r=R(\theta)} \rho^2 f(\rho) f(r)\, \mathrm{d}r\, \mathrm{d}\rho + \int_{\rho=R_{\min}}^{\rho=R(\theta)} \int_{r=R_{\min}}^{r=\rho} r^2 f(r) f(\rho)\, \mathrm{d}\rho\, \mathrm{d}r\right]$$

where M is a constant accounting for geometry and fluid properties, and r and ρ are the larger and smaller pore radii, respectively.

The closed-form solution for eq. [7] is not readily obtained in many cases. Childs and Collis-George (1950) suggested a summation form to evaluate the above integral. Marshall (1958) calculated the saturated coefficient of permeability from the above integral using equal water content intervals. Kunze et al. (1968) modified the model and applied it to the coefficient of permeability calculation for unsaturated soils. Nielsen et al. (1960) found that the calculation is significantly improved when the computed coefficient of permeability is matched to the measured value for the coefficient of permeability at saturation. Mualem (1974, 1976) derived an analytical solution of the equation and obtained the following form:

$$[8] \qquad k_r(\theta) = \int_0^{\theta} \frac{(\theta - \vartheta)}{\psi^2} \mathrm{d}\vartheta \bigg/ \int_0^{\theta_S} \frac{(\theta_S - \vartheta)}{\psi^2} \mathrm{d}\vartheta$$

where ϑ is a dummy variable of integration representing the volumetric water content.

Equation [8] is the most general form of Childs and Collis-George's (1950) type of statistical model. The integral form makes it possible to derive a closed-form solution for the relative permeability from a function for the soil–water characteristic curve.

Wyllie and Gardner (1958) modified Childs and Collis-George's (1950) model by assuming that the flow in a porous medium is controlled mainly by the joint interface between two adjacent thin layers that are imaginary and randomly connected by parallel capillaries. The modification leads to Burdine's (1953) model. Other investigators tested Childs and Collis-George's (1950) model from both analytical and experimental standpoints. Millington and Quirk (1961) suggested a correction factor of $[S_e^{\beta}]$ to account for the correlation between the pores of adjacent cross sections. A β value of 4/3 was specified. Kunze et al. (1968) and Jackson (1972) found that a β value of 1 was preferred.

Mualem (1976) developed a similar model to that of Childs and Collis-George (1950). Two imaginary parallel slabs normal to the flow direction and separated by a distance dx are considered. The distance dx is of the same order as the pore radii. Two assumptions were made

(*i*) there is no by-pass flow between the slab pores, and

(*ii*) the pore configuration can be replaced by a pair of capillary elements whose lengths are proportional to their radii (i.e., l_1/l_2 equal to r/ρ).

The coefficient of permeability is then proportional to r_e^2 equal to $r\rho$. A correlation factor, $G(R, r, \rho)$, was introduced to account for the partial correlation between r and ρ. To consider the eccentricity of the flow path, a tortuosity factor, $T(R, r, \rho)$, was used. $G(R, r, \rho)$ and $T(R, r, \rho)$ were further assumed to be power functions of the effective degree of saturation. The above assumptions lead to a new statistical model:

$$[9] \qquad k_r = S_e^{\alpha}\left[\int_0^{\theta} \frac{\mathrm{d}\theta}{\psi} \bigg/ \int_0^{\theta_S} \frac{\mathrm{d}\theta}{\psi}\right]^2$$

where α is an empirical constant with a statistically defined value of 0.5.

Mualem and Dagan (1978) proposed a general form of the statistical model as follows:

$$[10] \qquad k_r(\theta) = \int_{\substack{\text{over}\\\text{filled}\\\text{pores}}} J(r, \rho, R) F(r, \rho, R)\, \mathrm{d}r\, \mathrm{d}\rho \bigg/ \int_{\substack{\text{over}\\\text{all}\\\text{pores}}} J(r, \rho, R) F(r, \rho, R)\, \mathrm{d}r\, \mathrm{d}\rho$$

where $J(r, \rho, R)$ is the equivalent radius, and $F(r, \rho, R)$ is the areal probability.

The various statistical forms of the permeability function can all be derived based on eq. [10], with different assumptions about $J(r, \rho, R)$ and $F(r, \rho, R)$.

Relationship between different kinds of models

Different forms of the permeability function for a soil with an incompressible structure have been reviewed. The permeability functions can be correlated because of the relationship between the volumetric water content, degree of saturation, and soil suction. As a result, one form of the function can be transformed into another form using the existing relationships. For instance, the relative permeability function in terms of the effective degree of saturation, can be expressed in terms of soil suction based on the function of the soil–water characteristic curve proposed by Brooks and Corey (1964). Similarly, using the relationship between the effective degree of saturation and the volumetric water content, the relative permeability function with respect to the effective degree of saturation can be expressed as a function of the volumetric water content.

Fredlund and Xing (1994) recommended the following function for fitting the soil–water characteristic curve over the entire suction range up to 1 000 000 kPa:

$$[11] \qquad \theta = \left[1 - \frac{\ln(1 + \psi/C_r)}{\ln(1 + 10^6/C_r)}\right] \frac{\theta_S}{\{\ln[\exp(1) + (\psi/\alpha'')^{n''}]\}^{m''}}$$

Fig. 1. Air-entry value versus void ratio (data from Laliberte et al. 1966).

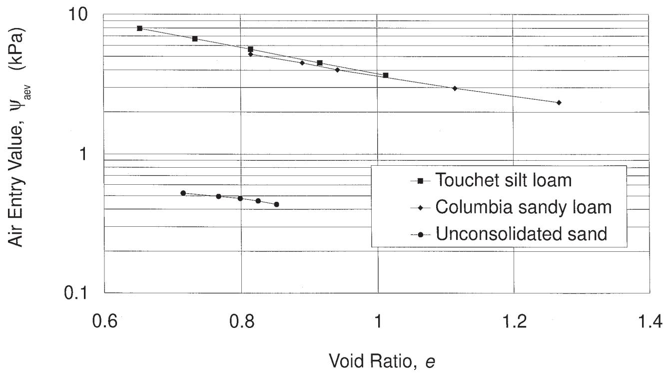
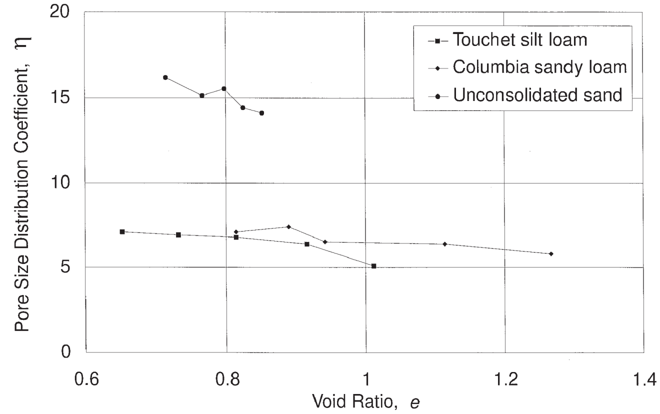


Fig. 2. Pore-size distribution coefficient, η, versus void ratio (data from Laliberte et al. 1966).



where exp (1) is the natural number (i.e., 2.71828), α'' is an experimental coefficient related to the air-entry value, C_r is an empirical constant (a typical value is 3 000 kPa), and m'' and n'' are empirical indices.

Equation [11] makes it possible to fit the soil–water characteristic curve without having to know the air-entry value and the residual volumetric water content (Fredlund and Xing, 1994). Substituting eq. [11] into the integral form of Childs and Collis-George's (1950) model, Fredlund et al. (1994) obtained a new relative permeability function,

$$[12] \qquad k_r = \int_{\ln\psi}^{\ln 10^6} \frac{\theta(e^y) - \theta(\psi)}{e^y}\theta'(e^y)\mathrm{d}y \Bigg/ \int_{\ln\psi_{aev}}^{\ln 10^6} \frac{\theta'(e^y) - \theta_s}{e^y}\theta'(e^y)\mathrm{d}y$$

where y is the dummy variable for the logarithm of soil suction.

Permeability function for a deformable saturated porous medium

The coefficient of permeability for a saturated soil can be defined as a function of the void ratio or porosity. Kozeny (1927) is believed to have been the first to derive the following permeability function for a saturated porous medium.

$$[13] \qquad k_s = \frac{C_1 n^3}{a_s^2}$$

where C_1 is the Kozeny constant, n is porosity, and a_s is specific surface area per unit volume of soil matrix.

Taylor (1948) derived an equation for the coefficient of permeability of a saturated soil as follows:

$$[14] \qquad k_s = C_2 \frac{\rho_w g}{\mu}\frac{e^3}{1+e}$$

where ρ_w is the density of water, g is the acceleration due to gravity, μ is viscosity, e is void ratio, and C_2 is a constant related to the soil–water system.

Equation [14] indicates that the k_s versus $[e^3/(1+e)]$ relationship is linear. Taylor (1948) presented experimental data to verify eq. [14]. In addition, it was pointed out that the log k_s versus e relationship is also close to a straight line. Consequently, two useful permeability functions for a saturated, deformable porous medium were obtained from Taylor (1948):

$$[15] \qquad \log k_s = \log k_{so} + b_5(e - e_o)$$

$$[16] \qquad k_s = k_{so} \frac{e^3}{1+e} \Bigg/ \frac{e_o^3}{1+e_o}$$

where k_{so} is the saturated coefficient of permeability at void ratio e_o and b_5 is an experimental index.

Based on experimental data obtained from a variety of soils, Lambe and Whitman (1969) concluded that the log k_s versus e relationship is approximately a straight line for most soils. The k_s versus $[e^3/(1+e)]$ relationship also indicates a linear form in some cases.

Permeability function for a deformable unsaturated porous medium

The coefficient of permeability for a deformable unsaturated soil has been investigated experimentally and theoretically. In most of these studies, the initial void ratios of the samples were obtained by compacting the soil rather than through loading and consolidation. Soils compacted at different water contents have different densities and different soil structures. The soil structure is particularly significant for compacted clays.

Previous studies

Many of the studies made on the influence of the degree of saturation and the void ratio on the permeability function do not have a theoretical basis. Rather, they are a presentation of observed data (e.g., Staple and Lehane 1954; Barden and Pavlakis 1971; Reicosky et al. 1981; Nimmo and Akstin 1988; Fleureau and Taibi 1994).

Some investigators have approached the problem from both a theoretical and an experimental standpoint. Mitchell et al. (1965) obtained the following permeability function for a deformable unsaturated soil based on Taylor's (1948) derivation of the permeability function for a saturated soil,

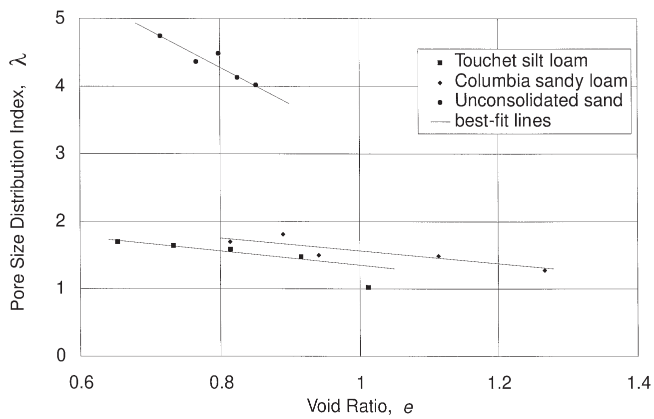


Fig. 3. Pore-size distribution index, λ, versus void ratio (data from Laliberte et al. 1966).

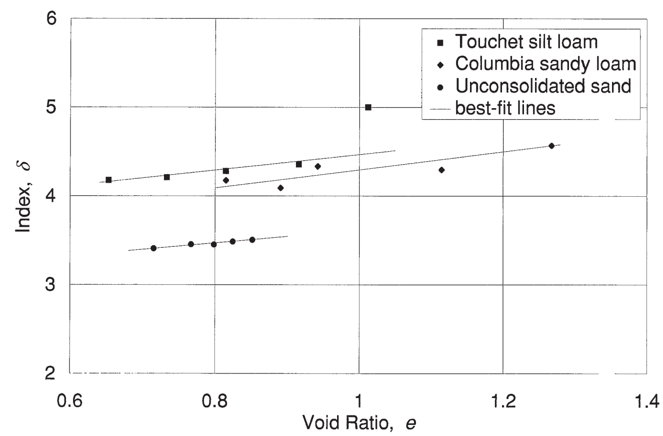


Fig. 4. Index, δ, in the macroscopic model versus void ratio (data from Laliberte et al. 1966).

[17] $$k = C_3 \frac{\rho_w g}{\mu} \frac{e^3}{1+e} S^3$$

where C_3 is a constant related to the soil–water system.

Experimental data obtained for a compacted clay suggested that the k versus S^3 relationship is approximately linear for degrees of saturation between 80 and 100% (Mitchell et al. 1965). However, the effect of changes in soil structure was not considered.

Laliberte et al. (1966) investigated the influence of soil density on the coefficient of permeability of unsaturated soils. A steady-state type test was used to measure the coefficient of permeability for three soils, namely, Touchet silt loam (GE3), Columbia sandy loam, and a sand. The clay fraction (<5 µm) for the preceding three soils are 15, 11, and 4%, respectively. The specimens were statically compacted. The volume changes for the specimens can be considered negligible during the permeability measurement and the difference in soil structure induced by the compaction may be neglected because of the small clay fractions in the soils. The study concluded that the density had a significant influence on the intrinsic permeability and the air-entry value of the soil. The pore-size distribution coefficient, η, and the pore-size distribution index, λ, were found to have little effect on the intrinsic permeability and air-entry value of the soil. The experimental results by Laliberte et al. (1966) have been re-analyzed and are presented in Figs. 1–4. The δ values were estimated from η and λ values using linear interpolation. It can be seen from Figs. 1–4 that the logarithm of the air-entry value, the pore-size distribution coefficient, the pore-size distribution index, and the index, δ, appear to be linearly related to the void ratio.

Lloret and Alonso (1980) suggested a permeability function for a deformable unsaturated soil. However, no data were presented to support the proposed equation, which has the following form:

[18] $$k = k(S, e_o)10^{(e-e_o)/b_6}$$

where b_6 is an experimental coefficient, and $k(S, e_o)$ is the permeability function at a void ratio of e_o.

Chang and Duncan (1983) suggested the following equation for the permeability function for a deformable unsaturated soil:

[19] $$k = k_{so} G_e H_s$$

where G_e is a factor related to void ratio, and H_s is a factor related to degree of saturation.

Singh (1965) provides a development of the factor, H_s based on a macroscopic model of unsaturated soil. A constant index, δ, of 3 was suggested in the study. Equation [15] or [16] was adopted for the factor G_e (Chang and Duncan 1983). However, evidence has indicated that the index δ is void-ratio dependent (Laliberte et al. 1966; Mualem and Dagan 1978; Huang 1994). Consequently, eq. [19] does not completely account for the influence of the void ratio on the coefficient of permeability.

Development of a new permeability function for a deformable unsaturated porous medium

Huang (1994) proposed a new general form of the permeability function for deformable unsaturated porous soil. The proposed function takes into account the influence of both the degree of saturation and the void ratio on the permeability function. The proposed permeability function can be expressed either in a macroscopic, an empirical, or a statistical form, as follows.

Macroscopic model for a deformable unsaturated porous medium

The index δ is assumed to be linearly related to the void ratio as illustrated in Fig. 4,

[20] $$\delta = \delta_o + c\,(e - e_o)$$

where δ_o is the index δ at a void ratio e_o, and c is an experimental parameter, defined as $\Delta\delta/\Delta e$.

A macroscopic model for a deformable unsaturated porous medium can be obtained by combining eqs. [5], [15], and [20]:

[21] $$k = k_{so} 10^{b(e-e_o)} S_e^{[\delta_o + c(e-e_o)]}$$

In eq. [21], the coefficient of permeability for a deformable unsaturated porous medium is expressed as a function of both the void ratio and the effective degree of saturation. A graphical representation of eq. [21] is presented in Fig. 5.

Fig. 5. Graphical representation of the macroscopic model for a deformable unsaturated porous medium.

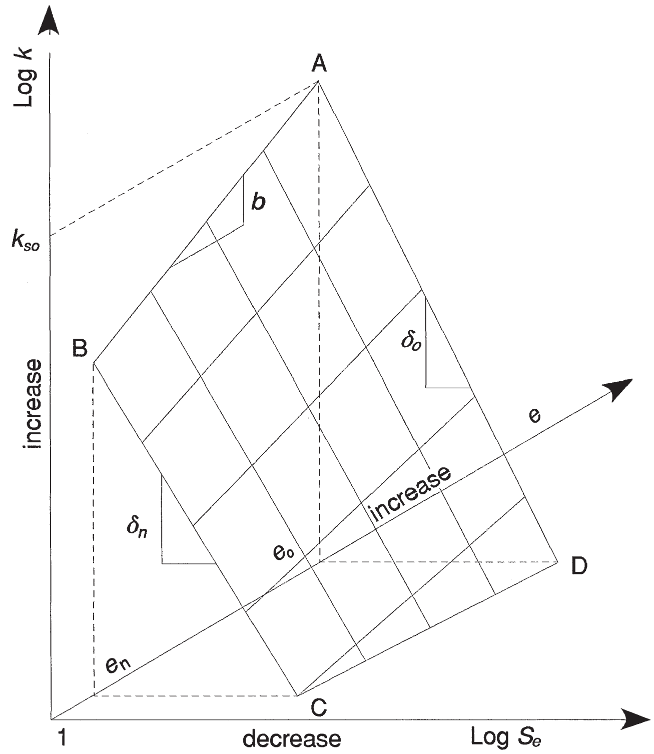
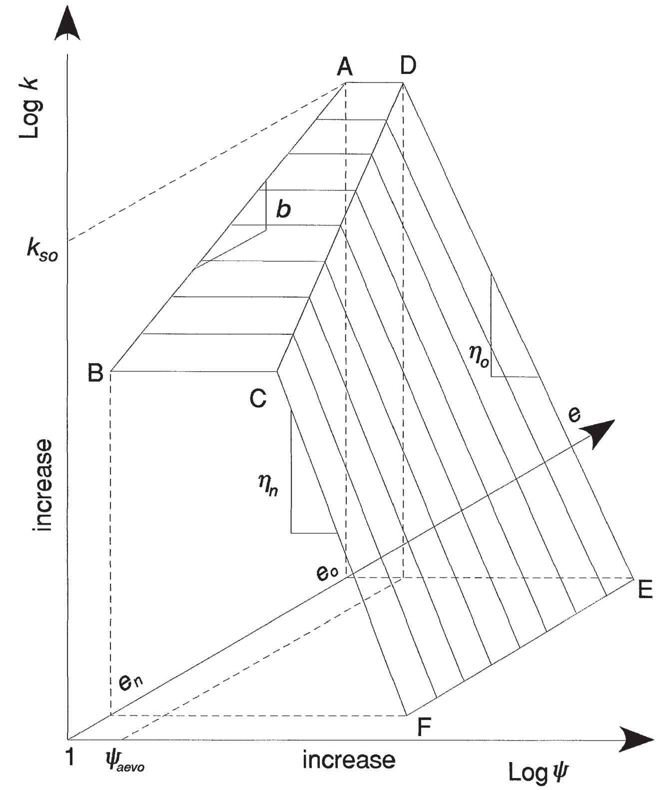


Fig. 6. Graphical representation of the empirical equation for a deformable unsaturated porous medium.



Empirical equation for a deformable unsaturated porous medium

Empirical permeability functions for a deformable unsaturated porous medium can be derived based on the empirical equations for the permeability function for nondeformable soils and assumptions with respect to the effect of changes in void ratio.

The logarithm of the air-entry value can be assumed to be linearly related to the void ratio ([Fig. 1]) as follows:

$$[22] \quad \log \psi_{\mathrm{aev}} = \log \psi_{\mathrm{aevo}} + a(e - e_{\mathrm{o}})$$

where ψ_{aevo} is the air-entry value at a void ratio of e_{o}, and a is an experimental parameter, defined as $\Delta\log\ \psi_{\mathrm{aev}}/\Delta e$.

Similarly, the pore-size distribution index, λ, can be assumed to be linearly related to void ratio (Fig. 3) as follows:

$$[23] \quad \lambda = \lambda_{\mathrm{o}} + d(e - e_{\mathrm{o}})$$

where λ_{o} is the pore-size distribution index at a void ratio of e_{o}, and δ is an experimental parameter, defined as $\Delta\lambda/\Delta e$.

Using eq. [3*b*], the soil–water characteristic curve for a deformable unsaturated porous medium can be described as follows:

$$[24a] \quad S_{\mathrm{e}} = 1 \qquad \text{for} \qquad \psi \le \psi_{\mathrm{aevo}} 10^{a(e-e_{\mathrm{o}})}$$

$$[24b] \quad S_{\mathrm{e}} = \left[\frac{\psi_{\mathrm{aevo}} 10^{a(e-e_{\mathrm{o}})}}{\psi}\right]^{[\lambda_{\mathrm{o}} + d(e-e_{\mathrm{o}})]}$$

for $\quad \psi > \psi_{\mathrm{aevo}} 10^{a(e-e_{\mathrm{o}})}$

A coefficient of permeability function for a deformable unsaturated porous medium in terms of soil suction can be obtained from eqs. [21]–[24]:

$$[25a] \quad k = k_{\mathrm{so}} 10^{b(e-e_{\mathrm{o}})} \qquad \text{for} \qquad \psi \le \psi_{\mathrm{aevo}} 10^{a(e-e_{\mathrm{o}})}$$

$$[25b] \quad k = k_{\mathrm{so}} 10^{b(e-e_{\mathrm{o}})} \left[\frac{\psi_{\mathrm{aevo}} 10^{a(e-e_{\mathrm{o}})}}{\psi}\right]^{\eta}$$

for $\quad \psi > \psi_{\mathrm{aevo}} 10^{a(e-e_{\mathrm{o}})}$

where η is defined as follows:

$$[26] \quad \eta = \lambda\delta = \lambda_{\mathrm{o}}\delta_{\mathrm{o}} + (c\lambda_{\mathrm{o}} + d\delta_{\mathrm{o}})(e - e_{\mathrm{o}}) + cd(e - e_{\mathrm{o}})^2$$

The last term of eq. [26], $cd(e - e_{\mathrm{o}})^2$, is negligible when the change in void ratio is small. The pore-size distribution coefficient versus void-ratio relationship can be inferred from eq. [26] to be approximately linear (see also data presented in Fig. 2). A graphical representation of eq. [25] is presented in Fig. 6.

Statistical model for deformable unsaturated porous media

Statistical models for a deformable unsaturated medium can be derived from the statistical models for the permeability function for nondeformable soils and the relationship for the effect of void ratio described previously.

The degree of saturation is superior to the volumetric water content in describing the permeability function for a deformable unsaturated porous medium (Huang 1994). The effective degree of saturation should be used as the variable to account for the mobile water. The statistical model for a

Table 2. Index properties for the soil used in the testing program (Rahardjo 1990).

G_s	w_L (%)	w_P (%)	PI (%)	Sand (%)	Silt (%)	Clay (%)	D_{10} (mm)	D_{30} (mm)	D_{60} (mm)
2.68	22.2	16.6	5.6	52.5	37.5	10	0.002	0.02	0.09

deformable unsaturated porous medium can be obtained from eqs. [8] and [15] as follows:

$$[27] \qquad k = k_{eo}10^{b(e-e_o)} \int_0^{S_e} \frac{n(S_e - \Sigma_e)}{\psi^2}\, d\Sigma_e \Bigg/ \int_0^1 \frac{n(1-\Sigma_e)}{\psi^2}\, d\Sigma_e$$

where Σ_e is the dummy variable of integration representing the effective degree of saturation.

Equation [27] is a general form of the statistical model for a deformable unsaturated porous medium. The porosity is not a constant for a deformable medium. The influence of the void ratio or porosity is reflected within the integrals in eq. [27]. The porosity, n, expressed as a function of soil suction is required to estimate the coefficient of permeability based on eq. [27].

The estimation of the coefficient of permeability using eq. [27] can be simplified by assuming that the volume change during the desaturation range (i.e., from the air-entry value to higher soil suctions) is negligible (Huang 1994). In other words, most of the volume changes occur prior to the soil suction reaching the air-entry value. A simplified statistical model for a deformable unsaturated porous medium can be derived based on the preceding assumption,

$$[28] \qquad k = k_{eo}10^{b(e-e_o)} \int_0^{S_e} \frac{(S_e - \Sigma_e)}{\psi^2}\, d\Sigma_e \Bigg/ \int_0^1 \frac{n(1-\Sigma_e)}{\psi^2}\, d\Sigma_e$$

Experimental program

An experimental program was conducted to determine the form of the permeability function for a deformable unsaturated soil (Huang et al. 1997).

Testing material

A silty sand from a Saskatchewan Department of Highway borrow pit was selected for testing. The index properties of the soil are shown in Table 2. Distilled water was added to the air-dried silty sand to prepare slurried specimens for the experimental program. The saturated coefficient of permeability and the air-entry value are the primary criteria for the selection of the soil for testing (Huang 1994).

Equipment

A triaxial permeameter was constructed for measuring the coefficient of permeability of the silty sand. The triaxial cell was a modification of the stress-controlled isotropic cell developed by Ho (1988). Details on the triaxial permeameter and the plumbing layout for the triaxial permeability test setup are presented in Huang et al. (1998). The confining pressure for the cell, the air pressure and the upper water pressure are controlled using pressure regulators and monitored using pressure transducers. The lower water pressure is controlled using a pressure regulator and indirectly monitored using a high precision differential pressure transducer. The differential pressure transducer is set to directly monitor the differential head applied to the soil specimen. The volumes of the inflow and outflow water are measured using two volume change indicators. The diffused air bubbles collected above the upper high air-entry disk and beneath the lower high air-entry disk are regularly flushed from the system. The volumes of the diffused air are collected and monitored using two diffused air volume indicators. The deformations of the soil specimen are measured using a "non-contacting displacement measuring system."[1]

The net normal stress and the matric suction of the soil specimen in the triaxial permeameter were independently controlled in order that the coefficient of permeability at different combinations of the stress state variables could be measured. Air leakage through the gaps between the rubber membrane and the cap as well as through the pedestal into the specimen are the primary concerns in the permeability tests conducted with this permeameter. Long term leakage observations suggested that with tight O-rings the permeameter could be used to measure coefficients of permeability as low as 5×10^{-11} m/s for a saturated soil, with an error of approximately ±18% error. For an unsaturated soil, the error becomes significantly lower. The air leaking into the unsaturated specimen mixes with the pore air of the specimen and has no effect on the water phase (Huang 1994).

The soil–water characteristic curves for the soil at different initial void ratios were obtained using a pressure plate cell.

Test program

Four groups of experiments were conducted to study the permeability function and the soil–water characteristic curve for the silty sand. Each group consisted of six different tests. Each test was performed on six different specimens. All the specimens were initially slurried. Different groups of tests were conducted by consolidating the specimens under various net normal stresses.

Flexible-wall permeability tests (FWPT)

Each of the six initially slurried specimens was assembled within a rubber membrane and isotropically consolidated under one of the designated net normal stresses. The coefficient of permeability of the specimen was measured at various matric suction values. The volume changes of the specimen during the permeability measurement were monitored. The first test in this group (FWPT1) was terminated at a matric suction of 50 kPa due to a failure of the lower high air-entry disk. Tests on the remaining five specimens were resumed after the faulty high air-entry disk was replaced.

[1] Manufactured by Kaman Sciences Corporation of Colorado Springs, Colorado.

Table 3. Summary of the volume–mass properties for the flexible-wall permeability tests.

Test No.	Initial void ratio, e_i	Consolidated void ratio, e_c	Final void ratio, e_f	Final water content, w_f (%)	Consolidation pressure, σ_c (kPa)
FWPT1	—	—	—	—	10
FWPT2	0.646	0.536	0.522	9.58	10
FWPT3	0.642	0.514	0.514	9.64	25
FWPT4	0.652	0.507	0.502	9.90	50
FWPT5	0.638	0.478	0.484	9.88	100
FWPT6	0.647	0.468	0.467	10.06	200

Table 4. Summary of the volume–mass properties for the rigid-walled permeability tests.

Test No.	Initial void ratio, e_i	Consolidated void ratio, e_c	Final void ratio, e_f	Final water content, w_f (%)	Consolidation pressure, σ_c (kPa)
RWPT1	0.649	0.531	0.523	10.46	12.5
RWPT2	0.643	0.521	0.515	10.51	25
RWPT3	0.651	0.501	0.499	10.82	50
RWPT4	0.637	0.488	0.481	10.12	100
RWPT5	0.648	0.462	0.468	11.47	200

Table 5. Summary of the volume-mass properties for pressure plate cell test Group No. 1.

Test No.	Initial void ratio, e_i	Consolidated void ratio, e_c	Final void ratio, e_f	Final water content, w_f (%)	Consolidation pressure, σ_c (kPa)
PPCT11	0.692	0.692	0.536	6.86	0
PPCT12	0.700	0.540	0.529	6.93	10
PPCT13	0.697	0.528	0.518	6.94	25
PPCT14	0.635	0.501	0.497	7.27	50
PPCT15	0.632	0.483	0.481	7.37	100
PPCT16	0.634	0.466	0.463	7.51	200

Table 6. Summary of the volume–mass properties for pressure-plate cell test Group No. 2.

Test No.	Initial void ratio, e_i	Consolidated void ratio, e_c	Final void ratio, e_f	Final water content, w_f (%)	Consolidation pressure, σ_c (kPa)
PPCT21	0.631	0.525	0.502	7.13	16
PPCT22	0.607	0.513	0.501	7.22	32
PPCT23	0.627	0.490	0.486	7.43	63
PPCT24	0.657	0.474	0.471	7.66	126
PPCT25	0.661	0.454	0.452	7.80	252
PPCT26	0.663	0.426	0.425	7.91	504

Rigid wall permeability tests (RWPT)

Each of the six initially slurried specimens was assembled into a stainless steel sample ring within a rubber membrane. The specimen was consolidated under one of the designated net normal stresses (K_o consolidation) and the coefficient of permeability of the specimen was measured at four different matric suction values. In this group of tests, the lateral displacement was negligible due to the use of the steel sample ring.

Pressure plate cell tests Group No. 1 (PPCT1)

Six initially slurried specimens were one-dimensionally preconsolidated under pressures corresponding to the confining pressures used in the flexible-walled permeability tests with the exception of the first test, PPCT11. In the first test, an unconsolidated slurried specimen was used. The specimens were completely unloaded prior to being placed into the pressure plate cells. The pressure plate cell tests were performed at different matric suction values.

Pressure-plate cell tests Group No. 2 (PPCT2)

Six initially slurried specimens were one-dimensionally preconsolidated under pressures determined from the net normal stresses used in the rigid-walled permeability tests. The specimens were completely unloaded prior to being moved to the pressure-plate cells. The pressure-plate cell tests were performed at different matric suction values. The normal stresses used in this group of tests were 1.26 times the net normal stresses used in the rigid-wall permeability tests (the factor, 1.26, was estimated from the compression index, C_c, and the swelling index, C_s). The normal stresses

Fig. 7. Coefficient of permeability versus matric suction obtained from flexible-walled permeability tests.

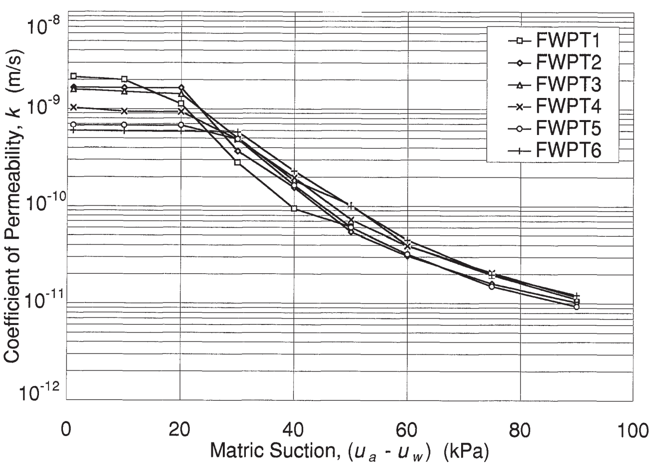


Fig. 8. Void ratio change versus matric suction obtained from the flexible-walled permeability tests.

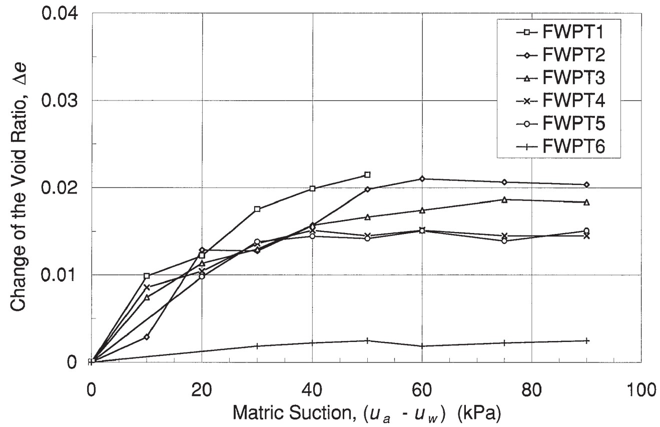


Fig. 9. Coefficient of permeability versus matric suction obtained from the rigid-walled permeability tests.

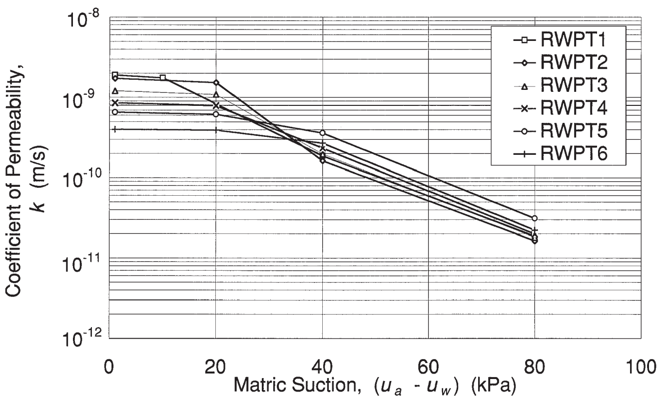
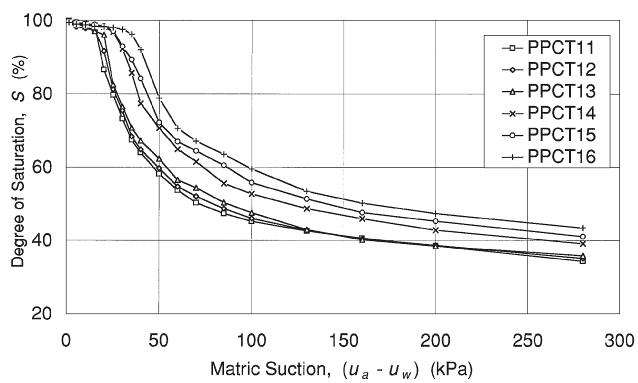


Fig. 10. Degree of saturation versus matric suction obtained from the pressure-plate cell tests Group No. 1.



for this group of pressure-plate tests were larger than those used in the tests using the rigid-wall permeability device. After unloading, the specimens swelled and the resulting void ratios were found to be close to the void ratios used in the rigid-wall permeability tests.

Test results

The volume–mass properties for the specimens in the four groups of tests are summarized in Tables 3–6, respectively.

The coefficients of permeability and the void-ratio changes from the flexible-walled permeability tests (FWPT) are plotted against matric suction in Figs. 7 and 8, respectively. The values of coefficient of permeability are approximately equal to the saturated coefficient of permeability when the matric suction is less than 20 kPa. The coefficient of permeability decreases rapidly as matric suction increases. Most of the void ratio change occurs between a matric suction of 0 and 40 kPa. The void ratio changes only slightly at matric suctions greater than 40 kPa.

Measurements for the coefficient of permeability are time-consuming, particularly when the specimens become unsaturated. As a result, the coefficient of permeability was measured at only four matric suction levels using the rigid-wall permeability test (RWPT). The permeability measurement results for the RWPT group of tests are plotted against matric suction in Fig. 9.

The soil–water characteristic curve for the silty sand can be obtained from the pressure-plate cell tests. The degree of saturation at any matric suction value can be calculated using the void ratio corresponding to that matric suction value (Huang 1994). The void-ratio changes were not monitored during the pressure-plate cell test. To estimate void-ratio changes, the void ratio versus matric suction relationship is assumed to be linear. Also, all the void-ratio changes are assumed to occur before the matric suction reached the air-entry value. The preceding assumptions were based on experimental evidence (Croney and Coleman 1954; Fredlund 1964; Fleureau et al. 1993).

The degrees of saturation from two groups of pressure-plate cell tests are plotted against matric suction in Figs. 10 and 11, respectively. The results show that the degree of saturation is close to 100% when suction is lower than the air-entry value. Above the air-entry value, the degree of saturation decreases rapidly with matric suction. Near the residual degree of saturation, the degree of saturation decreases slowly with respect to matric suction.

Analyses of test results

Equations [21], [25], and [28] show that the coefficient of permeability versus void ratio relationship is linear on a log

Fig. 11. Degree of saturation versus matric suction obtained from the pressure-plate cell tests Group No. 2.

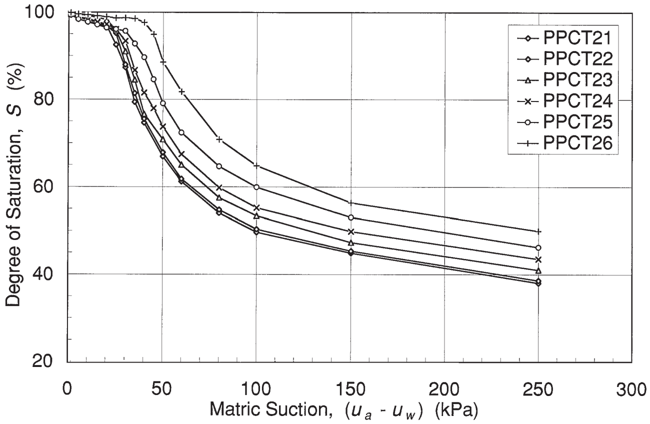


Fig. 12. Saturated coefficient of permeability versus void ratio for the flexible and rigid-walled permeability tests.

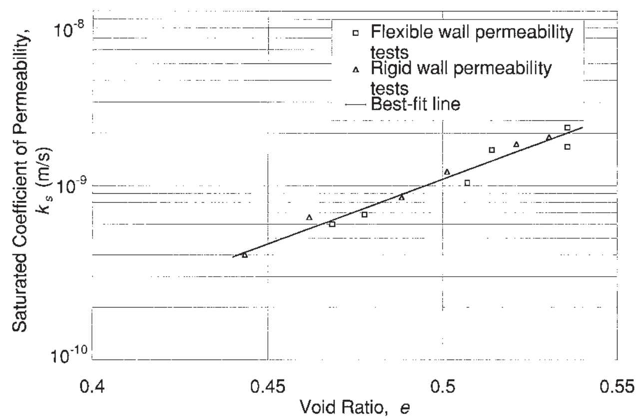


Fig. 13. Relative coefficient of permeability versus void ratio and best-fit lines for the flexible-walled permeability tests.

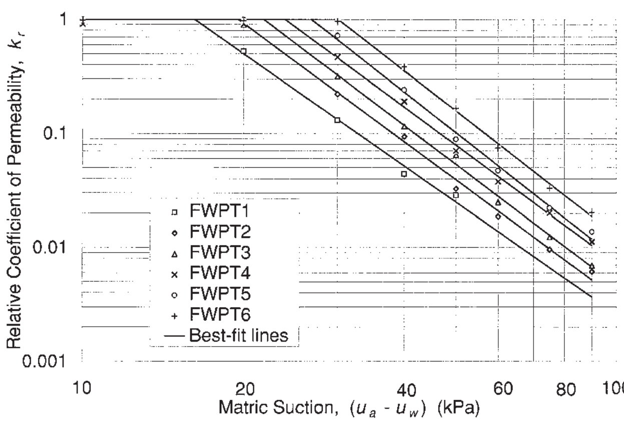


Fig. 14. Relative coefficient of permeability versus void ratio and best-fit lines for the rigid-walled permeability tests.

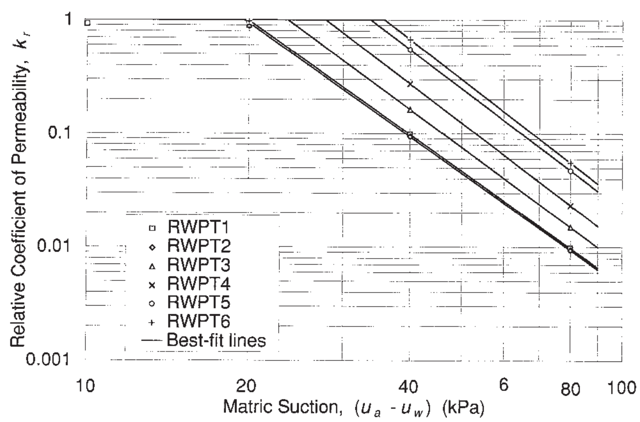
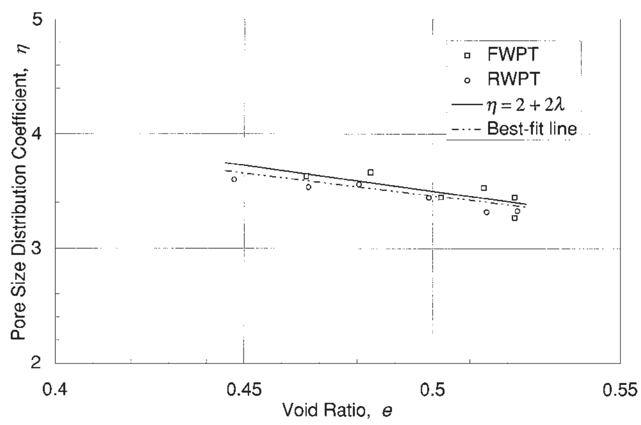


Fig. 15. Pore-size distribution coefficient, η, versus void ratio for the flexible and rigid-walled permeability tests.



k_s versus void ratio plot when the soil is saturated (i.e., S_e equals 100% or ψ is zero). The saturated coefficients of permeability obtained from the flexible-walled and the rigid-walled permeability tests are plotted against void ratio in Fig. 12. The experimental results confirm that the log k_s versus e relationship is approximately linear. The void ratio in Fig. 12 is referenced to the current void ratio at which the saturated coefficient of permeability was measured.

Equation [25] indicates that the relative permeability versus matric suction relationship is bilinear, with small changes in relative permeability for values of matric suction less than the air-entry value. The relative permeability obtained from both the flexible-walled and the rigid-walled permeability tests are plotted against matric suction in Figs. 13 and 14. The experimental results show that the relative permeability function can be approximated by a bilinear relationship on a log–log scale.

Equation [26] shows that the pore-size distribution coefficient is linearly related to the void ratio. The slopes of the best-fit lines in Figs. 13 and 14 represent the pore-size distribution coefficients at different void ratios. The pore-size distribution coefficients are plotted against void ratios in Fig. 15. The pore-size distribution coefficient versus void ratio relationship in Fig. 15 appears to be fairly linear. The pore-size distribution coefficient decreases with void ratio.

The residual degree of saturation represents the "immobile" portion of the water phase (Mualem, 1976) and must be estimated to determine the effective degree of saturation. At least six methods have been suggested for the estimation of the residual degree of saturation (Corey 1954; Brooks and Corey 1964; White et al. 1970; Mualem 1976; van Genuchten 1980; Chang and Duncan 1983). The least-squares method, suggested by Brooks and Corey (1964) and later by Mualem (1976), appears to be most suit-

Fig. 16. Residual degree of saturation, S_r, versus void ratio for the pressure-plate cell tests.

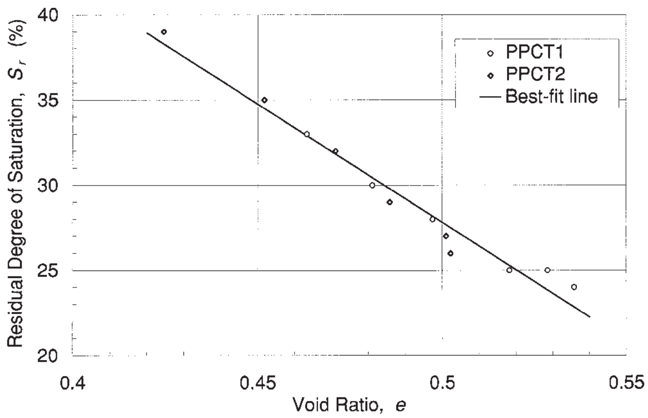


Fig. 17. Effective degree of saturation versus matric suction and best-fit lines for the pressure-plate cell tests Group No. 1.

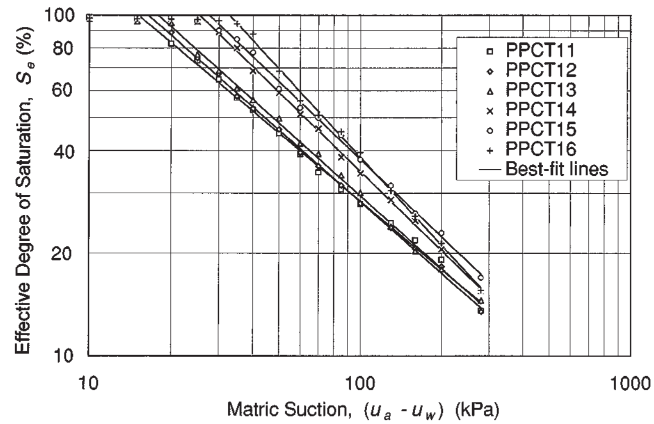


Fig. 18. Effective degree of saturation versus matric suction and best-fit lines for the pressure-plate cell tests Group No. 2.

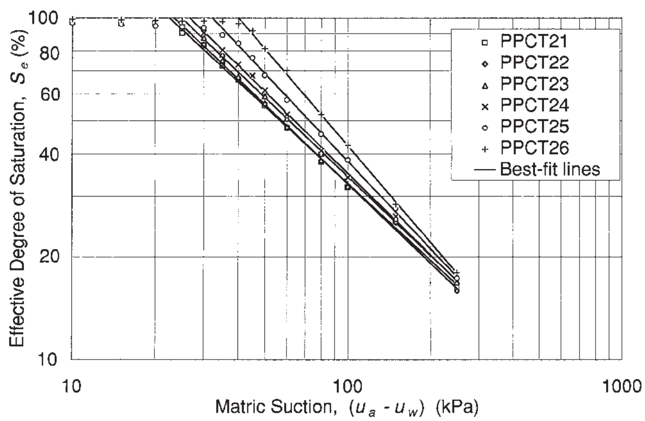
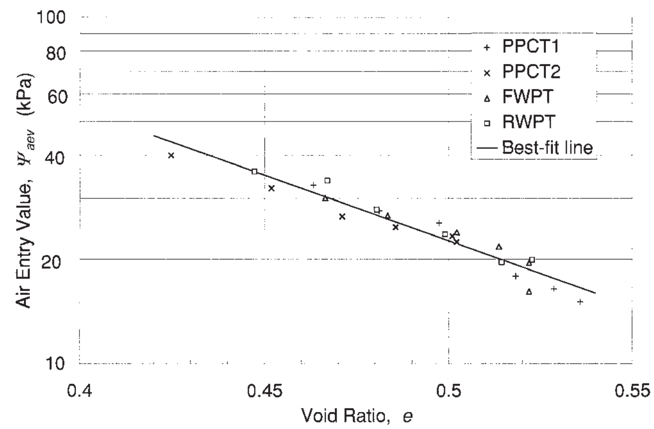


Fig. 19. Air-entry value versus void ratio for the permeability tests and the pressure-plate cell tests.



able from a practical standpoint. Using the least-squares method, the residual degree of saturation for the two groups of pressure-plate cell tests is estimated and plotted against void ratio in Fig. 16. The residual degree of saturation is shown to decrease with void ratio, indicating that the fraction of the immobile water increases as the volume of void decreases.

Equation [24] describes a soil–water characteristic curve for a deformable unsaturated soil in which the effective degree of saturation versus matric suction relationship is bilinear on a log–log scale. The bilinear curve has a flat portion at matric suctions below the air-entry value. The effective degree of saturation for the two groups of pressure-plate cell tests is presented against soil suction in Figs. 17 and 18, respectively. The results appear to confirm the predictions based on eq. [24].

The air-entry value is defined graphically as the intersection of the best-fit lines of the two linear segments of the soil–water characteristic curve (i.e., eq. [25]), or the permeability function (i.e., eq. [26]). The air-entry values obtained from all four groups of tests are plotted against void ratio in Fig. 19. The logarithm of the air-entry value versus void ratio relationship is approximately linear. The linearity of the logarithm of the air-entry value versus void ratio relationship is theoretically implied by eqs. [24] and [25].

The slopes of the best-fit lines for the soil–water characteristic curves shown in Figs. 17 and 18 represent the pore-size distribution index, λ, at different void ratios. The pore-size distribution index decreases linearly with void ratio (Fig. 20). The linear pore-size distribution index versus void ratio relationship can be approximated using eq. [23].

The index, δ, for the macroscopic model can be estimated from the pore-size distribution index, λ, and the pore-size distribution coefficient, η (eq. [26]). The index, δ, corresponding to the void ratio of the specimens in both the flexible-walled and the rigid-walled permeability tests can be obtained using linear interpolation. The index, δ, increases with void ratio (Fig. 21) and can be approximated by a linear relationship using eq. [26].

The void ratios did not change significantly during desaturation (i.e., from the air-entry value to higher suctions) and the final void ratios were used in determining the values of the parameters plotted in Figs. 15, 16, and 19–21. All the parameters were obtained within the desaturation range, except k_s (Huang 1994).

The coefficient of permeability of a deformable unsaturated soil can be estimated from the soil–water characteristic curve (i.e., eq. [28]). The permeability function for any particular void ratio can be calculated based on the corresponding soil–water characteristic curve and the saturated

Fig. 20. Pore-size distribution index, λ, versus void ratio for the pressure-plate cell tests.

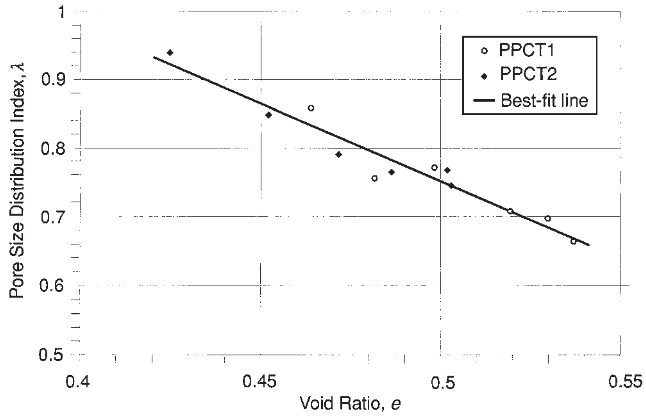
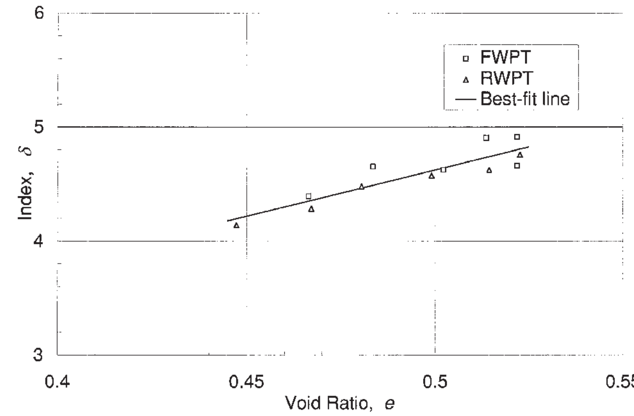


Fig. 21. Index, δ, versus void ratio for the flexible and rigid-walled permeability tests.



coefficient of permeability for the same void ratio. Substituting eq. [24] into eq. [28] gives the following equation:

$$[29a] \quad k = k_{so}10^{b(e-e_o)} \qquad \text{for} \qquad \psi \le \psi_{aev}$$

$$[29b] \quad k = k_{so}10^{b(e-e_o)}\left[\frac{\psi_{aevo}10^{a(e-e_o)}}{\psi}\right]^{2\lambda+2} \qquad \text{for} \qquad \psi > \psi_{aev}$$

where λ is the pore-size distribution index (i.e., $\lambda = \lambda_o + d(e - e_o)$).

A comparison of eq. [29] with eq. [24] shows that the pore-size distribution index, λ, can be related to the pore-size distribution coefficient, η, as follows:

$$[30] \quad \eta = 2\lambda + 2$$

Equation [30] is a theoretical relationship between the pore-size distribution index and the pore-size distribution coefficient. The relationship for eq. [30] is presented in Fig. 15, together with the pore-size distribution coefficients obtained from the flexible-walled and the rigid-walled permeability tests. The theoretical relationship given by eq. [30] is close to the best-fit line for the experimental data. It would appear to be reasonable to predict the coefficient of permeability of a deformable unsaturated soil using eq. [28], which utilizes the soil–water characteristic curve of the soil.

The experimental data indicate that the range of values for the parameters δ, η, λ, S_r, ψ_{aev}, and k_s for the tested void ratios are about 4.1 to 5.0, 3.2 to 3.7, 0.65 to 0.95, 0.25 to 0.40, 15 to 40 kPa, and 0.40×10^{-8} to 2.1×10^{-8} m/s, respectively. The void ratio does not significantly affect the parameters δ, η, λ and S_r. In engineering practice, average values for the parameters δ, η, λ and S_r, can be used for a soil without significant error in estimating the coefficient of permeability. The void ratio has a significant effect on the air-entry value, ψ_{aev}, and the coefficient of permeability at saturation, k_s. The air-entry value and the saturated coefficient of permeability for the corresponding void ratio should be used.

Conclusions

For a porous medium with an incompressible structure, the permeability function can be expressed in three different forms as follows:

(*i*) As empirical functions of soil suction or volumetric water content.

(*ii*) As macroscopic models based on the "effective" degree of saturation.

(*iii*) As statistical models in which the relative permeability is calculated from the soil–water characteristic curve.

The preceding three forms of the permeability functions for a nondeformable porous medium are related due to the relationship between the volumetric water content, degree of saturation, and soil suction. As a result, one form of model can be transformed to another form with the existing relations. A general form for the permeability function for a deformable soil should incorporate the influence of both the degree of saturation and the void ratio.

An experimental program has been conducted to determine the permeability function for a silty sand at various void ratios. The data obtained from the experimental program can be reasonably interpreted using the theory developed in this paper for the permeability function of a deformable unsaturated soil. The experimental data indicate that the parameters, δ, η, λ, and S_r, in the permeability function are not strongly dependent on the void ratio and can be taken as constant values in engineering practice. Other parameters, such as the coefficient of permeability at saturation, k_s, and the air-entry pressure, ψ_{aev}, are highly dependent on the void ratio so that the influence of the void ratio cannot be ignored.

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