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The use of finite element computed pore-water pressures in a slope stability analysis

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ABSTRACT: A key parameter in a stability analysis is the pore-water pressure. Traditionally, the pore-water conditions are described by piezometric lines or r_u parameters. These methods are not the best when the pore-water conditions are irregular and change with time. Numerical methods such as the finite element method can be used to compute the pore-water pressure distribution for any irregular conditions that may arise due to changes in boundary conditions. The problem that arises is how to use the vast amounts of data generated by a finite element analysis in a limit equilibrium stability analysis. This paper presents a method based on the interpolating or shape functions inherent in a finite element formulation. In conclusion, the interpolating function approach is shown to be a good method for incorporating finite element computed pore-water pressure conditions into a stability analysis.

1. INTRODUCTION

Finite element seepage analyses can compute the pore-water pressure distribution for heterogeneous ground conditions under both steady-state and transient boundary conditions. While such analyses give solutions to highly complex situations, they also at the same time invariably produce large amounts of data. In a seepage analysis, for example, the pore-water pressure (or total hydraulic head) is computed at each node in the finite element mesh. In a mesh with 1000 nodes, the pore-water pressure is then known at 1000 discrete points. The problem is how to use this large amount of data in a slope stability analysis. Only a fraction of the data may be in the region of the potential slip surface zone. Ultimately, the objective is to find from this large amount of data the pore-water pressure at the base of each slice in a stability analysis.

This paper presents a method based on the interpolating or shape functions inherent in a finite element formulation for extracting the appropriate pore-water pressure from the finite element results. The method is efficient and makes it possible to accurately use any complex, irregular pore-water distribution in a slope stability analysis.

2. INTERPOLATING FUNCTIONS

Finite element formulations are usually based on some assumed distribution of the field variable within the element. In a seepage analysis, the field variable is the total head. The distributions of the field variables are often described by what are known as interpolating or shape functions. Furthermore, the same functions can be used to describe the shape of a surface or to interpolate values within the element based on conditions at the nodes. For example, the hydraulic head (h) anywhere within the element is defined as,

$$h = \langle N \rangle \{H\} \qquad (1)$$

where $< N >$ is a vector of interpolating functions and $\{H\}$ is a vector of heads at the element nodes.

The interpolating (or shape) functions for a 4-noded quadrilateral element are (Bath, 1982):

$$\begin{aligned} N_1 &= \tfrac{1}{4}(1+r)(1+s) \\ N_2 &= \tfrac{1}{4}(1-r)(1+s) \\ N_3 &= \tfrac{1}{4}(1-r)(1-s) \\ N_4 &= \tfrac{1}{4}(1+r)(1-s) \end{aligned} \qquad (2)$$

where r and s are the local coordinates within the element. The local coordinates vary between -1 and +1, as illustrated in Figure 1.

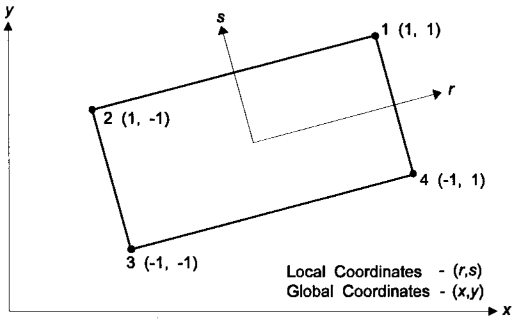

Figure 1 Local coordinates for a quadrilateral finite element

Different interpolating functions exist for elements of different shapes, and the functions are dependent on the presence of secondary nodes along the element sides. Only 4-noded quadrilateral interpolating functions are used in this paper to illustrate the technique.

The shape of the element can also be mapped to the global coordinate system with the same interpolating functions. That is, any local position within the element can be defined in terms of the global coordinates at the nodes. In equation form,

$$x = \langle N_1\, N_2\, N_3\, N_4 \rangle \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{Bmatrix} \qquad (3)$$

$$y = \langle N_1\, N_2\, N_3\, N_4 \rangle \begin{Bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \end{Bmatrix} \qquad (4)$$

where x and y are the global coordinate positions within the element, X and Y are the global coordinates at the element nodes, and N_1, N_2, N_3 and N_4 are the interpolating functions as defined in Equation 2.

The pore-water pressure U is known at the element nodes from the finite element analysis. For the stability analysis, however, we need the pore-water pressure at the base of each slice (Figure 2). The x-y coordinate position at the base of the slice is known. What is required is to establish the local coordinates (r,s) at the base of the slice. Once r and s are known, the pore-water pressure at the base of the slice can be computed from the equation,

$$u = \langle N \rangle \{U\} \qquad (5)$$

Let us say that the pore-water pressures at the corner nodes are $U_1 = 9$, $U_2 = 10$, $U_3 = 8$ and $U_4 = 7$. The pore-water pressure at a local coordinate of $r = 0.2$ and $s = 0.4$ can then be computed as follows:

$$\begin{aligned} N_1 &= \tfrac{1}{4}(1+0.2)(1+0.4) = 0.42 \\ N_2 &= \tfrac{1}{4}(1-0.2)(1+0.4) = 0.28 \\ N_3 &= \tfrac{1}{4}(1-0.2)(1-0.4) = 0.12 \\ N_4 &= \tfrac{1}{4}(1+0.2)(1-0.4) = 0.18 \end{aligned} \qquad (6)$$

$$u = 0.42*9 + 0.28*10 + 0.12*8 + 0.18*7$$
$$u = 8.80$$

The equations,

$$\begin{aligned} x &= \langle N \rangle \{X\} \\ y &= \langle N \rangle \{Y\} \end{aligned} \qquad (7)$$

can be used to compute the local coordinates r and s. The coordinates x and y at the slice base are known, as are the global coordinates X and Y. Since we have

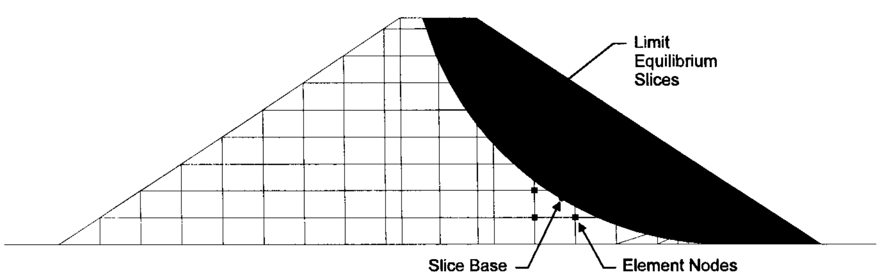

Figure 2 Finite element mesh with slip surface and slices

two equations with two unknowns, we can compute the values of r and s.

In summary, the procedure is,

- Identify the element which encompasses the base of the slice.
- Compute the local coordinates r and s at the position of x and y at the center of the slice base.
- Compute the pore-water pressure at the base of the slice using the interpolation functions and the known finite element computed heads at the nodes of the elements.

3. ELEMENT SEARCH

As noted above, the local coordinates r and s can be computed for any global coordinates x and y. When the absolute values of r and s are both less than or equal to 1.0, the global coordinate x-y is within the element. When the absolute value of either r or s is greater than 1.0, the slice base coordinate x-y is outside the element. This condition can be used to find the element that encompasses the base of the slice. When the x-y coordinate at the base of a slice corresponds to local coordinates between -1 and +1, the element encompasses the base of the slice.

Searching for the right element can be done efficiently by always starting the search from the most recent element that was found to encompass the base of a slice. Let us say that the most recent element number was 225. For the next slice, the search expands out from 225 by looking at 224, 226, 223, 227, 222, 228, 221, 229 and so forth. Since a finite element mesh numbering is sorted to minimize the memory requirements, this scheme makes it possible to find the element that encompasses the slice base by searching only within part of the finite element mesh. This is referred to as a bi-directional search procedure.

4. EMBANKMENT SEEPAGE EXAMPLE

Figure 3 shows the results of a simple finite element seepage analysis through a homogeneous earth embankment. The results were obtained using the commercially available software product known as SEEP/W (GEO-SLOPE, 1994). Shown on the cross-section are the equipotential lines and the water table. The finite element computed heads at each node can be used directly in the slope stability software known as SLOPE/W (GEO-SLOPE, 1995). The resulting Factor of Safety is 1.253, as illustrated in Figure 4.

The pore-water pressure conditions in a slope stability analysis are often described simply by defining the water table. The pore-water pressure at the base of each slice is then computed as the unit weight of the water times the vertical distance between the slice base and the water table. When the water table is extracted from the finite element analysis presented in Figure 3, the Factor of Safety for the same slip surface is 1.204.

The reason for the difference in Factor of Safety between 1.253 and 1.204 is the different pore-water pressure distribution along the slip surface, as depicted in Figure 5. Using the water table gives a pore-water pressure that is too high. This is due to using the vertical distance between the water table and the slice base, which is only correct where the equipotential lines are vertical (i.e., hydrostatic condition). Where the equipotential lines are curved, the vertical dimension is too great and leads to an over-estimation of the pore-water pressure. Using the finite element computed results directly in the slope stability analysis overcomes this problem.

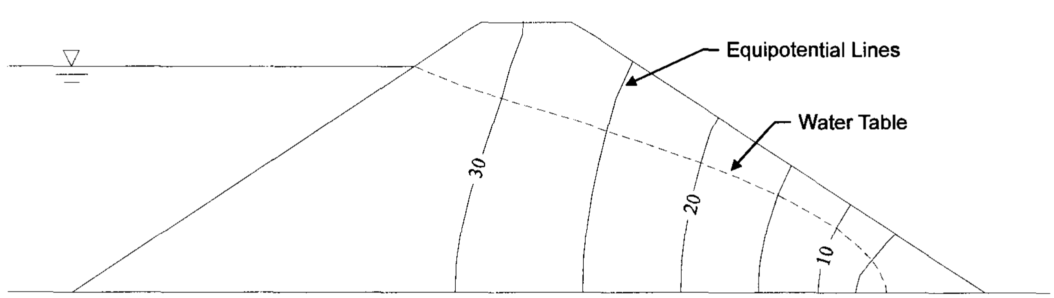

Figure 3 Steady-state analysis with equipotential lines and with water table

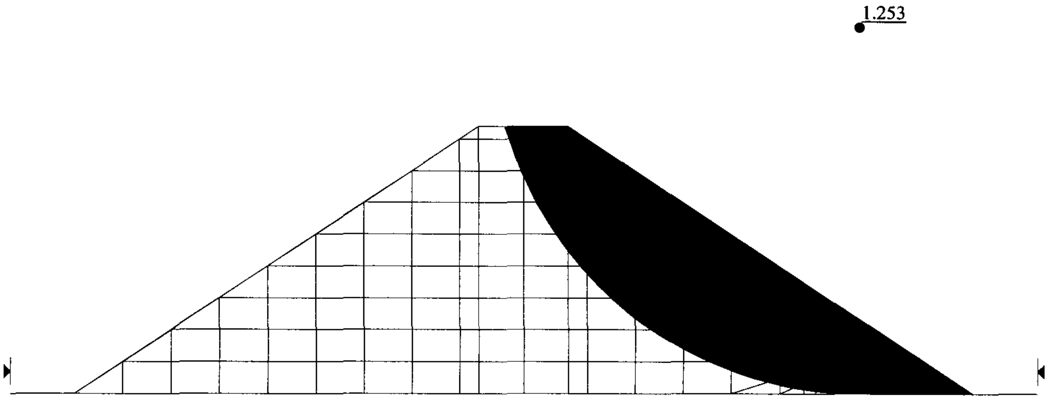

Figure 4 Stability with finite element computed pore-water pressures

Another advantage of using the finite element results directly in a stability analysis is that the negative pore-water pressures can be readily included. The negative pore-water pressures can be even more irregular than the positive pore-water pressures; consequently, it becomes almost mandatory to use the finite element results directly if the negative pore-water pressures are to be included in the stability analysis. For the examples presented in this paper, the negative pore-water pressures are included in the Factor of Safety calculations. This effect is evident in the pore-water distribution curves presented in Figure 5.

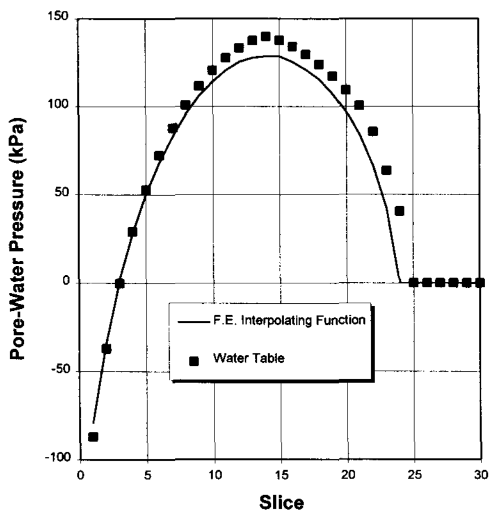

Figure 5 Comparison of pore-water pressure distributions

5. TRANSIENT SEEPAGE EXAMPLE

Using the water table to describe the pore-water pressure conditions in a slope stability analysis is manageable for steady-state seepage cases. Moreover, the technique errors on the safe side by computing pore-water pressures that are too high. Under transient conditions, however, the situation becomes more complex, making it cumbersome to specify the changing water table positions for each time increment. In this case, it is much more convenient to use the finite element results directly in the slope stability analysis. In addition, it leads to more accurate Factors of Safety.

Figure 6 shows the variable positions of the water table in an embankment after reservoir drawdown. Each of the water tables represents the conditions at a certain time after drawdown. The equipotential lines and water table for one of the time steps in the transient process are illustrated in Figure 7.

With the integration feature present in SLOPE/W and SEEP/W, it is possible to directly use the SEEP/W results for each time step in a SLOPE/W analysis. This makes it possible to readily assess the changes in Factor of Safety with time, as illustrated in Figure 8. Once the slope stability problem has been defined, it is necessary only to identify the SEEP/W output file for each time step to compute the Factors of Safety for the various times. SLOPE/W reads the selected SEEP/W file and uses the data in the stability calculations.

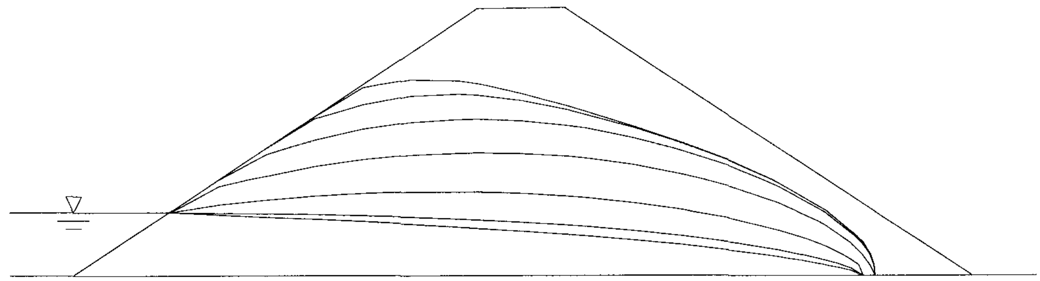

Figure 6 Variable positions of water table after reservoir drawdown

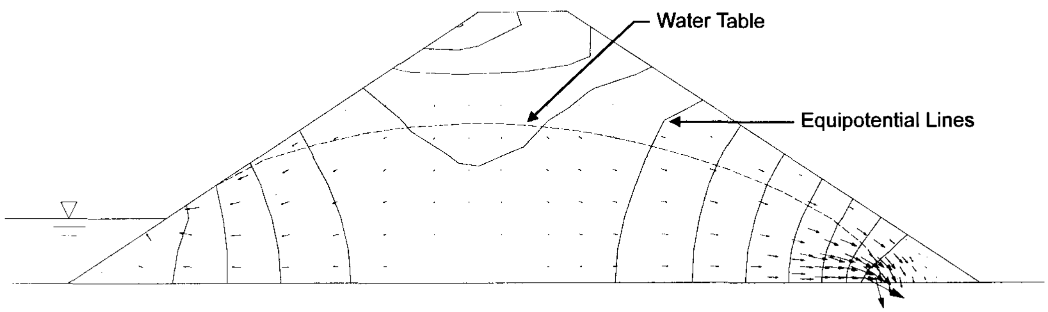

Figure 7 Equipotential lines & water table position at intermediate stage after reservoir drawdown

6. SPLINE INTERPOLATION COMPARISON

McClarty, Fredlund and Barbour (1991) have presented a scheme for using discrete point pore-water pressure data in a slope stability analysis. The scheme, based on a spline interpolation technique, is a form of a geostatistical analysis. Pore-water pressure is defined at selected points and the spline interpolation technique then uses the point data to estimate the pore-water pressure at the base of each slice in the stability analysis. A typical example is presented in Figure 9, where a selected number of nodal values from the case presented in Figure 3 are defined at the same *x-y* coordinates as the finite element nodes. The Factor of Safety is 1.277 when the spline interpolation method is used as compared to 1.253 when the finite element results are used directly.

The good agreement in Factor of Safety indicates that the spline interpolation technique gives good estimates of the correct pore-water pressure. The downside of the method is that it is cumbersome to extract only isolated values from the finite element results. Moreover, as McClarty, et. al. have pointed

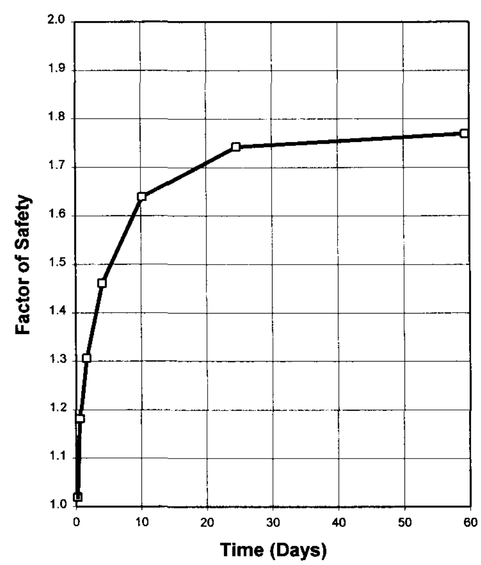

Figure 8 Factor of Safety as a function of time after reservoir drawdown

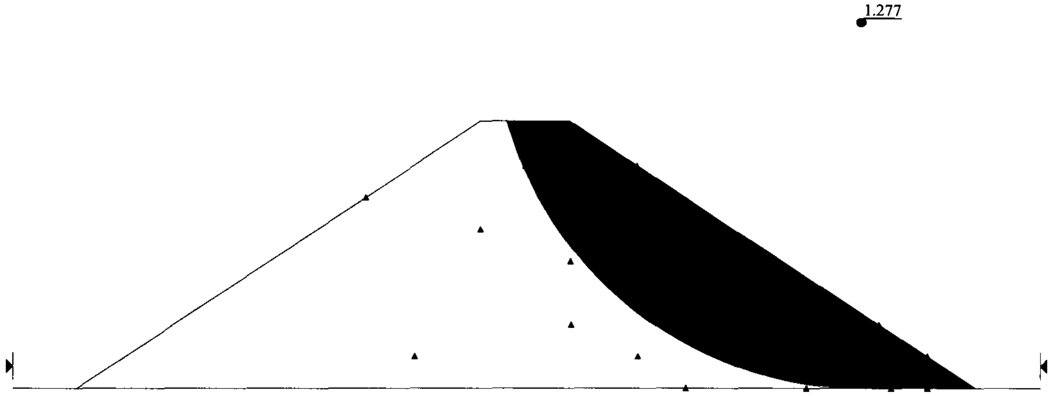

Figure 9 Stability Analysis using pore-water pressure defined at discrete points

out, the pore-water pressure estimates are dependent to some degree on the location and number of data points included in the spline interpolation. It is not practical to include all the finite element results due to computer processing and memory limitations. Consequently, only selected values can be used from the finite element analysis. The finite element interpolating function method presented in this paper does not suffer from these limitations.

The spline interpolation technique is an ideal scheme when limited, sparse data is available, as often happens with actual field measurements. In such cases, the method gives excellent estimates of the pore-water pressure at the base of each slice in the limit equilibrium stability analysis.

7. DISCUSSION AND CONCLUSION

The key to implementing the finite element interpolating function approach for use in a stability analysis is an efficient search routine. It is necessary to find the element that encompasses the slice base. This needs to be done for each slice within the potential sliding mass and needs to be repeated for each trial slip surface. Searching through the entire element array each time takes too much computing time and makes it impractical to apply the method, particularly when the finite element mesh is large. The bi-direction search method described above overcomes this problem and makes it practical to implement the interpolating function procedure.

The attractive feature about the finite element interpolating function scheme is that the pore-water pressure at the base of each slice is exactly as computed by the finite element analysis. The same interpolating functions used in the finite element formulation are used in the slope stability analysis. No averaging or smoothing of the data is required.

The use of finite-element computed pore-water pressures in a slope stability analysis opens the door to doing stability analyses where the pore-water pressures are highly irregular and change with time. A typical example is the infiltration of precipitation into a slope. The pore-water pressure, particularly in the unsaturated zone, can vary significantly throughout the cross-section. Such variability can be readily considered in a stability analysis with the finite element interpolating function method.

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