Constitutive modeling of a metastable-structured compacted soil

J.H.F. Pereira and D.G. Fredlund

Abstract: The influence of wetting-induced collapse on the mechanical and hydraulic properties of a residual gneiss soil compacted as a metastable-structured material was experimentally investigated. Volume changes and coefficient of permeability were investigated using a triaxial permeameter system where the stress state variables were independently controlled. The compacted specimens were consolidated isotropically. Measurements of total volume change, water content change, and coefficient of permeability were made at specified matric suction values following a wetting stress path. The experimental data were analyzed to define volume change and water permeability constitutive relationships for the metastable-structured soil. A constitutive model is proposed for predicting the volume change behavior of the compacted metastable-structured residual soil during wetting-induced collapse. The proposed constitutive model is applied to the prediction of the volume change behavior of a compacted metastable-structured soil under different external loading conditions. Prediction is compared to experimental results.

Key words: collapsible soil, metastable structured soil, unsaturated soil, numerical modeling, mechanical properties, hydrological properties.

Introduction

A collapsible soil is commonly referred to as a metastable-structured soil. An increase in pore-water pressure results in swelling for an unsaturated stable-structured soil, whereas an increase in pore-water pressure may cause a volume decrease for an unsaturated metastable-structured soil (Barden et al. 1969). The research literature is in agreement that collapse is a behavior that any unsaturated soil may undergo under particular conditions of stress and saturation (Barden et al. 1973; Lawton et al. 1991).

The collapse behavior of compacted and cohesive soils depends on several factors; namely, the percentage of fines (especially clay fraction), the initial water content, the initial dry density, and the energy and process used in compaction (Jennings and Burland 1962; Barden et al. 1973).

In Terzaghi's theory, the mechanical behavior of a saturated soil is governed by the principle of effective stress. Initial attempts to extend such a theory to unsaturated soils had limited success (Bishop and Blight 1963). In recent years, a more sound theory has been established by Fredlund and Morgenstern (1976). Such a theory is consistent with a multiphase, continuum mechanics approach and describe the mechanical behavior of an unsaturated soil as a function of two independent sets of stress variables; namely, the net normal stress $(\sigma - u_a)$ and the matric suction $(u_a - u_w)$ wherein u_a and u_w are the pore-air and the pore-water pressures in the soil voids. In this theory, the saturated condition is a special case where the effective stress (i.e., $(\sigma - u_a)$) becomes the governing stress state variable. The collapsing behavior of soils during saturation is one of the complex aspects to be developed through application of the unsaturated soil theory. The prediction of the performance of collapsing earth structures during saturation is one of the engineering problems depending upon these developments.

Current practice in geotechnique recognizes an unsaturated soil as a four-phase material composed of air, water, soil skeleton and contractile skin (Fredlund and Morgenstern 1976). Under this idealization and from a mechanical viewpoint, two phases can flow (i.e., air and water), and two phases come to equilibrium under imposed loads (i.e., soil skeleton and contractile skin).

Darcy's law has been used to describe the water flow through soils in both the saturated and unsaturated conditions (Freeze and Cherry 1979). The variation of the soil permeability with void ratio for saturated conditions is well established. For unsaturated soils, water flows through the pore spaces filled with water. Therefore, there is a rapid decrease of the soil permeability to water as the degree of saturation decreases (Gardner 1961).

Studies have been attempted to explain the collapse of soils during saturation as a result of changes in the effective stress and in the stress-strain relationships of the material (Lourens and Czapla 1987). Recently, studies have attempted to explain these phenomena by using the two independent stress state variables for unsaturated soil (Miranda 1988; Lloret and Ledesna 1993). However,

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these studies have experienced difficulties in reproducing available experimental results in terms of the stress-strain behavior of collapsing soils during saturation. This is particularly true for the condition of collapse of the soil structure under a constant vertical load and K_0-conditions. Under these conditions a collapsing soil undergoes an increase in mean net confining stress (Maswoswe 1985; Lawton et al. 1991). However, those studies have encountered difficulties to reproduce the increase in horizontal stress during saturation of a collapsing soil. Hence, there exists a need for a better understanding of the mechanical behavior of collapsing soils in view of current theories for unsaturated soils. The primary objective of this paper is to develop a theoretical understanding of the mechanical behavior of a collapsing soil during saturation by using the theory of consolidation for unsaturated soils as presented by Fredlund and Rahardjo (1993).

Mechanical behavior of collapsing soils

Matyas and Radhakrishna (1968) stated that the state of a soil element may be graphically represented as a point in a system of coordinate axis representing the state parameters. This point is called a state point and its displacement, when the element state changes, is called the state path. All the possible state paths would form the state surface of the soil. It was proposed that changes in the void ratio and degree of saturation of an unsaturated soil must be expressed as functions of the stress variables, $(\sigma - u_a)$ and $(u_a - u_w)$, forming constitutive and three-dimensional surfaces.

Continuity requirements for an unsaturated soil

The continuity requirement for an unsaturated soil, deforming under an applied stress gradient, can be expressed (Fredlund and Rahardjo 1993) as follows,

$$[1] \qquad \frac{\Delta V_v}{V_0} = \frac{\Delta V_w}{V_0} + \frac{\Delta V_a}{V_0} + \frac{\Delta V_c}{V_0}$$

where:

V_0 = initial overall volume of the soil element,
V_v = volume of soil skeleton voids,
V_w = volume of water phase,
V_a = volume of air phase, and
V_c = volume of the contractile skin (i.e., air-water interface).

By neglecting the volume changes in the contractile skin, the above relationship showed that only the volume changes associated with two phases must be measured, while the third can be computed. In practice, changes in void volume and water phase volume are usually measured. By using a rectangular Cartesian coordinate system and referencing deformation to an elemental volume, the total volumetric deformation, $d\varepsilon_v$, of an unsaturated soil element can be expressed as,

$$[2] \qquad d\varepsilon_v = d\varepsilon_x + d\varepsilon_y + d\varepsilon_z = dV_v / V_0$$

where:

$\varepsilon_x, \varepsilon_y, \varepsilon_z$ = deformations on the x-, y-, z- directions.

Assuming infinitesimal deformations on the soil element, the deformation variable associated with the water phase can be defined as dV_w / V_0.

Volume change behavior of unsaturated soils

Moduli and compressibility forms of the constitutive relationships have been used by researchers to illustrate discrete or incremental constitutive models. In these relationships, the deformation variables for total volume change and water volume change of the soil element are associated to changes in the stress state variables by means of elasticity modulus.

Assuming the soil as an incrementally isotropic, linear and elastic material, the moduli constitutive relationships have been presented (Fredlund and Morgenstern 1976), in accordance with the generalized Hooke's law, as follows (Fung 1965):

$$[3] \qquad d\varepsilon_{ij} = \frac{1}{E}[(1+\mu)d\sigma_{ij} - \mu d\sigma_{\alpha\alpha}\delta_{ij}] + \frac{1}{H}d(u_a - u_w)\delta_{ij}$$

where:

σ_{ij} = total stress tensor,
$\sigma_{\alpha\alpha} = \sigma_{ii} + \sigma_{jj} + \sigma_{kk}$,
ε_{ij} = total deformation tensor,
H = elasticity modulus for the soil structure relative to a change in $(u_a - u_w)$,
E = elasticity modulus for the soil structure associated to a change in $(\sigma - u_a)$,
μ = Poisson ratio for the soil, and
δ_{ij} = Kronecker delta.

A similar semi-empirical approach has been used to the water phase constitutive relationship:

$$[4] \qquad \frac{dV_w}{V_0} = \frac{1}{E_w}d(\sigma_{ii} - 3u_a) + \frac{1}{H_w}d(u_a - u_w)$$

where:

E_w = water volumetric change modulus associated with a change in $(\sigma - u_a)$, and
H_w = water volumetric change modulus associated with a change in $(u_a - u_w)$.

The above forms of the constitutive equations can be applied to general cases where non-linear stress versus strain relationships occur by means of incremental procedures.

In an alternative way to the moduli form, Fredlund and Rahardjo (1993) presented the compressibility equations, for soil structure and water phase for a triaxial stress state, as follows:

$$[5] \qquad dV_v / V_0 = m_1^s d(\sigma_m - u_a) + m_2^s d(u_a - u_w)$$

$$[6] \qquad dV_w / V_0 = m_1^w d(\sigma_m - u_a) + m_2^w d(u_a - u_w)$$

where:

$m_1^s = 3(1-2\mu)/E$, is the coefficient of total volume change with respect to mean net normal stress,

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$m_2^s = 3 / H$, is the coefficient of total volume change with respect to matric suction,
$m_1^w = 3 / E_w$, is the coefficient of water volume change with respect to mean net normal stress,
$m_2^w = 1 / H_w$, is the coefficient of water volume change with respect to matric suction, and
σ_m = mean net normal stress.

In the state surfaces defined by Fredlund and Rahardjo (1993), both compressive net normal stresses and matric suction were assumed as positive. Therefore, this implied negative signs for the four parameters m_1^s, m_2^s, m_1^w and m_2^w for a soil which undergoes volume decrease due to increases in any stress variable. It was stated that the four negative parameters characterize a stable soil. In turn, the stable soil swelled upon decrease in any stress variables. These sign conventions are also valid for the corresponding elasticity parameters.

Collapsing soils

Nowadays, the geotechnique recognizes the following facts regarding the mechanical behavior of compacted collapsing soils:

(*a*) Any type of soil compacted at dry of optimum conditions and at a low dry density may develop a collapsible fabric or metastable structure (Barden et al. 1973).
(*b*) A compacted and metastable soil structure is supported by microforces of shear strength (i.e., bonds) which are highly dependent upon capillary action. With the increase of the water content the bonds start losing strength and at a critical water content the soil structure collapses (Jennings and Knight 1957; Barden et al. 1973).
(*c*) There is a gradual increase in compressibility as well as a gradual decrease in shear strength of a collapsible soil during the saturation process (Jennings and Burland 1962; Barden et al. 1973),
(*d*) The soil collapse progresses as the degree of saturation increases. There is; however, a critical degree of saturation for a given soil above which negligible collapse will occur regardless of the magnitude of the pre-wetting overburden pressure (Booth 1977).
(*e*) The collapse is associated with localized shear failures rather than an overall shear failure of the soil mass (Maswoswe 1985).
(*f*) During wetting-induced collapse, under a constant vertical load and under K_0-oedometer conditions, a soil specimen undergoes an increase in horizontal stresses (Maswoswe 1985).
(*g*) From a phenomenological point of view, there is an increase in the Poisson ratio of the collapsing soil during saturation (Pereira 1996).
(*h*) Under a triaxial stress state, the magnitude of volumetric strain, resulting from a change in stress state or from wetting, depends on the mean normal total stress and is independent of the principal stress ratio (σ_a / σ_r). However, the individual components of volumetric strain (i.e., axial and radial strain) depend on the principal stress ratio. For a given mean normal total stress, the magnitude of axial collapse increases and the magnitude of radial collapse decreases with an increasing stress ratio (Lawton et al. 1991).

A soil collapse model must be able to properly reproduce the progress and magnitude of the soil collapse as a function of the two independent stress variables governing the mechanical behavior of unsaturated soils. It is worthy emphasizing that Lawton et al. (1991) observations provides valuable information for the modeling of the collapsing soil behavior since it relates the total volumetric wetting-induced collapse to the mean net normal total stress. In addition, it illustrates that depending on the applied stress ratio a collapsing soil can even undergo expansion to the direction of the minor total stress during saturation.

According to eq. [5], if the mean net normal stress is kept constant, the collapsible soil structure must undergo a volume decrease due to a decrease in its matric suction. Therefore, a collapsible soil has a positive compressibility modulus (i.e., m_2^s)associated with a change in matric suction (Fredlund and Rahardjo 1993). Depending on the initial degree of saturation and the reduction in porosity during the soil collapse, it is possible that the volumetric water content may decrease as the matric suction decreases. In this case, the water phase compressibility modulus associated with a change in matric suction is also positive, (i.e., m_2^w).

Equation 3 assumes isotropic mechanical properties for an unsaturated soil. This implies a positive value for the isotropic elasticity modulus for the soil structure relative to a change in matric suction (i.e., H) for a collapsible soil, since m_2^s is positive. An isotropic modulus, H, results in isotropic wetting-induced collapse of a soil element in response to a decrease in matric suction, independent on the total stress state applied on the soil element. Such a prediction contradicts the previously presented available experimental data reported by Maswoswe (1985).

Isotropic wetting-induced soil collapse can occur on a soil specimen under an isotropic total stress state (Lawton et al. 1991). In addition, Lawton et al. (1991) findings predict that during triaxial wetting-induced collapse a soil specimen undergoes anisotropic deformations which are functions of the applied anisotropic stress state. Therefore, a stress induced anisotropic modulus, H, appears to be a reasonable alternative to the theory of unsaturated soils to properly model the collapsing behavior of a soil during saturation. By using rectangular a Cartesian system the proposed stress induced anisotropic constitutive equations for the normal deformations are as follows:

$$[7] \qquad \varepsilon_x = \frac{(\sigma_x - u_a)}{E} - \frac{\mu}{E}(\sigma_y + \sigma_z - 2u_a) + \frac{(u_a - u_w)}{H_x}$$

$$[8] \qquad \varepsilon_y = \frac{(\sigma_y - u_a)}{E} - \frac{\mu}{E}(\sigma_x + \sigma_z - 2u_a) + \frac{(u_a - u_w)}{H_y}$$

$$[9] \quad \varepsilon_x = \frac{(\sigma_z - u_a)}{E} - \frac{\mu}{E}(\sigma_x + \sigma_y - 2u_a) + \frac{(u_a - u_w)}{H_z}$$

where:

$H_i = H / (1 + H\xi_i)$, is the elasticity modulus for the soil structure in the i-direction relative to a change in matric suction (i.e., $u_a - u_w$),

H = isotropic elasticity modulus (function of the mean net total stress) for the soil structure relative to a change in $(u_a - u_w)$,

ξ_i = stress induced anisotropic collapse factor in the i-direction (function of the stress ratios σ_i / σ_j and σ_i / σ_k) for the soil structure relative to a change in $(u_a - u_w)$, and

i, j, k = directions of a three-orthogonal coordinate system (e.g., x, y, z).

In addition, and a consequence of the assumption that the volumetric wetting-induced collapse is a unique function of the applied mean net stress, comparison between eq. [5] and eqs. [7] to [9] allows a definition of the following relationships for the soil parameters, m_2^s, H_i, ξ_i.

$$[10] \quad m_2^s = \frac{1}{H_x} + \frac{1}{H_y} + \frac{1}{H_z}$$

$$[11] \quad x_z + x_y + x_x = 0$$

Besides, it can also be concluded that ξ_i are non-linear soil parameters with values equal to zero for an isotropic stress state.

Flow law and hydraulic properties for unsaturated soils

The soil permeability to air remain significantly greater than the soil permeability to water for all water contents in a compacted soil (Barden and Pavlakis 1971). This study is related to the practical cases wherein the pore-air phase can be considered at constant atmospheric pressure.

Some unsaturated soils do not undergo significant changes in void ratio in response to changes in the stress state variables. For these soils, the water coefficient of permeability can be expressed as a sole function of their degree of saturation. Therefore, the soil-water characteristic curve has been used to derive semi-empirical relationships for the permeability to water, as a function of the soil matric suction, for such soils (Brooks and Corey 1964). Relationships between the soil permeability to the water and matric suction are strongly dependent on the stress path. However, hysteresis has relatively little influence on the relationships between the water coefficient of permeability and the degree of saturation for soils with a non-deforming soil structure.

A metastable soil may undergo a considerable rearrangement of its structure due to wetting-induced collapse. Therefore, the water coefficient of permeability for a collapsible soils must be investigated as a function of its volume mass properties (i.e., void ratio and degree of saturation). In engineering practice, a collapsible soil maintains a relatively open structure even for relatively low matric suctions. This means that the soil possesses a low degree of saturation and, consequently, a low permeability with respect to the water phase and a high permeability with respect to the air phase even at low levels of matric suction.

Physics involved in the problem of wetting of collapsing soils

Dakshanamurthy et al. (1984) extended the theory by Biot for consolidation of saturated soils to unsaturated soils. In their work, the air continuity equation was explicitly included as a continuous and compressible phase. Their basic physics equations were as follows.

(*a*) Equilibrium equations:

$$[12] \quad \sigma_{ij,j} + b_i = 0$$

where:

b_i = body forces.

(*b*) Air continuity equation:

$$[13] \quad \frac{\partial}{\partial t}[\rho_a n(1 - S + H_c S)] + \nabla[\mathbf{J}_a] = 0$$

where:

$\mathbf{J}_a$ = mass rate of air flowing across a unit area of the soil,

ρ_a = mass density of air,

n = porosity,

S = degree of saturation,

H_c = Henry's constant (nH_cS = air volume dissolved in the pore-water phase), and

$\Delta = \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k$, the divergence operator.

(*c*) Water continuity equation.

$$[14] \quad \frac{\partial(\rho_w \theta_w)}{\partial t} + \nabla(\rho_w \mathbf{v}_w) = 0$$

where:

ρ_w = mass density of water,

$\theta_w = nS$, the volumetric water content, and

$v_w = v_w^x i + v_w^y j + v_w^z k$, macroscopic velocity vector of water.

The coupled flow and stress equations involved in the behavior of a collapsing soil mass requires a numerical solution. In the present paper, the finite element method was used in the development of a numerical model dealing with a coupled solution. Comparison between experimental and numerical results are later illustrated. Details of the coupled solution involving water flow-mechanical equilibrium in soils are presented in Pereira (1996).

Table 1. Index properties of the soil.

Soil	Residual silty sand
Grain size distribution	Sand = 52%
	Silt = 35%
	Clay = 13%
Atterberg limits	Liquid limit, w_l = 29
	Plastic limit, w_p = 17
	Plasticity index, PI = 12
Specific gravity	G_s = 2.64
USCS	SW-SM; well-graded sand with silt

Laboratory testing results and soil modeling

The soil utilized in this research study is a residual silty sand derived from a granitic gneiss of the Ceara group in Northeast Brazil. Table 1 illustrates the index properties of the soil tested.

Laboratory tests were performed to define the constitutive relationships for the collapsing soil. Soil specimens were statically compacted at dry of optimum and low dry density conditions as compared to the AASHTO compaction energy. The collapsing soil specimens were compacted at a gravimetric water content of 10.5% (i.e., minus 4% dry of optimum conditions) and at a dry density of 14.75 kN/m^3 (i.e., compacted at 90% of the AASHTO compaction energy).

Double-oedometer tests in the range of vertical stresses from 0 to 800 kPa were performed on the compacted soil specimens. The soil presented low compressibility when loaded under unsaturated conditions. The soil specimens did not present any collapsing behavior when saturated under vertical stresses lower than 50 kPa. A vertical stress of 100 kPa produced soil collapse amounting to 3.0%. The volumetric collapse reached about 7.2% and 11% when loaded under vertical stresses of 200 kPa and 400 kPa, respectively.

A drying soil-water characteristic curve showed that the soil specimen started desaturating at a matric suction of about 3.0 kPa. The degree of saturation of the specimens dropped to values of less than 50% for suction values of about 60 kPa. The coefficient of permeability of compacted soil specimens at saturated conditions varied from $k_w = 1.5 \times 10^{-6}$ to $k_w = 4.0 \times 10^{-9}$ m/s, for applied vertical stresses from 25 kPa to 800 kPa. A relationship between the logarithm of the water coefficient of permeability versus the void ratio was non-linear. Low values for the permeability with respect to water would be expected for unsaturated soil specimens under suction values higher than 80 kPa. The method of Brooks and Corey (1964) was used to estimate the soil permeability of the unsaturated soil. A coefficient of permeability equal to 10^{-11} m/s was predicted at a matric suction of 80 kPa. At the as-compacted initial matric suction of about 370 kPa, the coefficient of permeability was predicted to be of about 10^{-13} m/s.

A triaxial permeameter system was utilized to the definition of the volume change constitutive relationships for the compacted collapsing soil. This equipment allowed for the performance of isotropic consolidation tests on unsaturated soils. The testings followed stress paths wherein the as-compacted soil specimen was firstly loaded and then gradually wetted (i.e., by reducing the applied matric suction). The coefficient of permeability could be measured using the triaxial permeameter using the constant head, steady-state, controlled head method (Pereira 1996).

Laboratory test results

To establish the constitutive relationships required, four triaxial permeability tests were conducted on four statically compacted collapsing soil specimens. Each soil specimen, at its as-compacted initial condition, was isotropically loaded under a given net normal confining pressure. The pore-air pressure was controlled in a drained mode. The specimens were allowed to consolidate at various steps of decreasing matric suction (i.e., following a wetting stress path under a constant net confining pressure) until saturation was reached. Applied net confining pressures of about 20 kPa, 50 kPa, 100 kPa and 200 kPa were utilized, respectively, for the four specimens tested. The changes in the total volume of the specimens were monitored during each step of the tests. The outflow and inflow of water to the specimen was also monitored in order to determine the changes in its water content. This allowed the computation of the soil coefficient of permeability at each applied matric suction. The specimen for the tests had a height of 44.8 mm and a diameter of 101.1 mm. The as-compacted soil specimens presented a void ratio of about 0.754 and a degree of saturation of about 36.7%.

A typical collapse behavior illustrated that a stress path, which follows decreasing matric suction at a given net confining pressure, shows that there are three distinct phases in the collapse mechanism. In the first phase (pre-collapse phase), at relatively high matric suctions, the soil does not collapse and only small deformations occur in response to a decrease in matric suction. In the second phase (collapse phase), at intermediate matric suctions, large deformations are observed in response to a decrease in matric suction. In the third phase (post-collapse phase), at low matric suctions, there is an absence of deformations as the matric suction is reduced to zero.

In terms of water volume changes, it was observed that, for the range of confining stresses used, the wetting curve is practically independent of the soil collapse. The influence of the soil collapse was only noticed as the soil approaches complete saturation. It was observed that, the lower the porosity of the collapsed soil, the less is the increase in water content in response to further decreases in matric suction.

The small amounts of water flow involved made it difficult to determine the influence of the gradual soil collapse on the water coefficient of permeability of the unsaturated soil specimens.

Soil modeling

The soil models were defined by using best-fit analyses in the search for continuous functional relationships that could capture the essential characteristics of the behavior of the soil as observed from the available data. A superior curve-fitting was obtained by using a five parameter logistic function. Equation [15] shows this function in terms of void ratio as a function of matric suction at a given net confining stress.

$$[15] \quad e = e_u + (e_s - e_u) \Big/ \left[1 + \left(\frac{(u_a - u_w)}{c}\right)^b\right]^a$$

where:

e_u = initial void ratio of a soil specimen under a given net confining stress,
e_s = void ratio at saturated conditions of a soil specimen under a given net confining stress,
c = matric suction value at the inflection point (i.e., middle point of the "collapse" phase),
b = slope parameter (i.e., slope of the "collapse" phase), and
a = the symmetry parameter which makes the logistic function asymmetric.

Equation [16] is the mathematical model, for net confining stresses higher than 45 kPa, for the void ratio state surface of the collapsing soil resulting from the best-fit analysis of the available data.

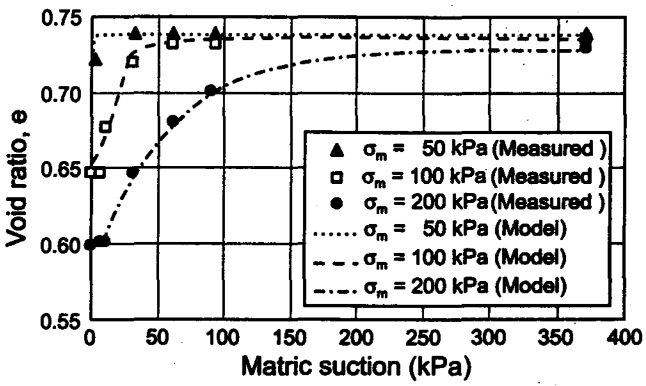
$$[16] \quad e = e_u + (e_s - e_u) \Big/ \left[1 + \left(\frac{(u_a - u_w)}{c}\right)^b\right]$$

where:

e_u = 0.76 - 0.0073 $\ln(\sigma^*)$,
e_s = 1.226 $(\sigma^*)^{-0.1359}$
$c = c_1(\sigma^*)^2 + c_2(\sigma^*) + c_3$,
$b = b_1(\sigma^*)^{b2}$,
$c_1 = 9.4 \times 10^{-4}$,
$c_2 = 7.46 \times 10^{-2}$,
$c_3 = -4.07$,
$b_1 = 49.01$,
$b_2 = -6.1 \times 10^{-1}$, and
σ^* = net confining stress.

Figure 1 illustrates the best-fit model in terms of void ratio versus matric suction relationships under different net confining stresses. The available experimental data are also illustrated in this figure and show a comparison of the best-fit surface provided by eq. [16]. The void ratio state surface allows for the evaluation of the compressibility parameter, m_1^s, which expresses (see eq. [5]) a relationship between the soil elasticity modulus (i.e., E) and the Poisson ratio (i.e., μ). Therefore, a Poisson ratio relationship is required in order to completely define the mechanical behavior of the soil structure.

Fig. 1. Void ratio best fit modeling.



A Poisson ratio equal to 0.3 may reflect the as-compacted conditions of a loosely compacted soil (Miranda 1988). However, changes in the structure of a metastable structured soil upon saturation, results in corresponding changes in the Poisson ratio (Maswoswe 1985).

In this research study, a first step consisted in the establishment of a relationship between the Poisson ratio and the mean normal stress of the collapsing soil at saturated conditions. Such a relationship was defined by combining the available triaxial and double-oedometer test results with the experimental observation that: "the volumetric collapse of a soil mass is truly a function of the acting mean total stress" (Lawton et al. 1991). Approximate values for mean total stress (i.e., σ_m) in the oedometer test at various values of vertical stresses (i.e., σ_v), were determined by selecting values of vertical stress for various void ratios on the oedometer curve. Then, the corresponding value of mean net stress on the saturated curve of the isotropic triaxial test was determined for the corresponding void ratio. A given combination of mean total stress and vertical stress allowed an evaluation of the horizontal stress (i.e., σ_h) on the oedometer test. Then, for a given mean confining stress, the Poisson ratio for the saturated collapsing soil (i.e., μ_s), was evaluated as follows:

$$[17] \quad \mu_s = (3\sigma_m - \sigma_v) / (\sigma_v + 3\sigma_m)$$

where

$\sigma_m = (\sigma_v + 2\sigma_h) / 3$, and
$\sigma_h = [\mu / (1 - \mu)]\sigma_v$.

The Poisson ratio for the collapsing soil at unsaturated conditions was determined as follows: *a*) at a given mean net stress (i.e., net confining stress) a constant Poisson ratio equal to 0.3 is assumed for the as-compacted soil condition; *b*) at a given mean total stress, it is assumed that the Poisson ratio increases with the soil collapse.

This implies an increase of the Poisson ratio when the matric suction decreases which reflects experimental evidence (Maswoswe 1985).

In this research study, the same relationship used to simulate the soil collapse versus matric suction (i.e., eq. [16]) is used to simulate the change in Poisson ratio in response to a change in matric suction. This assumption implies a variation of the Poisson ratio of the collapsing soil from an initial value of about 0.3 (i.e., at the as-compacted conditions) to a value calculated, by using eq. [17], when the soil reaches saturated conditions. Therefore, the Poisson ratio for the collapsing soil under unsaturated conditions is calculated as:

$$[18]\quad \mu = 0.3 + (\mu_s - 0.3) \Big/ \left[1 + \left(\frac{(u_a - u_w)}{c}\right)^b\right]$$

where:

μ_s = 0.092 ln(σ_m) – 0.021; the Poisson ratio equation for the saturated soil obtained by using eq. [17].

The degree of saturation state surface was also defined using a best-fit analysis. The logistic function provided the best-fit results of the available data. The mathematical model obtained is expressed as follows:

$$[19]\quad S = S_0 + (1 - S_0) \Big/ \left[1 + \left(\frac{(u_a - u_w)}{c}\right)^d\right]$$

where:

$S_0 = a + b\ln(\sigma^*)$,
$a = 0.354$,
$b = 3.65 \times 10^{-3}$,
$c = 7.91$, and
$d = 0.977$.

A phenomenological model for the degree of saturation already includes both the effect of changes in the soil structure (i.e., pore size changes) and the water inflow when the collapsible soil is saturating. Figure 2 shows the resulting best-fit model in terms of the degree of saturation versus matric suction relationship under different net confining stresses.

In terms of soil permeability to water versus stress state variables, the Brooks and Corey (1964) equation provided a satisfactory best-fitting relationship for the collapsing soil. A relationship between the coefficient of permeability at saturated conditions, k_s, and the net confining stress was previously defined and used as a constraint in the best-fit analysis. Equation [20] expressed the best-fit mathematical equation for the water coefficient of permeability.

$$[20]\quad k_w = k_p\left(\frac{\Psi_{cr}}{(u_a - u_w)}\right)^\lambda$$

where:

$k_w \le k_s$,
$k_p = -1.39 \times 10^{-7} + 6.259 \times 10^{-8} \ln(\sigma^*)$,
$k_s = 1.17 \times 10^{-6} - 1.8 \times 10^{-7} \ln(\sigma^*)$, is the coefficient of permeability for the soil at saturated conditions,
$\Psi_{cr} = 3.0$, and
$\lambda = 2.90$.

Fig. 2. Degree of saturation best-fit modeling.

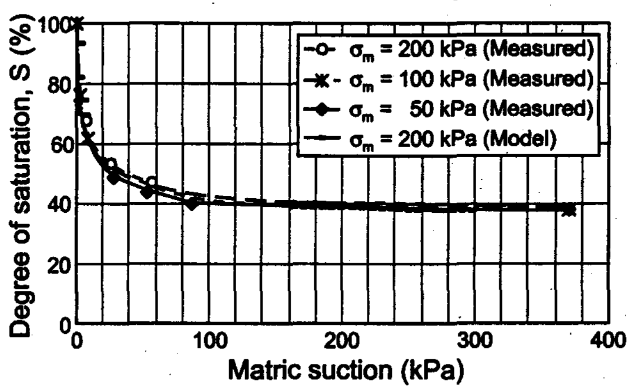


Fig. 3. Soil permeability best-fit modeling.

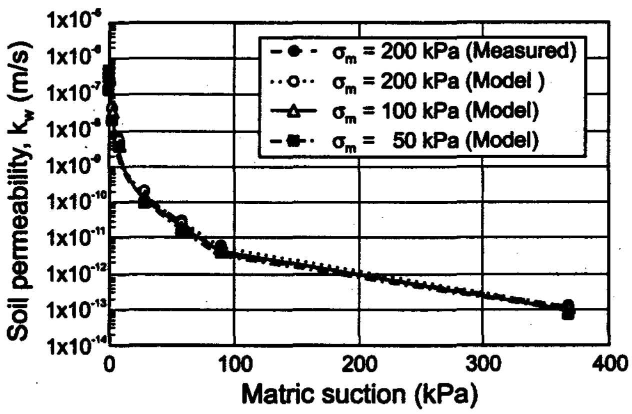


Figure 3 shows the best-fit results for the collapsing soil in terms of the water coefficient of permeability versus matric suction relationship for different net confining stresses. The figure also shows the available data for the net confining stress of 200 kPa, in order to show the accuracy of the predicting model.

Numerical modeling of a metastable-structured soil

A finite element model, developed in a computer program hereafter called COUPSO (Pereira 1996), was utilized to solve the coupled equations for the consolidation of unsaturated soils (i.e., eqs. [12] and [14]). The numerical solution takes into account the stress induced anisotropic constitutive relationships for a collapsing soil (i.e., eqs. [7] to [9]). In this paper, the ability of the model to handle soils that undergo collapsing behavior is firstly demonstrated by using results of laboratory tests presented by Maswoswe (1985). Then, the numerical model is applied to the analysis of the mechanical behavior of the residual silty soil compacted at metastable-structured conditions.

Maswoswe (1985) conducted tests on soil specimens compacted at metastable-structured conditions. Test SK2 is one of the tests that corresponded to a soil compacted to an initial void ratio of about 0.66. In this test, the soil specimen was loaded to a vertical stress of 190.4 kPa and

Table 2. SK2 test results from Maswoswe (1985).

Point	$\sigma_v - u_a$ (kPa)	$(u_a - u_w)$ (kPa)	$(\sigma_h - u_a)$ (kPa)	Void ratio (e)	$(\sigma_m - u_a)$* (kPa)
A	0.1			0.66	
B	190.4	250	50.9	0.65	97.4
C	190.4	220	80.4	0.61	117.0
D	190.4	150	89.6	0.56	123.2
E	190.4	0	110	0.45	136.8
F	220.4	0	120.9	0.45	154.0

*$(\sigma_m - u_a) = [(\sigma_v - u_a) + 2(\sigma_h - u_a)]/3$

then wetted under K_0 conditions. Table 2 shows the measured data from Maswoswe (1985).

The soil parameters required for the COUPSO computer program were defined by using the following procedures:

a) State surfaces for void ratio, *e*, and degree of saturation, *S*.

The available data in Table 2 were used to define a plane for the void ratio state surface of the collapsing soil. This plane was adjusted for the stress states corresponding to the initial point (i.e., B) and the final point (i.e., E) of the wetting-induced collapse. The void ratio state surface was established as follows:

[21] $$e = a_0 + a_1(\sigma_m - u_a) + a_2(\sigma_m - u_a)(u_a - u_w)$$

where:

$a_0 = 0.66$,
$a_1 = -1.53 \times 10^{-3}$, and
$a_2 = 5.70 \times 10^{-6}$.

For the degree of saturation state surface a relationship from Alonso (1993), based on the data by Maswoswe (1985), was used. The state surface for the degree of saturation was expressed as:

[22] $$S = 1 - m \tanh[n(u_a - u_w)]$$

where:

$m = 0.64$
$n = 5.38 \times 10^{-3}$ kPa^{-1}.

b) Poisson ratio, μ, coefficient of permeability, k_w, and factors ξ_i.

Since the horizontal stress increases when the soil collapse under K_0-conditions and at a constant vertical stress, the following linear relationship between the Poisson ratio, μ, and the matric suction was established based on the initial and final stress states (i.e., points B and E) of the wetting-induced collapse:

[23] $$\mu = \mu_s + d_0(u_a - u_w)$$

where:

$\mu_s = 0.37$,
$d_0 = (u_0 - u_{fs}) / (u_a - u_w)_{max}$,
$\mu_0 = 0.21$, and
$(u_a - u_w)_{max} = 250$ kPa.

The coefficient of permeability (i.e., k_w)was considered constant and equal to 1.36×10^{-8} m/s. The stress-induced anisotropic factors, ξ_i, were defined in a trial and error process, using the computer program COUPSO, in such a way that the wetting-induced collapse (i.e., from point B to point E) was exactly reproduced. In this process, equal values were used for the horizontal-direction anisotropic factors (i.e., ξ_x and ξ_z), simulating the confining oedometric conditions. The closed-form relationship ξ_y equal to minus $(\xi_x + \xi_z)$ (i.e., eq. [11]) defined the anisotropic factor in the vertical direction. The trial and error process defined horizontal-direction anisotropic factors ξ_x and ξ_z equal to minus 1.28.

Figure 4 presents a comparison between the results obtained using COUPSO and the measured values obtained by Maswoswe (1985). Figure 4 shows the capability of COUPSO to reproduce a wetting induced soil collapse where the vertical stress is kept constant. The intermediate values (i.e., points C and D) were calculated by keeping the previously defined soil parameters and changing the boundary conditions to reproduce the partial wetting collapse. Figure 4 shows that despite the assumptions involved, especially a constant value for the factors ξ_i, the numerical and experimental results are in good agreement. It is also illustrated that there is an increase in the mean net stress, as a result of an increase in the net horizontal stress during soil collapse as measured by Maswoswe (1985). These results show the importance of the stress induced anisotropic factors ξ_i, to the constitutive relationships for a collapsing soil.

A second application of COUPSO is related to the behavior of the metastable-structured compacted residual silty soil during saturation. The soil properties are based on the laboratory results previously discussed. The constitutive relationships for void ratio, *e*, Poisson ratio, μ, degree of saturation, *S*, and permeability to water for the collapsing soil are the previously presented equations [16], [18], [19] and [20], respectively.

The stress induced anisotropic factors ξ_i, (i.e., ξ_x, ξ_y and ξ_z) were defined in a trial and error process using the computer program COUPSO and the available double-oedometer tests. This trial and error procedure consisted of the reproduction of the wetting-induced (i.e., matric suction from 360 to 0 kPa) collapse of the soil specimen under K_0-conditions and under an applied vertical stress of 200 kPa. A value of about minus 2.60 was calculated

Fig. 4. Soil collapse during saturation.

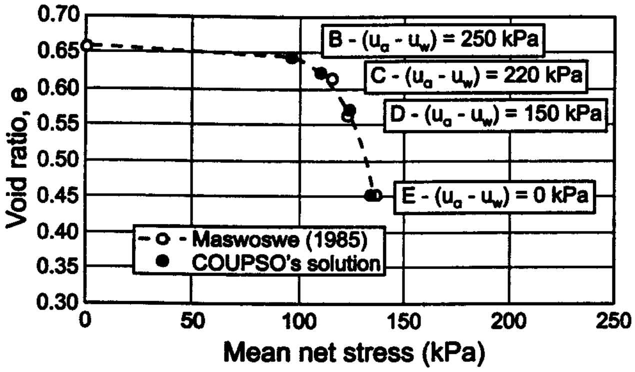
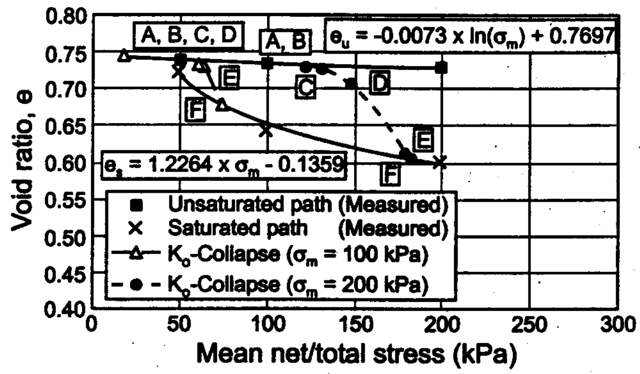


Fig. 5. Residual soil collapse during saturation.



for the anisotropic factors ξ_x and ξ_z from the above trial and error procedure.

The numerical analyses were performed by discretizing the soil specimen, which had a height equal to 0.02 meters, using a mesh of 5 equal axis-symmetric finite elements (Pereira 1996). The analysis was transient and required time steps varying from 5 to 0.5 seconds. Figure 5 illustrates the simulation of the wetting-induced soil collapse by using the above defined anisotropic factors. It shows the wetting induced soil collapse of residual silty soil specimens under K_0-loading conditions and under vertical loads of 100 and 200 kPa. Both the unsaturated and saturated loading paths were obtained from the triaxial permeameter tests.

The intermediate values (i.e., points B, C, D and E) correspond to values of matric suction of 200, 100, 50 and 10 kPa, respectively. These points were calculated by changing the boundary conditions to reproduce the partial wetting collapse. The results demonstrate the generality of the stress induced anisotropic collapse factors in simulating the collapsing soil behavior under different vertical loads.

In this wetting-induced collapse test, it is worth noting the monotonic stress path followed by the collapsing soil specimen. For this loading condition, the concept of stress state surfaces is fulfilled and reasonable predictions of changes in void ratio and degree of saturation of an unsaturated soil can be expected.

Conclusions

(1) The theory by Fredlund and Rahardjo (1993) for the consolidation of an unsaturated soil in its more generalized form (i.e., introducing a stress-induced anisotropic behavior of an unsaturated soil element in response to a change in matric suction), can be used to simulate the stress-strain behavior of a collapsing soil during saturation. The mechanical behavior of a stable soil is a special case of this more generalized theory.

(2) Additional studies on the anisotropic collapsing factors (i.e., ξ_i factors) are required. More research is needed to completely understand these factors and their influence on the stress-strain behavior of a metastable-structured soil.

(3) The numerical model COUPSO is able to simulate K_0-condition triaxial tests on metastable soils. The predicted results show reasonable accuracy to the measured data.

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