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MEASURED AND SIMULATED BEHAVIOR OF AN EXPANSIVE SOIL

F. Schuai, and D. G. Fredlund

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Comportamento Medido e Simulado de um Solo Expansivo

F.Schuai
Department of Civil Engineering, University of Saskatchewan, Saskatoon, Canada

D.G.Fredlund
Department of Civil Engineering, University of Saskatchewan, Saskatoon, Canada

ABSTRACT: In this paper, a theoretical model for describing the changes of soil volume, vertical total stress, and matric suction with time during various swelling oedometer tests is formulated. The proposed model has been used to simulate the results from various oedometer tests. The computed values are in good agreement with measured values.

RESUMO: Este trabalho apresenta um modelo teórico formulado para descrever variações de volume, de tensões totais verticais e de sucção mátrica com o tempo durante vários ensaios de expansão edométricos. Os valores calculados concordam bem com os medidos.

1. INTRODUCTION

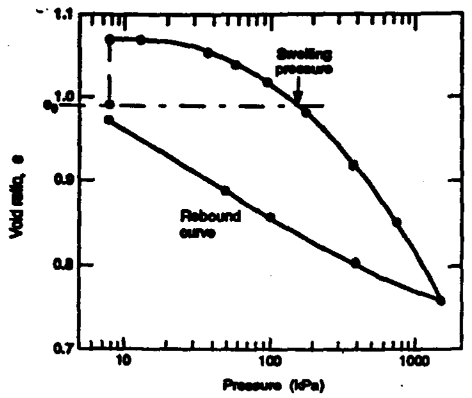
Expansive soils exhibit significant volumetric expansion upon wetting due to a decrease in the matric suction. A pressure will develop if an attempt is made to stop the swelling. Such pressures are sometimes sufficiently large to cause serious damage to structures. In order to predicate the pressure or amount of heave, numerous laboratory testing procedures have been proposed. These procedures generally involve the use of a one-dimensional consolidation apparatus (i.e., oedometer). Among them, the most commonly used testing procedures are: (i) Free Swell oedometer test, (ii) Loaded Swell oedometer test and (iii) Constant Volume oedometer test

Free Swell Oedometer Test Method

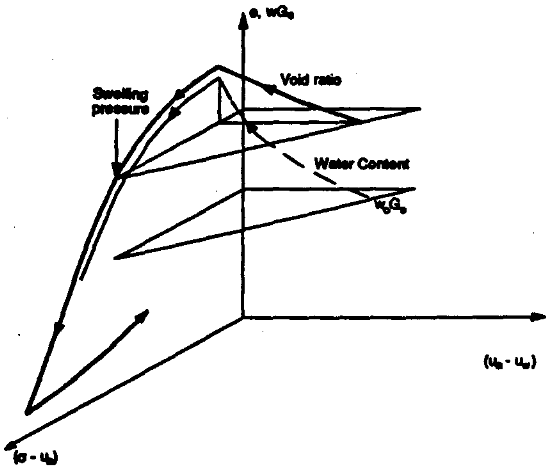
In the Free Swell oedometer test, the soil specimen is brought in contact with water and allowed to swell freely with a token load applied. Then the soil is gradually consolidated back to its original volume in the conventional manner (Fig. 1a). The swelling pressure is defined as the stress necessary to consolidate the specimen back to its original volume (Hardy, 1965; Sridharan, Rao and Sivapullaiah, 1986). The stress paths adhered to can be more clearly understood using a three-dimensional plot with the stress state variable forming the abscissas (Fig. 1b).

Loaded Swell Oedometer Test Method

In the Loaded Swell oedometer test, a number of "identical" specimens are subjected to different initial applied loads and allowed to swell freely. The resulting final volume changes are then plotted against the corresponding applied load or stresses. The stress corresponding to zero volume change is termed the swelling pressure (Skempton, 1961; Gizienski and Lee, 1965; Nobel, 1966; Matyas, 1969). The stress path followed is shown in Fig. 2.



a) Conventional "Free Swell" data plot

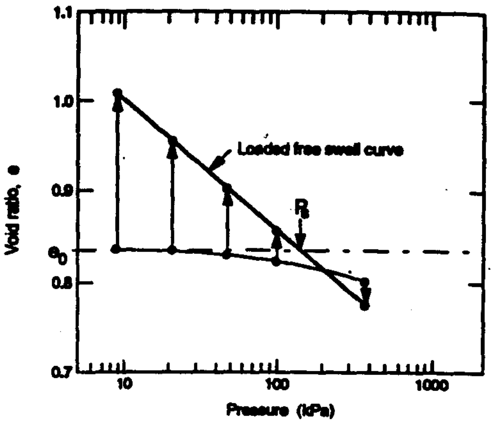


b) Three-dimensional stress path plot

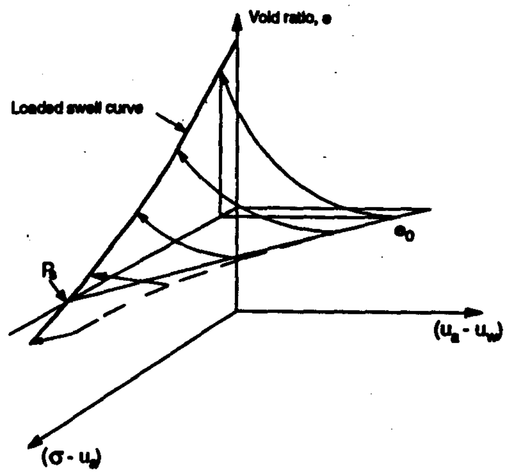
igure 1 Stress path representation for the "Free Swell" oedometer test (from Fredlund, 1995).

Constant Volume Oedometer Test Method

In the Constant Volume test procedure, a specimen is subjected to a token load and immersed in water. The specimen volume is maintained constant throughout the first part of the test by varying the load applied to the specimen, as required. This procedure is continued until there is no further tendency for swelling. The applied load at this point is refereed to as the "uncorrected" swelling pressure, P_s. The soil specimen is then further loaded and unloaded following the conventional oedometer test procedure. The test results are commonly plotted as shown in Fig. 3a. The actually stress paths followed during the test can be visualized using a three-dimensional plot of the stress state variables versus void ratio and water content (Fig. 3b)



(a) Two-dimensional plot



(b) Three-dimensional plot

Figure 2 Stress path followed in the Loaded Swell test.

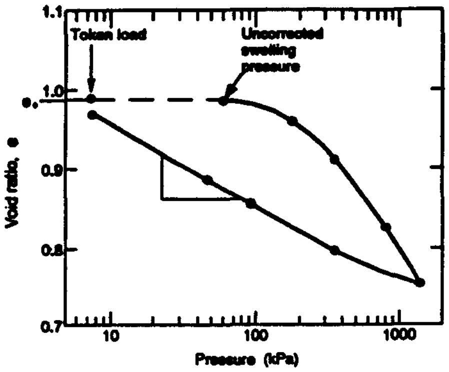
In order to eliminate sampling disturbance, Fredlund et al. (1980) defined a correction to the data to give a corrected swelling pressure.

A large amount of test results and experience involving these three methods have been reported. In contract, little attempt has been made to theoretically and analytically simulate the testing procedures.

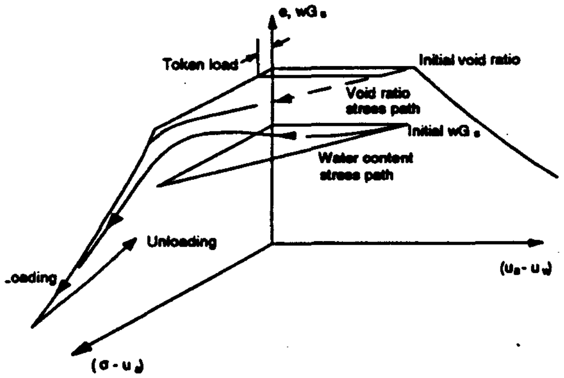
2. THEORETICAL MODEL

2.1 Processes Involved in Swelling Tests

Swelling oedometer tests primarily involve two processes; namely, the transient water flow process and the load-deformation process. The



a) Conventional "Constant Volume" data plot



b) Three-dimentional stress path plot

igure 3 Stress path followed in the "Constant Volume" oedometer test (from Fredlund, 1995).

load-deformation process describes the deformation response of the soil as a result of a change of load or matric suction and is governed by the equilibrium equation and the constitutive equations for unsaturated soils. For one-dimensional case, the equilibrium equation can be written in incremental form as follows:

$$\frac{\partial \Delta\sigma_z}{\partial z} = 0 \qquad [1]$$

where:

$\Delta\sigma_z$ = increment of total normal stress in the z-direction (i.e., vertical direction).

The constitutive equations for an unsaturated soil were proposed by Fredlund (1979) and are written as:
for soil structure:

$$\varepsilon_z = \frac{\Delta V_v}{V_0} = m_1^s \Delta(\sigma_z - u_a) + m_2^s \Delta(u_a - u_w) \qquad [2]$$

for water phase:

$$\frac{\Delta V_w}{V_0} = m_1^w \Delta(\sigma_z - u_a) + m_2^w \Delta(u_a - u_w) \qquad [3]$$

where:

ε_z = the strain in the z-direction
σ_z = total normal stress in the z-direction
V_0 = initial overall volume of the soil element
ΔV_v = change in the volume of soil voids in the soil element
m_1^s = the coefficient of volume change with respect to a change in net normal stress
m_2^s = the coefficient of volume change with respect to a change in matric suction
ΔV_w = change in the volume of water in the soil element
m_1^w = the coefficient of water volume change with respect to a change in the net normal stress
m_2^w = the coefficient of water volume change with respect to a change in matric suction

The transient water flow process is governed by the continuity equation which can be written as:

$$\frac{\partial}{\partial t}\left(\frac{V_w}{V_0}\right) = \frac{1}{\rho_w g}\frac{\partial}{\partial z}\left(k_w \frac{\partial u_w}{\partial z}\right) \qquad [4]$$

where:

k_w = coefficient of permeability with respect to the water phase in the z-direction (which is a function of the negative pore-water pressure)
z = elevation
u_w = pore-water pressure
ρ_w = density of water
g = gravitational acceleration

For some swelling tests (i.e., Constant Volume oedometer test) water flow process and load-deformation process are not independent. During the Constant Volume test, the total stress increases with respect to time. The increase in the total stress results in a tendency for the soil to decrease its volume and hence results in a decrease in the negative pore-water pressure. Alternatively, the water content increase due to transient water flow causes the negative pore-water pressure to decrease. The decrease in the negative pore-water pressure results in a tendency for the soil to increase in volume and consequently results in a further increase in the total stress because of the requirement to maintain a constant volume. Therefore, it is difficult to separate the two processes (i.e., the transient water flow process and the soil volume change process) during the Constant Volume test. In order to simulate various swell testing procedures, the simultaneous equations coupling the transient water flow process with the load-deformation process must be formulated.

2.2 Governing Differential Equation for Swelling Test

In order to simplify the derivation, the following assumptions are made: (1) isotropic soil, (2) infinitesimal strain, (3) linear constitutive relations for a small change in net normal stress or matric suction, (4) the permeability with respect to the air phase, k_a, is significantly greater than the permeability with respect to the water phase, k_w, which means that the pore-air pressure is always equal to the surrounding air pressure (i.e., $u_a = 0$). According to assumption (4) the net normal stress, $(\sigma_z - u_a)$, becomes equal to the total vertical stress, σ_z and matric suction, $(u_a - u_w)$, becomes equal to the negative pore-water pressure, $-u_w$. Then constitutive Eqs. 2 and 3 for an unsaturated soil become:

$$\varepsilon_z = \frac{\Delta V_v}{V_0} = m_1^s \Delta\sigma_z - m_2^s \Delta u_w \qquad [5]$$

$$\frac{\Delta V_w}{V_0} = m_1^w \Delta\sigma_z - m_2^w \Delta u_w \qquad [6]$$

2.2.1 *Differential Equation for Water Flow*

The continuity requirement for the water phase is given by Eq. 4 and the constitutive relation for the water phase of an unsaturated soil is given by Eq. 6.

Substituting Eq. 6 into Eq. 4 gives:

$$m_1^w \frac{\partial \sigma_z}{\partial t} - m_2^w \frac{\partial u_w}{\partial t} = \frac{1}{\rho_w g}\frac{\partial}{\partial z}\left(k_w \frac{\partial u_w}{\partial z}\right) \qquad [7]$$

Differentiating Eq. 5 with respect to time yields

$$\frac{\partial \varepsilon_z}{\partial t} = m_1^s \frac{\partial \sigma_z}{\partial t} - m_2^s \frac{\partial u_w}{\partial t} \qquad [8]$$

Equation 8 can be rewritten as:

$$\frac{\partial \sigma_z}{\partial t} = \frac{1}{m_1^s}\left(\frac{\partial \varepsilon_z}{\partial t} + m_2^s \frac{\partial u_w}{\partial t}\right) \qquad [9]$$

Substituting Eq. 9 into Eq. 7 gives:

$$\frac{m_1^w}{m_1^s}\frac{\partial \varepsilon_z}{\partial t} - \left(m_2^w - \frac{m_2^s m_1^w}{m_1^s}\right)\frac{\partial u_w}{\partial t} = \frac{1}{\rho_w g}\frac{\partial}{\partial z}\left(k_w \frac{\partial u_w}{\partial z}\right) \qquad [10]$$

Written in terms of displacement, the strain in the z-direction can be expressed as:

$$\varepsilon_z = \frac{\partial \delta_z}{\partial z} \qquad [11]$$

where:

δ_z = the displacement in the z-direction

Substituting the strain in Eq. 11 into Eq. 10 gives:

$$\frac{m_1^w}{m_1^s}\frac{\partial}{\partial t}\left(\frac{\partial \delta_z}{\partial z}\right) - \left(m_2^w - \frac{m_2^s m_1^w}{m_1^s}\right)\frac{\partial u_w}{\partial t} = \frac{1}{\rho_w g}\frac{\partial}{\partial z}\left(k_w \frac{\partial u_w}{\partial z}\right) \qquad [12]$$

Equation 12 can be rewritten as:

$$\alpha \frac{\partial}{\partial t}\left(\frac{\partial \delta_z}{\partial z}\right) - \beta \frac{\partial u_w}{\partial t} = \frac{1}{\rho_w g}\frac{\partial}{\partial z}\left(k_w \frac{\partial u_w}{\partial z}\right) \qquad [13]$$

where:

$$\alpha = \frac{m_1^w}{m_1^s}$$

$$\beta = m_2^w - \frac{m_2^s m_1^w}{m_1^s}$$

Equation 13 is the transient flow equation for the water phase in terms of displacement and negative pore-water pressure.

2.2.2 *Differential Equation for Soil Volume Change*

The constitutive relation for the volumetric strain of an unsaturated soil under one-dimensional case is given by Eq. 5 and the deformation equation for the one-dimensional case is given by Eq. 11. Substituting the strain in Eq. 11 into Eq. 5 allows the displacement to be written as:

$$\frac{\partial \delta_z}{\partial z} = m_1^s \Delta\sigma_z - m_2^s \Delta u_w \qquad [14]$$

Rearranging Eq. 14 results in

$$\Delta\sigma_z = \frac{1}{m_1^s}\frac{\partial \delta_z}{\partial z} + \frac{m_2^s}{m_1^s}\Delta u_w \qquad [15]$$

Substituting Eq. 15 into the equilibrium equation (i.e., Eq. 1) and differentiating it with respect to z gives:

$$\frac{\partial^2 \delta_z}{\partial z^2} = -m_2^s \frac{\partial u_w}{\partial z} \qquad [16]$$

Equation 16 is load-deformation equation for the swelling oedometer test.

2.2.3 *Simultaneous equations for swelling test*

The load-deformation equation and the transient water flow equation for the swelling tests are given by Eq. 16 and Eq. 13, respectively. Equation 13 is used to define the negative pore-water pressure, u_w, when solving for soil volume change in Eq. 16. The displacement, δ_z, required in Eq. 13 depends on the deformation given by Eq. 16. Equations 16 and 13 must be solved simultaneously.

The governing equations for swelling tests, Eq. 16 and Eq. 13, can be written as the form of simultaneous equation as follows:

$$\begin{cases} \dfrac{\partial^2 \delta_z}{\partial z^2} = -m_2^s \dfrac{\partial u_w}{\partial z} \\ \alpha \dfrac{\partial}{\partial t}\left(\dfrac{\partial \delta_z}{\partial z}\right) = \dfrac{1}{\rho_w g}\dfrac{\partial}{\partial z}\left(k_w \dfrac{\partial u_w}{\partial z}\right) + \beta \dfrac{\partial u_w}{\partial t} \end{cases} \qquad [17]$$

There are two equations in Eq. 17 with two unknowns (i.e., δ_z and u_w). These equations can be used to compute the negative pore-water pressure and displacement at various depths and times during the swelling process. The negative pore-water pressure change, Δu_w, and the displacement, δ_z, computed from Eq. 17 can be substituted into Eq. 15 to calculate the change in vertical stress, $\Delta\sigma_z$, applied to the soil surface during the test.

The relationships required to solve Eq. 17 are the compression or rebound curve (i.e., V_w/V_0 versus $(\sigma_z - u_a)$), the soil-water characteristic curve, shrinkage or swelling curve and the permeability function (i.e., $k_w(u_w)$).

3. EXPERIMENTAL RESULTS AND NUMERICAL SIMULATIONS

The governing differential equation describing the pore pressure and volume change behavior during various swelling tests (i.e., Eqs. 16 and 13) are nonlinear. The coefficients of permeability, k_w, the coefficients of soil volume change, m_1^s and m_2^s, and the coefficients of water volume change, m_1^w and m_2^w, vary with vertical position and time due to changes in total stress, and negative pore-water pressure. A closed-form solution is not available. Therefore, a computer program called "SWELL" was developed using the finite element method for predicting the pore-water pressure and volume change behavior during a swell test (Shuai, 1996).

The proposed theoretical model was used to simulate the results from several oedometer swelling tests (i.e., Free Swell oedometer test, Constant Volume oedometer test, and Loaded Swell oedometer test) on the compacted Regina Clay. The specimen used in the tests have a moulding water content of 26% and an initial void ratio of 0.96. Comparisons are given in this section of the paper between measured and predicted matric suction, vertical total stress and volume change values for the swelling oedometer tests.

Figure 4 shows the measured and computed deflection versus time curves for two 100 mm high specimens and two 20 mm height specimens in the Free Swell oedometer tests. A comparison between the computed and measured curves shows reasonable good agreement for the full length of the test for these specimens. The predicted total heaves are almost the same as those measured.

A comparison between the computed and measured matric suction profiles for the 100 mm high specimen is presented in Fig. 5. The matric suction was measured by placing Whatman No. 42 filter papers between each layers of specimen and measuring the water

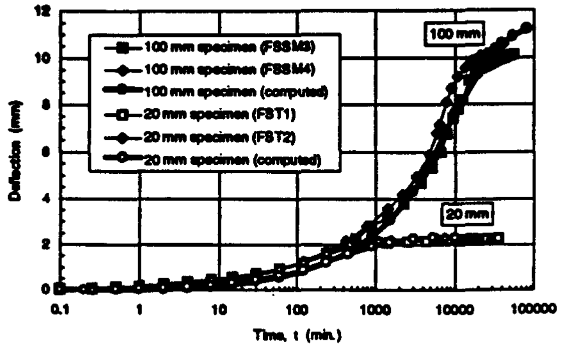


Figure 4 Computed and measured deflection versus time curves for Free Swell oedometer tests

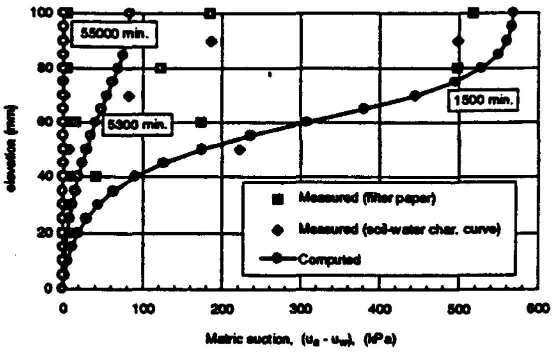


Figure 5 Computed and measured matric suction profiles for Free Swell oedometer test

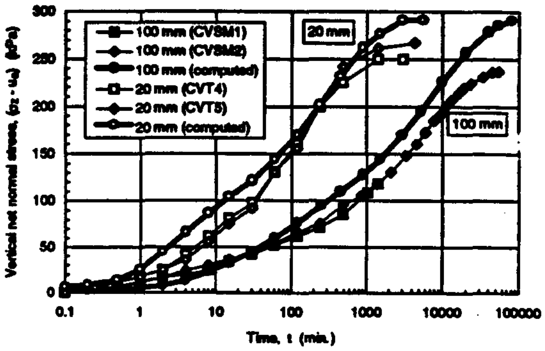


Figure 6 Computed and measured vertical normal stress versus time curves for Constant Volume oedometer tests

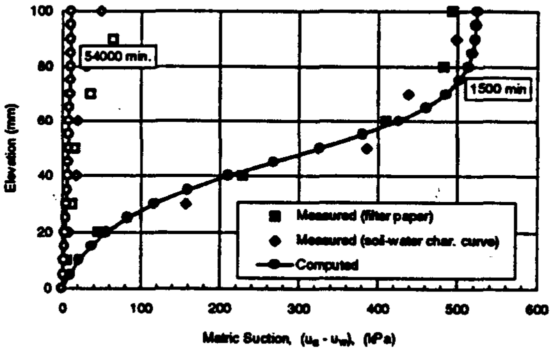


Figure 7 Computed and measured matric suction profiles for Constant Volume oedometer test

content of the filter papers after each test. Good agreement was found between the measured and computed matric suctions for the early (i.e., t = 1500 min.) and latter stages (i.e., t = 54700 min.) of the test. Some differences were noted during the medial stage (i.e., t = 5300 min.) of the test. The matric suctions predicted at the upper part of the specimen are somewhat lower than the measured suctions. The poor prediction could be attributed to the filter paper which were placed in the specimen. Since the water retentivity of the filter paper is much higher than that of the soil, the filter paper may absorb more water than the soil for a given decrease in matric suction. As a result, the filter papers in the soil slow the advance of the saturated zone resulting in a slower decrease of matric suction.

The measured and computed vertical total stress versus time curves for two 100 mm high specimens and two 20 mm height specimens in the Constant Volume oedometer tests are shown in Fig. 6. Good agreement was found between the computed and measured curves for the full duration of the test for all specimens.

The measured and computed profiles of matric suction are presented in Fig. 7. The correlation between the measured and computed matric suctions are good.

The measured and computed deflection versus time curves for a series of the Loaded Swell oedometer tests are shown in Fig. 8. The computed and measured deflection-time curves show good agreement when the applied load is higher than 300 kPa and lower than 20 kPa, but the correlation between the measured and computed deflection-time curves is not as good during the latter stages of the test when the applied load is lower than 300 kPa and higher than 20 kPa. The discrepancy could be contributed to the high ratio of horizontal to vertical stress (i.e., σ_h/σ_v) at the end of the test.

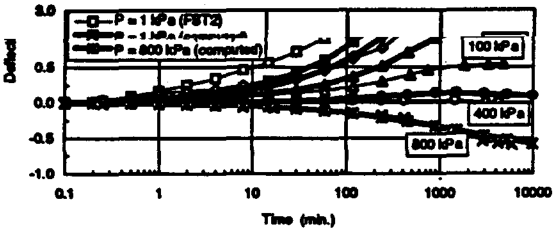


Figure 8 Computed and measured time-heave relationship for Loaded Swell oedometer tests.

Komornik and Zeitlen (1965) used a special oedometer to measure the lateral swelling pressure under different vertical loads. The results obtained indicated that the ratio of horizontal to vertical stress at the end of the test increased as the amount of swell increased. In other words, under a constant vertical stress, the mean normal stress applied to the soil increases at the end of the test. The test results obtained by Dakshanamurthy (1979) showed that total heave decreased with increasing mean normal stress. Therefore, the increase in the mean normal stress due to the increasing stress ratio (i.e., σ_h/σ_v) will decrease the total heave at the end of the Loaded Swell test. Since the increase in the ratio of horizontal to vertical stress has not been take into account in this theoretical simulation, it should be expected that the calculation results will over-estimate the total heave. The amount of the over-estimation will decrease as the applied load increase since the ratio of horizontal to vertical stress decrease with the increasing vertical applied load and decreasing amount of swell.

CONCLUSIONS

The swelling oedometer tests primarily involve two processes; namely, the transient water flow process and the load-deformation process. These processes are governed by following basic equations:

(i) The equilibrium equation for an element of soil;

(ii) The constitutive equation for unsaturated soils:

load-deformation process are not independent for some swelling tests (i.e. Constant Volume oedometer test). The simultaneous equations coupling the transient water flow process with the load-deformation process are required for simulation of swell testing procedures.

The simultaneous equations for describing the processes which occur during a swelling oedometer test are formulated. These equations are nonlinear with respect to position and time and should be solved using numerical method. The soil properties required in the theoretical model are the coefficient of permeability function (i.e., k_w); the coefficients of volume change (i.e., m_1^s and m_2^s) and the coefficients of water volume (i.e., m_1^w and m_2^w). These soil properties can be obtained using ordinary laboratory methods.

The proposed theoretical model was used to simulate the results from Free Swell oedometer test, Constant Volume oedometer test, and Loaded Swell oedometer test. In general, good agreement was found between the computed and measured values of volume change, vertical total stress and pore-water pressure. Some over-estimation of total heave was noted for Loaded Swell oedometer tests when the surcharge loads were significantly lower than the swelling pressure. This poor prediction is attributed to the increasing ratio of horizontal to vertical stress at the end of the test which is not possible to simulate using a one-dimensional model.

The work presented in this paper concentrated on the simulation of swell testing processes performed in the laboratory. However, the proposed theoretical model is also of value in prediction of insitu total heave or collapse, the swelling pressure and the rate of swell or collapse.

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