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INTERPRITATION OF UNDRAINED SHEAR STRENGTH OF UNSATURATED SOILS IN TERMS OF STRESS STATE VARIABLES

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Interpretation of Undrained Shear Strength of Unsaturated Soils in Terms of Stress State Variables

Interpretação da Resistência Não Drenada de Solos Não Saturados em Termos de Variáveis de Estado de Tensão

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ABSTRACT: The unsaturated soil, shear strength theory can be used to interpret the meaning of undrained shear strength in terms of two stress state variables, net normal stress, $(\sigma - u_a)$ and matric suction, $(u_a - u_w)$, assuming both linear and nonlinear failure envelopes with respect to matric suction. The interpretation of the theory is applied to the test results from glacial till specimens representing three different initial water content conditions (i.e., at optimum, dry and wet of optimum conditions), in both confined and unconfined conditions using conventional test procedures. This proposed framework can be recommended for interpreting the shear strength of unsaturated soils in undrained loading conditions for different soils.

RESUMO: A teoria de resistência ao cisalhamento de solos não saturados pode ser usada para interpretar o significado da resistência não drenada em termos de duas variáveis do estado de tensões, a tensão líquida normal, $(\sigma - u_a)$ e a sucção mátrica $(u_a - u_\omega)$, assumindo tanto envoltórias de ruptura linear ou não lineares em relação à sucção mátrica. A interpretação da teoria é aplicada a resultados de ensaios em amostras de till glacial representando três condições iniciais de teor de umidade (i.e., condições ótima, acima e abaixo da ótima), em condições confinada e não confinada usando procedimentos convencionais de ensaio. A estrutura proposta pode ser recomendada para interpretar a resistência de solos não saturados sob condições de carregamento não drenado para diferentes solos.

1. INTRODUCTION

Undrained total stress analyses (i.e., ϕ equals zero analysis) (Skempton 1948) are sometimes used in assessing the stability of embankments, and foundations located over saturated fine-grained soils. The design of pavements and the assessment of the ultimate bearing capacity of clays are other examples where the undrained shear strength of the soil is used for analysis purposes. The undrained shear strength is usually assumed to be one-half the unconfined compressive strength, c_u, for field specimens obtained from various depths. While this assumption is somewhat realistic for *saturated soils*, its usage is questionable for interpreting the undrained shear strength of *unsaturated soils*.

The independent contribution of matric suction towards the undrained shear strength is generally not consciously considered in most analyses. However, it is the matric suction which holds the soil specimen together during an unconfined compression test. The matric suction in the specimen tested in the laboratory is a function of the insitu pore-water pressure and the change in pore-water pressure resulting from unloading the soil during sampling. Hence, the measured undrained shear strength should be interpreted, taking into account the influence of matric suction, $(u_a - u_w)$.

The shear strength of unsaturated soils for undrained loading conditions is interpreted by using the stress state variable theory of Fredlund et al. (1978). The undrained shear strength can fundamentally be assessed in terms of stress state variables, $(\sigma - u_a)$, and matric suction, $(u_a - u_w)$. Relationships are developed based on simplifying assumptions to assess the contribution of matric suction towards the shear strength both in confined and unconfined, undrained loading conditions assuming linear and nonlinear failure shear strength envelopes. The proposed interpretation is applied to the test results on a glacial till compacted at three initial conditions (i.e., at optimum, dry and wet of optimum water contents).

2. UNDRAINED SHEAR STRENGTH OF UNSATURATED SOILS

A linear form for the shear strength equation was proposed by Fredlund, Morgenstern and Widger (1978) for unsaturated soil. The shear strength was written in terms of the stress state variables.

$$\tau_f = c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)\tan\phi^b \quad [1]$$

where:

τ_f	=	shear strength of an unsaturated soil
c'	=	effective cohesion of saturated soil
ϕ'	=	effective angle of shearing resistance for a saturated soil
ϕ^b	=	angle of shearing resistance with respect to matric suction
$(\sigma_n - u_a)$	=	net normal stress on the plane of failure, at failure
$(u_a - u_w)$	=	matric suction of the soil at the time of failure

The experimental behavior of the shear strength under drained conditions has been found to be nonlinear when the soil is tested over a large range of suctions (Gan et al. 1988, Vanapalli et al. 1996a). The equation proposed by Fredlund et al. (1978) can also be used to describe the nonlinear variation of shear strength of the soil with a frictional angle with respect to suction, $\tan\phi^b$, as a variable.

Undrained shear strength analyses are presented in this paper applying Fredlund et al. (1978) shear strength equation (i.e., Eq. 1). Effective shear strength parameters of the soil, along with the initial matric suction and the results from unconfined and confined compression tests are required for the analysis. The procedure focuses on the use of conventional soil testing techniques to provide a measure of the angle ϕ^b.

Changes in matric suction due to an applied total isotropic pressure can be computed from a knowledge of the initial conditions of the soil, using a marching forward technique. This procedure is detailed in Fredlund and Rahardjo (1993). This technique is not required if the matric suction condition is measured.

During undrained axial compression, the matric suction can increase, decrease or remain constant depending upon the *A* pore pressure parameter of the soil. The analysis in this paper are addressed in terms of the *B* pore pressure parameters. The influence of the *A* parameter is not considered, thereby assuming that changes in matric suction caused by axial loading (i.e., $d(\sigma_1 - \sigma_3)$) as negligible. The presented theory can be used for practical applications to determine the contribution of matric suction towards the undrained shear strength, ϕ^b.

Two methods of analysis are presented in this paper. The first procedure assumes that ϕ^b is a constant value, thereby assuming a planar failure envelope for shear strength. In the second procedure, the methodology is described for predicting the nonlinear variation of the angle, ϕ^b, as a function of matric suction, $(u_a - u_w)$.

2.1 Interpretation of Confined Compression Tests (Assuming a Planar Failure Envelope)

The failure conditions for a confined compression test are shown in Figure 1. When the saturated shear strength parameters are known, a tangent with a slope angle of shearing, ϕ', can be drawn to the Mohr's envelope. The intercept, OI, obtained from such a construction is equal to the summation of effective cohesion, c', and the product of matric suction, $(u_a - u_w)$, and $\tan\phi^b$. Thus from Figure 1:

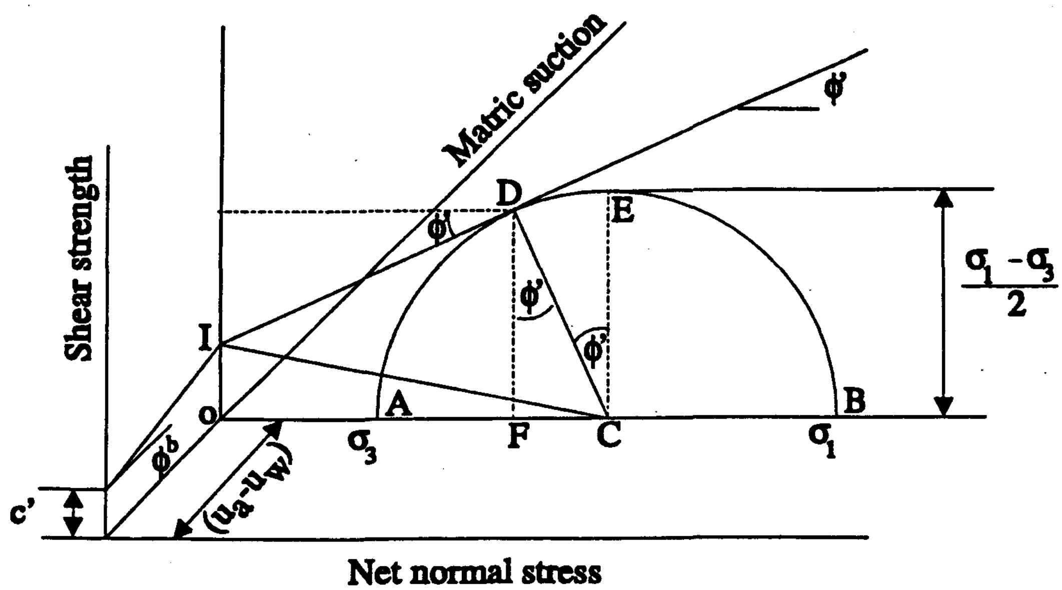

Figure 1. Three dimensional representation of confined compression test expressed in terms of stress state variables

$$OI = \{c' + (u_a - u_w)\tan\phi^b\} \qquad [2]$$

The shear strength, τ_f, for this loading condition is equivalent to DF from Figure 1. From the geometry of the diagram

$$\tau_f = DF = DC\cos\phi' = \left(\frac{\sigma_1 - \sigma_3}{2}\right)\cos\phi' \qquad [3]$$

For the shear strength analysis of triaxial test results Eq. [1] may be expressed as:

$$\tau_f = \{c' + (u_a - u_w)\tan\phi^b\} + (\sigma_n - u_a)\tan\phi' \qquad [4]$$

where:

$$\sigma_n = \left(\frac{\sigma_1 + \sigma_3}{2}\right) - \left(\frac{\sigma_1 - \sigma_3}{2}\right)\sin\phi'$$

A relationship for frictional angle, ϕ^b, is obtained by equating Eqs. [3] and [4].

$$\tan\phi^b = \frac{\{q_u(\cos\phi' + \sin\phi'\tan\phi')\}}{(u_a - u_w)} - \frac{\{(c_u + \sigma_3 - u_a)\tan\phi' - c'\}}{(u_a - u_w)} \qquad [5]$$

where:

$q_u = \left(\frac{\sigma_1 - \sigma_3}{2}\right)$, failure deviator stress from the undrained triaxial test

$(u_a - u_w)$ = is the matric suction in the specimen at failure condition

If we assume that the pore-air dissolves in the water of the specimen and the pore-air pressure, u_a, is equal to zero, Eq. [5] takes the form:

$$tan\phi^b = \frac{\{q_u(cos\phi' + sin\phi' tan\phi')\}}{(u_a - u_w)} - \frac{\{(c_u + \sigma_3)tan\phi' - c'\}}{(u_a - u_w)} \quad [6]$$

The shear strength contribution due to matric suction, $tan\ \phi^b$, for undrained loading conditions can be estimated using Eq. [6].

2.2 Interpretation of Unconfined Compression Tests (Assuming a Planar Failure Envelope)

Equation [6] can be applied to the unconfined compression tests also by setting the confining pressure to zero.

$$tan\phi^b = \frac{[c_u(cos\phi' + sin\phi' tan\phi')]}{(u_a - u_w)} - \frac{[c_u\, tan\phi' + c']}{(u_a - u_w)} \quad [7]$$

where:

$c_u = \sigma_1/2$ = unconfined compressive strength

For unconfined compression tests, the pore-air pressure can be assumed to be atmospheric and the results can be interpreted assuming a constant matric suction.

2.3 Interpretation of Unconfined and Confined Compression Tests (Assuming Nonlinear Failure Surface with respect to Matric Suction)

The nonlinear variation of the shear strength under drained loading conditions with respect to matric suction, can be estimated using the soil-water characteristic curve (drying) and the saturated shear strength parameters (Vanapalli et al. 1996a, Fredlund et al. 1996). This theory can also be used to predict the nonlinear variation of the angle, ϕ^b, using the unconfined and confined undrained shear strength test results which follow a trend of wetting.

The degree of saturation versus matric suction variation (i.e., the soil-water characteristic curve relationship) can be obtained from the triaxial test results at failure conditions. The degree of saturation changes in these specimens, without any change in water content. The rate at which suction contributes towards shear strength can be related to the area of water available within the voids (Vanapalli 1996a). With the compression of air voids, the available water content communicates soil suction as a stress state variable over a greater area for specimens subjected to higher confining pressures.

The soil-water characteristic curve generated from the undrained tests can be used for predicting the shear strength of unsaturated soils under undrained loading conditions. However, results from desiccator tests for higher suction range are necessary to have the entire range suction range of soil-water characteristic curve data (i.e., from 0 to 1,000,000 kPa). The variation of soil-water characteristic curve behavior at suctions greater than 3000 kPa suction will be similar for both drying and wetting conditions.

The shear strength contribution due to matric suction and cohesion is given as:

$$\tau_f = c' + (u_a - u_w)[(\Theta^\kappa)tan\phi'] \quad [8]$$

where:

Θ = normalized volumetric water content, defined as the ratio of volumetric water content, θ and volumetric water content, θ_S, at a saturation of 100%.

κ = fitting parameter used for obtaining a good correlation between the experimental and predicted shear strength values.

The soil-water characteristic curve can be mathematically represented as follows (Fredlund and Xing 1994).

$$\Theta = [C(\psi)]\left[\frac{1}{ln\left(e + \left(\frac{\psi}{a}\right)^n\right)}\right]^m \quad [9]$$

where:

θ = volumetric water content

θ_s = saturation volumetric water content

a = soil suction related to the air-entry value of the soil

n = soil parameter related to the slope at the inflection point on the soil-water characteristic curve

ψ = soil suction

m = soil parameter related to the asymmetry of the soil-water characteristic curve

θ_r = volumetric water content at residual conditions

e = natural number, 2.71828...

$C(\psi)$ = a correction function which forces the soil-water characteristic curve through a suction of 1,000,000 kPa at zero water content

The correction factor is defined as:

$$C(\psi)=\left[1-\frac{\ln\left(1+\frac{\psi}{C_r}\right)}{\ln\left(1+\frac{1{,}000{,}000}{C_r}\right)}\right] \qquad [10]$$

where: C_r = the soil suction corresponding to a residual water content, θ_r.

The degree of saturation, S, is also equal to the normalized volumetric water content, Θ.

Vanapalli et al. (1996a) and Fredlund et al. (1996) have shown that using the soil-water characteristic data and the saturated shear strength parameters the value of $tan\phi^b$ at any value of matric suction can be estimated by:

$$tan\phi^b = \frac{d\tau}{d(u_a-u_w)} = \left[(\Theta^\kappa)+(u_a-u_w)\frac{d(\Theta^\kappa)}{d(u_a-u_w)}\right]tan\phi' \qquad [11]$$

Vanapalli et al. (1996b) have shown that for drained direct shear tests in unsaturated conditions to soil suction has no influence on the angle of shearing resistance, ϕ'. Thus, the difference between the shear strength at failure conditions and the strength contribution due to net normal stress from the undrained triaxial tests can be assumed to be equivalent to the shear strength contribution due to matric suction and effective cohesion, c'. This can be expressed mathematically as below:

$$\tau_f-\{(\sigma_n-u_a)tan\phi'\}=c'+(u_a-u_w)tan\phi^b \qquad [12]$$

where:

$$\tau_f=\left(\frac{\sigma_1-\sigma_3}{2}\right)cos\phi'$$

$$\sigma_n=\left(\frac{\sigma_1+\sigma_3}{2}\right)-\left(\frac{\sigma_1-\sigma_3}{2}\right)sin\phi'$$

The validity of the assumptions made for this analysis can be checked by comparing the results obtained from [8] and [12].

Undrained tests follow the trends of wetting and under this condition gravimetric water content is constant in the specimens. However, the degree of saturation is increasing at a constant water content under the applied confining pressures.

3. TESTING PROGRAM

A glacial till obtained from Indian Head, Saskatchewan was used for the study. The soil was air-dried for several days, pulverized using a rubber mallet, and passed through a 2 mm sieve. The liquid limit, ω_L, and the plastic limit, ω_P, are 35.5% and 16.8% respectively. The percentages of sand, silt, and clay are 28, 42, and 30 respectively. The AASHTO standard compacted density was 1.80 Mg/m^3 with an optimum content of 16.3%. The specific gravity of soil was 2.73. The soil is classified as a CL.

Three initial water contents were selected for preparing the soil specimens representing dry of optimum (i.e., initial water content of 13% and a γ_d of 1.73 Mg/m^3), at optimum (i.e., initial water content of 16.3% and a γ_d of 1.80 Mg/m^3)and wet of optimum (i.e., initial water content of 19.2% and a γ_d of 1.77 Mg/m^3) conditions. Statically compacted specimens were used for the study. The details of testing procedures are available in Vanapalli (1994).

3.1 Matric Suction of Specimens

The measured initial matric suction values in the statically compacted specimens at different initial water content conditions along with other properties are summarized in Table 1.

Table 1
Matric Suction Values of Statically Compacted Specimens

Water Content (%)	Water Content Relative to Optimum	Dry Density, γ_d (Mg/m^3)	Void ratio, *e*	Degree of Saturation, *S* (%)	Matric Suction, (u_a-u_w) (kPa)
16.3	Optimum	1.80	0.52	86.0	152
13.0	Dry of Optimum	1.73	0.58	62.0	368
19.2	Wet of Optimum	1.77	0.54	97.0	68

3.2 Effective Shear Strength Parameters

The measured shear strength parameters at different initial contents and densities were essentially the same. The effective cohesion, c' was found to be equal to 15 kPa and angle of internal friction, ϕ' was equal to 23 degrees.

3.3 Unconfined Compression Tests

Three unconfined compression tests were conducted with identical initial conditions for the different water contents selected to obtain average values (see Table 2).

Table 2
Shear Strength Contribution due to Matric Suction, ϕ^b, from Unconfined Compression Tests Assuming a Planar Failure Envelope

Water Content Relative to Optimum	Matric Suction (kPa)	Unconfined compressive strength (kPa)	ϕ^b (degrees)
Wet of Optimum	68	64.3	22.1
Optimum	152	120.8	23.1
Dry of Optimum	368	180.5	15.8

Figure 2 shows typical unconfined compression test results conducted on specimens with a dry of density of 1.80 Mg/m^3 and a initial water content of 16.3% (reflecting the optimum water content condition of the soil). The initial matric suction in these specimen was 152 kPa. The specimens failed after reaching a peak strength at about 3 to 5% axial strain.

3.4 Relationship between the Undrained Shear Strength and the Applied Confining Pressure

Figure 3 shows the variation of shear strength with the applied confining pressure. A horizontal failure envelope can be observed for specimens tested with optimum initial water content conditions beyond a confining pressure of 400 kPa.

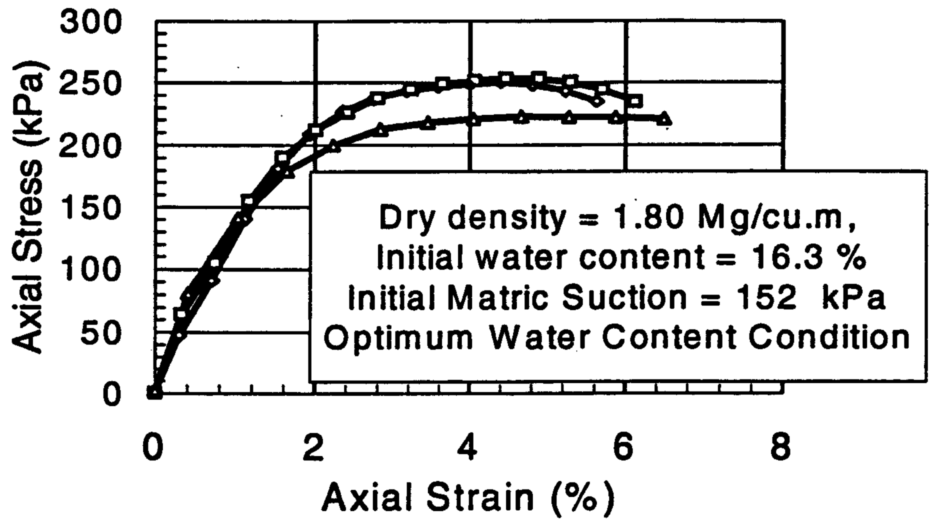

Figure 2. Stress versus strain relationships from unconfined compression tests

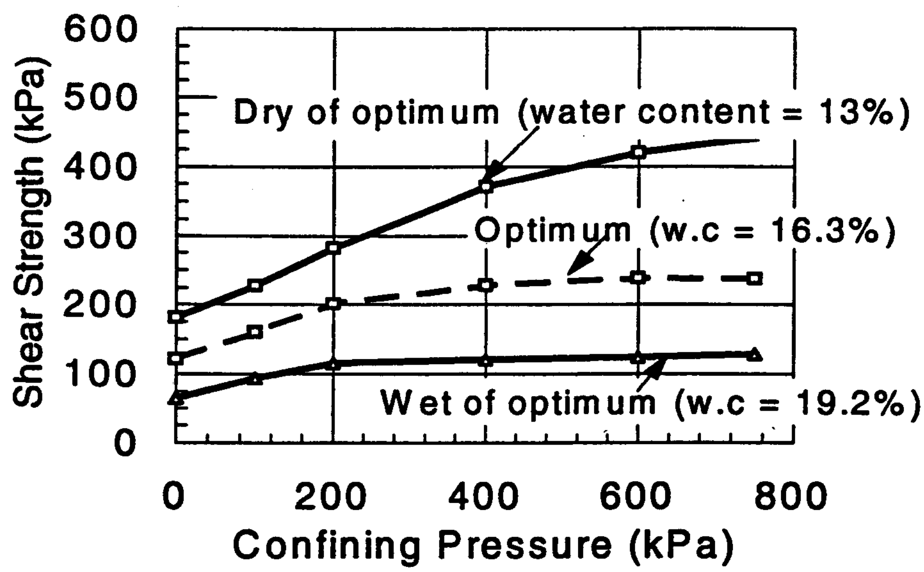

Figure 3. Variation of shear strength with the applied confining pressure

Similarly, wet of optimum water content specimens exhibited a horizontal failure envelope beyond 200 kPa. It can be inferred that the specimens were saturated and may have a suction close to zero. The measured and predicted degrees of saturation (using the theory of undrained pore pressure parameters) at these confining pressures were also close to 100% degree of saturation.

4.0 ANALYSIS OF UNCONFINED AND CONFINED TRIAXIAL TESTS FOR ϕ^b

4.1 ϕ^b from Unconfined Compression Tests Assuming a Planar Failure Envelope

The shear strength contribution due to matric suction, (i.e., ϕ^b) from the unconfined compression test results are summarized in Table 2 using Eq. [7]. The contribution due to matric suction in the dry of optimum conditions is lower than the contribution at optimum conditions. The average computed value of ϕ^b for the dry of optimum specimens was 15.8 degrees as compared to 23.1 degrees for specimens at optimum conditions. The undrained strength under dry of optimum conditions was higher than the strength at optimum conditions. However, the contribution of strength due to matric suction is lower. The initial matric suction in the dry of optimum specimens was 368 kPa compared to 152 kPa in the optimum specimens. The shear strength contribution due to matric suction, as shown through the value of ϕ^b, decreases with the increase in the matric suction. This is in agreement with the behavior observed in the drained shear strength for unsaturated soils for the same soil (Vanapalli et al. 1996b).

Similar to the optimum water content specimens, the specimens at wet of optimum conditions showed a higher value of ϕ^b. The average value of ϕ^b was equal 22.1 degrees and is near to the effective angle of friction, ϕ'. Theoretically, the value of ϕ^b for optimum conditions should be lower than the ϕ^b value for wet of optimum conditions. A difference of one degree between optimum and wet of optimum may be caused by natural variations in the specimens.

The value of ϕ^b approaches the value of ϕ' for the specimens tested with the optimum and wet of optimum conditions. The average initial degree of saturation of the specimens at wet of optimum conditions is high (i.e., 95.4%) and has a matric suction of 68 kPa. Thus, an average value of ϕ^b equals 22.1 degrees (for wet of optimum specimens) which is near to the effective angle of internal friction value, ϕ', for saturated conditions, appears to be reasonable.

4.2 Confined Compression Tests Assuming a Planar Failure Envelope

Table 3 shows the shear strength contribution due to matric suction in terms of ϕ^b using Eq. [6]. The results indicate that ϕ^b is increasing with an increasing confining pressure up to 200 kPa. With the increasing confining pressure, the degree of saturation increases which results in a decrease in matric suction, and hence ϕ^b should be increasing. At higher degrees of saturation, ϕ^b should approach and equal ϕ'. However, ϕ^b values higher than ϕ' were calculated for specimens tested at confining pressure values of 100 and 200 kPa. These calculations were based on predicted matric suction values rather than measured values.

Positive pore-water pressures were predicted for specimens subjected to confining pressures greater than 400 kPa. This is likely to be true due to the fact that the measured degrees of saturations for specimens failed at these confining pressures were 100% and the variation of confined compressive strength with the applied confining pressure, is horizontal (Figure 3). The angle ϕ^b values predicted for higher confining pressures are close to ϕ'. Similar trends in the results were obtained for specimens tested at wet of optimum conditions. Specimens tested with dry of optimum conditions are not discussed as defined failure conditions were not measured.

4.3 Nonlinear Failure Envelope (Using the Soil-Water Characteristic Curve - Wetting)

The degree of saturation versus matric suction relationship obtained from the testing program along with the osmotic dessicator data are shown as the wetting soil-water characteristic curve in Figure 4.

Using this soil-water characteristic curve data and the saturated shear strength parameters c' equal 15 kPa and ϕ' equal 23 degrees, the variation of shear strength with matric suction is predicted using Eq. [18]. A value of κ equal to 2.2 has been used for predicting the shear strength for the same soil under drained loading conditions with different net normal stresses and initial water content conditions (Vanapalli et al. 1996a).

Table 3

ϕ^b from Confined Compression Tests Assuming a Planar Failure Envelope for Optimum Water Content Specimens Using Equation [6]

Confining pressure (kPa)	Predicted pore-water pressure (kPa)	Confined compressive strength (kPa)	ϕ^b (degrees)
0	-152.0	120.8	23.1
100	-74.4	159.6	32.9
200	-21.4	178.1	40.1
400	90.0	227.9	20.7
600	268.9	237.9	22.7
750	382.1	236.8	24.8

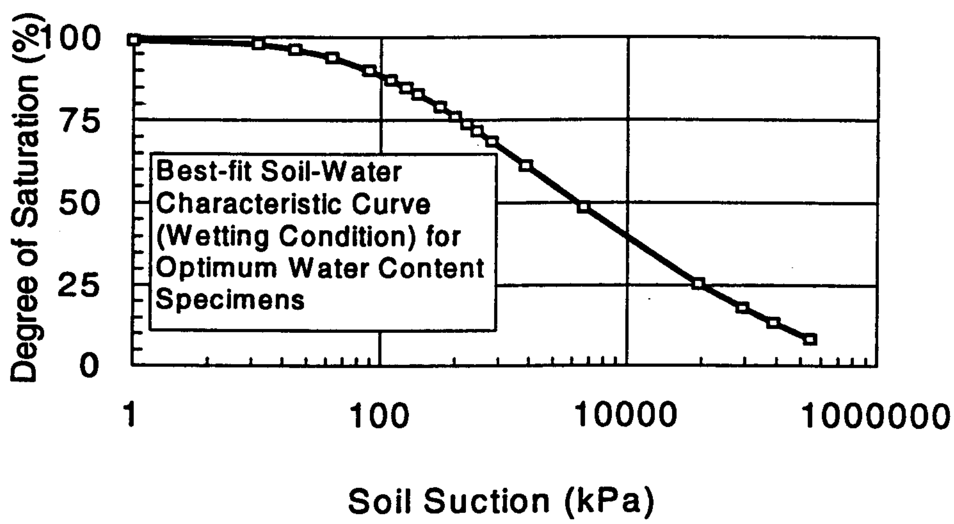

Figure 4. Relationship between the degree of saturation versus soil suction from undrained tests and desiccator test data

This value has given good correlations between the predicted and measured shear strength values using the soil-water characteristic curve (drying) and the saturated shear strength parameters. It appears that κ

value is independent of the structure induced to the soil due to initial water content, loading and drainage conditions. A value of κ equal to 2.2 is hence used for predicting undrained shear strength.

Figure 5 shows the contribution of shear strength due to matric suction obtained by subtracting the contribution of net normal stress from the total strength (Eq. 21) and the shear strength prediction using the wetting soil-water characteristic curve data and the saturated shear strength parameters (Eq. 18). There is a reasonably good comparison between the measured and predicted shear strengths using a value of κ equal to 2.2.

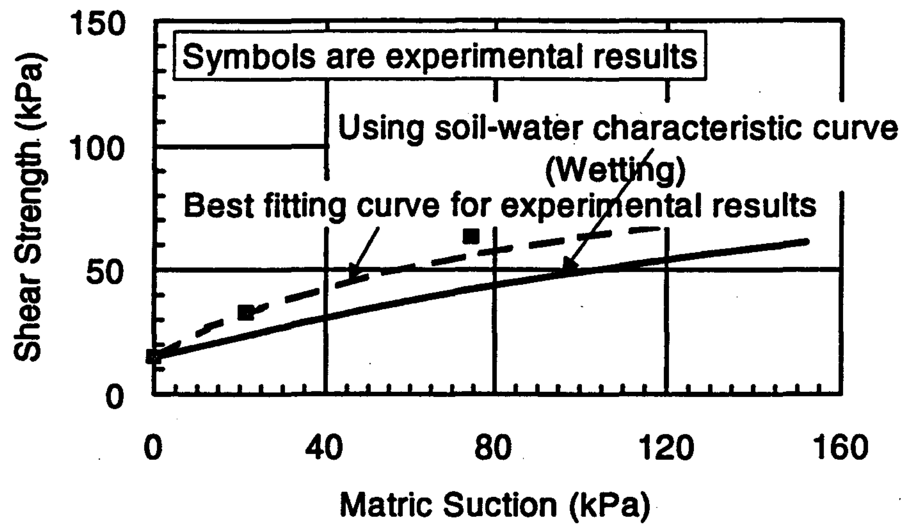

Figure 5. Relationship between the shear strength and matric suction

Comparative studies made along similar lines for specimens tested with dry of optimum conditions did not yield comparable results. It is likely that the predicted matric suctions were not the available matric suctions. This can be substantiated from the comparisons made between the predicted degree of saturation using the theory of undrained pore pressure parameters and measured degrees of saturations for specimens after failure conditions. Measured degrees of saturation were higher than the predicted values of degrees of saturation. It appears that matric suctions and degrees of saturations of these specimens were undergoing significant changes during the shearing stage.

Thus, the assumption of neglecting matric suction changes during the shearing stage of the specimen may be acceptable for the specimens at optimum and wet of optimum conditions but not for specimens with dry of optimum initial conditions.

5. SUMMARY

A framework for the interpretation of undrained shear strength of unsaturated soils using stress state variable theory proposed by Fredlund et al. (1978) is presented and applied to the results on glacial till specimens. The results appear to be satisfactory. The presented theory can be used for practical applications to determine the contribution of matric suction towards the undrained shear strength, ϕ^b. Further studies are however required to examine the proposed framework in this research paper on different soils.

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