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Abstract

A non-linear model for unsaturated soils is presented. The model involves 13 parameters covering shear strength, deformation of soil structure and water volume change. All the parameters have either physical or geometrical significance, and can be determined using laboratory tests. Only two types of triaxial tests are required to determine the 13 parameters, namely, 1) the triaxial shear test with controlled-suction and 2) the triaxial shrinkage test with controlled-net mean stress. The model is used to predict the suction changes for a constant water content test. Good agreements are obtained.

Introduction

Non-linear models for unsaturated soil are based on either a total stress-strain relationship or an incremental stress-strain relationship. The former uses the concept of state surfaces, relating void ratio and degree of saturation to net mean stress and soil suction [1,2] or expresses the state surfaces in explicit mathematical equations [3,4]. The latter formulates the constitutive relationship in incremental form [3,5] or mixed-form [6,7]. A new nonlinear model for unsaturated soil is presented.

Theoretical Basis

The linear constitutive relationships under triaxial conditions for an unsaturated soils [5] can be written in an incremental form as follows:

$$d\varepsilon_1 = [d(\sigma_1 - u_a) - 2\mu_t d(\sigma_3 - u_a)]/E_t \quad + \quad ds/H_t \qquad [1]$$

$$d\varepsilon_3 = [-\mu_t d(\sigma_1 - u_a) - (1-\mu_t) d(\sigma_3 - u_a)]/E_t \quad + \quad ds/H \qquad [2]$$

$$d\varepsilon_w = dp/K_{wt} \quad + \quad ds/H_{wt} \qquad [3]$$

where E_t = tangent Young's modulus for soil structure, μ_t = tangent Poisson's ratio, H_t = tangent volumetric modulus for the soil structure with respect to change in matric suction, K_{wt} = tangent water volumetric modulus associated with a change in net mean stress, H_{wt} = tangent water volumetric modulus associated with a change in matric suction, ε_1 = axial (or major) strain, ε_3 = radial (or minor) strain, ε_w = water volume change, u_a = pore-air pressure, s = suction (i.e., $u_a - u_w$), σ_1 = major principal stress, σ_3 = minor principal stress and p = net mean stress (i.e., $[(\sigma_1 + 2\sigma_3)/3] - u_a$).

In a triaxial test where, σ_3 and s are maintained constant, Eqs. 1 and 2 give:

$$E_t = d(\sigma_1 - u_a)/d\varepsilon_1 = d(\sigma_1 - \sigma_3)/d\varepsilon_1 \qquad [4]$$

$$\mu_t = -d\varepsilon_3/d\varepsilon_1 \qquad [5]$$

The tangent bulk modulus, K_t, is given by:

$$K_t = dp/d\varepsilon_v = E_t/[3(1-2\mu_t)] = d(\sigma_1 - \sigma_3)/(3d\varepsilon_v) \qquad [6]$$

where $\varepsilon_v = \varepsilon_1 + 2\varepsilon_3$ (i.e., volumetric strain)
Meanwhile, Eq. (3) gives

$$K_{wt} = dp/d\varepsilon_w = d(\sigma_1 - \sigma_3)/(3d\varepsilon_w) \qquad [7]$$

In a triaxial test where matric suction increases while net mean stress is maintained constant, Eqs (1), (2) and (3) reduce to:

$$H_t = 3ds / d\varepsilon_v \quad [8]$$

$$H_{wt} = ds / d\varepsilon_w \quad [9]$$

In addition, the Mohr-Coulomb failure criterion for unsaturated soils is:

$$(\sigma_1 - \sigma_3)_f = \left[2(c' + s_f \tan\phi^b)\cos\phi' + 2(\sigma_3 - u_a)\sin\phi'\right] / (1 - \sin\phi') \quad [10]$$

Eqs. (1) to (10) form the basis for the determination of the parameters of the proposed model using laboratory tests.

Laboratory tests

Two series of laboratory tests have been conducted; namely, 1.) triaxial shrinkage test at constant net mean stress, and 2.) triaxial drained shear test at constant net cell pressure, $(\sigma_3 - u_a)$, and constant matric suction, s.

A total of 4 triaxial shrinkage tests were performed, and the adopted net mean stresses were 5, 50, 100 and 200 kPa, respectively, A total of 14 triaxial drained shear tests were conducted. The adopted suctions were 0, 50, 100, 200 and 300 kPa. The adopted net mean stresses were 100, 200 and 300 kPa.

The soil used in the study was an unsaturated compacted loess from Shanxi province, China. The compacted specimens had a dry density of 1.70 g/cm^3, a water content of 17.2%, a suction of 20 kPa and a void ratio of 0.6.

The modified triaxial apparatus and the details of the tests are presented elsewhere [8,9].

Determination of Model Parameters

All the parameters involved in the model and the changes in the parameter with stress state are evaluated using the results of the laboratory tests on the compacted loess.

1. Strength parameters

Values of 3.7 kPa, 32.7° and 18.2° were obtained for the shear strength parameters, c', ϕ' and ϕ^b, respectively, for the compacted loess [9].

2. Initial Young's Modulus

The $(\sigma_1 - \sigma_3)$ versus ε_a relationship for the compacted loess can be put into the following hyperbolic form:

$$(\sigma_1 - \sigma_3) = \varepsilon_a / (a + b\varepsilon_a) \quad [11]$$

where a = reciprocal of the initial tangent modulus, E_i; b= the reciprocal of the asymptotic value of the ultimate deviator stress $(\sigma_1 - \sigma_3)_{ult}$.

The log (E_i/p_{atm}) versus log $\{(\sigma_3 - u_a)/p_{atm}\}$ relationship for the compacted loess is shown in Fig 1. The equation for the curves corresponding to each suction value can be described using the following equation:

$$E_i = kp_{atm}[(\sigma_3 - u_a)/p_{atm}]^n \quad [12]$$

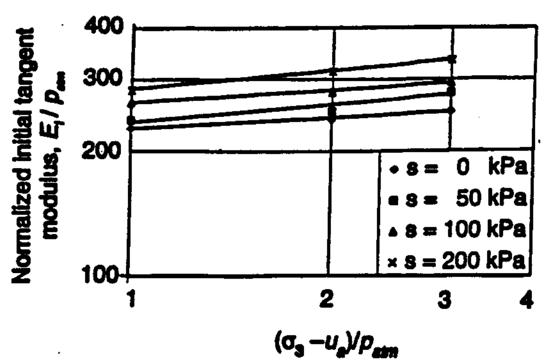

Fig. 1 Normalized initial tangent modulus versus normalized net mean stress relationship for the compacted loess.

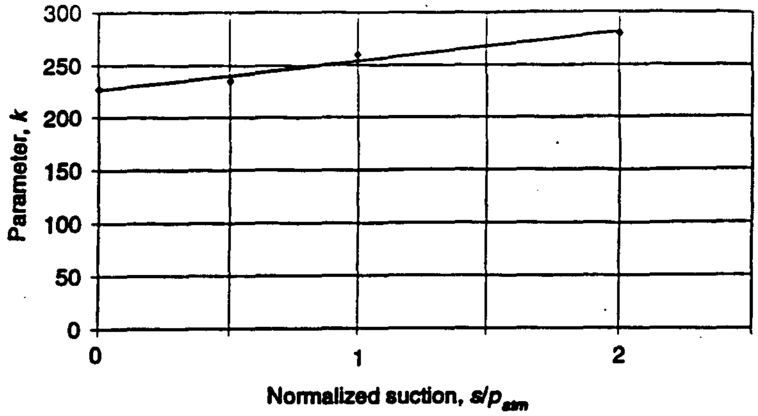

Fig. 2 Parameter k versus normalized suction relationship for the compacted loess.

Where p_{atm} is the atmospheric pressure and k and m are dimensionless parameters. The value of k is numerically equal to the value of (E_i/p_{atm}) when $(\sigma_3 - u_a)/p_{atm}$ is equal to 1, and m is the slope of the line in Fig. 1.

The value of m varies slightly with suction and can be taken as a constant value of 0.12 for the compacted loess. The value of k changes significantly with suction. The relationship between k and s/p_{atm} (Fig. 2) can be represented as:

$$k = k^{\circ} + m_1(s/p_{atm}) \qquad [13]$$

where k° is the value of k when suction is equal to zero, m_1 is a dimensionless constant which denotes the slope of the line in Fig. 2. The values of k° and m_1 are 227 and 27.8, respectively for the compacted loess.

3. Failure Ratio, R_f

The parameter, R_f, is defined as:

$$R_f = (\sigma_1 - \sigma_3)_f / (\sigma_1 - \sigma_3)_{ult} \qquad [14]$$

The value of R_f was found to decrease slightly with suction. For simplicity R_f can be taken as a constant value of 0.83 for the compacted loess.

4. Tangent Young's Modulus, E_t

From Eqs. 4, 11 and 14, E_t can be expressed as:

$$E_t = (1 - R_f L)^2 E_i \qquad [15]$$

where L is referred to as the stress level or fraction of strength mobilized and is given by:

$$L = (\sigma_1 - \sigma_3)/(\sigma_1 - \sigma_3)_f \qquad [16]$$

Substituting Eqs. 10, 12, 13, 14, and 16 into Eq. 15, the tangent Young's modulus for any stress condition is given by:

$$E_t = \left[1 - \frac{R_f(1-\sin\phi')(\sigma_1 - \sigma_3)}{2(c' + s_f \tan\phi^b) + 2(\sigma_3 - u_a)\sin\phi'}\right]^2 (k^{\circ} + m_1 s) p_{atm} \left(\frac{\sigma_3 - u_a}{p_{atm}}\right)^m \qquad [17]$$

5. Tangent Bulk Modulus for the Soil Structure, K_t

The Poisson's ratio, μ_t, in the Duncan-Chang's model [10] was replaced by the tangent bulk modulus in the model proposed by Duncan et al [11]. The value of K_t was calculated at the point on the stress strain curve where the stress level is equal to 70%, i.e.,

$$K_t = [(\sigma_1 - \sigma_3)/(3\varepsilon_v)]\,\big|_{l=70\%} \qquad [18]$$

The effect of net cell pressure on K_t was found to be small and K_t can be taken as a constant for each suction. The linear relationship between tangent bulk modulus for the soil structure and normalized suction, s/p_{atm} (Fig. 3) is given by:

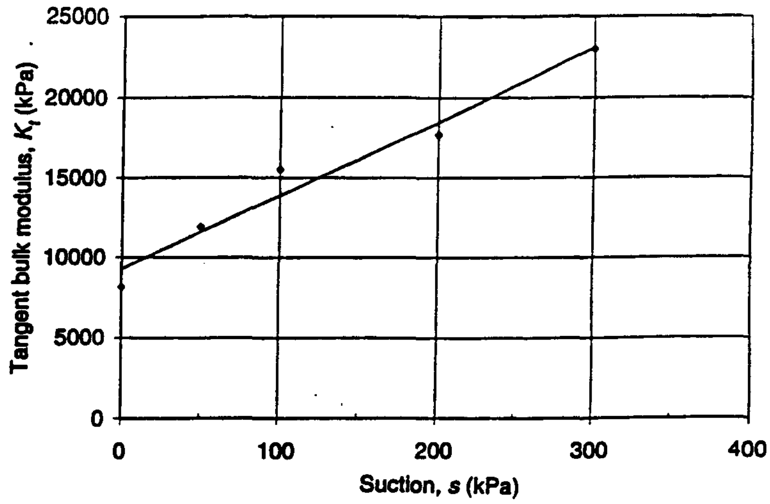

Fig. 3 Tangent bulk modulus versus suction relationship for the compacted loess.

$$K_t = K_t^o + m_2 s \qquad [19]$$

where K_t^o and m_2 are the intercept and slope of the line in Fig.3, respectively. The values of K_t and m_2 are equal to 9.2 kPa and 46, respectively for the compacted loess.

6. Tangent Water Modulus Associated with Changes in Net Mean Stress, K_{wt}

The parameter, K_{wt}, can be determined from Eq. (11). An alternate procedure for determining K_{wt} is as follows:

The *w-p* relationship from a triaxial shear test at a constant suction and different net cell pressures (Fig. 4) shows that the water content either stays constant or drops significantly after failure. The data points for various net cell pressures fall into a narrow band (Fig. 4). The difference between the data point and the best-fit straight line for each net mean stress is less than 0.3%. The slopes of the lines for various suction, $\beta(s)$, are found to be close to each other. The average value of the slopes, $\beta(s)$, is 2.92×10^{-5}/kPa.

The water volume change, ε_w, is related to the water content [9] as follows:

$$w = w_o - (1 + e_o)\varepsilon_w / G_s \qquad [20]$$

Differentiating Eq. (20) with respect to net mean stress gives:

$$K_{wt} = 1/\lambda_w(s) = -(1 + e_o)/(G_s \beta(s)) \qquad [21]$$

where $\lambda_w(s) = d\,\varepsilon_w/dp$.

The negative sign on the right hand side of Eq. 21 indicates that water content decreases with drainage. For the compacted loess with a void ratio, e, equal to 0.6 and G_s equal to 2.72, the value of K_{wt} is 20 MPa.

7. Tangent Volumetric Modulus for Soil Structure Associated with Changes in Matric Suction, H_t, and Tangent Water Volumetric Modulus Associated with Changes in Matric Suction, H_{wt}

The values for the parameters, H_t and H_{wt}, were obtained from the results of the triaxial shrinkage tests as follows:

$$H_{wt} = ds / d\varepsilon_w = \ln 10(s + p_{atm})/(\lambda_w(p)) \qquad [22]$$

$$H_t = 3ds / d\varepsilon_v = 3 \ln 10(s + p_{atm})/(\lambda_s(p)) \qquad [23]$$

where $\lambda_w(p)$ is a constant equal to 15.0% for the compacted loess. The value of $\lambda_s(p)$ increases with p as follows:

$$\lambda_s(p) = \lambda_s^o(p) + m_3 \log[(p + p_{atm})/p_{atm}] \qquad [24]$$

where $\lambda_s^o(p)$ is the value of $\lambda_s(p)$ when p is zero, and m_3 is a constant. For the compacted loess, values of 0.026and 0.093 were obtained for $\lambda_s^o(p)$ and m_3, respectively.

8. Summary of Model Parameters

Eleven parameters (i.e., c', ϕ', ϕ^b, m, m_1, m_2, m_3, k^o, K_t^o, and $\lambda_s^o(p)$ and R_f) are associated with the soil structure. Two parameters (i.e., K_{wt} (or $\lambda_w(s)$ or $\beta(s)$ and $\lambda_w(p)$ are associated with the water phase.

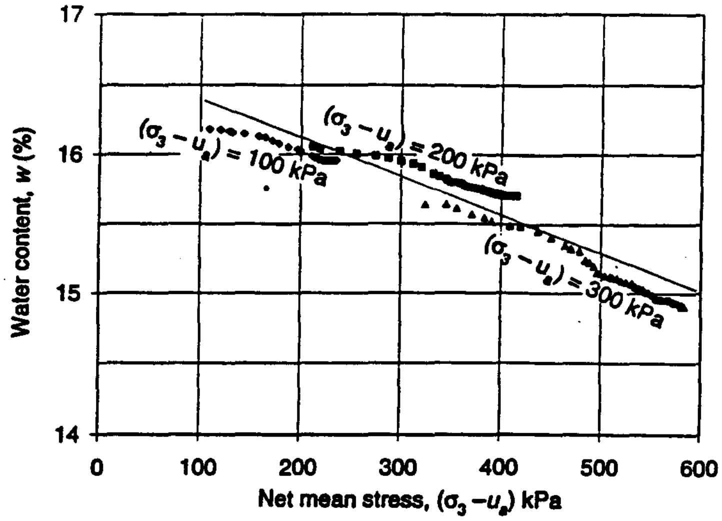

Fig. 4 Water content changes during triaxial shear test on the compacted loess at constant suction, s equal to 50 kPa.

Under saturated conditions (i.e., s = 0), the parameters ϕ^b, m_1, m_2, m_3, $\lambda_t^o(p)$ and $\lambda_w(p)$ are equal to zero and K_{wt} is equal to K_t^o. The model reduces to the Duncan-Chang's model with six parameters, c', ϕ', m, k^o, R_f and K_t^o.

Application: Constant Water Content Triaxial Test

In a constant water content triaxial test, the pore-air is drained (or maintained constant) while the pore-water is undrained. The suction changes during the constant water content triaxial test can be predicted using the model just presented.

Letting Eq. 3 equal to zero and integrating the two sides of Eq. 3 results in:

$$\ln[(s_1 + p_{atm})/(s_2 + p_{atm})] = (p_2 - p_1)/\Omega \qquad [25]$$

where $\Omega = (K_{wt}\lambda_w(p)/\ln 10)$ is a constant (the subscripts 1 and 2 represent the initial and final points on the stress strain curve or the first and second point for any segment on the stress strain curve, respectively.

The suction changes during a constant water content isotropic compression test (Fig. 5) [1] and constant water content triaxial tests (Fig. 6) [12] are predicted using Eq. 25. The Ω values for results presented in Fig. 5 and Fig. 6 were 637 and 666, respectively. Reasonable values were predicted using Eq. 25.

Conclusions

1. A nonlinear model for unsaturated soils is developed. The model presented has the Duncan-Chang's model as a special case when the soil is saturated.
2. A total of 13 parameters are involved in the new model. Eleven of the parameters are associated with the soil structure and two parameters are associated with the water phase. All the parameters have either physical or geometrical significance, and can all be determined using laboratory tests.
3. Only two types of triaxial tests are needed to determine all 13 parameters, namely, triaxial shear test with controlled suction and triaxial shrinkage test.
4. Predicted suction changes using the proposed model compared well with available data from constant water triaxial tests.

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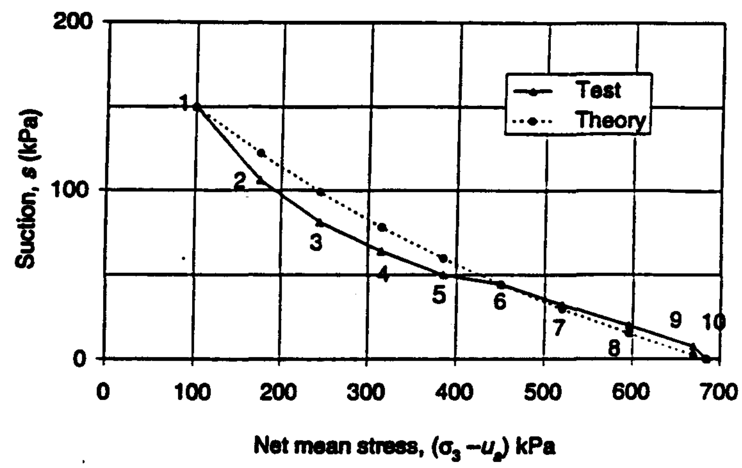

Fig. 5 Comparison of model prediction and experimental data of suction versus net mean stress relationship for Selset clay under constant water content, isotropic compression test (data from Bishop and Blight, 1963).

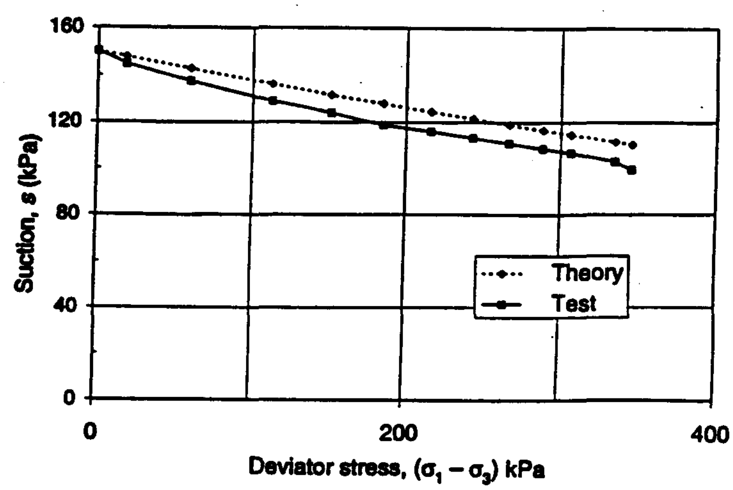

Fig. 6 Comparison of model prediction and experimental data of suction versus deviator stress relationship for a copper mill tailings (a gray silty fine sand) under constant water content triaxial test (data from Drumright, 1989).